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Debt, Agency Costs, and Industry Equilibrium

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ABSTRACT

We show that risk characteristics of projects' cash flows are endogenously determined by the investment decisions of all firms in an industry. As a result, in reasonable settings, financial structures which create incentives to expropriate debtholders by increasing risk are shown not to reduce value in an industry equilibrium. Without taxes, capital structure is irrelevant for individual firms despite its effect on the equityholders' incentives, but the maximum total amount of debt in the industry is determinate. Allowing for a corporate tax advantage of debt, capital structure becomes relevant but firms are indifferent between distinct alternative debt levels.

THE EFFECT OF FIRMS' financial structures on subsequent investment and output decisions has recently received considerable attention in the finance literature. The central insight that equityholders control the firm's investment and production decisions and in general do not maximize the value of a levered firm has been applied to the analysis of risk shifting, underinvestment, and firms' abilities to enter into optimal contracts with their customers.[1] Existing models of firms' financial structures and their effect on investment decisions are generally based on a single-firm framework and do not take into account the relationship between project cash flows and the investment decisions of all firms in the industry.[2] In this paper we allow the distribution of investment projects within an industry to adjust and show that this generates new insights and reverses several key results of single-firm agency cost models. The new predictions conform with well documented

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[1]The original contributions are Jensen and Meckling (1976) and Myers (1977). See Green (1984, 1986), Haugen and Senbet (1981), and Williams (1987) for an analysis of the risk shifting incentive of debt and perquisites consumption. Titman (1984) examines the effect of financial leverage on contracts with customers. For a survey of this literature up to the early eighties see the monograph by Barnea, Haugen, and Senbet (1985).

[2]One exception are the contributions on industries in which firms behave strategically by Allen (1986), Brander and Lewis (1986), and Maksimovic (1986, 1988).

empirical evidence which cannot be explained by single-firm agency models. In particular, we derive the following main results.

First, we demonstrate that the risk of a project's cash flows is endogenously determined by the investment decisions of all firms in the industry. As the number of firms adopting a given production technology increases, the price of the good sold more closely reflects their production cost. Thus, these firms become better hedged against changes in the cost of production and generate less risky cash flows.[3] Since technologies with higher expected costs of producing output are adopted by fewer firms, this implies that they exhibit riskier cash flows than low-cost technologies.

Second, our analysis shows that, in reasonable settings, financial structures which create incentives to shift risk, and which appear to reduce the value of the firm when viewed in isolation, do not lower value when the distribution of projects in the industry is taken into account. In equilibrium the aggregate amount of debt in an industry adjusts, inducing investment decisions, such that the values of low-risk and high-risk projects are equal. As a result, although an individual firm's financial structure can influence investment decisions, it does not affect firm value.

Third, when we allow for a tax advantage of debt, the resulting capital structure equilibrium is asymmetric. Some firms issue low amounts of debt, foregoing debt-related tax shields but committing to the subsequent choice of the less risky project with higher pre-tax cash flows. Other firms issue large amounts of debt, capturing large tax benefits but creating incentives to subsequently choose the riskier project with lower pre-tax cash flows. In equilibrium, individual firms are indifferent between a low leverage ratio and choice of the project with higher expected pre-tax cash flows or a higher leverage ratio and choice of the project with lower expected pre-tax cash flows. The aggregate number of firms choosing a particular financial structure is endogenous.

Fourth, we demonstrate that the principal determinants of firms' financial structures such as the corporate tax rate not only have a direct effect on firms' decisions but also change the equilibrium distribution of investment projects in the industry. This affects the cash flows generated by alternative projects and thus, indirectly, firms' capital structure choices. For example, an increase in the corporate tax rate leads to an increase in the number of firms choosing the riskier investment since it supports a higher debt level. This changes the risky project's cash flow distribution and, as we demonstrate, tends to lower the project's debt capacity.

Incorporating industry equilibrium conditions into the analysis explains empirical findings which are considered to be inconsistent with static agency theories in a single-firm framework. First, in contrast to existing agency models, our model predicts that even within industries debt ratios will vary widely. This accords well with empirical observations by, for example, Rem-

[3]Throughout the remainder of this paper we use the terms investment project and production technology interchangeably.

mers, Stonehill, Wright, and Beekhuesen (1974) and Bradley, Jarrell, and Kim (1984).[4] Second, our model with corporate taxes predicts a negative relationship between profitability as measured by earnings before interest and taxes (EBIT) and leverage. Firms with high debt levels realize higher debt tax shields but choose projects with lower EBIT than firms with low debt levels. This prediction is consistent with numerous empirical studies such as Titman and Wessels (1988) and Long and Malitz (1985).[5] Finally, our analysis suggests that firms with high debt levels choose technologies with risky cash flows. The empirical evidence on this relationship is less determinate. While Castanias (1983) and Baskin (1989) find that firms' debt levels and the riskiness of cash flows are positively related, Titman and Wessels (1988) and Bradley, Jarrell, and Kim (1984) find an insignificant or a negative relationship.

Our results on agency costs of debt can be compared to those of Miller (1977) on taxes. In Miller's model firms are indifferent between alternative capital structures because in equilibrium the corporate tax advantage of debt is offset by its personal tax disadvantage. In our model the corporate tax advantage of debt is offset by the disadvantage of subsequent distortions of equityholders' incentives. Thus, before making their investment decisions individual firms are indifferent between alternative capital structures, but the aggregate amount of debt in the industry is determined in equilibrium.[6]

The analysis in this paper is also related to the contributions on the effect of financial structure on product market decisions by Allen (1986), Brander and Lewis (1986), and Maksimovic (1986, 1990). In this literature it is demonstrated that in imperfectly competitive industries, where firms act strategically in the product market, the choice of financial structures affects firms' optimal output levels. Our paper derives the distribution of financial structures within an industry in which firms behave as price-takers. The model also builds on the literature on input substitution under uncertainty by Blair (1974), Abel (1983), and de Meza (1986). In particular, de Meza (1986) models the effect of cost uncertainty on firms' technology choices in an industry equilibrium. We extend this literature by addressing the conflicts of interest between classes of security holders and the effect of financial structure on the equilibrium.

[4]Bradley, Jarrell, and Kim (1984) find that when leverage ratios of non-regulated firms are regressed on SIC codes, the R^2 of these regressions is less than 0.25. The observation that apparently similar firms exhibit a wide variation of capital structures has also been made by Myers (1984, p. 589). Alternative explanations for this variation are provided by Myers (1984) and by Fischer, Heinkel, and Zechner (1989) in a dynamic model of optimal capital structure.

[5]Other corroborating studies include Chudson (1945), Arditti (1967), Carleton and Silberman (1977), Nakamura and Nakamura (1982), Kester (1986), Baskin (1989), and Myers (1990). Kim and Maksimovic (1990) find in a study of the airline industry that firms with high financial leverage use more input per unit of output, and are therefore less efficient, than firms with low debt levels.

[6]A major difference between our framework and that of Miller (1977) is that in Miller's model investment decisions are fixed exogenously whereas in our model firms make their investment choices subsequent to their capital structure decisions.

In Section I we introduce the model and show that financial structure is irrelevant in a competitive equilibrium. The effect of a corporate tax advantage of debt is analyzed in Section II, and we summarize our findings in Section III. The proofs of all propositions are contained in the Appendix.

I. The Model Without Taxes

A. Overview

In this section we analyze the effect of financial structure on subsequent investment and production decisions in the absence of taxes. As in the single-firm case, a firm's financial structure generally affects equityholders' incentives to choose between alternative investment projects. However, when the decisions of other firms in an industry are taken into account, the effect of asset substitution on firm value differs from that in a single-firm framework. This occurs because, as de Meza (1986) demonstrates, it may be optimal for different firms to use projects with different risk profiles. In such cases financial structure affects investment decisions but may not alter firm value.

To see this, assume that there exist two alternative production technologies with the same expected costs but whose cost shocks are imperfectly correlated.[7] Suppose that, initially, all firms choose the same technology. Then competitive behavior implies that the price of the output will be highly correlated with this technology's cost shocks. Now consider a single firm's incentives to adopt the alternative technology. The price of the good sold will not be highly correlated with this deviating firm's cost shocks. In those states in which the deviating firm's production cost is lower than that of its rivals, it can increase its production and hence its profit. By contrast, in states in which the deviating firm has higher production costs, it will find it optimal to decrease production. The option to adjust the production quantity implies that profits are convex in the difference between prices and costs. Since, as discussed above, this difference is more variable for the deviating firm, its *expected* profit is higher. The option feature becomes less valuable if more firms deviate and adopt the alternative technology. In equilibrium the number of firms choosing each technology adjusts until their expected values are equated and an individual firm is indifferent between alternative technologies. This intuition generalizes to cases in which production technologies have different expected costs. The number of firms adopting each project again adjusts until their net present values are the same. However, the riskiness of projects' cash flows differs.

[7]Imperfect correlation of production costs may occur for a variety of reasons. Examples are difference in exposure to energy prices of alternative technologies, uncertainties about a new production process, imperfect and segmented input markets, or the need to enter into long-term contracts, if, say, one of the investments employs unionized labor. For a detailed example of the use of different technologies in the same industry see Henderson's (1988) discussion of the manufacture of photolithographic alignment equipment used in the production of microchips.

Now consider how the firm's project choice is affected by its financial structure. The equityholders of firms with risky debt have an incentive to choose the riskier investment. If there are too many firms with risky debt, the value of the riskier investment will fall below that of the less risky project. This creates incentives for some firms to reduce their debt levels and subsequently invest in the less risky project. In equilibrium the distribution of capital structures must adjust until no firm can increase its value by altering its equityholders' incentives to invest so that each individual firm is indifferent between alternative financial structures.[8] In the remainder of this section we introduce the model and formalize the above intuition.

B. The Model

We assume an industry in which a large number, n, of competing firms produce a homogeneous good, each firm perceiving the price as given. There are three dates. At time t_0 firms' original owners choose a financial structure consisting of equity and pure discount debt with face value D. At time t_1 equityholders select an investment project which determines the (possibly stochastic) cost of production at time t_2. The investment decision and the subsequent cost of production are not publicly observable and thus, cannot be contracted upon.[9]

After all uncertainty has been resolved, equityholders decide how much to produce at time t_2.[10] The output is sold and the resulting cash flows are distributed to security holders. To simplify the exposition we make the standard assumptions of risk neutrality and a zero interest rate. We analyze the problem recursively and begin with the production decision at time t_2.

B.1. The Time t_2 Production Decision

Each firm is assumed to be a price taker. There is no uncertainty remaining at time t_2 and equityholders face the following maximization problem:

$$\max_{q} pq - C(q, P), \tag{1}$$

[8] The main result here is that, in equilibrium, risky debt does not lower firm value. However, in the absence of taxes, there are no incentives for firms to issue debt either. We demonstrate in Section II how our analysis extends to the case where taxes provide a motive for firms to issue debt.

[9] For tractability we model uncertainty in a very simple way so that, in the model presented below, firms' cash flows could be used to infer the equityholders' investment decisions. This would allow agents to write forcing contracts where equityholders commit to a specific investment decision. In general, however, the cash flow distributions of alternative investment projects will have overlapping supports and therefore investors cannot perfectly infer the equityholders' preceding investment decisions from the final cash flows. Forcing contracts are therefore ruled out in the following analysis. More complex contracts contingent on final cash flows are discussed in Section III.

[10] We make the assumption that *all* uncertainty is removed before time t_2 only for expositional clarity. It is important, however, that equityholders receive some signal regarding the cost of production before they decide on the quantities produced. This will in general be sufficient to give rise to the option feature discussed above.

where $C(q, P)$ denotes the cost of producing q units of the good if project P is in place. The market price of the good, p, depends on the total quantity of the good produced, Q, and is given by

$$p = a - bQ. \tag{2}$$

B.2. The Time t_1 Investment Decision

The investment choice at time t_1 affects the costs of production at time t_2. We assume that these costs are not perfectly correlated across different technologies. To keep the analysis tractable each firm has access to one of only two investment projects, $P \in \{NS, S\}$, each of which requires an investment of I_P and enables the firm to produce the good.[11] The costs of production are *stochastic* for project S whereas they are *non-stochastic* for project NS.[12] The stochastic costs of project S are assumed to be the only source of uncertainty. There are two equiprobable states of nature and the marginal cost of producing with project S, $MC(S)$, in each state is[13]

$$MC(S) = \begin{cases} k - h + \gamma q \text{ in state } L \\ k + h + \gamma q \text{ in state } H. \end{cases} \tag{3}$$

We refer to states L and H as the "low-cost" and the "high-cost" state, respectively, since the marginal cost of technology S is low in state L and high in state H.[14]

For investment project NS the marginal cost function is state independent:

$$MC(NS) = k + \gamma q. \tag{4}$$

From equations (3) and (4) it follows that the technology with the lower initial investment, I_P, has lower expected total costs of producing any given quantity of the good.[15] We therefore refer to this technology as the efficient

[11]Equityholders may also be able to affect the risk of future cash flows by other means. For example, they could make risky side-bets by taking positions in financial markets. As long as the claims against the firm resulting from such side-bets are junior to the bondholders' claim, equityholders have no incentives to take such actions. Thus, if the bond indenture specifies me-first rules, exploitation of bondholders can only occur by substituting assets and altering the production technology.

[12]It is not important for our analysis to assume one technology with non-stochastic production costs. All principal results still obtain as long as the production costs of the alternative technologies are not perfectly correlated.

[13]The intuition underlying the results below extends to cases with many states or many technologies. Analyzing these more general cases increases the notational complexity considerably and does not generate significant new insights.

[14]We have also investigated the case in which there exists one non-stochastic cost technology and several stochastic cost technologies. If the stochastic cost technologies' marginal costs of producing are perfectly correlated and are given by equation (3) with different h parameters, then only two technologies are employed in equilibrium. The non-stochastic cost technology is always chosen. Which one of the stochastic cost technologies is chosen depends on their respective required capital outlays and h parameters.

[15]We have also analyzed the case where the intercept term k of the marginal cost function was allowed to differ across projects. In this case the lower-cost project has a lower k. This alternative specification of differences in the efficiency of the projects generates results analogous to the ones reported below.

technology and to the alternative, that is the technology with the higher initial investment outlay, as the inefficient technology.

Define π_P^θ as the firm's profit in state $\theta \in \{H, L\}$ given that project P is in place. Profits differ across states for two reasons. First, the stochastic production costs have a direct effect on the profits of those firms with project S. Second, the stochastic production costs affect the price of the good sold and thus make the profits of firms with project NS risky as well.

The value of the firm's equity as a function of the investment choice is given by

$$E(D, S) = {}^1\!/_2\{\max[\pi_S^L - D, 0] + \max[\pi_S^H - D, 0]\} - I_S \tag{5a}$$

$$E(D, NS) = {}^1\!/_2\{\max[\pi_{NS}^L - D, 0] + \max[\pi_{NS}^H - D, 0]\} - I_{NS} \tag{5b}$$

where D is the firm's face value of debt and π_P^θ denotes the firm's profit, derived in the Appendix.

The equityholders choose the investment project that maximizes the value of their securities and thus solve the following problem at time t_1:

$$\max_{P \in \{S, NS\}} E(D, P). \tag{6}$$

Inspection of equations (5a) and (5b) reveals that the choice of investment projects generally depends on the firm's debt level. We now endogenize the capital structure decision at time t_0.

B.3. The Time t_0 Capital Structure Decision

At time t_0 the firms' owners will choose a financial structure that maximizes the total value of the firm, taking into account the investment choice implied by the capital structure decision. Thus the equityholders' maximization problem at time t_0 is given by

$$\max_D V(P(D)) = E(D, P(D)) + B(D, P(D)) \tag{7}$$

where $V(P(D))$, $E(D, P(D))$ and $B(D, P(D))$ denote the values of the firm and its equity and debt securities, respectively, which depend on the face value of debt and the firm's project choice. The latter is itself a function of the firm's debt level.

C. Financial Structure and Industry Equilibrium

In an industry equilibrium the number of firms choosing each project adjusts until the expected profits from the two projects are equal.[16] In this section we derive the distribution of projects and characterize the equilibrium financial structures which support it. More specifically, we proceed as follows. (i) We first derive the equilibrium number of firms choosing projects S and NS. (ii) For this equilibrium distribution of projects we characterize the

[16] If not, some firms can increase their values by changing their capital structures and thereby altering the equityholders' investment decisions.

riskiness of cash flows generated by each project. (iii) We then derive equityholders' project choice as a function of the firm's debt level; and, finally, (iv) we characterize the equilibrium distributions of firms' debt levels that support investment decisions derived in (i).

C.1. The Equilibrium Distribution of Project Choices

As discussed above, if production costs are not perfectly correlated, a project can be employed in equilibrium even if it requires a higher initial investment. As long as there is one state in which the project has lower production costs than the alternative project, it may be profitable for some firms to adopt it. In equilibrium, the number of firms choosing each project will adjust such that the higher investment outlay of the project will just be offset by the higher profits in the state(s) in which its production costs are lower. The equilibrium number of firms choosing projects S and NS is derived in Proposition 1 by finding the number of firms which equates the total values of the two projects.[17] Define $I^* \equiv I_S - I_{NS}$.

Proposition 1: *If the difference between the required investments is sufficiently small then $\hat{f}$ firms choose project S and the remaining $n - \hat{f}$ firms choose project NS where*

$$\hat{f} = \frac{(h^2 - 2\gamma I^*)(bn + \gamma)}{2bh^2}.$$

If $I^* > \frac{h^2}{2\gamma}$, then all firms choose project NS and if $I^* < \frac{h^2}{2\gamma}\frac{\gamma - nb}{\gamma + nb}$, then all firms choose project S.

For the distribution of projects derived in Proposition 1 the projects' expected cash flows, net of the initial investment, are equal.[18] In general, however, the riskiness of these cash flows differs across technologies and, therefore, equityholders of levered firms are not indifferent between projects. Thus, in order to analyze equityholders' investment decisions, we first need to determine the riskiness of the cash flows generated by the alternative investments.[19]

C.2. The Riskiness of Project Cash Flows

In contrast to the analysis in a single-firm context, the riskiness of a project is endogenous and depends on the equilibrium number of firms choosing each investment project. To understand this result intuitively, suppose that a given firm faces an unexpectedly high cost of production. If most other firms in the industry have chosen the same production technology

[17] de Meza (1986) derives a similar result for the case in which the initial investment is zero.

[18] In general the distribution of projects derived in Proposition 1 does not maximize consumer surplus. It can be shown that consumer surplus is maximized if all n firms choose technology S. For a discussion of consumers' gains and price uncertainty see Waugh (1944).

[19] Throughout the remainder of the paper we focus on the case in which each technology is chosen by at least one firm.

and therefore face the same shock in production costs, then the higher production costs will be reflected in a higher price of the goods sold. This reduces the sensitivity of the firm's profits to stochastic changes in production costs. By contrast, if the firm is the only firm or one of few firms in the industry affected by this increase in costs, prices will not reflect the higher production costs and the firm's profits will fall significantly. The same intuition holds for the alternative case where a firm's cost of producing a given quantity of the good is lower than that of its competitors. If this firm is one of few firms realizing this cost advantage, then its profits will be large. If the firm shares this cost advantage with a large number of other firms, then the positive effect on profits will be smaller. Thus, the relative magnitudes of projects' cash flows depend on the total number of firms choosing each project. It follows from Proposition 1 that, as a project becomes relatively less cost efficient due to an increase in the initial investment, I_P, it is chosen by fewer firms. Together with the above discussion this implies that the maximum cash flow from the inefficient project exceeds the maximum cash flow from the efficient project and the minimum cash flow from the inefficient project is lower than the minimum cash flow from the efficient project. This result is stated formally for $I^* > 0$ in Lemma 1.

Lemma 1: *In equilibrium* $\pi_S^L > \pi_{NS}^H > \pi_{NS}^L > \pi_S^H$.

The ranking of profits in Lemma 1 implies a direct relationship between the risk of net cash flows, defined as profits net of initial investment, and efficiency. Less cost-efficient projects exhibit riskier net cash flows. By contrast, low-cost projects are chosen by more firms and therefore generate less risky net cash flows. This is so even if the unit cost of production associated with this project is more variable than that of the alternative project. The next proposition formalizes this result.

Proposition 2: *In equilibrium the net cash flows of the inefficient project are riskier in the sense of Rothschild and Stiglitz* (1970) *than the net cash flows of the efficient project.*

We can now analyze how an individual firm's financial structure affects the equityholders' investment incentives, taking as given the equilibrium distribution of projects derived in Proposition 1. Proposition 2 implies that an asset substitution problem exists when projects are not equally cost efficient, i.e., if $I_S \neq I_{NS}$, and we focus on this case. To simplify the analysis we assume throughout the remainder of the paper that $I_S > I_{NS} = 0$, i.e., that technology S is less efficient.[20]

[20]For the general case where $I_{NS} > 0$, the debt levels of firms choosing project NS have to satisfy an additional constraint: the expected payments to equityholders must exceed the cost of the project, I_{NS}. Assuming $I_{NS} = 0$ allows us to drop this constraint. In a previous version of the paper we have also derived the results for the case where $I^* < 0$. As shown in Proposition 2, technology NS is then riskier. In that case all the following results still obtain with technologies S and NS interchanged.

C.3. Project Choice as a Function of the Debt Level

Since in equilibrium the expected profits from each investment project are equal, equityholders only choose the less risky project if the debt claims are riskless under both investment choices. If debt is risky, equityholders strictly prefer the riskier project. Thus, there exists an upper bound on the debt level that is consistent with the subsequent choice of the low-risk investment project. The relationship between a firm's debt level and the equityholders' incentives to choose a particular project is derived in the following proposition. Define $D^{\max} = \pi_S^L - 2I_S$.

Proposition 3: *Equityholders of firms with riskless debt, i.e., $D \leq \pi_S^H$, are indifferent between projects S and NS. Equityholders of firms with face values of debt $\pi_S^H < D \leq D^{\max}$ strictly prefer project S and their debt is risky. Equityholders of firms with debt face values greater than $D^{\max}$ do not invest.*

The effect of a firm's financial structure on the equityholders' investment incentives is illustrated in Figure 1. The total value of the firm as a function of the face value of debt is given by the horizontal line $V(D, S) = V(D, NS)$.

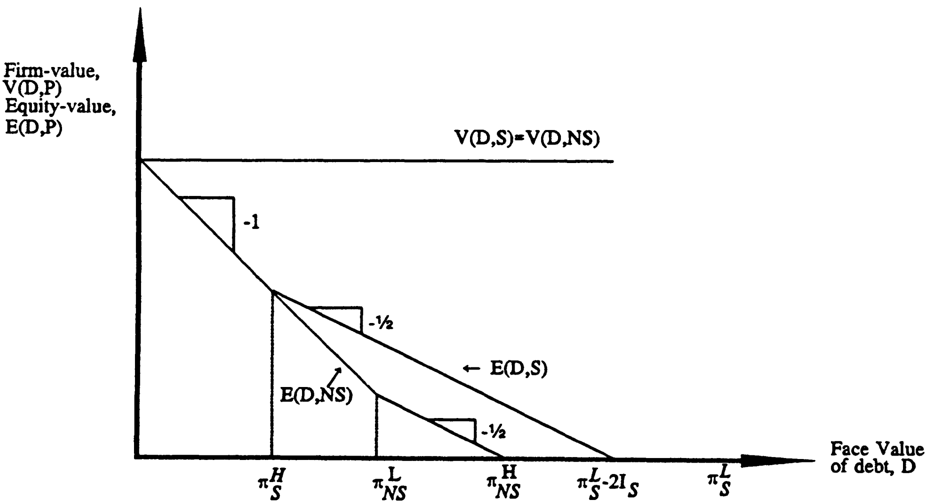

Figure 1. Firm value and equity value as a function of debt, D, and investment choice, $P \in \{S, NS\}$ for the case without taxes and $I_S > I_{NS} = 0$. I_S, I_{NS} denote the initial capital outlays for projects S and NS, respectively; D denotes the face value of debt; $V(D, P)$, $E(D, P)$ are, respectively, the total value of the firm and the value of equity, as a function of its debt level, D, and its project choice, P; π_P^θ denotes project P's profit in state θ, where $\theta \in \{H, L\}$ and the two states are equally likely. The ordering of the cash flows is derived in Lemma 1 and in the proof of Proposition 2. As the face value of debt increases, the value of the equity decreases with a slope of -1 if the debt is riskless, while it decreases with a slope of $-{}^1\!/_2$ if the debt is risky. For project S, debt is risky if D exceeds π_S^H. For project NS debt is risky if D exceeds π_{NS}^L. Thus, for $D > \pi_S^H$, the equity value for project S exceeds the corresponding equity value for project NS. Equityholders of firms with $D > \pi_S^H$ therefore prefer project S. In equilibrium, the total value of firms adopting project S or NS is equal.

The equity value is a decreasing function of the debt level. For debt levels below π_S^H, debt is riskless irrespective of the investment choice and thus the equity values are the same for both investment projects. For debt levels between π_S^H and π_{NS}^L debt is still riskless for project NS but already risky for project S. Thus, over this range, $E(D, S)$ decreases less rapidly than $E(D, NS)$. Finally, for debt levels greater than π_{NS}^L, debt is risky for both investment projects and the slopes of $E(D, S)$ and $E(D, NS)$ are the same. For debt levels greater than or equal to π_{NS}^H the equity only has value if project S is chosen. If the debt level exceeds $\pi_S^L - 2I_S$, equityholders do not have an incentive to invest. Thus, if the face value of debt is less than or equal to π_S^H, equityholders are indifferent between projects S and NS. For debt face values greater than π_S^H, equityholders find it in their own best interest to choose project S.[21]

C.4. The Equilibrium Distribution of Financial Structures

We next analyze the equilibrium distributions of debt levels in the industry such that at time t_0 no firm can increase value by altering its financial structure and thereby affecting equityholders' subsequent investment incentives. As we have shown in Proposition 3, equityholders of those firms with risky debt outstanding have an incentive to subsequently choose the less efficient and riskier investment. If there are too many firms with risky debt, the expected profits of the less efficient technology are reduced below those of the alternative technology. This creates incentives for some firms to reduce their debt levels, thereby inducing the subsequent choice of the efficient technology. In equilibrium the distribution of debt levels will result in the first best distribution of projects at time t_1 described in Proposition 1. In this case the net present values of the two projects are equal and the value of an individual firm does not depend on its financial structure.[22] The equilibrium is characterized in Proposition 4.

Proposition 4: *In equilibrium not more than $\hat{f}$ firms choose risky debt levels, i.e., $D > \pi_S^H$. An individual firm's choice of a debt level in the range $0 \leq D \leq D^{\max}$ affects its future investment decision but not the value of the firm.*

According to Proposition 4 the number of firms with risky debt outstanding is bounded. This implies that there exists an upper bound on the aggregate amount of debt firms issue in equilibrium. This upper bound is given by $\hat{f}D^{\max} + (n - \hat{f})\pi_S^H$.

In this section we have demonstrated that first best investment decisions can be achieved even if some firms issue risky debt, creating incentives for equityholders to choose the riskier project. The specific distribution of pro-

[21] In the model we do not formally allow equityholders to invest in more than one production technology. However, even if we allow firms to do this, it can be shown that it is not sequentially rational for equityholders of firms with risky debt to employ both technologies simultaneously.

[22] As stated in Proposition 4 below, debt levels are irrelevant for $0 \leq D \leq D^{\max}$. If $D > D^{\max}$ equityholders do not invest at time t_1 and the firm value is zero.

jects and financial structures depends on the assumed model structure, i.e., linear demand and marginal cost curves and that only the constant in the marginal cost function is affected by the state. However, the result, that firms are indifferent between alternative financial structures as long as the efficiencies of the technologies only differ within a range, will generally be robust with respect to these assumptions. In the next section we extend the analysis and introduce a motive for firms to issue debt by allowing for a corporate tax advantage of debt.

II. The Model with Corporate Taxes

In Section I the only potential effect of the capital structure decision on firm value was by altering investment incentives. In this section we generalize the analysis and introduce corporate taxes. With taxes the value of the firm is affected by the amount of debt-related tax shields and, as a result, the firm's capital structure is no longer completely irrelevant. We show, however, that firms are indifferent between (i) restricting the amount of debt to a critical level and subsequently investing in a project that maximizes before-tax profits and (ii) choosing a high debt level and investing in a less efficient project with lower before-tax profits. In equilibrium the lower before-tax profit from the "inferior" investment is exactly offset by the interest tax deduction. Thus, although the complete irrelevance result derived above no longer holds with taxes, firms are still indifferent between distinct alternative financial structures.

To simplify the exposition, we do not consider personal taxes and assume that total payments to debtholders are deductible from the corporate tax base.[23] Let τ denote the corporate tax rate. Then the quantity produced at time t_2 is obtained by

$$\max_q [pq - C(q, P)](1 - \tau).$$

Thus, the optimal output produced at time t_2 is unaffected by the proportional corporate tax. The equity values at time t_1 are now given by

$$E(D, S) = (1 - \tau)\{{}^1\!/_2[\max[\pi_S^L - D, 0] + \max[\pi_S^H - D, 0]] - I_S\} \quad (10a)$$

$$E(D, NS) = (1 - \tau)\{{}^1\!/_2[\max[\pi_{NS}^L - D, 0] + \max[\pi_{NS}^H - D, 0]]\} \quad (10b)$$

where π_P^θ denotes the profits before corporate tax and before debt payments and where it is assumed that the initial investment outlay can be depreciated

[23] In a more realistic framework, the tax benefit of debt would only be realized on the interest portion of the debt payments. As shown by Talmor, Haugen, and Barnea (1985) and Zechner and Swoboda (1986), the distinction between interest and principal becomes important when tax shields can be lost due to depreciation and/or other non-cash charges. In the context of our model, making only interest payments tax deductible would not alter the following results.

immediately for tax purposes so that the after-tax investment outlay is given by $(1 - \tau)I_S$.[24]

Equilibrium at time t_0 requires that no firm can increase its value by changing its capital structure. In the absence of a corporate tax advantage of debt, the differential cash flows generated by alternative investment projects must compensate investors for the difference in the initial capital outlay. With corporate taxes investors also take into account differences in the tax shields generated by alternative investments. These differences affect firms' after-tax profits and therefore alter the equilibrium distribution of projects in an industry. The following proposition derives the equilibrium number of firms choosing the risky and the riskless investment projects.[25]

Proposition 5: *The number of firms choosing investment project S, $\hat{f}^\tau$, is given by*

$$\hat{f}^\tau = \hat{f} + \frac{\gamma E[\Delta TS](bn + \gamma)}{bh^2(1 - \tau)},$$

where $\hat{f}$ denotes the number of firms choosing project S in the absence of taxes and ΔTS is the difference in the corporate tax shields generated by the two alternative investment projects:

$$E[\Delta TS] = E[TS(D_S)] - E[TS(D_{NS})],$$
$$E[TS(D_S)] = {}^1/_2\tau\{\min[\pi_S^L, D_S] + \min[\pi_S^H, D_S]\} \text{ and}$$
$$E[TS(D_{NS})] = {}^1/_2\tau\{\min[\pi_{NS}^L, D_{NS}] + \min[\pi_{NS}^H, D_{NS}]\}.$$

It is apparent from Proposition 5 that the number of firms choosing a particular technology can be higher or lower than in the no-tax case, depending on the sign of the difference in the expected tax shields. In particular, if the less efficient project realizes larger tax shields, the number of firms adopting it will be greater than that without taxes.

[24] There are several ways of modeling the tax shields arising due to the initial investment outlay. Equations (10a) and (10b) reflect our assumption that the investment, I_P, can be depreciated and leads to an immediate tax refund to equityholders of τI_P. Alternatively, one could assume that the tax benefit from deducting the investment is realized at time t_2. In this case, at time t_2, the tax authorities reduce the firm's tax liabilities by τI_P. (This might lead to negative tax payments, especially for firms with technology S in state h. "Negative tax payments" can occur in a world in which losses can be carried back and/or tax shields can be sold to other corporations.) Then equityholders of firms choosing technology S pay the cost of the investment, but only receive the benefits of its tax deductibility in the no-default states, i.e., in state L. The corresponding equity value is then given by $E(D, S) = {}^1/_2\{\pi_S^L - D - \tau(\pi_S^L - D - I_S)\} + {}^1/_2 \times 0$. Thus, the fact that the tax deductibility of I_S does not benefit the equityholders in state H increases the expected after-tax investment cost of technology S. While this would affect the specific numbers of firms choosing each project, modeling this effect does not qualitatively alter any of our results in Propositions 5 and 6.

[25] As in the previous section we focus on interior equilibria where $0 < \hat{f}^\tau < n$.

Next we can again relate a firm's financial structure to its investment choice. In the presence of a tax advantage of debt and holding investment decisions constant, the initial owners of a firm prefer to be fully debt financed. However, high debt levels are inconsistent with the subsequent choice of the less risky project. As a result, we obtain an asymmetric equilibrium in which some firms are highly levered and the equityholders of these firms choose the risky investment project. Equityholders of other firms will choose lower debt levels, thereby committing to the less risky project. The following proposition derives this result formally.

Proposition 6: *Firms which choose the less efficient technology issue debt with a higher face value than firms choosing the efficient technology. Specifically,* $D_S = \pi_S^L - 2I_S$, $D_{NS} = \pi_{NS}^L + \pi_{NS}^H - \pi_S^L + 2I_S$ *and* $D_S > D_{NS}$.

The optimal debt levels derived in Proposition 6 have the following properties. Firms choosing project S issue risky debt.[26] For these firms the optimal face value of debt is $D_S = \pi_S^L - 2I_S$, the highest debt level consistent with subsequent investment. If the face value of debt exceeds this value, then equityholders would "underinvest" and neither project would be chosen. The debt of firms with face values D_{NS} is riskless if project NS is chosen but risky if project S is adopted.[27] At the debt level D_{NS} defined in Proposition 6, equityholders are exactly indifferent between exploiting debtholders by choosing technology S and choosing the more efficient technology NS. For any debt level below D_{NS}, firms would unnecessarily forego debt tax shields.

Two Corollaries follow from Proposition 6.

Corollary 1: *Firms choosing the inefficient technology issue debt with a higher market value.*

Corollary 2: *Firms choosing the inefficient technology realize higher expected tax shields than firms choosing the efficient technology*: $E[TS(D_S)] > E[TS(D_{NS})]$.

Thus, firms that choose the risky project have higher amounts of debt outstanding and also realize higher expected tax shields. This result together with the expression for $\hat{f}^\tau$ in Proposition 5 implies that taxes increase the number of firms that choose the inefficient technology. In equilibrium the higher value of debt tax shields is offset by the fact that the profits before interest and taxes of the inefficient technology are lower.

A. Capital Structure Effects of Changing Corporate Tax Rates

The distributions of investment projects and firms' equilibrium financial structures depend on exogenous parameters such as the corporate tax rate, τ, and the uncertainty of the production cost captured by the parameter, h. In

[26] It is shown in Lemma 2 in the Appendix that $\pi_S^L - 2I_S > \pi_{NS}^H$. Since the profit definitions imply that $\pi_{NS}^H > \pi_S^H$, debt with a face value of $D_S = \pi_S^L - 2I_S$ is risky.

[27] This is true since, as shown in Lemma 2 in the Appendix, $\pi_{NS}^H - \pi_S^L + 2I_S < 0$, implying that $D_{NS} = \pi_{NS}^L + \pi_{NS}^H - \pi_S^L + 2I_S < \pi_{NS}^L$, i.e., debt with this face value is riskless if project NS is chosen.

an industry equilibrium, changing each one of these parameters has two effects. First, there is a direct effect on firms' optimal capital structures, holding the investment decisions of other firms constant. Second, a change in a parameter affects the equilibrium distribution of projects and thereby their risk characteristics. This second effect on optimal capital structure cannot be captured by models focusing on single firm. In this section we examine the interaction of both effects.

As an example consider the effect of an increase in the corporate tax rate on firms' financial structures. Holding the distribution of before-tax cash flows of projects S and NS constant, this change in the corporate tax rate increases the difference in the value of the tax shields generated by the two projects. Each individual firm then has an incentive to raise the face value of debt to $D = D_S$, and choose project S. This corresponds to the first effect discussed above. However, as an increasing number of firms do this, they alter the distribution of projects in the industry and thereby the projects' cash flows. In the new equilibrium, more firms choose technology S, but individual firms are again indifferent between the two alternative projects. Since in the new equilibrium more firms choose project S, the cash flows generated by this production technology are less risky, i.e., its highest profit, π_S^L, decreases and its lowest profit increases. Since the face value of debt issued by firms choosing technology S, $D_S = D^{\max}$, depends on the highest profit, and is equal to $\pi_S^L - 2I_S$, it decreases with the corporate tax rate. The opposite is true for the debt level of firms which plan to select technology NS, D_{NS}. The intuition for this result relies on the fact that the less risky project S's cash flows are, the smaller are the gains to equityholders from shifting to this project. This raises the debt level that can be chosen while still committing to the low risk project. This implies a positive relationship between corporate taxes and the optimal debt level D_{NS}. Thus, if the distribution of firms' investment decisions is endogenous, then there is no simple relationship between corporate taxes and optimal debt levels. While some firms will increase their debt levels as a response to higher tax rates, other firms lower the face value of debt issued. These results are formally stated in Proposition 7.

Proposition 7:

(a) The number of firms adopting the inefficient project increases with the corporate tax rate: $d\hat{f}^\tau / d\tau > 0$.
(b) The optimal debt face value of firms choosing the inefficient project decreases with the corporate tax rate and the optimal debt face value of firms choosing the efficient project increases with the corporate tax rate: $dD_S / d\tau < 0$ *and* $dD_{NS} / d\tau > 0$.

III. Conclusions

This paper shows that the analysis of important dimensions of the agency problems associated with debt requires a multi-firm framework. In an industry equilibrium the riskiness of a project's cash flows is determined endogenously and depends on the investment decisions of all firms. In equilibrium those investment projects with higher expected costs of producing output are chosen by fewer firms and generate riskier cash flows. Although debt provides incentives for asset substitution, we provide a reasonable setting where individual firms are indifferent between alternative financial structures as long as the difference in efficiency of the alternative technologies does not exceed a given bound. In equilibrium the number of firms with high leverage ratios adjusts so that initial firm owners are indifferent between the choice of a high debt level and subsequent investment in the risky project and a low debt level and subsequent investment in the low risk project. We show that, in the absence of a tax advantage of debt, capital structure affects investment decisions but not the value of the firm. The latter result provides an analog to Miller's (1977) tax equilibrium for the case of agency costs.

If we allow for a tax advantage of debt, individual firms are no longer indifferent between all alternative financial structures. In the absence of agency costs, firms would now wish to become fully debt financed. However, large amounts of debt create incentives to subsequently invest in the riskier project. As a result, some firms find it optimal to limit their debt issues and thus commit to invest in the less risky project. In equilibrium the cash flow from the low risk project compensates firms for the foregone tax shields. Individual firms are indifferent between issuing the maximum amount of debt consistent with the subsequent choice of the less risky project and becoming highly levered and choosing the riskier project.

The model of financial structure in this paper provides several empirical predictions. First, our model suggests that, even when the financial structure affects the equityholders' incentives to invest, firms in the same industry are indifferent between alternative financial structures. This is consistent with empirical findings indicating that apparently similar firms in the same industry exhibit diverse financial structures.

Second, as discussed above, the riskiness of an investment project depends on the decisions of all firms in the industry. A firm adopting a technology chosen by most of its competitors is partly hedged against shocks in its production costs since they will be reflected in the price of the goods sold. This natural hedge is not available for deviant firms. Thus, the cash flows of firms adopting a technology which is chosen by a minority of firms should be more volatile.

Third, we demonstrate that changes in the corporate tax rate can have a surprising effect on firms' optimal debt level because the value of tax shields influences the equilibrium distribution of projects in the industry. Increasing the corporate tax rate makes the project with the higher debt capacity more attractive and, consequently, it is chosen by more firms. This tends to lower the highest cash flows generated by this project and thus decreases the debt capacity of firms choosing this project. Thus, firms that choose the risky project decrease their debt levels as the corporate tax rate increases. By contrast we show that firms choosing the low risk project are able to increase their debt level as a response to an increase in the tax rate.

Fourth, the model suggests a link between technology choice and financial structure. Within an industry, firms that adopt the technology chosen by the majority of firms generate higher expected earnings before interest and taxes and are less levered than firms that deviate and adopt a technology which is only chosen by few firms.

The analysis of this paper has focused on the choice between alternative production technologies. Our analysis of an industry equilibrium can also be extended to explore the effect of financial structure on other operating decisions. For example, consider an industry in which the relative demand for high quality and low quality product lines is uncertain. Then there may exist an equilibrium in which some firms produce high quality lines and others produce low quality lines. Firms' financial structures may affect equityholders' decision whether to produce high or low quality goods, but for the individual firm the choice between high leverage and low leverage will be a matter of indifference.

Appendix

Derivation of Firms' Profits: Firms maximize profits in each state by producing until marginal cost is equal to the price of the good. The profits are then equal to the difference between revenues and costs at that quantity. For example, in state L a firm with technology S will produce the quantity q_S^L such that $p_L = k - h + \gamma q_S^L$, where p_L denotes the price of the good in state L. The firm's profit in this state is then given by

$$\pi_S^L = p_L q_S^L - \int_0^{q_S^L} MC(S, L)\, dq.$$

Substituting for q_S^L and $MC(S, L)$ and integrating yields $\pi_S^L = (p_L - k + h)^2/2\gamma$. Similarly, we obtain

$$\pi_S^H = \frac{(p_H - k - h)^2}{2\gamma}; \; \pi_{NS}^L = \frac{(p_L - k)^2}{2\gamma}; \; \pi_{NS}^H = \frac{(p_H - k)^2}{2\gamma},$$

where p_H denotes the price of the good in state H.

Proof of Proposition 1: The total firm values for projects S and NS are given by

$$V(S) = {}^1\!/_2[\pi_S^L + \pi_S^H] - I_S \tag{A1}$$

and

$$V(NS) = {}^1\!/_2[\pi_{NS}^L + \pi_{NS}^H] - I_{NS}. \tag{A2}$$

In equilibrium the values of the two projects must be equal. Equating expressions (A1) and (A2) yields

$$h[p_L - p_H] + h^2 = 2\gamma I^*$$

or

$$p_L - p_H = 2\gamma I^*/h - h, \tag{A3}$$

where $I^* = I_S - I_{NS}$. Let the equilibrium number of firms choosing investment project S be $\hat{f}$. The prices in each state can be determined by equating the aggregate output in each state with the quantity demanded, yielding

$$p_L = \frac{a\gamma + b(nk - h\hat{f})}{bn + \gamma} \tag{A4}$$

$$p_H = \frac{a\gamma + b(nk + h\hat{f})}{bn + \gamma}. \tag{A5}$$

Subtracting the expression for p_H from the expression for p_L, we obtain

$$p_L - p_H = \frac{-2bh\hat{f}}{bn + \gamma}. \tag{A6}$$

Equating expressions (A3) and (A6), we solve for $\hat{f}$

$$\hat{f} = \frac{(h^2 - 2\gamma I^*)(bn + \gamma)}{2bh^2}. \tag{A7}$$

For the above solution to be feasible we need $0 < \hat{f} < n$. These inequalities lead to the following condition

$$\frac{h^2}{2\gamma} > I^* > \frac{h^2}{2\gamma}\frac{\gamma - nb}{\gamma + nb}.$$

If $I^* > \dfrac{h^2}{2\gamma}$ then all firms choose project NS and if $I^* < \dfrac{h^2}{2\gamma}\dfrac{\gamma - nb}{\gamma + nb}$ then all firms choose project S.

Proof of Lemma 1: Assume that $I^* > 0$. Since it follows directly from the profit definitions that $\pi_{NS}^H > \pi_{NS}^L$, we only need to show that $\pi_{NS}^L > \pi_S^H$ and $\pi_S^L > \pi_{NS}^H$. Substituting for π_{NS}^L and π_S^H, the first inequality can be written as

$$\frac{(p_L - k)^2}{2\gamma} > \frac{(p_H - k - h)^2}{2\gamma}.$$

For this to hold requires $p_L - p_H > -h$. Substituting for p_H and p_L from (A4) and (A5) we obtain

$$\hat{f} < \frac{bn + \gamma}{2b}.$$

Substituting for $\hat{f}$ from (A7) we obtain the condition that

$$\hat{f} = \frac{(h^2 - 2\gamma I^*)(bn + \gamma)}{2bh^2} < \frac{bn + \gamma}{2b},$$

which holds for $I^* > 0$. The proof of the second inequality, $\pi_S^L > \pi_{NS}^H$, is analogous.

Proof of Proposition 2: We prove this proposition for $I^* > 0$. We demonstrate that, net of differential capital investments, the cash flows of project S are a mean preserving spread of cash flows of project NS. Define $\overline{\pi}_P^\theta$ as the net cash flows of project P in state θ, $\overline{\pi}_P^\theta \equiv \pi_P^\theta - I_P$. Recall that in equilibrium expected net cash flows are equal:

$$^1/_2\left[\overline{\pi}_S^H + \overline{\pi}_S^L\right] = {}^1/_2\left[\overline{\pi}_{NS}^H + \overline{\pi}_{NS}^L\right]. \tag{A8}$$

We have shown above that $\pi_S^H < \pi_{NS}^L$ which, for $I_S > I_{NS}$, implies $\overline{\pi}_S^H < \overline{\pi}_{NS}^L$. Thus, the equilibrium condition (A8) requires that $\overline{\pi}_S^L > \overline{\pi}_{NS}^H$ and, as a result, the net cash flows of the inefficient project S are a mean preserving spread of the net cash flows of project NS and are therefore riskier in a Rothschild and Stiglitz sense. The proof for $I^* < 0$ is analogous.

The following lemma will be used in the proof of Proposition 3.

Lemma 2: *Assume that* $I_S > I_{NS} = 0$. *Then the following inequality holds in equilibrium*: $\pi_S^L - \pi_{NS}^H > 2I_S$.

Proof: For $I_S > I_{NS} = 0$ it follows from Lemma 1 that $\pi_S^H < \pi_{NS}^L$. Then, for equation (A8) to hold, this requires that $\pi_S^L > \pi_{NS}^H + 2I_S$.

Proof of Proposition 3: Proposition 2 and Lemma 2 imply that $\pi_S^H < \pi_{NS}^L < \pi_{NS}^H < \pi_S^L - 2I_S$. Then, if $D \le \pi_S^H$ debt is riskless. Since in equilibrium $V(D, S) = V(D, NS)$, equityholders of firms with riskless debt are indifferent between projects S and NS. For $\pi_S^H < D \le D^{\max}$, debt is risky if project S is chosen. For this case Proposition 2 implies that $E(D, S) > E(D, NS)$. Thus, for these debt levels equityholders prefer the risky investment project S. For $D > D^{\max}$, investing in technology S is a negative net present value project for the equityholders and, thus, for these debt levels they do not invest.

Proof of Proposition 4: Assume that f firms choose debt levels $D \ge \pi_S^H$ and that $f \le \hat{f}$. We start by showing that this distribution of financial structures is an equilibrium. Conjecture the following investment decisions. The f firms with risky debt (i.e., $D \ge \pi_S^H$) outstanding all choose project S. $n - \hat{f}$ firms with riskless debt (i.e., $D < \pi_S^H$) choose project NS and $\hat{f} - f$ firms with riskless debt choose project S. The total number of firms choosing project S is therefore equal to $\hat{f}$ which implies that $V(D, S) = V(D, NS)$. For this distribution of projects, it follows from Proposition 3 that the above conjectured investment decisions are sequentially rational for equityholders. There is no incentive for any firm to change its capital structure or investment decision.

Next we prove that if more than $\hat{f}$ firms choose debt levels $D \ge \pi_S^H$, there does not exist an equilibrium. It follows from Proposition 1 that the values of the two projects are equal if and only if $\hat{f}$ firms adopt technology S and $n - \hat{f}$ firms adopt technology NS. Proposition 3 implies that, with this distribution of projects, equityholders of firms with $D > \pi_S^H$ have an incentive to select project S. If more than $\hat{f}$ firms have such high debt levels, then this contradicts the equilibrium condition in Proposition 1 that only $\hat{f}$ firms choose project S. Thus, if more than $\hat{f}$ firms choose debt levels $D \geq \pi_S^H$, the values of the two projects cannot be equal and some firms lower their debt levels to induce a change in the subsequent investment decision.

We have now demonstrated that not more than $\hat{f}$ firms issue risky debt and that in equilibrium $\hat{f}$ firms choose project S and $n - \hat{f}$ firms choose project NS. In this case the values of the two projects are equal but the investment decision of an individual firm depends on its debt level according to Proposition 3.

Proof of Proposition 5: In equilibrium $V(D_S, S) = V(D_{NS}, NS)$:

$$\begin{aligned} &{}^1\!/_2\left[\pi_S^H + \pi_S^L - 2I_S\right](1-\tau) + E\left[TS(D_S)\right] \\ &\quad = {}^1\!/_2\left[\pi_{NS}^H + \pi_{NS}^L\right](1-\tau) + E\left[TS(D_{NS})\right]. \end{aligned} \tag{A9}$$

The first term on the left-hand side of equation (A9) denotes the after-tax net-present value of an all-equity financed firm choosing project S. The second term reflects the corporate tax savings due to issuing debt with a face value of D_S. A similar interpretation can be given to the right-hand side of equation (A9) for firms choosing project NS.

Substituting the definitions of firms' pre-tax profits and simplifying yields

$$p_L - p_H = \frac{2\gamma\left(I_S(1-\tau) - E[\Delta TS]\right)}{h(1-\tau)} - h \tag{A10}$$

where ΔTS is defined in the statement of the Proposition. Since taxes do not affect the production decision, equation (A6) also holds with taxes. Equating the expressions for $p_L - p_H$ from equations (A6) and (A10) yields

$$\hat{f}^\tau = \frac{\left(h^2(1-\tau) + 2\gamma\left(E[\Delta TS] - I_S(1-\tau)\right)\right)(bn+\gamma)}{2bh^2(1-\tau)}. \tag{A11}$$

Substituting the expression for $\hat{f}$ from Proposition 1 and noting that $I_{NS} = 0$ yields the result.

The following Lemma will be used in the proof of Proposition 6.

Lemma 3: *In equilibrium the following condition holds*: $\pi_S^L - \pi_{NS}^H > 2I_S$.

Proof: There are two cases to consider depending on whether $E[\Delta TS]$ is greater or less than zero.

Assume first that $E[\Delta TS] < 0$. Then $(E[\Delta TS] - I_S(1-\tau)) < 0$ which implies from (A10) that $p_L - p_H > -h$. Using the definitions of firm profits, it then follows that $\pi_S^H < \pi_{NS}^L$. Inspection of equilibrium condition (A9) reveals that the inequalities $\pi_S^H < \pi_{NS}^L$ and $E[\Delta TS] < 0$ imply that $\pi_S^L - \pi_{NS}^H - 2I_S > 0$.

Second, assume that $E[\Delta TS] > 0$. To construct a contradiction, assume that $\pi_S^L - \pi_{NS}^H - 2I_S < 0$. Then firms adopting the NS project can be fully debt financed without creating an incentive to switch to project S, so that now $D_{NS} = \pi_{NS}^H$ and $D_S = \pi_S^L - 2I_S$. Then

$$E[\Delta TS] = {}^1\!/_2\tau\left[\left(\pi_S^H + \pi_S^L - 2I_S\right) - \left(\pi_{NS}^L + \pi_{NS}^H\right)\right]. \tag{A12}$$

However, using the condition $V(D_S, S) - V(D_{NS}, NS)$, equation (A12) implies that $E[\Delta TS] < 0$, contradicting the initial assumption that $E[\Delta TS] > 0$. Thus $\pi_S^L - \pi_{NS}^H - 2I_S > 0$.

Proof of Proposition 6: We first conjecture that $\hat{f}^\tau$ firms choose project S and $n - \hat{f}^\tau$ firms choose project NS. Then we verify this conjecture and show that for this distribution of projects, $\hat{f}^\tau$ firms issue debt with face value $D = D_S$ and subsequently choose project S and $n - \hat{f}^\tau$ firms issue debt with face value $D = D_{NS}$ and subsequently choose project NS.

We first prove that $D_S = \pi_S^L - 2I_S$. This is the highest feasible debt level for firms that choose project S.[28] Note that, holding the project choice constant, the tax advantage of debt implies that $dV(D, P)/dD > 0$. Thus, if it is incentive compatible, i.e., if equityholders of firms with debt levels $D = \pi_S^L - 2I_S$ invest in project S, it is optimal at time t_0 to set $D_S = \pi_S^L - 2I_S$. Since it has been shown in Lemma 3 that the highest profit of technology NS, π_{NS}^H is less than $\pi_S^L - 2I_S$, equityholders of firms with debt levels $D_S = \pi_S^L - 2I_S$ have no incentive to switch to technology NS. Thus it is optimal for $\hat{f}^\tau$ firms to set $D_S = \pi_S^L - 2I_S$ and subsequently select project S.

Next we prove that $D_{NS} = \pi_{NS}^L + \pi_{NS}^H - \pi_S^L + 2I_S$. We first conjecture that

$$\pi_{NS}^L > D_{NS} > \pi_S^H. \tag{A13}$$

Then incentive compatibility requires that $E(D_{NS}, NS) \geq E(D_{NS}, S)$,

$$\frac{1-\tau}{2}\left(\pi_{NS}^L + \pi_{NS}^H\right) - D_{NS}(1-\tau) \geq \frac{1-\tau}{2}\left(\pi_S^L - D_{NS}\right) - I_S(1-\tau)$$

or that

$$D_{NS} \leq \pi_{NS}^L + \pi_{NS}^H - \pi_S^L + 2I_S.$$

Since, for a given investment choice, $dV/dD > 0$, the above condition must hold as an equality at the optimal debt level. Thus the result that $D_{NS} = \pi_{NS}^L + \pi_{NS}^H - \pi_S^L + 2I_S$ follows from the conjecture in (A13). We must therefore prove that the conjecture is fulfilled in equilibrium. The first inequality in (A13) states that

$$\pi_{NS}^L > D_{NS} = \pi_{NS}^L + \pi_{NS}^H - \pi_S^L + 2I_S$$

[28]Higher face values of debt would imply that equityholders cannot expect to recover their initial investment. For such debt levels equityholders would therefore default immediately.

or

$$\pi_S^L - \pi_{NS}^H - 2I_S > 0.$$

This has been shown to hold in Lemma 3.

The second inequality in (A13) states that

$$D_{NS} = \pi_{NS}^L + \pi_{NS}^H - \pi_S^L + 2I_S > \pi_S^H$$

or

$$\pi_{NS}^L + \pi_{NS}^H - \left[\pi_S^L + \pi_S^H - 2I_S\right] > 0.$$

In equilibrium the firm values must be equal:

$$\pi_{NS}^L + \pi_{NS}^H - \left[\pi_S^L + \pi_S^H - 2I_S + \frac{2E[\Delta TS]}{1-\tau}\right] = 0. \tag{A14}$$

Thus, if $E[\Delta TS] > 0$, then the second inequality in (A13) holds. Therefore we will now show that $E[\Delta TS] > 0$. It follows from the definitions of D_S and D_{NS} that

$$E\left[TS(D_S)\right] = {}^1\!/_2\tau\left[\pi_S^H + \pi_S^L - 2I_S\right]$$
$$E\left[TS(D_{NS})\right] \le {}^1\!/_2\tau\left[\pi_{NS}^H + \pi_{NS}^L\right].$$

To obtain a contradiction, assume that $E[\Delta TS] \le 0$. Then the equilibrium condition (A14) implies that

$$\pi_S^L + \pi_S^H - 2I_S \ge \pi_{NS}^L + \pi_{NS}^H.$$

But this inequality implies that

$$E\left[TS(D_S)\right] - E\left[TS(D_{NS})\right] > 0,$$

which contradicts our initial assumption that $E[\Delta TS] \le 0$. Thus, $E[\Delta TS] > 0$ and the second inequality in (A13) holds. As a result $D_{NS} = \pi_{NS}^L + \pi_{NS}^H - \pi_S^L - 2I_S$. From (A13) it follows that $D_S > D_{NS}$.

Proof of Corollary 1: In equilibrium, $V(D_S, S) = V(D_{NS}, NS)$. Thus, to prove the corollary, it is sufficient to show that $E(D_S, S) < E(D_{NS}, NS)$. It follows from the definition of D_S and D_{NS} that

$$E(D_S, S) = 0$$
$$E(D_{NS}, NS) = \frac{1-\tau}{2}\left[2\left(\pi_S^L - 2I_S\right) - \left(\pi_{NS}^L + \pi_{NS}^H\right)\right] > 0,$$

where the inequality follows from Lemma 3 that $\pi_S^L - 2I_S > \pi_{NS}^H$, and the profit definition implying that $\pi_{NS}^H > \pi_{NS}^L$.

Proof of Corollary 2: The proof of Corollary 2 is contained in the proof of Proposition 6 above.

Proof of Proposition 7: Before proving the proposition we first derive the expressions for $E[\Delta TS]$ in terms of p_H and p_L. The definition of $E[\Delta TS]$ in Proposition 5 and the definitions of D_S and D_{NS} in Proposition 6 imply that

$$E[TS(D_S)] = {}^1\!/_2\tau[\pi_S^L - 2I_S + \pi_S^H]$$
$$E[TS(D_{NS})] = \tau[\pi_{NS}^L + \pi_{NS}^H - \pi_S^L + 2I_S]$$

and, therefore,

$$E[\Delta TS] = \tau\left[\frac{3\pi_S^L}{2} + \frac{\pi_S^H}{2} - \pi_{NS}^L - \pi_{NS}^H - 3I_S\right].$$

Substituting the expressions for the profits we get

$$E[\Delta TS] = \frac{\tau}{4\gamma}\{(p_L - k)^2 - (p_H - k)^2 + 6(p_L - k)h + 4h^2 - 2(p_H - k)h\} - 3\tau I_S. \qquad \text{(A15)}$$

We first show that $d\hat{f}^\tau/d\tau > 0$. From Proposition 5 the expression for $\hat{f}^\tau$ is given by

$$\hat{f}^\tau = \frac{bn + \gamma}{2b} + \frac{\gamma(E[\Delta TS] - I_S(1 - \tau))(bn + \gamma)}{bh^2(1 - \tau)}.$$

Differentiating with respect to the corporate tax rate yields

$$\frac{d\hat{f}^\tau}{d\tau} = \frac{\gamma(bn + \gamma)}{bh^2(1 - \tau)^2}\left[E[\Delta TS] + (1 - \tau)\frac{dE[\Delta TS]}{d\tau}\right]. \qquad \text{(A16)}$$

To determine the sign of $d\hat{f}/d\tau$ we need to sign $E[\Delta TS]$ and $\dfrac{dE[\Delta TS]}{d\tau}$. In Corollary 2 it is shown that $E[\Delta TS] > 0$. Differentiating (A15) with respect to the corporate tax rate we obtain

$$\frac{dE[\Delta TS]}{d\tau} = \frac{\tau}{4\gamma}\left[2(p_L - k)\frac{dp_L}{d\tau} - 2(p_H - k)\frac{dp_H}{d\tau} + 6h\frac{dp_L}{d\tau} - 2h\frac{dp_H}{d\tau}\right] + \frac{1}{\tau}E[\Delta TS]. \qquad \text{(A17)}$$

From the expressions for p_L and p_H, (A4) and (A5), we obtain

$$\frac{dp_L}{d\tau} = -bh\frac{d\hat{f}^\tau/d\tau}{bn + \gamma} = -\frac{dp_H}{d\tau}. \qquad \text{(A18)}$$

Substituting for $\dfrac{dp_L}{d\tau}$ and $\dfrac{dp_H}{d\tau}$ from (A18) into (A17) and then solving (A16) yields $d\hat{f}^\tau/d\tau > 0$.

Proof that $dD_S/d\tau < 0$: Substituting the definition for π_S^L into the expression for D_S derived in Proposition 6 we obtain

$$D_S = \frac{(p_L - k + h)^2}{2\gamma} - 2I_S.$$

Differentiating with respect to the corporate tax rate yields

$$\frac{dD_S}{d\tau} = \frac{1}{\gamma}(p_L - k + h)\frac{dp_L}{d\tau}.$$

Substituting for $dp_L/d\tau$ from (A18) and recognizing that $d\hat{f}^\tau/d\tau > 0$,

$$\frac{dp_L}{d\tau} = \frac{-bhd\hat{f}^\tau/d\tau}{bn + \gamma} < 0 \tag{A19}$$

which implies that $\dfrac{dD_S}{d\tau} < 0$.

Proof that $dD_{NS}/d\tau > 0$: Substituting the definition for π_{NS}^L, π_{NS}^H, and π_S^L into the expression for D_{NS} derived in Proposition 6 and simplifying we obtain

$$D_{NS} = \frac{(p_H - k)^2}{2\gamma} - \frac{(p_L - k)h}{\gamma} - \frac{h^2}{2\gamma} + 2I_S.$$

Differentiating with respect to the tax rate yields

$$\frac{dD_{NS}}{d\tau} = \frac{1}{\gamma}\left[(p_H - k)\frac{dp_H}{d\tau} - h\frac{dp_L}{d\tau}\right] > 0,$$

where we have used the results that $dp_H/d\tau > 0$ and $dp_L/d\tau < 0$ from expressions (A18) and (A19).

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