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Author(s): Lilian Ng
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LILIAN NG*

ABSTRACT

This paper examines an asset pricing model in which the Sharpe-Lintner CAPM and the zero-beta CAPM are special cases. The model allows the ratio of expected market risk premium to market variance, the conditional expected excess returns, and the risks to change over time. The results are found to be sensitive to the choice of the portfolio formation techniques. Significant time variability is shown in the conditional expected excess asset returns and risks and also in the reward-to-risk ratio.

THE INCREASING EVIDENCE OF time variation in expected returns and risks has prompted many researchers to reexamine the empirical implications of various asset pricing models. For example, Gibbons and Ferson (1985), Keim and Stambaugh (1986), Campbell (1987), and Ferson, Kandel, and Stambaugh (1987) examine financial valuation models in the conditional form that allow expected returns to vary over time. Bollerslev (1987), French, Schwert, and Stambaugh (1987), and Ng (1990) allow both conditional expected returns and variances to be time varying. Recently, a number of empirical papers have attempted to model the time variation in conditional expected asset returns, variances, and covariances in tests of the single-period CAPM of Sharpe (1964) and Lintner (1965).[1] For example, Harvey (1989a) applies Hansen's (1982) generalized method of moments (GMM), and Schwert and Seguin (1990) use the Glejser weighted least-squares estimation approach that is robust to heteroskedasticity for size-sorted portfolio returns, and both studies provide evidence against the Sharpe-Lintner CAPM. However, Bollerslev, Engle, and Wooldridge (1988) and Bodurtha and Mark (1991) employ the autoregressive conditional heteroskedasticity (ARCH) method and find support for the single-period CAPM.

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[1] Earlier studies such as Gibbons and Ferson (1985) and Ferson, Kandel, and Stambaugh (1987) have examined the conditional CAPM by allowing time variation in expected returns, but assuming the covariances of asset returns to be constant over time.

My current paper tests a model in which the Sharpe-Lintner CAPM and the zero-beta CAPM are special cases. The main thrust of this empirical study is to provide an integrated framework that tests (i) whether the market proxy portfolio is on the conditional mean-variance frontier, (ii) whether the cross-sectional relation between asset risk premia and their covariance risks is linear or proportional, and (iii) whether the ratio of expected market risk premium to the conditional market variance is constant through time. This study focuses on multivariate tests that allow expected asset returns, the conditional variances, and the covariances of asset returns to change over time. The conditional covariance matrix of asset returns is assumed to follow a multivariate generalized ARCH (GARCH) process with the conditional correlation matrix of asset returns being constant over time.

The present study conducts multivariate tests with two different sets of test assets. The first set consists of stock portfolios based on ranked security market betas, while the other is based on firm size. In the empirical literature, these two portfolio formation techniques have been widely used in studies of financial valuation models. Of particular interest is that the selection of these test assets facilitates the comparison of results with earlier tests of the CAPM such as those of Fama and Macbeth (1973), who employed beta-ranked portfolios, and recent tests which are based on size-formed portfolios. More important, the use of different grouping criteria provides information about the sensitivity of the test results to different partitioning of the data.

The first section develops a general asset pricing model that allows for time variation in both expected returns and risks. Section II presents the empirical methodology. Section III describes the data used in this study. The results of the multivariate models are reported in Section IV, followed by a concluding section.

I. Conditional Asset Prices

Let $\{R_{it}, i = 1,\cdots, N\}$ denote the rates of return on risky assets from time $t-1$ to t; R_{mt} is the return on the market portfolio. Let R_{zt} be the rate of return on the minimum-variance zero-beta portfolio, whose return is assumed to be uncorrelated with R_{mt} and $R_{it} \forall i$. Then, the equilibrium relations among expected rates of return on assets in excess of the return on the zero-beta portfolio can be described as follows.

$$E[(R_{it} - R_{zt}) \mid \Psi_{t-1}] = \lambda_{ot}\text{cov}(R_{mt}, R_{it} \mid \Psi_{t-1}), \tag{1}$$

where λ_{ot} is a scalar related to the aggregate relative risk aversion of the economy. The expectation operator E, the covariance of asset returns with the market, and λ_{ot} are conditioned on the information Ψ_{t-1} available in the market at time $t-1$. The asset pricing relation (1) is a generalization of the one-period zero-beta CAPM developed by Black (1972). From the equilibrium pricing relation (1), it follows that

$$E[(R_{mt} - R_{zt}) \mid \Psi_{t-1}] = \lambda_{ot}\text{var}(R_{mt} \mid \Psi_{t-1}); \tag{2}$$

that is, the expected return on the market in excess of the zero-beta rate is proportional to λ_{ot}.

Models in (1) and (2) assume that there is no riskless asset. Suppose there is a riskless asset. Replacing R_{zt} in (1) by the risk-free rate R_{ft} essentially yields the well-known Sharpe-Lintner CAPM and is consistent with Merton's (1980) analysis of expected market risk premia under constant relative risk aversion (CRRA). Under the assumption that the variance of the change in wealth is much larger than the variance of the change in state variables, or the first partial derivative of the investor's optimal consumption function with respect to his wealth is much larger than the first partial derivative of the investor's optimal consumption function with respect to the state variables, Merton (1980) derives equation (2), where $\lambda_{ot} = \lambda_o$ is a constant. He interprets λ_o as the Arrow-Pratt measure of aggregate relative risk aversion for the economy. Thus, the risk premium is strictly positive when investors' utility functions are increasing and strictly concave.

Suppose λ_{ot} is stable over time, and the expected market return in excess of the zero-beta rate in relation (2) is linear in its variance. Rewriting and combining the modified equations (1) and (2) yield

$$E[(R_{it} - R_{zt}) \mid \Psi_{t-1}] = \frac{[\delta_o + \lambda_o \text{var}(R_{mt} \mid \Psi_{t-1})]\text{cov}(R_{mt}, R_{it} \mid \Psi_{t-1})}{\text{var}(R_{mt} \mid \Psi_{t-1})} \tag{3}$$

Relation (3) can be interpreted as a variant of the zero-beta CAPM where the constant parameter δ_o represents transaction costs arising from the difference in riskless lending and borrowing rates. The model also parallels Litzenberger and Ramaswamy's (1979) after-tax version of the CAPM if δ_o represents dividend yields. A nonzero δ_o can therefore arise because of incremental effects, such as dividend yields or transaction costs that are not explicitly incorporated in the theoretical model, and of any nonzero covariance of the excess return on the zero-beta portfolio with the market variance (that is, $\text{cov}(R_{zt} - R_{ft}, \sigma^2_{mt}) \neq 0$).

Suppose the expected return on the minimum-variance zero-beta portfolio is constant over time and the riskfree rate is observable at time $t - 1$. In matrix notation, (3) can be reexpressed as

$$E[r_t \mid \Psi_{t-1}] = \alpha_o \iota + (\delta_o + \lambda_o w'_{t-1}\Omega_t w_{t-1})(w'_{t-1}\Omega_t w_{t-1})^{-1}\Omega_t w_{t-1}, \tag{4}$$

where α_o is a scalar, $E[r_t \mid \Psi_{t-1}]$ is the vector of expected excess returns on N assets from time $t - 1$ to t, ι is an $(N \times 1)$ vector of ones, w_{t-1} is the $(N \times 1)$ vector of asset value weights at the beginning of the period. The mean vector and covariance matrix Ω_t of these excess asset returns are conditioned on the information Ψ_{t-1} available in the market at time $t - 1$.

The asset pricing relation in (4) predicts the cross-sectional relation between expected excess asset returns and their conditional excess return covariances with the market, $\Omega_t w_{t-1}$. The parameter α_o therefore represents the expected zero-beta risk premium, or it can be informally interpreted as Jensen's (1969) measure. The slope coefficient of $\Omega_t w_{t-1}$ is the ratio of conditional expected market return in excess of the zero-beta rate to the conditional market volatility. Thus, (4) can be viewed as the zero-beta CAPM with its slope coefficient of $\Omega_t w_{t-1}$ dependent upon the level of conditional market volatility.

If $\alpha_o = 0$, then the riskfree rate equals the zero-beta rate. If $\delta_o = 0$, then the price of risk, or the ratio of expected market return in excess of the zero-beta rate to the conditional market variance, would be a constant. Finally, if $\lambda_o = 0$ and $\delta_o = 0$, then risk is not rewarded, implying that expected asset returns in excess of the zero-beta return are stable over time.

In this study, relation (4) forms the principal focus of the empirical work presented in subsequent sections. Estimation and hypothesis testing of variants of this general asset pricing relation are discussed in Section IV.

II. The Econometric Framework

A. An Empirical Model

We start by assuming that the realized excess returns are unbiased estimates of investors' conditional expected excess returns. Thus, relation (4) can be reexpressed in terms of random excess returns as

$$r_t = \alpha_o \iota + \left(\delta_o + \lambda_o w'_{t-1}\Omega_t w_{t-1}\right)\left(w'_{t-1}\Omega_t w_{t-1}\right)^{-1}\Omega_t w_{t-1} + \varepsilon_t \qquad (5)$$
$$\left(\varepsilon_t \mid \Psi_{t-1}\right) \sim N\left(0, \Omega_t\right),$$

where ε_t denotes the column vector of the differences between realized excess returns and expected excess returns. To introduce cross-sectional variation in the parameters to be tested, the system of equations in (5) can be rewritten as

$$r_t = \alpha + \left(\delta + \lambda w'_{t-1}\Omega_t w_{t-1}\right)\left(w'_{t-1}\Omega_t w_{t-1}\right)^{-1}\Omega_t w_{t-1} + \varepsilon_t \qquad (6)$$
$$\left(\varepsilon_t \mid \Psi_{t-1}\right) \sim N\left(0, \Omega_t\right),$$

where α is the $(N \times 1)$ vector of portfolio-specific intercepts, and δ and λ are the $(N \times N)$ *diagonal* matrices with portfolio-specific slope coefficients along the diagonal.[2] Equation (6) differs from (5) in that the parameters α, δ, and λ are allowed to vary across portfolios but are constant over the test period (see Brown and Weinstein (1983)). This approach complements the tests of

[2] The model in relation (6) is also similar in spirit to the heteroskedastic excess-return model examined by Schwert and Seguin (1990). Using our notation, the empirical model they tested can be expressed as follows:

$$r_t = \alpha + \left(\delta + \beta w'_{t-1}\Omega_t w_{t-1}\right)\left(w'_{t-1}\Omega_t w_{t-1}\right)^{-1} r_{mt} + \varepsilon_t,$$

where $\beta = \Omega_t w_{t-1}(w'_{t-1}\Omega_t w_{t-1})^{-1}$. Although this formulation is closely related to relation (6), it does not examine the price of aggregate risk. The model, however, conditions on r_{mt}. In their study, Schwert and Seguin (1990) test and reject the hypothesis that $\alpha = 0$.

Bollerslev, Engle, and Wooldridge (1988), who assume that the slope coefficient of covariance risk is constant and that $\delta = 0$. They further constrain a priori the slope coefficients λ to be equal across the test assets.

Tests of three asset-pricing hypotheses as special cases of (6) are performed. First, the conditional mean-variance efficiency of the market proxy portfolio is tested as the hypothesis that the alpha, lambda, and delta parameters are constant across assets. The second hypothesis involves the test of the conditional Sharpe-Lintner CAPM which further implies that the alphas be jointly zero. The joint null hypothesis of $\alpha = 0$ is tested against the alternative hypothesis that $\alpha \neq 0$. Finally, the test of the Merton (1980) model suggests that the sum of the delta and alpha coefficients in (6) should be jointly zero across all assets; otherwise, the reward-to-risk ratio is a function of the conditional market variance.

In this paper, tests of restrictions implied by the theoretical models are based on conditional moments. Tests incorporating changing conditional moments tend to be more powerful. For example, Hansen and Richard (1987) have shown that omitting conditioning information in tests of the one-period CAPM may lead to misleading inferences about the mean-variance efficiency of the market proxy portfolio. To implement tests of the above hypotheses, the dynamics of the variance-covariance structure in (6) must be specified, and these dynamics are examined next.

B. Covariance Dynamics

The dynamics of the conditional covariance matrix of asset innovations in relation (6) are assumed to follow a multivariate GARCH process, building upon the work of Bollerslev (1990). The appealing feature of this approach is that it captures empirical regularities found in stock returns. For example, Mandelbrot (1963) and Fama (1965) have shown that the unconditional distribution of first differences of the logarithm of stock prices generally have fatter tails than a normal distribution and that the error variances tend to cluster together.

This multivariate setting takes into account the cross-sectional correlations in residuals as well as the conditional heteroskedasticity. Hence, if the system of equations in (6) is estimated simultaneously, then full efficiency can be attained. Consistent with Bollerslev, Engle, and Wooldridge (1988) and Schwert and Seguin (1990), we assume that the innovation vector ε_t follows a simple multivariate GARCH(1,1) process. The time-varying conditional covariance matrix can therefore be parametered as follows.

$$
\begin{aligned}
&(\varepsilon_t \mid \Psi_{t-1}) \sim N(0, \Omega_t) \\
&\mathrm{vech}(\Omega_t) = a + b\,\mathrm{vech}(\varepsilon_{t-1}\varepsilon'_{t-1}) + c\,\mathrm{vech}(\Omega_{t-1}),
\end{aligned}
\tag{7}
$$

where vech$(\cdot)$ denotes the column-stacking operator of the lower portion of a symmetric matrix, ε_t is an $(N \times 1)$ vector of innovations, a is a $(\tfrac{1}{2}N(N+1) \times 1)$ parameter vector, and b and c are $(\tfrac{1}{2}N(N+1) \times \tfrac{1}{2}N(N+1))$ matrices of constant parameters. The multivariate GARCH(1,1) specification in (7) has ($\frac{1}{2}N^2(N+1)^2 + \frac{1}{2}N(N+1)$) parameters in the conditional variances and covariances. For a ten-portfolio multivariate GARCH(1,1) model, the number of parameters to be estimated would be 6105. It is practically impossible for any existing numerical algorithm to simultaneously estimate so many parameters. Thus, for tractability, reasonable restrictions on the parameters are necessary. In Bollerslev, Engle, and Wooldridge (1988), the conditional covariance is assumed to depend on its own lagged covariance and the cross-product of past forecast errors. In their formulation, the number of parameters for a ten-portfolio model would be 165—a number still too large to estimate.[3] To alleviate this problem, we consider an approach similar to that of Bollerslev (1990) in which the correlation matrix of excess portfolio returns are assumed to be constant over time. Schwert and Seguin (1990) have shown that the time-series behavior of conditional variances from this specification provides a good description of the monthly stock data is used.

We let ω_{ijt} denote the ijth element in Ω_t, and ρ_{ij} the ijth element in the conditional correlation matrix R, which is assumed to be constant. D_t is the diagonal matrix with standard deviations $\sqrt{h_{it}}$ along the diagonal. Thus, the model of the conditional heteroskedasticity for each conditional variance is given by

$$h_{it} = a_i + b_i \varepsilon_{it-1}^2 + c_i h_{it-1}, \quad i = 1,\cdots,N, \tag{8}$$

with constant conditional correlations ρ_{ij},

$$\omega_{ijt} = \rho_{ij}\sqrt{h_{it}}\sqrt{h_{jt}}, \quad j = 1,\cdots,N, \quad i = j+1,\cdots,N,$$

where $-1 < \rho_{ij} < 1$.

The conditional covariance matrix Ω_t defined by (7) and (8) can be stated as

$$\Omega_t = D_t R D_t. \tag{9}$$

The time-varying matrices Ω_t, $t = 1,\cdots,T$, are positive definite almost surely if and only if R is positive definite for all t, and $h_{it} \geq 0$ almost surely for all i. For finite variance and stationarity, it is necessary to impose that $a_i > 0$, $b_i \geq 0$, $c_i \geq 0$, and $b_i + c_i < 1$ for all i.

The log-likelihood function can be represented as follows.

$$\ln L(\Phi) = \text{const} - \tfrac{1}{2}\sum_t \ln|\Omega_t| - \tfrac{1}{2}\sum_t \varepsilon_t'\Omega_t^{-1}\varepsilon_t, \tag{10}$$

where Φ contains the unknown parameters in ε_t and Ω_t. To evaluate (10), an $(N \times N)$ matrix inversion for every t is required. However, by a direct substitution of equations (8) and (9) into (10), the log-likelihood function can

[3] In a ten-portfolio model, there are 55 variance and covariance equations. Since each equation has three unknown parameters, the total number of parameters to be estimated would be 165.

be rewritten as

$$\ln L(\Phi) = \text{const} - \tfrac{1}{2}\sum_t \sum_i \ln h_{it} - \frac{T}{2}\ln|R| - \tfrac{1}{2}\sum_t \left(\varepsilon_t' D_t^{-1}\right) R^{-1}\left(D_t^{-1}\varepsilon_t\right). \tag{11}$$

In contrast to (10), equation (11) is far easier to manage because the latter involves only an $(N \times N)$ *diagonal* matrix inversion for every period t. All maximum-likelihood estimates are obtained by maximizing the log-likelihood function in (11) using the Davidon-Fletcher-Powell algorithm. Under the standard regularity conditions, the parameter estimates and their standard errors are consistent and asymptotically normal. Hypothesis testing is therefore conveniently accomplished via standard likelihood ratio tests.

III. Data Description

The current study uses monthly realized returns on all common stocks traded in the New York Stock Exchange during the period January 1926 through December 1987. Data were obtained from the Center for Research in Security Prices (CRSP) at the University of Chicago. Two methods of constructing portfolios are adopted. The first grouping technique ranks assets according to their market betas and divides them into ten groups. Using the first 5 years of monthly data (January 1926–December 1930), the beta coefficient of each security is estimated using a market model regression with the CRSP value-weighted index as a proxy for the market portfolio. Next, securities are ranked on the basis of their estimated beta coefficients and then grouped into ten value-weighted portfolios. The entire procedure is then repeated after dropping the first year of data and moving forward until the last year of data (1987) is reached. For this approach, only stocks with a complete set of returns on the CRSP monthly return file for the 5-year estimation period and for the 1 year after the estimation period are included in the portfolios.

The other formation technique, grouping securities based on their market values of equity, stems from evidence of earlier empirical works demonstrating the significant relation between firm size and stock return (see Banz (1981) and Reinganum (1981)). This methodology sorts securities into portfolios based on their market equity values at the end of each year (beginning December 31, 1926) from the smallest to the largest, and then assigns securities to each decile portfolio for the following year. The market value of a security is defined as its market price multiplied by the number of shares outstanding. Each portfolio therefore gets an equal number of securities, with smallest firms assigned to Portfolio 1 and largest firms to Portfolio 10. Any remaining securities after dividing them by 10 are assigned equally to the first and last portfolios; otherwise the last portfolio gets the additional security if the number remaining is odd. The returns from the decile portfolios are value-weighted, with the weights changing every month. Relative market values add up to one across the ten portfolios at each observation. Since this grouping technique begins with December 1926, the number of observations is more than that of beta-ranked portfolios.

Note that in both grouping techniques, the market-value weight of each decile portfolio in every observation is known. With decile portfolio weights, the rates of return on the ten decile portfolios (beta-ranked as well as size-sorted) span the rates of return on the market, thereby allowing the conditional expected returns on the CRSP value-weighted proxy to be generated within the model. The conditional market variances and market betas of individual portfolios can be estimated simultaneously.

The monthly risk-free rate is the holding period return on the shortest Treasury bill with at least 30 days to maturity. This rate is then subtracted from each of the stock portfolio returns to compute a monthly excess portfolio return.

IV. Estimation and Empirical Results

Tests of the parameter restrictions on the general CAPM in equation (6) are performed using two test assets, namely beta-ranked portfolios and size-sorted portfolios. Standard likelihood ratio tests are employed, and the results are contained in Table I. Using monthly returns from beta-ranked portfolios for the entire sample period January 1931–December 1987, the chi-square test statistic of 25.52, distributed asymptotically χ^2 with 27 degrees of freedom, is not significant at conventional levels. For the two subperiods, the cross-sectional tests also yield the same results. These results provide no evidence against the null hypothesis that the parameters α, δ, and λ are constant across all portfolios. However, using returns from size-sorted portfolios for the overall sample period January 1927–December 1987, the chi-square test statistic is 58.26, indicating that the parameters are different across size-formed portfolios. The test statistics for the two subperiods, 1931–1957 and 1958–1987, remain significant at the 5% level. Given this observation, separate tests on the equality of each parameter across these portfolios are performed for the whole estimation period, and the results are also reported in Table I. The tests results do not reject that the α parameters or the δ parameters are jointly zero, but they do reject that the δ and λ parameters are the same across decile size-formed portfolios. The chi-square test statistic also rejects the hypothesis of $\alpha = \delta = 0$ at the 5% significance level.

To check the adequacy of the multivariate GARCH(1,1) specification, some diagnostic tests for serial correlations in the conditional first and second moments of the resulting residuals are conducted. We perform Ljung and Box (1978) portmanteau tests for up to the twelfth order serial autocorrelation in the resulting standardized residuals $\varepsilon_{it}/\sqrt{h_{it}}$ and squared standardized residuals $\varepsilon_{it}^2/h_{it}$. These test statistics are distributed asymptotically chi-square with twelve degrees of freedom. The test statistics for serial correlations in standardized residuals range from 2.94 to 8.35 using beta-ranked

Table I

Cross-Sectional Tests of the General Capital Asset Pricing Model

This table reports likelihood ratio tests of the parameter restrictions on the multivariate GARCH(1,1) model (6), using monthly excess returns to ten beta-ranked portfolios and ten size-sorted portfolios. Data are obtained from the CRSP return files for the period 1927–1987. The test statistics are asymptotically chi-square distributed, and their p-values and degrees of freedom are reported accordingly. Relation (6) dictates a linear relation between portfolio excess returns r_t, and their covariance risks $\Omega_t w_{t-1}$ where Ω_t is an $(N \times N)$ covariance matrix of portfolio excess returns and w_{t-1} is an $(N \times 1)$ vector of portfolio value weights at the beginning of the period. The conditional covariances are assumed to be proportional to the product of the conditional standard deviations $\sqrt{h_{it}}$ and $\sqrt{h_{jt}}$ as defined in equation (8) where ρ_{ij} is the unconditional correlation coefficient of the residuals from (6).

$$r_t = \alpha + \left(\delta + \lambda w'_{t-1}\Omega_t w_{t-1}\right)\left(w'_{t-1}\Omega_t w_{t-1}\right)^{-1}\Omega_t w_{t-1} + \varepsilon_t \tag{6}$$
$$\varepsilon_t \mid \Psi_{t-1} \sim N(0, \Omega_t)$$

where

$$h_{it} = a_i + b_i \varepsilon^2_{it-1} + c_i h_{it-1} \tag{8}$$
$$\Omega_{ijt} = \sqrt{h_{it}}\,\rho_{ij}\sqrt{h_{jt}}, \qquad i, j = 1, 2, \cdots, 10.$$

Sample period	Null hypothesis	χ^2	p-value	Number of degrees of freedom
Beta-ranked portfolios				
1/31–12/87	$\alpha = \alpha_i, \lambda = \lambda_i, \delta = \delta_i$	25.52	0.545	27
1/31–12/58	$\alpha = \alpha_i, \lambda = \lambda_i, \delta = \delta_i$	21.30	0.772	27
1/59–12/87	$\alpha = \alpha_i, \lambda = \lambda_i, \delta = \delta_i$	18.45	0.889	27
Size-sorted portfolios				
1/27–12/87	$\alpha = \alpha_i, \lambda = \lambda_i, \delta = \delta_i$	58.26	0.0004	27
1/27–12/56	$\alpha = \alpha_i, \lambda = \lambda_i, \delta = \delta_i$	49.19	0.006	27
1/57–12/87	$\alpha = \alpha_i, \lambda = \lambda_i, \delta = \delta_i$	42.14	0.032	27
1/27–12/87	$\alpha = 0$	11.4	0.3272	10
1/27–12/87	$\delta = 0$	31.43	0.0005	10
1/27–12/87	$\lambda = \lambda_i$	23.67	0.0049	9
2/27–12/87	$\delta = \delta_i$	23.29	0.0053	9
1/27–12/87	$\alpha = \delta = 0$	19.19	0.038	10

portfolios, and from 5.59 to 10.60 using size-formed portfolios. For resulting squared standardized residuals, the chi-square test statistics vary from 5.41 to 13.59 using beta portfolios, and from 3.35 to 10.35 using size portfolios. None of these portmanteau test statistics are significant at conventional levels, indicating no evidence of first or second order dependence in the resulting conditional standardized residuals. The results suggest that residuals from the estimated models are well-behaved and that the parsimonious multivariate GARCH(1,1) models provide adequate descriptions of the monthly stock returns.

The two portfolio formation techniques give rise to different conclusions. The asset pricing model in relation (4) cannot be rejected when beta-ranked portfolios are employed as test assets, but is rejected when size-formed portfolios are used.[4] The evidence based on beta portfolios is consistent with the earlier study by Fama and Macbeth (1973). They employed a similar grouping procedure. Results using size portfolios are in accord with recent evidence obtained by Ferson, Kandel, and Stambaugh (1987), Harvey (1989a), and Schwert and Seguin (1990). These authors used size portfolios in their tests of the CAPM. It is plausible that grouping securities based on firm size yields more powerful tests. The greater dispersion in the parameter estimates for size portfolios is consistent with the joint hypothesis that these parameters are related to factors such as dividend yield and transaction cost, and that size is a better proxy for these factors. It is also plausible that the value weights w_{it-1} are less informative for beta portfolios.

In contrast, however, Bodurtha and Mark's (1991) findings using a shorter time-series of monthly size-sorted portfolio returns (1926–1985) do not reject the CAPM. The difference in the results may be attributed to the variance dynamic process and methodology employed in their study. They use GMM to estimate a model with variances of stock returns following an ARCH process with lag 3. In comparison with the ARCH formulation, the GARCH(1,1) process has a more flexible lag structure and a longer memory.

Based on results in Table I, the parameters in model (6) are constrained accordingly. Tables II and III report maximum likelihood estimates of the restricted multivariate GARCH model using beta-ranked portfolios and size-sorted portfolios, respectively. The asymptotic standard errors are shown in parentheses. In Table II, the expected compensation for risk λ is estimated to be 6.1, and the positive sign of this coefficient is consistent with the theoretical formulation. The magnitude of the λ estimate is close to 7.02 reported in Ferson (1989), who uses the CRSP value-weighted index as a proxy for the market portfolio and employs GMM to estimate a model with time-varying covariances. Table III shows that the magnitude of this coefficient is significant and larger across size portfolios, ranging from 6.4 to 8.6. The λ coefficient of the smallest size portfolio is about the largest, while for the largest size portfolio it is smallest. Although the coefficients tend to decrease from the smallest portfolio to the largest portfolio, the descending pattern is not monotonic.

Table II reports a significant α estimate of 0.0052. The evidence supports the general CAPM but rejects the Sharpe-Lintner CAPM. The maximum-likelihood estimate of δ, based on its asymptotic standard error, is significantly different from zero, indicating that the price of risk varies with market volatility. The estimated value of the sum of α and δ, a magnitude of -0.0083, is significant at the 5% level. Thus, the expected market risk

[4] A recent study by Lo and MacKinlay (1990) shows that expected returns of portfolios sorted by some empirically motivated characteristic of securities bias tests of financial asset pricing models. Their analytical calculations and Monte Carlo simulations provide evidence showing that portfolios which are formed using data-motivated characteristic such as the market value of equity may reject a 5% test with probability one under the null hypothesis.

Table II

Maximum Likelihood Estimates of the Constrained Multivariate GARCH Model Using Ten Beta-Ranked Portfolios for the Period January 1931–December 1987.[a]

This table reports the estimates of the multivariate GARCH(1,1) model (5), using monthly excess returns to ten beta-ranked portfolios. Data are obtained from the CRSP returns files for the period 1927–1987. Relation (6) dictates a linear relation between portfolio excess returns r_t and their covariance risks $\Omega_t w_{t-1}$ where Ω_t is an $(N \times N)$ covariance matrix of portfolio excess returns and w_{t-1} is an $(N \times 1)$ vector of portfolio value weights at the beginning of the period. The conditional covariances are assumed to be proportional to the product of the conditional standard deviations $\sqrt{h_{it}}$ and $\sqrt{h_{jt}}$ as defined in equation (8) where ρ_{ij} is the unconditional correlation coefficient of the residuals from (5).

$$r_t = \alpha_o \iota + (\delta_o + \lambda_o w'_{t-1}\Omega_t w_{t-1})(w'_{t-1}\Omega_t w_{t-1})^{-1}\Omega_t w_{t-1} + \varepsilon_t \tag{5}$$
$$\varepsilon_t \mid \Psi_{t-1} \sim N(0, \Omega_t),$$

where

$$h_{it} = a_i + b_i \varepsilon_{it-1}^2 + c_i h_{it-1}, \tag{8}$$
$$\Omega_{ijt} = \sqrt{h_{it}}\,\sigma_{ij}\sqrt{h_{jt}}, \qquad i, j = 1, 2, \cdots, 10.$$

	α_o	$\delta_o \times 10^2$	λ_o	$a_i \times 10^3$	b_i	c_i
Lowest	0.0052	−1.3500	6.0916	0.1372	0.1353	0.7563
	(0.0015)	(0.2300)	(0.5745)	(0.0162)	(0.0096)	(0.0183)
r_2				0.1876	0.1073	0.7848
				(0.0191)	(0.0075)	(0.0152)
r_3				0.2390	0.0844	0.7997
				(0.0290)	(0.0062)	(0.0183)
r_4				0.3811	0.1007	0.7425
				(0.0328)	(0.0071)	(0.0179)
r_5				0.5014	0.1162	0.7048
				(0.0414)	(0.0082)	(0.0198)
r_6				0.4971	0.1005	0.7383
				(0.0392)	(0.0068)	(0.0162)
r_7				0.4610	0.0926	0.7648
				(0.0313)	(0.0053)	(0.0124)
r_8				0.4993	0.0965	0.7648
				(0.0366)	(0.0058)	(0.0132)
r_9				0.7511	0.1279	0.6924
				(0.0492)	(0.0072)	(0.0154)
Highest				1.1638	0.1496	0.6521
				(0.0801)	(0.0094)	(0.0185)

[a] Standard errors in parentheses.

premium is said to be linearly related to the conditional market variance with a negative intercept of −0.0083, which is inconsistent with Merton's (1980) result on expected market risk premiums under CRRA. However, the negative intercept is consistent with the results obtained by Bollerslev, Engle, and Wooldridge (1988) and Harvey (1989b).

Table III

Maximum Likelihood Estimates of the Constrained Multivariate GARCH Model Using Ten Size-Sorted Portfolios for the Period January 1927–December 1987.[a]

This table reports the estimates of the constrained multivariate GARCH(1,1) model of (6), using monthly excess returns to ten size-sorted portfolios. Data are obtained from the CRSP returns files for the period 1927-1987. Relation (6′) dictates a proportional relation between portfolio excess returns, r_t, and their covariance risks, $\Omega_t w_{t-1}$, where Ω_t is an $(N \times N)$ covariance matrix of portfolio excess returns and w_{t-1} is an $(N \times 1)$ vector of portfolio value weights at the beginning of the period. The conditional covariances are assumed to be proportional to the product of the conditional standard deviations $\sqrt{h_{it}}$ and $\sqrt{h_{jt}}$ as defined in (8), where ρ_{ij} is the unconditional correlation coefficient of the residuals from (6′).

$$r_t = (\delta + \lambda w'_{t-1}\Omega_t w_{t-1})(w'_{t-1}\Omega_t w_{t-1})^{-1} + \varepsilon_t \qquad (6')$$

$$\varepsilon_t \mid \Psi_{t-1} \sim N(0, \Omega_t)$$

where

$$h_{it} = a_i + b_i\varepsilon^2_{it-1} + c_i h_{it-1}, \qquad (8)$$

$$\Omega_{ijt} = \sqrt{h_{it}}\rho_{ij}\sqrt{h_{jt}}, \qquad i, j = 1, 2, \cdots, 10.$$

	$\delta_i \times 10^2$	λ_i	$a_i \times 10^3$	b_i	c_i
Smallest	−1.3900	8.3741	0.6469	0.1968	0.7405
	(0.1700)	(0.7221)	(0.0416)	(0.0094)	(0.0099)
r_2	−1.5690	8.5623	0.4981	0.1538	0.7790
	(0.1400)	(0.6498)	(0.0247)	(0.0071)	(0.0075)
r_3	−1.5710	8.4080	0.5840	0.1464	0.7605
	(0.1200)	(0.5779)	(0.0208)	(0.0052)	(0.0058)
r_4	−1.5100	8.2761	0.4582	0.1317	0.7854
	(0.1200)	(0.5636)	(0.0176)	(0.0051)	(0.0058)
r_5	−1.3100	7.4369	0.4454	0.1311	0.7774
	(0.1090)	(0.5421)	(0.0143)	(0.0049)	(0.0055)
r_6	−1.5500	7.9486	0.3130	0.1091	0.8182
	(0.1100)	(0.5594)	(0.0126)	(0.0035)	(0.0061)
r_7	−1.4500	7.5001	0.4074	0.1233	0.7740
	(0.1104)	(0.5208)	(0.0189)	(0.0054)	(0.0079)
r_8	−1.2800	7.3482	0.2891	0.1090	0.8081
	(0.1112)	(0.5438)	(0.0154)	(0.0050)	(0.0076)
r_9	−1.2400	7.0782	0.2908	0.1021	0.7986
	(0.1200)	(0.5479)	(0.0162)	(0.0051)	(0.0084)
Largest	−1.0600	6.3946	0.2356	0.0924	0.8099
	(0.1600)	(0.6417)	(0.0172)	(0.0056)	(0.0102)

[a] Standard errors in parentheses.

Bollerslev, Engle, and Wooldridge (1988) argue that the negative expected excess return on the market portfolio may be attributed to the preferential tax treatment on capital gains. Because of a lower tax on capital gains over most of the sample, investors had the incentive to hold the market portfolio even when its gross expected excess return is negative. The negative intercept could be an artifact of approximating a nonlinear relation with a linear function. It is plausible that the negative intercept may imply that the true relation between expected market risk premium and its ex ante variance is in fact convex, and trying to fit a straight line between them may result in the line intersecting the negative segment of the y-axis.[5]

The estimates of α and δ imply that Merton's (1980) reward-risk ratio is positively associated with market volatility. This is consistent with findings provided by Harvey (1989b), and Kandel and Stambaugh (1990). Unlike the current study, both of these studies employ financial variables that can explain a portion of the time-series behavior of expected market returns that is not explained by the conditional market variance. Using these instrumental variables, Harvey (1989b) finds that the reward-risk ratio is time varying. Like Kandel and Stambaugh (1990), he also shows that there is a negative correlation between the level of the reward-to-risk ratio and the business cycle. The magnitude of the ratio is high when the business cycle is at its trough and low when the business cycle is at its peak.

The δ and λ estimates, however, are found to vary across portfolios formed on the basis of firm's market capitalization.[6] Notice that the first derivative of returns with respect to δ, on the average, is about 1.7 times larger than the first derivative of returns with respect to $\lambda_i \sigma_{mt}^2$, suggesting that changes in δ would have a greater impact on expected portfolio returns than changes in λ. Given the estimate of σ_{mt}^2 is approximately 0.003, the spreads in the two parameters (δ_i and $\lambda_i \sigma_{mt}^2$) are 0.011 for the smallest size portfolio and 0.009 for the largest size portfolio. The average spread for each size portfolio is about 1%. Thus, the effect of a 1% change in δ on the required rate of return would be approximately 1.7 times larger than that of a 1% change in $\lambda \sigma_{im}^2$.[7] This perhaps suggests that the variation in δ is more important than the variation in λ, and thus the incremental effect of changes in δ on expected portfolio returns in not inconsequential. As Table III indicates, the magnitude of the δ parameter is larger for smallest firms than for largest firms; the difference is about 0.33%. This is consistent with the joint hypothesis outlined in Section I that these parameters are related to differences in dividend yields and transaction costs and that these factors are more closely associated with firm size than with beta.

V. Conclusion

This paper reports the results of multivariate tests of a conditional capital asset pricing model that allows time variation in the conditional expected

[5] Although it is possible that some conditional expected excess asset returns are negative, Harvey (1989b) argues that it is also possible that the expectation generating mechanism is misspecified.

[6] These findings are consistent with Harvey's (1989a) results; he attributes this observation to the *clientele effect*. However, he also notes that it seems puzzling that investors who are most risk averse would prefer to hold the riskiest stocks.

[7] I wish to thank the referee for making this observation.

asset returns, asset variances, and covariances. The time-varying covariance matrix of asset returns is assumed to follow a multivariate GARCH process. This approach suggests a model of expected returns conditional on the information set that includes current and past conditional variances and covariances and past squared forecast errors. The econometric procedure used permits a simultaneous estimation of conditional market variances and individual portfolio market betas within the model.

Empirical results based on the pooled time series and cross-section of beta-ranked portfolio returns do not reject the conditional mean-variance efficiency of the market proxy portfolio. The findings also indicate that the ratio of expected excess market return to the conditional market variance, or the reward-to-risk ratio, is positively correlated with the level of the conditional market variance.

When tests are based on ten size-sorted portfolios, however, the tests reject the model. The evidence is consistent with the results of Harvey (1989a) and Schwert and Seguin (1990) but contradicts those of Bollerslev, Engle, and Wooldridge (1988), Bodurtha and Mark (1991), and of such earlier tests of the CAPM as those of Fama and Macbeth (1973). Except in Bodurtha and Mark (1989), these results indicate that size-sorted portfolios lead to greater dispersion of the parameters tested. The cross-sectional variation in the parameter estimates is perhaps attributed to some incremental effects not explicitly incorporated in the model. Thus, size-sorted portfolios may provide a better proxy for these factors than beta-ranked portfolios.

REFERENCES

Arrow, Kenneth, 1970 *Essays in the Theory of Risk-Bearing* (Markham Publishing Company, Chicago).

Banz, Rolf, 1981, The relationship between return and market value of common stocks, *Journal of Financial Economics* 9, 3-18.

Black, Fisher, 1972, Capital market equilibrium with restricted borrowing, *Journal of Business* 45, 3-18.

Bodurtha, James and Nelson Mark, 1991, Testing the CAPM with time varying risks and returns, *Journal of Finance* 46, 1485-1505.

Bollerslev, Tim, 1987, A conditionally heteroskedastic time series model for speculative prices and rates of return, *Review of Economics and Statistics* 69, 542-546.

——, 1990, Modeling the coherence in short run nominal exchange rates: A multivariate generalized ARCH model, *Review of Economics and Statistics* 72, 498-505.

——, Robert F. Engle, and Jeffrey Wooldridge, 1988, A capital asset pricing model with time varying covariances, *Journal of Political Economy* 96, 116-131.

Brown, Stephen J. and Mark I. Weinstein, 1988, A new approach to testing asset pricing models: The bilinear paradigm, *Journal of Finance* 38, 711-743.

Campbell, John Y., 1987, Stock returns and the term structure, *Journal of Financial Economics* 18, 373-400.

Fama, Eugene, 1965, The behavior of stock market prices, *Journal of Business* 38, 34-105.

—— and James Macbeth, 1973, Risk, return and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607-636.

Ferson, Wayne, 1989, Changes in expected security returns, risk, and the level of interest rates, *Journal of Finance* 44, 1191-1217.

——, Shmuel Kandel, and Robert F. Stambaugh, 1987, Tests of the asset pricing with time-varying risk premiums and market betas, *Journal of Finance* 42, 201-220.

French, Kenneth, G. William Schwert, and Robert F. Stambaugh, 1987, Expected stock returns and volatility, *Journal of Financial Economics* 19, 3-30.

Gibbons, Michael and Wayne Ferson, 1985, Tests of asset pricing models with changing expectations and an unobservable market portfolio, *Journal of Financial Economics* 11, 217-236.

Hansen, Lar P., 1982, Large sample properties of generalized method of moment estimators, *Econometrica* 50, 1029-1054.

——— and Scott F. Richard, 1987, The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models, *Econometrica* 55, 587-614.

Harvey, Campbell, 1989a, Time varying conditional covariances in tests of asset pricing models, *Journal of Financial Economics* 24, 289-317.

———, 1989b, Is expected compensation for market volatility constant through time?, Working paper, Duke University.

Jensen, Michael C., 1969, Risk, the pricing of capital assets, and the valuation of investment portfolios, *Journal of Business* 42, 167-247.

Kandel, Shmuel and Robert F. Stambaugh, 1990, Expectations and volatility of consumption and asset returns, *Review of Financial Studies* 3, 207-232.

Keim, Donald B. and Robert F. Stambaugh, 1986, Predicting returns in the stock and bond markets, *Journal of Financial Economics* 17, 357-390.

Lintner, John, 1965, The valuation of risky assets and the selection of risky investments in the portfolios and capital budgets, *Review of Economics and Statistics* 47, 13-37.

Litzenberger, Robert H. and Khrisna Ramaswamy, 1979, The effect of personal taxes and dividends on capital asset prices, *Journal of Financial Economics* 7, 163-196.

Ljung, Greta and George E. P. Box, 1978, On a measure of lack of fit in time-series models, *Biometrika* 65, 297-303.

Lo, Andrew and A. Craig MacKinlay, 1990, Data snooping biases in tests of financial asset pricing models, *Review of Financial Studies* 3, 431-467.

Mandelbrot, Benoit, 1963, The variation of certain speculative prices, *Journal of Business* 36, 394-419.

Merton, Robert C., 1980, On estimating the expected return on the market: An exploratory investigation, *Journal of Financial Economics* 8, 323-361.

Ng, Lilian K., 1990, Expected risk premium and implied market volatility, Working paper, University of Texas at Austin.

Pratt, John J., 1964, Risk aversion in the small and in the large, *Econometrica* 32, 122-136.

Reinganum, Marc, 1981, Misspecification of capital asset pricing, *Journal of Financial Economics* 9, 19-46.

Schwert, G. William and Paul Seguin, 1990, Heteroskedasticity in stock returns, *Journal of Finance* 45, 1129-1155.

Sharpe, William, 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425-442.