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Order Form and Information in Securities Markets

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ABSTRACT

This paper examines the effects of price-contingent orders on security prices. We show that a market maker who knows the type and composition of trades will set larger spreads and adjust prices faster than if price-contingent orders were not allowed. Because traders have rational expectations over the book, we demonstrate that uncertainty over order type reduces the variance of prices but with a corresponding loss in price informativeness. We also show that the sequence property of price-contingent orders increases the probability of large price movements. This distinction between variance and episodic price volatility has important policy implications.

INDIVIDUALS AND INSTITUTIONS USE price-contingent orders to limit their losses resulting from a fall in the price of a stock they hold. These orders may be explicit stop (stop-loss) orders, they may arise from portfolio insurance, or they may simply be the result of deciding in advance to sell if the price hits a certain point. One question that naturally arises is: Does the use of such orders affect the performance of the market?

In this paper we examine the effect of order form on security prices. We show that the ability to trade via alternative order forms can induce some traders to alter their trading behavior, leading to changes in the informativeness of the market order flow. This changes the stochastic process of prices, even when market participants know the type of order submitted. We show that a market maker who knows the type and composition of trades will set larger spreads and will adjust prices faster than he would if price-contingent orders were not allowed. We also demonstrate that uncertainty over the order type can actually **reduce** the variance of the price process from what it would have been if order type were known. These price process effects dictate that the trading mechanism itself can affect security prices.

By focussing on the role of order form, our research demonstrates how a simple "book" affects security prices. Cohen, Maier, Schwartz, and Whitcomb (1981) and O'Hara and Oldfield (1986) studied the effect of market and limit orders on market performance but only in the absence of information consid-

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erations. In a recent paper, Rock (1989) analyzed the interaction between risk-averse market makers and limit order traders. His analysis focussed on when limit orders will transact given that in his model, market makers have more information than limit order traders but are subject to inventory problems.

Our analysis of stop orders in a market with asymmetrically informed traders focusses on a different question. We are interested in understanding how order form affects the transmission of information into security prices. As we demonstrate, this has important implications for the efficiency of the market. We show that introducing a book of stop orders improves the efficiency of the market maker system in that it increases the speed of price adjustment to the strong-form efficient value. In a floor broker system with no active market makers, the efficiency effects are more intriguing. We show that allowing price-contingent orders results in a lower price variance than in the market maker system but also a corresponding worsening of the efficiency of price adjustment across time. These dual effects suggest that policies introduced to "improve" market performance by reducing variance may simultaneously impede market efficiency by slowing price adjustment.

One area where our results may be of immediate interest is to the growing debate surrounding the effect of order strategies on market performance. Brennan and Schwartz (1989) demonstrate in a one-period representative investor model that portfolio insurance can increase market volatility. Grossman (1988) makes a similar volatility argument in a model where synthetic puts convey less information than the use of actual put options for portfolio insurance. Gennotte and Leland (1990) demonstrate how, with risk-averse agents, hedging-related supply shocks can lead to a large increase in security price volatility when the extent of such hedging is unknown to some agents. In this "price pressure" model, a fall in price resulting from a hedging-induced supply shock causes uninformed market participants to limit their participation in the market, thereby removing liquidity and magnifying the price movement.

Our work extends and modifies these results in several important ways. Because we develop a multiperiod trading model, we are able to demonstrate the important difference between the effects of order form on price variance and on episodic price volatility. We show that uncertainty over the types of orders lowers the conditional variance of prices relative to what it would be if order types were known. This result, the opposite of the effect predicted by Gennotte and Leland (1990), arises because our traders have rational expectations about the size and structure of the book.[1] These expectations cause floor brokers to move prices less on average than they would if they knew the type of order transacting.

[1]While Gennotte and Leland use a rational-expectations framework, they do not allow traders to have expectations about the order flow. Consequently, traders irrationally believe such orders do not exist. This causes traders to consistently underestimate the extent of liquidity trading, and if trades cannot be observed, volatility increases. As we show in our model, if expectations are incorporated such effects do not arise.

Yet, while variance is lower, large deviations of prices are more likely to occur in markets with price-contingent orders. These price movements occur because traders use transactions as signals of potential new information and, since stop orders make sequences of sell orders more likely, prices may consistently fall to reflect the possibility of new, adverse information. In retrospect, traders may be wrong, but at any time the market price is efficient with respect to publicly available information. In our model, price-contingent strategies have opposite effects on the variance of prices and on the probability of large deviations of prices because such strategies change the flow of information. As we demonstrate, these information flow effects alone are sufficient to explain how seemingly "irrational" prices can prevail in a market with rational traders.

Our paper is organized as follows. In Section I, we develop a model of security trading that incorporates both market and stop orders and analyze how such orders affect trader behavior. Section II then analyzes how the book of stop orders affects the behavior of prices in a market maker system where the extent and composition of the book is known. Section III considers this same issue in a floor broker system where the extent and composition of price-contingent orders may not be known. Section III also includes a numerical example of how sequences of stop orders may induce large episodic price volatility. The paper concludes in Section IV with a discussion of the policy implications of our work.

I. A Model of Security Trading

We consider a market in which buyers and sellers trade an asset whose value is represented by a random variable V. Each trade involves a specific quantity of the asset which, for simplicity, we normalize to one unit. We consider two market-clearing protocols, one involving an active market maker and the other involving floor brokers. The difference between the two systems will be explained shortly. To focus on the information effects of trading, we assume all market participants are risk neutral and act competitively.

In this market there are some traders who have better information about the value of the asset. Specifically, some of the traders observe a signal s about V; this signal can take one of two values, low and high.[2] The probability that this signal is low, denoted Prob $\{s = L\}$ is δ, with Prob $\{s = H\} = 1 - \delta$. Let $\underline{V} = E[V \mid s = L]$ and $\overline{V} = E[V \mid s = H]$. The other traders, the market maker, and any floor brokers do not know the signal, but they do know the information structure. Thus, their initial unconditional expectation of the asset's value is $V^* = \delta\underline{V} + (1 - \delta)\overline{V}$.

Trading in this market may involve either market orders or stop orders. In a market order, a trader buys or sells one unit of the asset at the prevailing

[2] In our model informed traders do not know in advance of an information event that they will be the ones to receive information. Consequently, every market participant faces uncertainty over whether other traders are currently trading on information which they have not received.

quote. This quote is set by the market maker if he clears trades or by floor brokers if the market maker is inactive. In either case, a trader can submit an order to a central exchange. If the order is a market order it is placed in a queue and is cleared sequentially. Stop orders involve a very different structure. Each stop-loss order specifies a price at which one unit of stock is to be sold and each stop-buy order specifies a price at which one unit of the stock is to be bought. By definition, a stop-loss (-buy) order will not be accepted at a price better (worse) than or equal to the last trade price.[3] The market maker places each stop order in the "book" until the market price moves to the level specified in the order. At that point, the stop order is executed.

There are several characteristics of stop orders that may be important for our analysis. First, although each stop order specifies a price, it may not always execute at that price. By NYSE rules, a trade at price p triggers (or "elects") any stop-loss orders on the book at price p or above. These orders then transact at the next sale price, which may be lower than the specified price.[4] Second, stop orders get priority of execution over market orders. Hence, although there may be market orders in the queue, any appropriate stop orders are executed first. If there are multiple stop orders at a given price, they are executed in the order they were placed in the book. Third, only the market maker knows the composition of the book. Although all market participants know stop orders exist, they do not know how many (if any) there are nor do they know the prices at which they were placed. This means that market participants cannot tell whether any transaction arose from a market order or from a stop order.

In the market we consider there is a group of potential traders who will become informed of any signal. These traders may trade through either market or stop orders. However, an informed trader who checks the quote will always prefer to use a market order. The reason why can be demonstrated by a simple example. Suppose that the trader has seen a low signal and so wants to sell. He can sell a unit via a market order at the current bid or he can submit a stop-loss order at some price below the current bid. The trader knows that the stop order will eventually be hit but that he will sell for less than the current bid. Selling at the higher price is obviously better so he will use a market order. This argument is summarized in the following Lemma.

[3] Orders to sell above the current quote are known as limit sell orders. Orders to sell when the price falls to a specified price below the current quote are stop-loss orders. Orders to buy the stock when the price rises to the specified level above the current quote are stop-buy orders. Stop-loss orders are far more widely used than stop-buy orders.

[4] Our protocol for clearing stop orders is that used on the NYSE. There is another type of stop order known as a stop-loss limit order that clears in a different way. A stop-loss limit order must execute at its stated price, but if market prices fall through that point the order is put in the book as a limit order and, hence, may not execute at all. Since this type of stop order is rarely used, we do not consider it in our analysis.

LEMMA 1: *When his turn to trade arrives, an informed trader who can trade one unit via a market or a stop order will use a market order.*

Proof: (All proofs are given in the Appendix.)

There are also uninformed potential traders in the market. Some of these traders use market orders and some may use stop orders. There may be myriad factors affecting each trader's order form choice, but, assuming traders are rational, they cannot believe that they can make positive expected profits from stop orders. That is, they cannot "front run" or free ride on informed trading. The reason for this is that uninformed traders are in effect trading with risk-neutral agents (market makers or floor brokers) who have at least as much information as they do. The quote is the quote setter's expected value of the asset at the time of trade, so at the time an uninformed trader submits a stop order it has zero expected value. If the order is not hit, it has a zero return, and, if it is hit, it trades at the current market price which is the current expected value of the asset.

LEMMA 2: *At the time an uninformed trader submits a stop order his expected profit from that order is zero.*

Proof: (See Appendix)

Since neither informed traders acting on information nor uninformed traders hoping to "piggyback" on information will use stop orders, it follows that stop orders will only be used for noninformation related reasons such as price protection. In the analysis that follows we incorporate these characteristics by assuming that stop orders are submitted by uninformed, purely discretionary traders. Alternatively, market orders are submitted by traders who need immediate liquidity. These traders may be acting on information or they may be uninformed traders facing liquidity needs. This characterization is similar to that of Rock (1989) and captures the differing benefits offered by alternative order forms.

Market clearing in our model can occur in two different ways. First, if the market maker is active, he receives all orders, knows the structure of the book, sets bid and ask quotes, and clears all trades. We determine how prices evolve in this market in Section II. It may be the case, however, that in some market settings the size and composition of any contingent orders are not known. Consequently, we consider in Section III a market in which the structure of the book is not known when prices are set. This could occur in a market maker system if traders pursue price-contingent order strategies (such as portfolio insurance) that do not explicitly involve the book. It could also occur in a market in which floor brokers quote bids and asks and orders are matched to the best quote. In this system, a computer (or inactive market maker) keeps track of orders, and floor brokers know only whether an order is a buy or a sell and not whether it originated as a market or a stop order. For semantic ease, we use this floor broker depiction to characterize how factors such as access to the book affect the behavior of security prices.

What is important is to emphasize that in either market clearing protocol the book does not provide any signal of the underlying information. However, while the composition of the book is not correlated with the asset's value, this does not mean that the book is irrelevant. Because traders attempt to infer information from the pattern of trades, knowing which trades are stop orders is valuable in itself. In addition, the ability to trade via stop orders may affect the total volume of trading, so the existence of the book may introduce changes in the order flow as well.

In our model, only the market maker knows the actual composition of the book. Other traders and floor brokers know that a book can exist, however, and so must form expectations about its size and structure. These expectations reflect the probability distributions traders attach to parameters of the order flow, and so are "rational," given publicly available information.

This modeling approach is a significant departure from that taken by Gennotte and Leland (1990). They assume that any trader who does not directly observe the price-contingent order flow believes that such orders do not (indeed, cannot) exist. In their framework, traders are irrational as they do not incorporate expectations about order flow into their behavior, and so are simply using the wrong model. Our approach captures the more believable scenario that trader's expectations, while rational, may be incorrect because they do not know the exact outcome of the order process. As we demonstrate, this focus provides important policy insights that do not arise in models which ignore trader rationality.

To model the order process we assume that when an information event occurs fraction μ of the orders come from informed traders and fraction $1 - \mu$ come from uninformed traders. The uninformed are split into two groups; fraction γ are potential sellers and fraction $1 - \gamma$ are potential buyers. Some fraction of each of these groups of uninformed traders will use stop orders if such orders are permitted. We let w^s (w^B) be the fraction of uninformed sellers (buyers) who use stop orders. As stop-loss orders are used more frequently than stop-buy orders, we impose the restriction $w^s \geq w^B$. The market maker knows these fractions because he has seen the history of stop and market orders. The book is private information to the market maker so other traders may not have had the opportunity to learn these fractions. From their point of view w^s and w^B are random variables with joint distribution $g(w^s, w^B)$ on $[0, 1] \times [0, 1]$.

Finally, we need to say how the uninformed traders who would use stop orders behave if stop orders are not allowed. We let $\eta w^s(\eta w^B)$, $0 \leq \eta \leq 1$, be the fraction of uninformed sellers (buyers) who do not participate in the market if there is no book. This construction allows us to capture possible volume effects introduced by the existence of the book. For example, if $\eta > 0$, allowing stop orders increases trading volume because some traders would otherwise not participate in the market. Alternatively, if $\eta = 1$, allowing stop orders does not affect trading volume but does change the composition of trades as some traders shift from market orders to stop orders. Other traders do not know the book, so from their point of view it is random. They know that there may be an initial book consisting of old stop orders. We let $\theta^s(x, y, n)$ be the probability of n old (submitted prior to the current day) stop-loss orders in the interval of prices (x, y), for $x, y \in [\underline{V}, \overline{V}]$, for each n.[5] The function $\theta^B(x, y, n)$ is defined symmetrically.

During the trading day the book changes in two ways. First, some stop orders are hit and are removed from the book. Second, new stop orders may be submitted. Any such orders come from uninformed traders and so do not affect the market makers beliefs. We let $g^s(p \mid (b_t, a_t))$ be the distribution of stop-loss order prices, for $p \in [\underline{V}, b_t)$, for stop orders submitted at date t. Similarly, $g^B(p \mid (b_t, a_t))$, for $p \in (a_t, \overline{V}]$, is the distribution of stop-buy order prices. We assume that θ^s, θ^B, g^s, and g^B are continuous in prices.

Having defined the structure of trading, we now consider how prices are determined. In the next section, we analyze this price-setting problem in a market with an active market maker. The price-setting problem in a broker market is analyzed in Section III. One factor common to both settings is that, because of our assumption of risk neutrality, inventory does not affect pricing decisions. A risk-neutral dealer or broker adjusts for inventory only if there is some carrying or borrowing cost; since we consider price behavior within a trading day we ignore such costs.

II. Market Makers and Security Prices

At the beginning of the trading day, the market maker must determine initial quotes for buying and selling one unit of the asset. While it would be tempting for the market maker to simply "pick off" old stop-sell orders by setting a low opening price, this violates the clearing rule that requires a market sale at p to trigger any stops at p or above. Consequently, the first trade of the day must be a market order. What the market maker does not know, however, is whether this market order comes from an informed trader or simply reflects uninformed liquidity trading. Because this affects profit, the market maker must take the potential for informed trading into account in determining quotes. Moreover, we assume that the market maker's quotes must yield zero expected profit. This assumption can be justified by appealing to competitive forces or to institutional constraints imposed on the market maker.

[5]For example, the book could be modeled as arising from the independent actions of N traders, each of whom chooses with probability β to enter a stop order. If a trader enters an order, he does so at a price p selected according to the density $f(p)$ on $[\underline{V}, b_0)$ where b_0 is the last trade price. This implies that the number of stop orders in any interval (b, b_0), $\underline{V} \le b \le b_0$, is distributed binomially with

mean $N\beta \int_b^{b_0} f(p)\, dp$ and variance $N\beta(\int_b^{b_0} f(p)\, dp) \cdot (1 - \beta \int_b^{b_0} f(p)\, dp)$.

Then $\theta(x, y, n) = \binom{N}{n} (\beta \int_x^y f(p)\, dp)^n \cdot (1 - \beta \int_x^y f(p)\, dp)^{N-n}$, $n \le N$.

Clearly, $\theta(x, y, n)$ is continuous in x and y for all $n \le$ N.

To determine his quotes, therefore, the market maker must calculate the expectation of the asset's value, conditional on the type of trade. Before trading begins, the market maker believes that $\Pr\{V = \underline{V}\} = \delta$. The market maker also knows that the composition of the informeds' trading depends on their information, with good news eliciting buy orders and bad news causing traders to sell. A market order to sell, therefore, may indicate that the signal was low or it may simply reflect an uninformed trader's liquidity needs.

We assume that the market maker is a Bayesian who updates his beliefs based on the trade that occurs. Let these conditional expectations be given by $b_0 = E[V \mid S_0]$ and $a_0 = E[V \mid B_0]$, where b_0 is the bid quote, a_0 is the ask quote, and S_0 and B_0 represent the event that the first market order is a sell or a buy, respectively. These can be written as

$$b_0 = \underline{V}\delta(S_0) + \overline{V}(1 - \delta(S_0)), \quad \text{and} \tag{1}$$

$$a_0 = \underline{V}\delta(B_0) + \overline{V}(1 - \delta(B_0)), \tag{2}$$

where the term $\delta(S_0)$ is the conditional probability of $s = L$ given a market sell, and $\delta(B_0)$ is defined similarly for a market buy. By Bayes rule, if stop orders are allowed, the updating formula for δ, given a trade of S_0, is:

$$\delta(S_0) = \frac{\delta \Pr\{S_0 \mid \underline{V}\}}{\delta \Pr\{S_0 \mid \underline{V}\} + (1 - \delta)\Pr\{S_0 \mid \overline{V}\}}$$

$$= \frac{\delta[\mu + (1 - \mu)\gamma(1 - w^s)]}{[\delta\mu + (1 - \mu)\gamma(1 - w^s)]} \tag{3}$$

If stop orders are not allowed, a similar expression results with w^s replaced by ηw^s. That is, stops attract fraction w^s of the uninformed potential sellers to the book, but not allowing stops causes fraction ηw^s to not participate in the market. We present the analysis for the market with stop orders, the analysis without stop orders is similar. A similar expression can be derived for $\delta(B_0)$. By inspection, $\delta(S_0) > \delta$ if $\mu > 0$ and $\delta \in (0, 1)$. Thus, a market maker who is uncertain about V increases the probability he attaches to $V = \underline{V}$, given that a trader wants to sell to him. The size of this adjustment depends on the fraction of trades believed to be from informed traders μ and on the propensity of the uninformed to use market sell orders $\gamma(1 - w^s)$.

The price the market maker quotes can be found by substituting from Formula (3) into equation (1):

$$b_0 = \frac{\underline{V}\delta[\mu + (1 - \mu)\gamma(1 - w^s)] + \overline{V}[(1 - \mu)\gamma(1 - w^s) - \delta(1 - \mu)\gamma(1 - w^s)]}{[\delta\mu + (1 - \mu)\gamma(1 - w^s)]}$$

$$= V^* - \frac{\sigma_V^2}{[\overline{V} - \underline{V}]}\left[\frac{\mu}{\delta\mu + (1 - \mu)\gamma(1 - w^s)}\right] \tag{4}$$

where σ_V^2 is the variance of the conditional expectation of V.

As equation (4) indicates, provided there is any potential for information-based trading, the bid is set below V^*. Since informed traders profit at the market maker's expense, the latter protects himself by buying at a discount to the asset's prior expected value. By a similar derivation, the ask a_0 is set above V^*, causing a spread to develop in the market maker's prices. The larger the threat of informed trading (i.e., μ), the greater is the spread. Equation (4) also indicates that the greater the variability of the underlying value of the asset (i.e., $\underline{V}$ or $\overline{V}$), the more quotes diverge from V^*. This reflects the greater uncertainty a market maker faces when price variability is high.[6]

Of particular interest is how the book of stop orders affects these opening quotes. Although the first trade is known to be a market order, the book still influences prices through its effects on the probability of an information-based market order. Specifically, if stop orders are permitted, some traders who would previously have submitted market orders may now submit stop orders. Allowing stop orders may also induce traders to enter the market who would not have done so if they could only submit market orders. In our model, these effects are captured by the η variable. It is easy to demonstrate that if $\eta < 1$ the existence of a book increases the probability that a market sale is from an informed trader because some uninformed use the book who would otherwise not participate. As the following proposition indicates, this causes the market maker to set different quotes than he would if the book did not exist.

PROPOSITION 1: *If $\eta < 1$, allowing stop orders lowers the initial bid and raises the initial ask.*

That the book can induce price effects even when the type of order is known, the size of the book is known, and the orders in the book are known to be uncorrelated with information illustrates the important role that the very existence of the book plays in the price process. By providing an alternative trading mechanism, the book can divert order flow and hence change the information content of the remaining trades. This causes prices to differ from what they would have been in the absence of a book, even though no new information on the value of the asset exists.

A related issue is how the individual orders **per se** affect prices. To determine this, we need to analyze the market maker's subsequent quotes. As this quote-setting problem is the same throughout the trading day, we outline the general solution. We let t index trade times in the trading day. Between trade times, new stop orders may have arrived; but they arise only from the uninformed and do not cause quote revision or trade. Let H^*_t be the history of trades (buy (B) or sell (S)), and trade types (market (M) or stop (C)) up to and including time t for $t = 0, 1, \ldots$ in the trading day. So $H^*_0 = (S_0, M_0)$ if there is a sell at $t = 0$ or (B_0, M_0) if there is a buy and so

[6] The idea that information can cause a spread to develop was first demonstrated by Copeland and Galai (1983). Easley and O'Hara (1987) characterize the factors influencing the size of the spread when traders can be informed.

on. The updated probability on $\underline{V}$ at time t, given H^*_{t-1} and the trade at t, is given by Bayes rule as in equation (3). Using the new beliefs the market maker sets bid and ask quotes according to equations (1) and (2).

One question that obviously arises is: How do executed stop orders affect these quotes? Although the initial trade was a market order, as trades occur throughout the day the movement of prices may cause orders on the book to be triggered. Because stop orders are submitted by uninformed traders and the market maker knows which orders are stops, it is easy to demonstrate that the market maker's beliefs do not change if a stop order transacts, so his trading prices remain unaffected. Thus, executed stop orders trade at the current expected value of the asset.[7] In this market setting, therefore, individual stop orders do not affect prices, but the collection of aggregate stop orders (the book) does.

While prices are now different than they would have been in the absence of stop orders, it is not immediately obvious whether this is a good development. Certainly uninformed and informed traders using market orders are initially worse off, because the market maker sets "worse" initial trading prices. But the path of subsequent prices may be above or below where it would have been depending upon the actual sequence of market orders and stop orders.

To address these issues we analyze the stochastic process of transaction prices. At each date, quotes and thus transaction prices, are conditional expectations of value given the market maker's information about the history of trades. The price process is, thus, a martingale with respect to the market maker's information. It is also easy to see that prices converge to the appropriate value, $\underline{V}$ if $s = L$ or $\overline{V}$ if $s = H$. That is, prices become strong form efficient.[8] These results hold regardless of the existence of a book.

A book does, however, affect the fine details of the stochastic process, in particular, the speed of convergence. If strong form efficiency is a desirable property for security prices, then the faster prices converge to strong form values the more desirable is the price process. Focussing on the speed of adjustment as a measure of market performance was first suggested by Diamond and Verrecchia (1987) in their analysis of the effect of short sale constraints on market behavior. For our analysis here, if allowing a book increases the speed of convergence, then we say that it improves the market.

As quotes and thus transaction prices are linear in beliefs, the rate of convergence of prices is the same as the rate of convergence of beliefs. To

[7]Because only the market maker sees the book, a question arises as to whether he can exploit this information to increase profits. Specifically, suppose the market maker knows that the next trade will be a stop. He could quote a low price, execute the stop, and make a nonzero profit. However, if the subsequent trade turns out to be a market buy, then the stop will have executed at the lowest price of the day. Since this is observable, the exchange can enforce prohibitions against the market maker "squeezing" the stop order traders. Note that the trading rule we use does not permit the market maker to extract additional rents.

[8]The martingale property of prices and convergence to strong form efficiency was shown in similar models by Glosten and Milgrom (1985) and Easley and O'Hara (1987).

analyze the rate of convergence of beliefs we need a few definitions. First, as executed stop orders are merely cleared with market orders and do not effect prices, we analyze the speed of convergence in terms of the number of market trades. Let τ index market trades or equivalently price adjustments. For each signal $\Psi \in \{L, H\}$ define the probability on market trades $p^{\Psi} = (p^{\Psi}(B), p^{\Psi}(S))$, with each term giving the probability of the trade, given signal Ψ. The entropy of p^H relative to p^L is defined by

$$I_{p^L}(p^H) = p^L(B)\log\left(\frac{p^L(B)}{p^H(B)}\right) + p^L(S)\log\left(\frac{p^L(S)}{p^H(S)}\right). \tag{5}$$

The entropy of p^L relative to p^H is defined by the obvious expression.

As the market maker is a Bayesian, his beliefs converge exponentially to the truth. Suppose that a low signal has occurred. Then the probability on a low signal at time τ, δ_τ, converges almost surely to one at exponential rate $I_{p^L}(p^H)$. So prices converge almost surely to $\underline{V}$ at exponential rate $I_{p^L}(p^H)$. Thus the effect of a book on the rate of convergence is given by its effect on relative entropy.

PROPOSITION 2: *Prices converge almost surely to their strong form efficient value at an exponential rate. If signal Ψ occurs, this exponential rate is $I_{p^\Psi}(p^{\Psi'})$ for $\Psi' \neq \Psi$. Further, suppose $w^s \geq w^B \geq [(1-\mu)\gamma/(\mu + (1-\mu)\gamma)]w^s$ and $\eta < 1$. Then allowing a book increases the rate of convergence.*

Proposition 2 shows that how quickly prices adjust to information depends on the order forms allowed. When allowing a book of stop orders reduces the propensity of the uninformed to use market orders, there will be, on average, fewer price adjustments needed to reach the new equilibrium value than if there were no book. Since stop orders clear without changing prices, whether this adjustment also requires a smaller trading volume depends on the structure of the book. This convergence result does not mean, however, that quotes are always "better" at each point in time. Since the market maker moves quotes more in response to market orders, he can be fooled into moving quotes in the wrong direction by an uninformed trade. Nonetheless, he will learn the truth faster, and hence quotes and prices will adjust faster when a book exists.[9]

Our results on price adjustment, therefore, suggest that allowing price-contingent orders may improve the efficiency of the market.[10] An important assumption of our analysis thus far, however, is that the market maker

[9]Note that we have defined the speed of convergence in terms of market order trades. This seems appropriate because stop orders are simply cleared with the market orders and so do not transact separately.

[10]One aspect of stop orders we have not discussed is their effect on the market maker's inventory position. Since the market maker takes the buy side in every executed stop order, his inventory grows as a result of these orders. If there are multiple stops submitted at some price, inventory can become quite large. Provided the dealer is risk neutral and there are no limitations on his ability to finance his position, this is of no consequence. If there are constraints, however, stop orders can introduce difficulties into market clearing.

knows the size and composition of the book. In the case of stop orders, this is certainly a reasonable assumption. But there are other price-contingent order strategies (such as portfolio insurance) that do not explicitly involve the market maker's book. Consequently, while the market maker may know the traders are following such order strategies, he may not know the extent of the orders or their particular price composition. Indeed, a similar problem arises in any market setting in which the price-setting agent (or agents) do not have complete information on the nature of the order flow.

In the next section we address this question of how uncertainty over the size and composition of price-contingent orders affects market prices. To distinguish this from the market maker setting analyzed thus far, we investigate how stop orders and the book affect the behavior of security prices in a market with active floor brokers and an inactive market maker.

III. Brokers and Security Prices

Suppose floor brokers take over the dealer's function of setting bid and ask quotes. Floor brokers are similar to the market maker in that they are risk neutral and are uninformed of any signal of the asset's value. They differ from the market maker in that they may not know the propensity of traders to use the book (w^s, w^B), and they do not know whether any individual order is a stop order or a market order. Each broker is assumed to act competitively and to maximize expected profit.

In this market, the market maker brings orders to the floor but does not take a position himself.[11] The market maker announces an order (buy or sell) and solicits bids if it is a sell order and asks if it is a buy order. Each broker must then determine his quote. If many brokers quote the same price, one is chosen at random to complete the transaction. It is easy to demonstrate that in the absence of a book this floor broker system and the market maker system are identical. Because the price-setting agents in each market have the same information, they set the same prices. We now consider how stop orders and the book affect this price-setting process.

To facilitate comparison with our earlier analysis, we first consider the initial trade of the day. As before, brokers know this particular trade will be a market order. However, unlike before, they may not know the propensity of traders to use the book or, as will matter in the determination of subsequent prices, where orders are placed in the book. Since the size of the book affects the probability of an information-based market order, each broker must incorporate his expectations about uninformed trader behavior into his pricing problem.

As was the case for a market maker, the initial prices the floor brokers quote will be the conditional expectation of the asset's value, given the type

[11] As both the market maker and brokers are risk neutral, and the market maker has superior information, it is important that the market maker not take any position. Otherwise brokers would know any trade they get is one the market maker does not want and thus one they should not take. This issue is addressed in more detail in Rock (1989).

of trade. These quotes are given by equations (1) and (2). However, because there is uncertainty over the propensity of traders to use the book, the floor broker's updated beliefs differ from those of the market maker. In the case of a sale (S_0) the floor broker's posterior probability of a low signal is

$$\delta^F(S_0) = \Pr{}^F(\underline{V} \mid S_0) = \int f(\underline{V}, w^s, w^B \mid S_0)\, dw^s\, dw^B, \tag{6}$$

where $f(\underline{V}, w^s, w^B \mid S_0)$ is the posterior probability on $(\underline{V}, w^s, w^B)$, given S_0. By Bayes Rule this probability is

$$f(\underline{V}, w^s, w^B) = \frac{\delta(w^s, w^B)\Pr(S_0 \mid \underline{V}, w^s, w^B)}{\int \left[\delta \Pr(S_0 \mid \underline{V}, w^s, w^B) + (1-\delta)\Pr(S_0 \mid \overline{V}, w^s, w^B)\right] \cdot g(w^s, w^B)\, dw^s\, dw^B}. \tag{7}$$

Typically the floor brokers and the market maker will set different initial bid and ask prices. Because the floor brokers' beliefs can differ from the market maker's, it follows that prices can deviate as well. However, these price deviations do not display any systematic bias as would occur if market participants were always simply "wrong" about the underlying structure of one model. As the following proposition demonstrates the floor broker's initial quotes are the conditional expected values of the market maker's initial quotes.

PROPOSITION 3: *The conditional expected values (conditioned on trade types) of the market maker's initial quotes are the floor broker's initial quotes.*

What is interesting about this result is that despite the fact that floor brokers know the type of trade (a market order) they do not generally set the same quotes as the market maker. The reason is that the book creates more uncertainty for the floor broker than it does for the market maker. Earlier we showed that the book affected the market maker's quotes because it changed the probability of information-based market orders. Now, that same "informed order" effect exists, but the floor broker faces added uncertainty over the use of the book. This "book" uncertainty causes the floor broker's quotes typically to differ from those of the market maker, but does not **per se** introduce a bias into prices.

This unbiased relationship between quotes in the two types of markets will occur whenever both floor brokers and the market maker can identify the trade type. Obviously, this holds for the first trade of the day regardless of whether it is a buy or a sell. It also holds at any point during the day when the direction of trade reverses. For example, an order to sell, following one or more buys, must be a market order. This follows because the buys will have raised quotes (or left them unchanged), and this cannot trigger a stop order to sell. At such a point, the market maker and floor brokers would quote the same price if they all knew the propensity to use the book. If floor brokers did not know this exact propensity, then their quotes would be their expectations of the market maker's quotes.

For many (if not most) trades, however, the floor brokers will have less information than the market maker for yet another reason: They will not know the trade type. To isolate the effect of this trade type uncertainty, suppose that the propensity of traders to use the book (w^s, w^B) is known to all traders. Although floor brokers now know w they do no know the actual structure of the book and so will often be uncertain about whether a trade arises from a market order or a stop order. In particular, in any sequence of all buys or all sells every trade following the first trade in the sequence could either be a market trade or a stop order that has been hit.

A market maker knows whether any particular trade is a stop or a market and does not adjust quotes in response to stops. Floor brokers do not know the trade type and so make an adjustment following every trade which is independent of the actual trade type. This results in floor brokers changing their quotes at times when a market maker would have made no change. But this same "trade type" uncertainty results in floor brokers adjusting their quotes less than the market maker does in response to actual market orders.

To see the effects of these two offsetting forces on the variability of quotes, suppose that the brokers and market maker have the same conditional expectation of V at some date t (i.e., the same δ_t). Let $h^s(h^B)$ be the probability that a stop loss (buy) order is hit by the adjustment of the bid (ask) price between $t - 1$ and t. The market maker knows whether there is a stop order that will be hit, but an h is the probability from the floor brokers' point of view of these events. Implicit in this structure is an assumption that the probability of a stop order being hit by the market maker or the floor brokers is the same when they have equal conditional expectations of V.

Now at data t there are three events that can occur. First, a stop order may be hit. This event does not affect the market maker's beliefs or quotes, but it does change the floor brokers' beliefs and quotes. Second, if no stop is hit, a market order may arrive. This event will cause the market maker's quotes to change by more than the floor broker's quote change. Third, if no stop is hit, a new stop order may arise. This event has no effect on anyone's beliefs, and in fact it cannot be observed by floor brokers. This is irrelevant as our timing is by orders that trade—either market orders or crossed stop orders.

The market maker's quotes are conditional expectations of asset value with respect to his information. Similarly, the floor brokers' quotes are expectations with respect to their information. If $h^B > 0$ or $h^S > 0$ and $h^B + h^S < 1$ the market maker's information is finer than the floor brokers' information, and the conditional variance of his quotes is thus greater than that of the floor brokers.

PROPOSITION 4: *Suppose w^s and w^B are known to all traders and the probability of a stop order being hit (h^s, h^B with at least one strictly positive and their sum less than one) is the same for the market maker and the floor brokers. If at date t the market maker and floor brokers have the same beliefs, then the conditional variance of the market maker's quotes is greater than the conditional variance of floor brokers' quotes.*

An important result of our analysis is that price variability is greater when order types are known (the market maker system) than unknown (the floor broker system). One reason for its significance is that it is the opposite of the conventional wisdom offered by other researchers. For example, Gennotte and Leland (1990) demonstrate that price volatility is *lower* in their model when price contingent orders are observed than when they are unobserved. What generates their result is that traders are not allowed to incorporate expectations of the price-contingent order flow into their behavior. Thus, when a large unobserved hedging-based order flow arises, traders have no choice but to believe it is information-related, and prices fall accordingly. In the Gennotte-Leland model, volatility is higher because traders irrationally ignore the possibility of price-contingent orders.

In our model, traders may, on occasion, underestimate the extent of price-contingent trading, with prices falling more than seems justified. (We address this issue further later in the paper). But is also the case that traders may overestimate such price-contingent trading, causing prices to be above where they should be. As Proposition 4 demonstrates, if expectations are considered, the overall effect of uncertainty over order type is to reduce price variability.

That prices are more variable in the market maker system where order form is observable, however, is not necessarily undesirable. In a single trading-period framework like that of Brennan and Schwartz (1989) or Gennotte and Leland (1990) the only characteristic of price that can be ascertained is its volatility. In the multiperiod framework considered here, however, it is also possible to analyze the speed with which prices reflect information. To some extent, there is a tradeoff between these two factors that makes simple policy prescriptions based on volatility unwise.

To illustrate this tradeoff between volatility and the informativeness of prices, consider how prices adjust to information. Starting from the beginning of the day the most stable price process simply keeps prices fixed, while the least stable process is the one-step move that adjusts to $\underline{V}$ or $\overline{V}$ immediately. If the move is made to the correct (full-information efficient) price this process seems desirable in spite of its greater variability. Alternatively, if the move is in the wrong direction the lack of stability comes with no benefit. This suggests that volatility per se may not be a useful concept; what matters is where prices move to and not merely how much they move.

Generally, suppose we have two stochastic processes of asset prices each of which is formed by a sequence of conditional expectations of asset value with respect to some increasing sequence of information sets. Then both processes are martingales with respect to their respective sequences of information sets. In line with the usual ordering of efficiency concepts we say that the first process is *more efficient* than the second if the information that it is based on is no coarser than the second processes' information at each date and is finer at some date. By this criterion the market maker system produces a more efficient price process than does the floor broker system.

PROPOSITION 5: *In both the market maker and floor broker systems, transaction prices form a Martingale (with respect to past prices). But if $w^s > 0$ or*

$w^B > 0$, then the market maker price process is more effecient that the floor broker price process.

The results in Proposition 5 suggest that uncertainty over the types of orders submitted introduces unwanted inefficiency into the market clearing process. Although this uncertainty results in less price variability, it also causes prices to deviate longer from full-information values. If efficiency is a goal for market design, then this suggests that price-contingent order strategies per se are not the problem. Rather, it is the uncertainty over their size and structure that can lead to undesirable outcomes. We return to this issue further in Section IV.

While our discussion thus far has focussed on the variance and speed of adjustment of various price processes, it is also true that markets may exhibit differing periodic price behavior. In particular, it may be that allowing different order forms can affect the potential for large price movements unrelated to information. Indeed, the market's behavior in October 1987 is perhaps best viewed as such a periodic episode; recent research (see Grossman (1989) and Schwert (1990)) on volatility in the market at that time suggests no overall general increase in variance. Analyzing such periodic price behavior requires an analysis of how sequences of orders can affect market behavior. Indeed, because market clearing rules give stop orders priority over market orders, it would seem that such sequence effects may be important. In the single period models analyzed by previous researchers, the interaction between the sequence of orders and price movements cannot be addressed. In the multiperiod framework developed here, however, these market movements can be analyzed.

In both the market maker system and the floor broker system, a random sequence of liquidity-motivated sell orders causes prices to drop. If these orders, in turn, trigger price-contingent orders, then how much prices fall depends on whether the extent and composition of these orders is known. If, as may have been the case in the October 1987 market crash, the extent of price-contingent orders is underestimated, then prices may appear to behave "irrationally," falling far more than is justified by any change in the fundamental value of the asset. What causes this price movement is the combination of what we have referred to as "book" uncertainty and "order-type" uncertainty. Because floor brokers can underestimate the extent of price-contingent orders, prices can potentially fall more in the floor broker system than would occur with the same order flow in the market maker system.[12]

[12]An interesting issue we have not addressed is the effect that contingent order strategies have on other traders. In particular, since contingent orders generate large price swings, an uninformed owner of the stock may find the value of his portfolio highly variable. If the trader is risk neutral and faces no potential liquidity needs, this variability is irrelevant. But, suppose the trader does face short-run constraints as, for example, does a portfolio manager evaluated over some specific time interval. Then, knowing that others follow contingent strategies may induce the trader to follow them as well. We caution, however, that this is merely a conjecture. To provide definitive conclusions on the uninformeds' trading behavior requires a formal model of their decision-making, which we have not provided. Nonetheless, the potential for contingent trading strategies to beget contingnet strategies is certainly an intriguing possibility.

This sequence effect of trade-type uncertainty can best be illustrated in a simple example. Suppose that the only stop orders are to sell and that ten percent of uninformed sellers use such orders. Further, suppose that the uninformed are equally likely to buy or sell, that the signal is equally likely to be high or low, and that ten percent of all traders are informed. If we normalize the range of full information efficient prices to [0, 1], then the opening bid from either a floor broker or a market maker is 0.445. If the first trade of the day is a sale, all stop orders at prices at or above 0.445 are hit, and in the market maker system they trade at 0.445. The market maker's bid then falls to 0.391.

In the floor broker system stop orders that are hit by an opening sale do matter. To keep the example simple, we assume that floor brokers believe that each of the next several trades is equally likely to be generated by a stop or a market order. In this case if the first trade of the day is a sale all stops at or above 0.445 are triggered. But the floor broker's bid for the next unit to be sold falls to 0.428, so the first stop order trades at this price. Now all stops at or above 0.428 have been triggered. The stop in this group having the highest priority trades at the next bid which is 0.41. This process continues until finally no stops are triggered by the price at which the last stop was traded. A sequence of seven such triggered stops following an opening sale will cause the bid to fall to 0.31.

If the floor broker's expectations about the structure of the book are correct, such an event is not very likely on a given day; it happens about 0.8 percent of the time. However, over many days the chance of this fall of prices to a level 20 percent below the market maker's quote on the same day is large.

Alternatively, if the floor brokers believe that there are no price-contingent orders then their prices fall much faster. For example, the same sequence of seven stops following an opening sale would cause the floor brokers bid to fall to 0.12 if these stops are unanticipated. Again, this price drop actually has a probability of only 0.8 percent on any given day, but over many days this fall to 70 percent below the market makers quote is likely to occur occasionally.

IV. Policy Implications and Conclusions

The important influence that order form exerts on market prices should not be surprising. It is through the process of trading that prices evolve, so how trades occur must be important in the price formation process. What may be surprising is the pervasiveness of this order form effect. As we demonstrate, even when the type of trade and the size of the book are known, the ability to trade via an alternative order form changes the stochastic process of prices.

In our analysis, we have identified four effects on the price process that arise from alternative order forms. First, by providing an alternative trading venue, a book of price-contingent orders changes the informativeness of the market order flow. This causes spreads to be larger than they would have been in the absence of the book, but it also increases the speed with which prices adjust to new information. Second, uncertainty over the extent of the book (or price-contingent orders) can cause price-setting agents to set different prices then they would have if the book were known. However, while prices may differ, we also demonstrated that these prices will be equal to conditional expected values and hence will not be biased. Third, we demonstrated how uncertainty over any individual order's type (either stop or market) actually reduces the variance of prices compared to prices when order type is known. This reduced volatility, however, is accompanied by a loss in the informativeness (and hence efficiency) of prices. Finally, we demonstrated how, if order types are not observable, sequences of price-contingent orders can induce large price movements unrelated to changes in underlying information.

Our analysis of these order form effects provides a number of insights into important issues in security market design. In particular, much recent debate has focussed on the effect of order form on market volatility. As we have demonstrated, there are really two issues to consider here. If volatility is meant to mean the variance of prices, then uncertainty over order form can actually reduce this variance if traders are rational. If, instead, volatility is meant to mean the potential for large episodic price movements unrelated to information, then unobserved price-contingent orders can increase such volatility. The recent research findings that market price variance was actually generally lower prior to the high volatility on the day of the crash is exactly consistent with the prediction of our analysis.

This ability of order form to affect market prices in the absence of changes in a security's fundamental value illustrates the important role of the trading mechanism in the process of price formation.[13] Indeed, following the market crash, several proposals to ban portfolio insurance and other contingent order strategies were suggested as a means to improve the stability of the market. Certainly, contingent order strategies can introduce noise into the trading process, causing prices potentially to diverge from fundamental values. We have also demonstrated, however, that prices can be "better" in markets with contingent orders because such orders can increase the speed with which prices converge to new information. This suggests that price-contingent orders per se are not the problem; if difficulties arise it is because of the inability of the market clearing system to identify correctly the nature of trades.

This difficulty underlies the frequently mentioned market reform to require disclosure of any outstanding contingent orders (see Grossman (1988)). Our model suggests that, to the extent prices are affected by traders who do not know the book, this would improve the stability of the market. As we

[13]This trading mechanism effect arises even when trades are organized off the floor of the exchange. Burdett and O'Hara (1987) demonstrate that similar price effects arise in the syndication process typically used for block trades or trades involving large amounts of stock. Since blocks usually transact at lower prices, traders learning of the trade attempt to profit from the block by entering limit buy orders or by selling the stock short. These actions, in turn, can cause prices to fall in anticipation of the block, even though there is no new information on the value of the stock.

have shown, the price variability introduced by uncertainty over the extent and structure of the book impedes the market's ability to adjust to new information.

Our analysis of order form in this paper can be viewed as a step in understanding how the trading process affects security price behavior. There are, however, several aspects of our model that deserve comment. Because we focus on stop orders we can separate the effects of order form from those of information on security prices. In the case of portfolio insurance the same separation seems likely to occur. But, for other order forms, this separation is unlikely to be accurate. As a result, the book may also provide a signal of information, thus playing an even larger role in the price-setting process.

A second issue relates to the type of orders we allow. We explicitly analyze only market orders and the simplest contingent order form, the stop order. One obvious extension to our work is to analyze the many other order forms used in security trading. For example, market-at-close orders have played a prominent role in the market's price behavior on "triple witching" days. An even more important order form is the limit order. Since limit order traders are often described as "competing" with the market maker, research into this order form seems particularly promising.

One final factor that should be noted is that our analysis considers only the information effects of order form and not other important effects such as inventory. In the case of stop orders, such inventory effects may be significant because, unlike limit orders, stop orders take liquidity from the market rather than supply it. To the extent that different order forms impose large inventory costs on market makers, the stability of prices may be affected, and, hence, our policy recommendations on stability should be interpreted cautiously. If inventory effects do arise, however, it is likely that they will serve to magnify the effects of order form on security price behavior demonstrated here.

Appendix

Proof of Lemma 1: We consider the case of a low signal. The profit from a market order to sell at bid b_t is $b_t - \underline{V}$. The profit from a stop loss order at price p is $p - \underline{V}$, as the trader knows that the order will be hit. However, stop loss orders will not be accepted at or above the current bid. So the profit from market order exceeds the profit from a stop order.

Proof of Lemma 2: We consider the case of a stop loss order entered at price p. Suppose this order is hit at some time t. Then the random variable giving profit, conditional on this event, is $\pi_t = p_t - V$. The price p_t is $E[V \mid I_t^m]$ where I_t^m is the market maker's information at t. So, conditional on the trader's information at date t, I_t, expected profit is

$$E[.\pi_t \mid I_t] = E\big[E[V \mid I_t^m] - V \mid I_t\big] = 0$$

as $I_t \subseteq I_t^m$. Then clearly the unconditional expected profit from a stop loss order at price p is zero. The same reasoning applies to any stop order.

Proof of Proposition 1: We consider the initial bid, the analysis of the initial ask is symmetric. The initial bid when stop loss orders are allowed is given by equation (4). Call this bid b_0^B. If stop loss orders are not allowed the initial bid is given by (4) with w^s replaced by ηw^s. Call this bid b_0^{NB}. Calculation shows that if $1 > \eta$, $b_0^B - b_0^{NB} < 0$. So, allowing stop orders lowers the initial bid.

Proof of Proposition 2: We prove the proposition for a low signal realization, the argument given a high signal is symmetric. Let $B_\tau(S_\tau)$ be the number of market buys (sells) to time τ. By Bayes Law

$$\log\left(\frac{1-\delta_\tau}{\delta_\tau}\right) = \log\left(\frac{1-\delta_0}{\delta_0}\right) + B_\tau \log\left(\frac{p^H(B)}{p^L(B)}\right) + S_\tau \log\left(\frac{p^H(S)}{p^L(S)}\right).$$

So, by the strong law

$$\frac{1}{\tau}\log\left(\frac{1-\delta_\tau}{\delta_\tau}\right) \underset{\text{a.s.}}{\rightarrow} p^L(B)\log\left(\frac{p^H(B)}{p^L(B)}\right) + p^L(S)\log\left(\frac{p^H(S)}{p^L(S)}\right)$$
$$= -I_{p^L}(p^H) < 0.$$

Thus, $(1-\delta_\tau)/\delta_\tau$ converges almost surely to zero at exponential rate $-I_{pL}(p^H)$.

Now by the bid equation for time τ,

$$\begin{aligned} b_\tau - \underline{V} &= \delta_\tau \overline{V} + (1-\delta_\tau)\overline{V} - \underline{V} \\ &= (1-\delta_\tau)\left(\overline{V} - \underline{V}\right). \end{aligned}$$

So, by the convergence result on beliefs, $B_\tau - \underline{V}$ converges almost surely to zero at exponential rate $I_{pL}(p^H)$.

To show the comparative dynamics result on allowing a book we need the expression for $I_{pL}(p^H)$, viz.

$$\frac{\mu + (1-\mu)\gamma(1-\eta w^s)}{\mu + (1-\mu)\left[\gamma(1-\eta w^s) + (1-\gamma)(1-\eta w^B)\right]}\log\left[\frac{\mu + (1-\mu)\gamma(1-\eta w^s)}{(1-\mu)\gamma(1-\eta w^s)}\right]$$
$$+\frac{(1-\mu)(1-\gamma)(1-\eta w^B)}{\mu + (1-\mu)\left[\gamma(1-\eta w^s) + (1-\gamma)(1-\eta w^B)\right]}$$
$$\cdot\log\left[\frac{(1-\mu)(1-\gamma)(1-\eta w^B)}{\mu + (1-\mu)(1-\gamma)(1-\eta w^B)}\right].$$

Here $\eta = 1$ if there is a book and $\eta = \alpha < 1$ if no book is allowed.

Calculation shows that

$$\frac{\partial I_{p^L}(p^H)}{\partial \eta} = \frac{(1-\mu)^2\gamma(1-\gamma)(w^B - w^s) + \mu(1-\gamma)w^B}{[\mu + (1-\mu)[\gamma(1-\eta w^s) + (1-\gamma)(1-\eta w^B)]]^2}$$
$$\cdot\left(\log\left(\frac{\mu + (1-\mu)\gamma(1-\eta w^s)}{(1-\mu)\gamma(1-\eta w^s)}\right)\right.$$
$$\left.+\log\left(\frac{\mu + (1-\mu)(1-\gamma)(1-\eta w^B)}{(1-\mu)(1-\gamma)(1-\eta w^B)}\right)\right)$$
$$+\frac{\mu(1-\mu)[(1-\mu)\gamma(1-\gamma)(w^s - w^B) + \gamma\mu w^s]}{[\mu + (1-\mu)[\gamma(1-\eta w^s) + (1-\gamma)(1-\eta w^B)]] \cdot [\mu + (1-\mu)(1-\gamma)(1-\eta w^B)](1-\mu)\gamma(1-\eta w^s)}.$$

Under the conditions given in the proposition this derivative is positive. So allowing a book increases the rate of convergence to full information efficient prices.

Proof of Proposition 3: We prove the claim for the bid, the argument for the ask is symmetric. Since the initial bid is determined by equation (3) it is sufficient to consider $\delta^m(S_0) = \Pr(\underline{V} \mid S_0, w^s, w^B)$ and $\delta^F(S_0) = \Pr(\underline{V} \mid S_0)$. The floor brokers' posterior marginal probability on $\underline{V}$ is $\int f(\underline{V}, w^s, w^B \mid S_0), dw^s\, dw^B$ where $f(\underline{V}, w^s, w^B \mid S_o)$ is the joint posterior on $(\underline{V}, w^s, w^B)$ conditional on a sale at time 0, i.e., (S_0). Alternatively, the posterior marginal probability on $\underline{V}$ can be written $\int \Pr(\underline{V} \mid S_0, w^s, w^B) g(w^s, w^B \mid S_0)\, dw^s\, dw^B$, where $g(w^s, w^B \mid S_0)$ is the posterior on (w^s, w^B) conditional on S_0. But by the definition of $\delta^m(S_0)$ this is $\int \delta^m(S_0) g(w^s, w^B \mid S_0)\, dw^s\, dw^B$, the conditional expected value of the market maker's posterior probability on $\underline{V}$.

Proof of Proposition 4: Since bids and asks are determined by equations (2) and (3), which are linear in beliefs, it is sufficient to compare conditional variances of beliefs. Let δ_t be the common posterior probability of $V = \underline{V}$ at date t.

We first consider the market maker's beliefs. If a stop order is hit, an event with probability $h = h^s + h^B$, the market maker's beliefs are unchanged. If a market sale (S_t^m) occurs at t, beliefs adjust to

$$\delta_{t+1}^m(S_t^m \mid \delta_t) = \frac{\delta_t(\mu + (1-\mu)\gamma(1-w^s))}{\delta_t\mu + (1-\mu)\gamma(1-w^s)},$$

and this event has probability $(1-h)[\mu\delta_t + (1-\mu)\gamma(1-w^s)]X$, where $X = [\mu + (1-\mu)(\gamma(1-w^s) + (1-\gamma)(1-w^B))]^{-1}$ is the inverse of the probability of a market order. If a market buy (B_t^m) occurs at t, beliefs adjust to

$$\delta_{t+1}^m(B_t^m \mid \delta_t) = \frac{\delta_t(1-\mu)(1-\gamma)(1-w^B)}{(1-\delta_t)\mu + (1-\mu)(1-\gamma)(1-w^B)},$$

and this event has probability $(1 - h)[(\mu(1 - \delta_t) + (1 - \mu)(1 - \gamma)(1 - w^B)]X$.Calculation shows that $E^m[\delta_{t+1}^m \mid \delta_t] = \delta_t$ and that the conditional variance is

$$\operatorname{Var}^m[\delta_{t+1}^m \mid \delta_t] = \left[\frac{(1-h)\big(\delta_t(1-\delta_t)\mu\big)^2}{\delta_t\mu + (1-\mu)\gamma(1-w^s)} + \frac{(1-h)\big(\delta_t(1-\delta_t)\mu\big)^2}{(1-\delta_t)\mu + (1-\mu)(1-\gamma)(1-w^B)}\right]X.$$

For the floor brokers, if a sale (S_t) occurs at t, beliefs adjust to

$$\delta_{t+1}^F(S_t \mid \delta_t) = \frac{\delta_t\big(h^s + (1-h)\big(\mu + (1-\mu)\gamma(1-w^s)\big)X\big)}{(1-h)\big(\mu\delta_t + (1-\mu)\gamma(1-w^s)\big)X + h^s},$$

and this event has probability $(1 - h)[\mu\delta_t + (1 - \mu)\gamma(1 - w^s)]X + h^s$. If a buy (B_t) occurs at t, beliefs adjust to

$$\delta_{t+1}^F(B_t \mid \delta_t) = \frac{\delta_t\big(h^B + (1-h)(1-\mu)(1-\gamma)(1-w^B)X\big)}{(1-h)\big(\mu(1-\delta_t) + (1-\mu)(1-\gamma)(1-w^B)\big)X + h^B},$$

and this event has probability $(1 - h)[\mu(1 - \delta_t) + (1 - \mu)(1 - \gamma)(1 - w^B)]X + h^B$.Calculation shows that $E^F[\delta_{t+1}^F \mid \delta_t] = \delta_t$ and that the conditional variance is

$$\operatorname{Var}^F[\delta_{t+1}^F \mid \delta_t] = \frac{\big((1-h)\mu\delta_t(1-\delta_t)\big)^2}{(1-h)\big(\mu\delta_t + (1-\mu)\gamma(1-w^s)\big) + h^s X^{-1}} + \frac{\big((1-h)\mu\delta_t(1-\delta_t)\big)^2}{(1-h)\big(\mu(1-\delta_t) + (1-\mu)(1-\gamma)(1-w^B)\big) + h^B X^{-1}}.$$

Inspection shows that term by term $\operatorname{Var}^m[\delta_{t+1}^m \mid \delta_t] \geq \operatorname{Var}^F[\delta_{t+1}^F \mid \delta_t]$ with strict inequality if $h^B > 0$ or $h^s > 0$ and $h^B + h^s < 1$.

Proof of Proposition 5: The proof of the martingale property is given in the proof of Proposition 4. The efficiency comparison is immediate from the definition of more efficient and the observation that the market maker knows whether each trade is based on a market or a stop order.

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