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A Comparison of Forward and Futures Prices of an Interest Rate-Sensitive Financial Asset

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ABSTRACT

This paper focuses on contractual distinctions as an explanation for the price divergence between futures and forward contracts. Specifically, it investigates the effect of marking-to-market on the observed price differences using the pricing model described in Cox, Ingersoll, and Ross (CIR) (1981, *Journal of Financial Economics* 9, 321–346). Using previously unavailable data, this paper employs Eurodollars, an interest rate-sensitive financial asset, to test the CIR model. Unlike prior empirical studies, test results support both the weak prediction concerning the sign of the average price difference and the stronger prediction that specific covariances explain the variation in the price differences.

FUTURES CONTRACTS AND FORWARD contracts are fundamentally similar. Both contracts are agreements between two parties to trade a specific good or asset at a future date at a price determined today. These agreements facilitate the transfer of risk between contracting parties, permitting the creation of hedging strategies.

In spite of their conceptual similarity, the implementation of these two contracts is quite different. Participants in futures markets trade highly standardized contracts by open outcry on organized exchanges. Clearing associations for the exchanges control default risk by assuming the opposite side of each transaction, guaranteeing payment. To reduce traders' incentive to default, the exchanges mark positions to market by settling contract gains and losses daily.

In contrast to the futures market, the market for forward contracts is mostly an over the counter market. In the absence of a clearinghouse, trader reputation helps control default risk. Unlike futures contracts, no marking-to-market occurs: the parties in the contract make no payments until maturity of the contract. The forward market for financial interest rate instruments departs even further from the futures market format: a trader can use the cash market for interest rate instruments to establish a forward position. For example, a trader can secure a forward position for a three-month T-bill beginning three months in the future by buying a T-bill with a six-month maturity and selling a T-bill with a three-month maturity.

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While the market structures of forward and futures contracts differ, many academics view the contracts as equivalent instruments whose prices should be identical in an efficient market. This perception has led to empirical studies of the pricing differences between futures and forward contracts. Investigations of T-bill futures and forwards, for example, reveal significant price differences, whereas investigations of foreign exchange futures and forwards do not. Some academic studies interpret these price discrepancies as evidence of market inefficiency[1], others attribute the differences to taxes and transactions costs.[2]

Recent theoretical work suggests, however, that contractual distinctions may generate price divergence between futures and forward contracts, even in an efficient market with no transactions costs. More precisely, Cox, Ingersoll, and Ross (1981) (CIR) argue that the marking-to-market of futures contracts creates price discrepancies between futures and forward contracts.[3] This daily resettlement, they contend, affects the *timing* of the cash flows between opposite sides in a futures contract. The difference in payment stream between futures and forward contracts in turn creates price divergence. They use arbitrage arguments to show that the price differences increase with the covariance of the futures price changes with riskless bond price changes. French (1983) uses copper and silver data to directly test the CIR model and its role in futures and forward price differences. These data provide only limited support for the model: the futures-forward price differences have the sign predicted by CIR, but the model cannot successfully explain the variation in the magnitude of these differences. French ascribes this failure to measurement error.[4]

Because the price effect of marking-to-market varies with the aforementioned futures-riskless bond covariance, the importance of marking-to-market in explaining the futures-forward price differences depends upon the underlying commodity or asset traded. The CIR marking-to-market effect will be more difficult to detect when this covariance is small. The covariance for the foreign exchange futures used by Cornell and Reinganum (1981) is typically quite small, on the order of 1.00×10^{-7} or less. The covariance of the copper and silver futures used by French is also small and of the same magnitude. The small size of the futures-riskless bond covariance for these underlying instruments suggests both that marking-to-market is not an important contributor to the futures-forward price difference for these assets, and that instruments with a larger covariance should offer a more powerful test of the CIR model.

[1] See Elton, Gruber, and Rentzler (1984) and Kolb and Gay (1985).

[2] See Capozza and Cornell (1979), Cornell and Reinganum (1981), Kawaller and Koch (1984), Gendreau (1985), and Allen and Thurston (1988).

[3] CIR (1981) also consolidate the results of Jarrow and Oldfield (1981), Sundaresan (1980), Richard and Sundaresan (1981), and French (1981).

[4] Park and Chen (1985) also test the effect of marking-to-market and, like French, find that the CIR model correctly predicts the sign of the futures-forward price difference. They do not, however, attempt to test the stronger CIR prediction about the variation in these differences.

This paper uses previously unavailable Eurodollar futures and forward data to test the CIR propositions. Eurodollars are more heavily traded than T-bills, have active futures and forward markets with low transactions costs, and are interest rate sensitive with a relatively large futures-riskless bond covariance, so that marking-to-market should generate a larger price difference.[5] Using Eurodollars also avoids some of the exchange rate issues, timing problems, and difficulties with different maturity structures that contributed to the measurement errors in French's study. The data provide much stronger evidence for the CIR propositions. The results support French's finding that the futures-forward price differences have the sign predicted by the CIR model. The data also uphold the more stringent CIR prediction concerning the variation in these price differences. This evidence suggests that market inefficiencies or imperfections are not the only explanation for the differences in futures and forward prices. Rather, the source of the observed futures-forward price differences may depend upon the underlying instrument. Even in an efficient market with low transactions costs, marking-to-market may be an important contributor to the futures-forward price difference if the covariance between fluctuations in futures prices and riskless bond prices is large.

The paper consists of five sections. Section I describes the effect of marking-to-market on futures and forward prices in the context of the CIR model and its propositions. Section II discusses Eurodollar futures and forward data. Section III comprises an examination of Eurodollar futures and forward behavior, and results of the CIR tests. Section IV concludes.

I. The Cox, Ingersoll, and Ross Model of Futures and Forward Prices

As described above, a futures contract resembles a forward contract that is settled daily and replaced by a new forward contract. Daily resettlement, CIR argue, leads futures prices to depend upon the correlation between spot prices and interest rates, while forward prices do not. CIR focus exclusively on the forward-futures price divergence created by this difference in payment streams, and assume there are no transactions costs, taxes, or default, and that individuals can borrow or lend at the same nominal rate.

CIR's propositions assert that the sign and the magnitude of the futures-forward price difference depend upon the covariance of the futures, forward, and spot prices with the riskless bond price. These propositions yield the testable implications described below. Each proposition has a weak implication, which predicts the sign of the average futures-forward price difference, and a strong implication, which predicts that the magnitude of this difference will depend upon whether the futures-forward price difference increases or decreases with the covariance of the futures, forward, and spot prices with

[5] The covariance for Eurodollar futures and T-bill cash prices is higher than the covariance between T-bill futures and T-bill cash prices, in spite of the high correlation between T-bill futures and cash prices, because Eurodollar futures prices are more variable than T-bill futures prices.

the riskless bond price. Note that CIR's propositions do not predict an exact magnitude for the futures-forward price difference without applying an intertemporal evaluation model.

Define the following variables:

$F(t, T)$ = Futures price at time t for contract maturing at time T,
$f(t, T)$ = Forward price at time t for contract maturing at time T,
$B(t, T)$ = Riskless bond price at time t for bond maturing at T,
$P(t)$ = Spot price at time t of underlying asset,
$\text{cov}(F(w), B(w))$ = Local covariance of the percentage changes in F with the percentage changes in B (var($\cdot$), and corr($\cdot$) defined similarly).

Using this notation, the testable implications of the CIR propositions are:

(I1) If the covariance between the price changes of futures and riskless bonds is always positive, then the forward price exceeds the futures price. In addition, the futures-forward price difference is a decreasing function of the market's expectation of the futures-riskless bond price change covariance. That is,

$$\text{cov}\big(F(w,T), B(w,T)\big) > 0 \quad \forall w \in [t,T] \Rightarrow F(t,T) < f(t,T)$$

$$\frac{\partial\big[F(w,T) - f(w,T)\big]}{\partial\big[\text{cov}\big(F(w,T), B(w,T)\big)\big]} < 0.$$

(I2) If the difference in the variance of bond price changes and the covariance between the spot price changes and the bond price changes is always negative, then the futures price is less than the forward price. Also, if the correlation between spot and bond prices changes is positive, then the futures-forward price difference is a decreasing function of the market's expectation of the variance in spot price changes. That is,

$$\text{var}\big(B(w,T)\big) - \text{cov}\big(P(w), B(w,T)\big) < 0 \quad \forall w \in [t,T] \Rightarrow F(t,T) < f(t,T)$$

$$\text{corr}\big(P(w), B(w,T)\big) > 0 \Rightarrow \frac{\partial\big[F(w,T) - f(w,T)\big]}{\partial\big[\text{var}\big(P(w,T)\big)\big]} < 0.$$

(I3) If the forward-bond covariance is positive, the futures price is less than the forward price, and the futures-forward price difference is a decreasing function of the expected forward-bond covariance. That is,

$$\text{cov}\big(f(w,T), B(w,T)\big) > 0 \quad \forall w \in [t,T] \Rightarrow F(t,T) < f(t,T)$$

$$\frac{\partial\big[F(w,T) - f(w,T)\big]}{\partial\big[\text{cov}\big(f(w,T), B(w,T)\big)\big]} < 0.$$

II. Eurodollar Futures and Forwards

A Eurodollar deposit is a dollar-denominated deposit in a bank outside the U.S. The London Interbank Offer Rate (LIBOR) is the rate at which London banks offer Eurodollars to their most credit worthy customers (other large banks). Banks are the biggest users of the Eurodollar markets: Morgan Stanley reports that bank-to-bank deals accounted for 76% of 1984's total Eurodollar transactions of $2.2 trillion.[6]

The forward market for Eurodollars is similar to the T-bill forward market: banks, importers, and exporters swap Eurodollar assets and liabilities of different maturities in the Eurodollar cash market to create forward positions. As in the forward market, commercial banks are active users of the Eurodollar futures market.

Eurodollar futures trade on the Chicago Mercantile Exchange (CME) in a quarterly contract cycle with cash settlement based upon the three-month LIBOR rate at expiration. The futures prices used in this paper are the daily settlement prices that determine the variation margin posted by futures holders as positions are marked to market.[7] I construct Eurodollar forward prices from the spot LIBOR yield curve, following the standard method used to derive T-bill forward prices.[8] The LIBOR rates are the latest available from Telerate at the close of the futures market. For example, equation (1) provides the three-month Eurodollar rate, three months forward, for comparison with a nearby Eurofuture contract with exactly three months remaining until expiration:

$$\left(1 + \frac{183}{360} r_6\right) = \left(1 + \frac{91}{360} r_3\right)\left(1 + \frac{92}{360} r_{\text{for}}\right) \tag{1}$$

where r_6 is the six-month LIBOR rate (matures in 183 days); r_3 is the three-month LIBOR rate (matures in 91 days); and r_{for} is the forward rate (the current rate for a 92-day maturity deposit beginning three months from today). Thus, to create a long forward position, an investor buys (lends) the six-month LIBOR and sells (borrows) the three-month LIBOR. As the time to expiration of the futures contract changes, the maturity length of the interest rates in the above equation must change to obtain the comparable forward rate.

The Eurodollar placements market is a flexible-date market in which participants can trade any length maturity, but historical data for LIBOR rates of odd maturities are not available. To have enough data for meaningful comparison, I estimate a daily quadratic LIBOR yield curve from the available 1-, 2-, 3-, 6-, and 12-month rates. I then use the fitted values from these regressions to calculate the daily Eurodollar forward price. Table I displays the summary statistics for these 1232 OLS regressions. The average

[6] See Morris (1985).

[7] Goldman Sachs provided daily data for Eurodollar futures prices for the first three nearby contracts, the 1-, 2-, 3-, 6-, and 12-month LIBOR rates, and the 1-, 2-, 3-, 4-, 5-, 6-, 9-, and 12-month T-bill rates from March 1982 to June 1987.

[8] For example, comparing T-bill futures to T-bill forward prices constructed from the yield curve, see Capozza and Cornell (1979), Rendleman and Carabini (1979), Elton, Gruber, and Rentzler (1984), Kawaller and Koch (1984), Gendreau (1985), Kolb and Gay (1985), and Kamara (1988).

standard error is approximately 5 basis points; using a cubic does not reduce the average standard error significantly.

One potential problem with estimating forward values from the fitted yield curve is that if banks must pay more to borrow Eurodollar deposits with odd length maturities, the fitted values are downwardly biased estimates of the true values. To check for this bias, I use less commonly quoted 4- and 5-month rates available for only a limited period of time during the sample period (10/86–7/87) from DRI. I compute the daily yield curve as above (using only the 1-, 2-, 3-, and 6-month maturities) and compare the actual four (or five) month rates with the fitted four (or five) month rates. If a liquidity preference for the common 1-, 2-, 3-, and 6-month maturities exists, then the mean difference should be positive.

Table II shows the mean difference in the actual and fitted values. The standard errors are adjusted for serial correlation induced by overlapping

Table I

Summary Statistics for Daily LIBOR Yield Curve Regressions:

$$\textbf{Yield}_t = \alpha + t \cdot \beta_1 + t^2 \cdot \beta_2 + \varepsilon_t, \qquad t = \textbf{days to maturity}$$

Yield curve estimates based upon LIBOR 1-, 2-, 3-, 6-, and 12-month rates. The yield curve regressions are quadratic in time to maturity; the curve is estimated once each day for a total of 1232 regressions (March 1982–June 1987). Goldman Sachs provided daily LIBOR data.

	α	β_1	$\beta_2\,(\times 10^5)$	Std. Err. of Estimate	Adj. R^2	R^2
Mean Coeff.	9.0697	0.0030	−0.2565	0.0498	0.8759	0.9739
Std. Error of Mean	0.0624	0.0001	0.0236	0.0022	0.0076	0.0038

Table II

Testing for Bias in Yield Curve Estimations: Actual Rates–Fitted Rates

Estimates a daily yield curve using only 1-, 2-, 3-, and 6-month LIBOR rates. The fitted values for the four and five month rates from the yield curve regressions are then compared to the actual four and five month LIBOR rates available from DRI for a limited time during the sample period (October 1986–July 1987). The mean difference in the actual and fitted four and five month rates will equal zero if the yield curve regressions produce unbiased estimates of the actual four and five month rates. Standard errors are adjusted for serial correlation induced by overlapping observations using Hansen and Hodrick (1980)'s method.

	Actual Four Month Rate-Fitted Four Month Rate	Actual Five Month Rate-Fitted Five Month Rate
Sample mean	0.0063	0.0060
Standard error	0.0063	0.0050
t-stat for mean = 0	1.0002	1.2096
Number of observations	183	183

observations (see Hansen and Hodrick (1980)). The t-statistics indicate that the mean does not differ significantly from zero, for both one-tailed and two-tailed tests. The results do not reveal any liquidity preference by the banks, and suggest that if a bias exists, it is quite small (less than one basis point as measured by the sample means in Table II).

III. Empirical Analysis

A. Eurodollar Futures and Forward Characteristics

Table III summarizes the daily percentage changes during March 1982–June 1987 of the forward and futures rates, measured as $\log[r_t/r_{t-1}]$, where r_t represents the rate at time t. The *nearby* contract refers to the contract with three or fewer months remaining until expiration, the *second nearby* refers to the contract with three to six months until expiration, and the *third nearby* refers to the contract with six to nine months until expiration.

Table III reveals that, in general, the forward yields are more volatile than the futures yields. Also, volatility of the forward contracts increases with time to expiration. One reason for the higher volatility of the forwards is that LIBOR rates which determine the forward prices move in increments of $^1/_{16}$ (equivalent to 6.25 basis points), while futures prices move in finer increments of one basis point. Measurement error in forward yields introduced by the yield curve regressions could also contribute to the higher forward volatilities. Because the yield curve regressions rely upon 1-, 2-, 3-, 6- and 12-month LIBOR rates, the fit for maturities between 6 and 12 months might not be as precise as the fit for lesser maturities. Finally, yearly volatility patterns do not suggest that any specific segmentation of the data is necessary.

B. Testing the Weak Predictions of the Cox, Ingersoll, and Ross Model

The weak implications of the CIR propositions predict the sign of the futures-forward price difference based upon the sign of the covariance of the futures, forward, or spot price changes with the riskless bond price changes. I construct a monthly series of futures-forward price differences and covariances to test these weak predictions. Specifically, the tests employ the forward-futures price difference from the third Monday of every month (five days prior to this date if the month is an expiration month, as the futures and forward prices must be equal on the expiration day. Alternatively, omitting the days close to contract maturity does not affect any test results).

The covariance series use the monthly sample covariances, so approximately 21 (the average number of trading days in a month) observations contribute to each sample covariance estimate. To convert the quoted interest rates into prices, I use the exchange's formulas. The covariance estimates employ T-bill prices to represent the riskless bond prices: the first nearby covariance estimates use the 2-month T-bill price, the second nearby

Table III

Daily Percent Changes in Futures and Forward Eurodollar Yields

Percent change in yields defined as $\log(r_t/r_{t-1})*100$. The statistics only include dates when observations for both futures and forwards exist. The nearby contract has three or fewer months remaining until expiration, the second nearby contract has three to six months until expiration, and the third nearby contract has six to nine months until expiration. Data provided by Goldman Sachs.

	Futures						Forwards				
Year	Mean	Std Dev	*t*-stat	Skewness	Kurtosis	*N*	Mean	Std Dev	*t*-stat	Skewness	Kurtosis
Panel A: Nearby contract											
1982	−0.196	1.726	−1.560	−0.231	0.593	188	−0.104	1.651	−0.867	−0.688	1.622
1983	−0.006	1.108	−0.086	0.428	0.914	240	0.028	1.395	0.317	0.175	2.362
1984	−0.091	1.004	−1.335	−0.080	3.982	218	−0.100	1.135	−1.307	−0.707	6.168
1985	−0.022	1.255	−1.246	0.027	1.922	200	−0.074	1.368	−0.765	−0.285	2.194
1986	−0.128	1.062	−1.718	−0.472	1.130	202	−0.023	1.094	−0.298	−0.210	2.173
1987	0.165	1.010	1.736	0.714	3.278	113	0.004	1.729	0.025	0.174	1.780
1982–87	−0.060	1.223	−1.676	−0.119	2.120	1161	−0.046	1.379	−1.142	−0.256	2.788
Panel B: Second nearby											
1982	−0.139	1.586	−1.205	0.151	0.706	188	0.013	1.733	0.100	−0.197	1.146
1983	0.022	1.180	0.287	0.280	0.412	237	0.046	1.487	0.482	−0.184	2.334
1984	−0.020	1.160	−0.153	0.495	2.958	217	−0.011	1.357	−0.118	0.244	4.736
1985	0.051	1.486	0.486	0.323	2.296	199	−0.008	1.561	−0.069	0.241	1.100
1986	−0.140	1.236	−1.606	−0.027	0.606	202	−0.067	1.296	−0.405	−0.416	3.282
1987	0.179	1.237	1.520	0.376	1.086	110	0.066	1.988	0.350	0.329	1.415
1982–87	−0.019	1.322	−0.488	0.232	1.572	1153	0.008	1.540	0.175	0.032	2.257
Panel C: Third nearby											
1982	−0.119	1.451	−1.118	0.538	1.190	185	−0.060	2.094	−0.388	−0.270	0.659
1983	0.027	1.068	0.388	0.186	0.364	238	0.067	1.654	0.623	0.125	0.974
1984	−0.014	1.128	−0.176	0.158	1.650	211	0.005	1.395	0.051	−0.163	2.574
1985	0.037	1.405	0.369	0.078	1.712	196	0.097	1.598	0.851	0.161	0.806
1986	−0.131	1.256	−1.472	0.187	0.874	198	−0.105	1.404	−1.057	−0.136	2.602
1987	0.220	1.270	1.770	0.407	1.114	104	0.196	1.990	1.006	0.088	1.208
1982–87	−0.013	1.261	−0.339	0.252	1.353	1132	0.022	1.673	0.435	−0.045	1.520

covariance estimates use the 4-month T-bill price, and the third nearby covariance estimates use the 6-month T-bill price.

Table IV displays the sample means, variances, and covariances for the monthly observations. The standard errors and t-statistics of the monthly covariances and variances are corrected for the third-order serial correlation detected in the sample. The standard errors and t-statistics of the futures-forward price difference are corrected for serial correlation induced by overlapping observations. The price difference observations overlap because they measure the monthly difference in futures and forward prices for a three-month LIBOR deposit at a specific future date. That is, the futures and forward contracts mature at fixed quarterly intervals, while the futures-forward price difference is measured at finer monthly intervals. To correct the standard errors for this overlap, I use the technique outlined in Hansen

Table IV

Monthly Futures-Forward Price Differences and Covariances of Futures and Forward Price Changes with Riskless Bond Price Changes: Tests of the Weak CIR Predictions

Log(F/f) is the percent difference in the futures and forward prices. Cov(F, B) is the sample covariance of the daily percentage change in the futures and T-bill prices over the month prior to the matching log(F/f) observation. Var(B)-cov(P, B) is the sample variance of the daily percentage change in the T-bill prices minus the sample covariance of the daily percentage change in the LIBOR and T-bill prices over the month prior to the matching log(F/f) observation. Cov(f, B) is the sample covariance of the daily percentage change in the forward and T-bill prices over the month prior to the matching log(F/f) observation. All t-statistics and standard errors are adjusted for serial correlation induced by overlapping observations using Hansen-Hodrick (1980) modified to reflect the type of overlap. Means and standard errors are multiplied by 10^2.

	Price Difference log(F/f)	cov(F, B)	var(B)-cov(P, B)	cov(f, B)
	Panel A: First nearby			
Mean	−0.022	0.008	0.022	0.003
Std error of mean	0.037	0.004	0.004	0.002
t-Stat for mean = 0	−0.579	2.297	5.114	1.950
N	63	63	63	63
	Panel B: Second nearby			
Mean	−0.237	0.009	0.148	0.004
Std error of mean	0.038	0.003	0.063	0.002
t-Stat for mean = 0	−6.176	3.226	2.346	2.354
N	63	63	63	63
	Panel C: Third nearby			
Mean	−0.478	0.008	−0.012	0.002
Std error of mean	0.078	0.002	0.003	0.000
t-Stat for mean = 0	−6.135	4.074	−3.827	3.146
N	63	63	63	63

and Hodrick (1980), modified to reflect the specific type of overlap induced by futures and forward contracts. The Appendix discusses the details of this method.

The results in Table IV generally support the weak predictions of the CIR model. CIR implies (see (**I1**)) that if the covariance between the futures price changes and the bond price changes (cov(F, B) in Table IV) is positive, the futures-forward price difference (log(F/f) in Table IV) is negative. Cov(F, B) is positive and differs significantly from zero for all three contracts. Under prediction (**I1**), then, log(F/f) should be negative. Table IV shows that log(F/f) is negative for all three contracts, and differs significantly from zero for the second and third nearby contracts.

The CIR model also implies (see (**I2**)) that if the difference between the riskless bond variance and the covariance of the spot and bond prices (var(B)-cov(P, B) in Table IV) is always negative, then the futures-forward price difference should also be negative. The initial condition of var(B)-cov(P, B) < 0 holds only for the third nearby contract, so the test of the second CIR proposition must be restricted to this contract. Log(F/f) is negative and significant for this contract, supporting the weak prediction of (**I2**).

Finally, the CIR model implies (see (**I3**)) that if the forward-riskless bond covariance (cov(f, B) in Table IV) is positive, the futures price will be less than the forward price. Again, the results are consistent with the CIR model: cov(f, B) is positive and differs significantly from zero for each contract; log(F/f) is negative for each contract and differs significantly from zero for the second and third nearby contracts.

The CIR model has the additional implication that the absolute value of futures-forward price difference increases with the time to expiration. The mean futures-forward price differences in Table IV are consistent with this prediction: the price difference for the first nearby contract is greater than the second nearby, which in turn is greater than the third nearby.

This increase in the futures-forward price difference with time to maturity potentially complicates the analysis because the mean price difference may not be stable. To understand how the instability of the price difference affects the results, I first conduct Wald tests for stability of the mean by dividing the sample into 3 parts, based upon time to expiration (the tests employ variance/covariance matrices adjusted for serial correlation induced by overlapping observations). The test results indicate that the mean of the portion of the sample with the greatest time to expiration differs significantly from the means of the sample with the next greatest and the least time to expiration. To correct for this instability, I divide each futures-forward price difference by time remaining to expiration. Wald tests on the transformed price difference fail to reject the null of equality of the means, so I repeat the tests performed in Table IV for the transformed variable. The results for this stabilized price difference are almost identical to those in Table IV and are therefore not reported.

Table IV shows that the results from the weak tests are consistent with the CIR model: the mean futures-forward price difference has the predicted sign.

The CIR propositions, however, relate the futures-forward price difference observed today to covariances or variances expected over the remainder of the contract. Rather than examining the mean, the next tests explicitly account for the evolution of covariance and variance expectations. Further, the CIR model predicts how the futures-forward price difference varies in response to changes in these covariances and variance-covariance differences. The strong tests of the CIR model regress the futures-forward price differences against the covariances and variances, providing a sharper test than the sign tests. A final advantage of the strong tests is that they allow different tax treatments, transactions costs, market structure, default risk, and restrictions to international capital mobility to contribute to Eurodollar futures-forward price difference. These factors do not affect the validity or the interpretation of the regression tests unless the factors vary systematically with the covariances and variances.

C. Testing the Strong Predictions of the Cox, Ingersoll, and Ross Model

The CIR propositions indicate that the futures-forward price difference is a decreasing function of the market's expectation of the futures-riskless bond covariance, the forward-riskless bond covariance, and the spot price variance over the remaining life of the contract. The tests of the strong predictions of the CIR model regress the futures-forward price difference against the various series of expected covariances and variances. The dependent variable in each OLS regression is the futures-forward price difference. The explanatory variable is the expected covariance of the daily percent changes in the futures and the riskless bond prices, the variance of the daily percent change in the LIBOR price, or the covariance of the daily percent changes in the forward and the riskless bond prices.

The expected covariances and variances are not directly observable; to estimate these variables, I fit an ARIMA model to each monthly series or sample covariances and variances, and forecast the market's expectation using the time series models. This method assumes that the time series model that appropriately described a series in the past continues to describe the series in the future. The ARIMA models which best fit the variance and covariance series are AR1 or AR2 models, with the exception of the third nearby forward-bond covariance. This series appears to be a white-noise series; no time-series model seems to adequately fit it. I then use the estimated model to forecast the expected covariance or variance. For example, if there are three months remaining until expiration, I take an average of the one-step ahead, two-step ahead, and three-step ahead forecasts to arrive at the expected covariance for the next three months.

To convert the estimates from daily covariances and variances to covariances over the remaining life of the contract, I multiply each covariance or variance by the number of trading days remaining until expiration. Thus, a covariance calculated with two months remaining until maturity is multiplied by 42, whereas a covariance estimated with one month left until maturity is multiplied by 21. (This method is not strictly correct unless the price diffusion process has the property that the variance over a period of $2t$ days is twice the variance over a t-day period. The familiar log-normal diffusion process has this property.) Finally, I adjust the OLS standard errors for the serial correlation induced by overlapping futures-forward price difference observations, using the modified Hansen-Hodrick approach described earlier.

Table V displays the results for the nine regressions. The CIR propositions predict that the futures-riskless bond covariance (cov(F, B) in Table V), the forward-riskless bond covariance (cov(f, B) in Table V), and the spot price variance (var(P) in Table V) over the remaining life of the contract are negatively related to the futures-forward price difference. The evidence in Table V supports the strong implications of the CIR model: each coefficient has the predicted sign.[9] If one excludes the white-noise third nearby forward-bond covariance series, six of eight coefficients are significantly less than zero at the 5% level, using a one-sided t-test. The two series with coefficients that are not significantly less than zero at the cov(F, B) and the var(P) series for the first nearby contact. One explanation for this failure is that because the CIR model predicts that the futures-forward price difference will increase with time to maturity, the contract with the least time to maturity will have the smallest futures-forward price difference and thus be most susceptible to measurement error.

To test whether these results are sensitive to using an ARIMA model to estimate expected covariances, I tried two additional methods of estimating the expected variances and covariance series. The first method employs the last month's realized covariance as an estimate of the covariance over the remaining contract life. The second method uses the realized covariances and variances as proxies for the expected covariances and variances and requires that the forecasting errors are orthogonal to the sample covariances and variances. The results from these two additional specifications are similar to results reported in Table V. Under both methods, all coefficients have the sign predicted by the CIR propositions. In addition, estimating expected covariances with the previous month's covariance results in eight out of nine coefficients that are significantly less than zero; estimating expected covariances using the realized sample covariances yields coefficients significantly less than zero in four of nine cases.

One reason for the insignificant coefficients using the second approach is that this approach makes the strong assumption that market participants have perfect knowledge about the future. If, however, they are not very good at predicting the future, forecast errors will tend to be large and the large forecast errors may prevent the coefficients from attaining significance. The other methods of estimating the expected covariances do not assume that the

[9] One reason for the low R^2's in Table V is that the present value operator in the CIR propositions produces nonlinear relations, while the regression tests are linear.

Table V

OLS Regressions of Futures-Forward Price Differences Against Expected Covariances and Variances: Tests of the Strong CIR Predictions

$$\log\left(\frac{F}{f}\right)_t = \alpha + \beta \cdot \mathrm{cov}(\ \cdot\ )_t + \varepsilon_t$$

Log(F/f) is the percent difference in the futures and forward prices. Cov(F, B) is the expected covariance of the daily percentage change in the futures and T-bill prices. Var(B) is the expected variance of the daily percentage change in the T-bill prices. Cov(f, B) is the expected covariance of the daily percentage change in the forward and T-bill prices. All standard deviations and t-statistics are adjusted for overlapping observations using Hansen/Hodrick (1980), modified to reflect the type of overlap. Covariances and variances are estimated by fitting an ARIMA model to each variance or covariance series, and using this time-series model to forecast expected covariances and variances.

	Explanatory Variable		
	cov(F, B)	var(P)	cov(f, B)
	Panel A: First nearby		
$\hat{\alpha}$	−0.056	−0.012	0.002
t-Stat	(−1.283)	(−0.205)	(0.031)
$\hat{\beta}$	−0.013	−0.075	−0.525
t-Stat	(−0.188)	(−1.132)	(−1.832)
R^2	0.001	0.026	0.055
N	61	61	62
	Panel B: Second nearby		
$\hat{\alpha}$	−0.159	0.074	−0.166
t-Stat	(−2.609)	(0.684)	(−3.081)
$\hat{\beta}$	−0.094	−0.167	−0.217
t-Stat	(−1.971)	(−3.229)	(−2.327)
R^2	0.074	0.163	0.097
N	61	62	62
	Panel C: Third nearby		
$\hat{\alpha}$	−0.088	−0.079	−0.367
t-Stat	(−0.636)	(−0.392)	(−0.666)
$\hat{\beta}$	−0.273	−0.102	−0.385
t-Stat	(−3.287)	(−2.140)	(−0.184)
R^2	0.188	0.086	0.001
N	62	62	62

market has knowledge of the future, and may therefore be the more plausible candidates for the underlying true expectations process.

D. The Impact of Measurement Error on the Cox, Ingersoll, and Ross Tests

Measurement errors in both the forward prices and the expected covariances and variances could potentially affect the interpretation of the above regression results. Under conditions reasonable to both forward prices and the expected covariances and variances, however, the regression coefficients are biased towards zero. Specifically, if the covariance or variance (denoted x_t) is measured with error $u_t \sim N(0, \sigma_u^2)$ and the futures-forward price difference (y_t) is measured with error $v_t \sim N(0, \sigma_v^2)$, then plim $\hat{\beta} = \beta/\{1 + (\sigma_u^2/\sigma_x^2)\}$ if the error vectors u and v are uncorrelated and independent of the true values of x and y. These conditions result in t-statistics biased toward zero.

The measurement error in the futures-bond covariance and the spot price variance derive from the inability to observe the market's expectation of these variables, while the forward price measurement error results from imprecise measurement of the LIBOR yield curve. These two types of measurement error are unrelated to each other and are likely to be uncorrelated. If this inference is correct, then the above conditions hold for the futures-bond covariance and the spot price variance, so the coefficient estimates for these variables will be biased toward zero.

A more complicated situation arises with the regressions involving the forward-bond covariance, because this variable suffers from both types of measurement error. The measurement error resulting from estimating the market's expectation of the forward-bond covariance is uncorrelated with the forward price measurement error if one assumes that the forward price measurement error results from the inadequacy of a quadratic to describe the LIBOR yield curve. More precisely, if the quadratic does not fail in a systematic manner (and Table II provides evidence it does not) then today's measurement error should be uncorrelated with past or future errors. Therefore, today's forward price error should be uncorrelated with the forward-bond covariance error (since estimation of these expectations employs past observations), and the t-statistics will be biased toward zero.

Thus, knowing how measurement errors occur in this sample enables one to sign the bias of the estimated coefficients. In this case the estimates are biased toward zero. This specific bias strengthens the support for the CIR model.[10]

V. Conclusions

CIR, using arbitrage arguments, model the difference in futures and forward prices created by the daily resettlement feature of futures contracts. By avoiding assumptions about consumer utility functions and risk preference, arbitrage models employ weaker assumptions than equilibrium models. The weaker assumptions may make empirical verification of arbitrage models

[10] Another approach to determining the severity of the measurement error problem is to use the futures-forward price difference as the explanatory variable and the variances and covariances as the dependent variables. Some would argue that estimating these reverse regressions minimizes the errors-in-variables problem by using the series with the greatest measurement error as the dependent variable. The results from the reverse regressions are almost identical to the results from Table III; no results switch from significant to insignificant, or vice versa.

easier, but abstractions from reality such as no transactions costs and frequent portfolio rebalancing may also introduce empirical deviations from the model. Whether these simplifications prevent validation of the theoretical model is an empirical question.

Previous empirical work supports the weak CIR prediction concerning the sign of the futures-forward price difference, but fails to find support for the stronger CIR prediction concerning the variation in the futures-forward price difference. The Eurodollar data used in this paper provides a more powerful test of the CIR model. The test results support both the weak and the strong CIR predictions because the contribution of marking-to-market to the observed futures-forward price difference depends upon the underlying asset. Eurodollars have a higher covariance with the riskless bond price so the marking-to-market effect is a larger component of the futures-forward price difference.

Appendix: Correcting for the Overlapping Structure of Futures-Forward Price Differences

As discussed in the text, because futures contracts follow a quarterly cycle with fixed expiration dates, monthly observations of the futures-forward price difference will overlap and be serially correlated. This overlapping structure is similar, but not identical, to the structure outlined in Hansen and Hodrick (1980). Specifically, if the first observation is the first month of the quarter, the first observation is correlated with the second and third observations, but not the fourth or higher observations. Instead of a band-diagonal covariance matrix, the futures-forward price difference covariance matrix is block diagonal, with block size of three (monthly observations, contract expiration every 4 months).

The appropriate modification of the asymptotic covariance matrix constrains the off-block elements to be zero, and estimates the block using Hansen-Hodrick's procedure. Not all of the OLS residuals, however, contribute to this estimation. For instance, to estimate the covariance component for first-order correlation, one includes all $\hat{u}_t \hat{u}_{t+1}$ (where the $\hat{u}$ represent the OLS residuals) except the residuals for those observations which one knows are uncorrelated. In the above example, $\hat{u}_1 \hat{u}_2$ and $\hat{u}_2 \hat{u}_3$ are included, but $\hat{u}_3 \hat{u}_4$ is not. Mathematically,

$$\hat{\sigma}_{12} = \frac{1}{N} \sum_{t=1}^{N} \hat{u}_t \hat{u}_{t+1} \qquad t = 1, 2, 4, 5, 7, 8, 10 \cdots$$

where σ_{12} represents a typical term in the variance-covariance matrix.

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