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Animal Spirits, Margin Requirements, and Stock Price Volatility

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ABSTRACT

A simple overlapping generations model is used to characterize the effects of initial margin requirements on the volatility of risky asset prices. Investors are assumed to exhibit heterogeneous preferences for risk-bearing, the distribution of which evolves stochastically across generations. This framework is used to show that imposing a binding initial margin requirement may either increase or decrease stock price volatility, depending upon the microeconomic structure behind fluctuations in economy-wide average risk-bearing propensity. The ambiguous effect on volatility similarly arises when the source of heterogeneity is noise trader beliefs.

BEYOND THEIR PRUDENTIAL FUNCTIONS, the effects of initial margin requirements on securities markets are not well understood. Many have argued, based upon empirical findings, that the level of margin requirements has no discernable impact on stock price behavior.[1] Nevertheless, the view that high margin requirements dampen speculative excesses and reduce excess price volatility remains popular. The simultaneous existence of such views highlights the uncertainty surrounding this issue. In fact, there exists no rigorously articulated theory of the interaction between initial margin requirements and stock prices and their returns' characteristics.

In this paper we construct a simple dynamic model useful for characterizing the effects of initial margin requirements on security prices and returns. To analyze the impact of margins, we introduce investor heterogeneity into an otherwise simplified overlapping generations (OLG) economy.[2] The OLG models are constructed so that each generation displays heterogeneous preferences or beliefs, the distribution of which evolves stochastically across generations.

In each of two heterogeneous taste models we consider there are two types of agents, one of which is more risk tolerant than the other. In one model, the

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[1]The results of Moore (1966), Largay and West (1973), Schwert (1989), Kupiec (1989), and Hseih and Miller (1990) support the impotency of initial margin regulations. Hardouvelis (1988, 1990) supports the view that high margin requirements reduce destabilizing speculation.

[2]We use a stripped-down version of the Samuelson (1958) model, similar to that used by De Long, Shleifer, Summers, and Waldman (1989) in their study of the effects of noise trader risk.

risk-tolerance coefficients of the two types remains fixed across generations, but the proportion of agents "born" of each type is stochastic. In the other, the proportion of agents of each type is unvarying, but the risk-tolerance coefficient of the less risk-averse agent is stochastic across generations. In both models, fluctuations in the "average" level of risk aversion contributes to asset price volatility. While their microstructures clearly differ, the two models are shown to exhibit identical representative agent characterizations.

We then impose a margin requirement that binds on the less risk-averse investor in some states of the world. Our principle result is that imposing a binding initial margin requirement may either increase or decrease stock price volatility. The direction of that effect depends critically upon the structure of the heterogeneity presumed to underlie fluctuations in the average investor's risk-bearing propensity.

In the economy with a stochastic proportion of risk-tolerant agents, margin requirements raise volatility. Here margins tend to restrain risk-tolerant investors' effective demand when the average economy-wide risk-tolerance is low, that is, margins restrict the risky asset positions of those agents willing to buy up the risky asset as its price falls. In the alternative model, where the proportion of risk-tolerant agents is fixed but their level of tolerance is stochastic, margins are more likely to bind on the risk tolerant investors in the high price state, that is, when their tolerance for risk is highest. Here, margins lower effective demand in the high price state, thereby lowering volatility.

The principal results are shown to generalize to models where investors are assumed to have heterogeneous expectations similar to the "noise trader" beliefs of De Long, Shleifer, Summers, and Waldman (1989). It is again shown that imposing margin requirements may either increase or decrease volatility. A binding margin requirement is likely to lower volatility if the beliefs of noise traders move together. Alternatively, when optimistic and pessimistic noise traders coexist but the proportion of optimists is stochastic, margins may raise volatility. Here, margins tend to bind on optimistic and rational traders when fewer optimists are around and the risky asset price is low.

In the first section, the two models of time-varying investor heterogeneity are constructed, and equilibrium prices in the absence of portfolio constraints are characterized. In Section II, the effects of margin requirements on portfolio positions and stock price volatility are analyzed. In Section III, we demonstrate that similar conclusions can be drawn when investors are characterized by heterogeneous (and stochastic) expectations but identical tastes. Our results are summarized and conclusions are drawn in Section IV.

I. A Simple Model of Stochastic Heterogeneity

A. Assumptions

We begin by laying out a simple two-period representative agent overlapping generations economy that serves as a useful anchor for the analysis to follow. A cohort of identical investor-consumers, characterized by either high or low risk tolerance, enters the market in each period and invests its entire endowment in shares of a risky asset and a risk-free asset. In the following period, they sell their assets to the new young generation, consumer their final wealth, and exit the market.

Every cohort has N agents, each of whom is endowed with a unit of wealth. There also exist N shares of a risky asset, or "stock", which produce a stochastic dividend in each period. The per share dividend is independently and identically distributed with mean $\bar{d}$ and variance σ_d^2. Agents may also invest (or borrow) at an exogenously given risk-free rate of interest $R_f > 0$. Portfolios are chosen so as to maximize expected utility which is assumed to be linear in the mean and variance of old age wealth $\tilde{W}$:[3]

$$EU = \overline{W} - \Gamma_i \sigma^2. \tag{1}$$

Risk tolerance Γ_i^{-1} varies across cohorts; in particular, it is assumed to follow a symmetric two-state Markov process, where the state is identified by the risk tolerance of the young generation. The high risk tolerance state is labeled state 1, that is, $0 < \Gamma_1 < \Gamma_2$. Let θ be the probability of remaining in the same state from one period to the next and $(1 - \theta)$ the probability of changing states.

B. Portfolio Allocation and Equilibrium

Let P_j represent the risky asset's price in state j, $j \in \{1,2\}$. Let $\overline{P}_j$ and σ_{Pj}^2 represent the mean and variance of its price in the following period, when the current state is j. If agent i purchases X_{ij} shares of the risky asset in state j, then the mean and variance of i's old-age wealth are

$$\begin{aligned} \overline{W}_{ij} &= \left(1 - X_{ij}P_j\right)\left(1 + R_f\right) + X_{ij}\left(\overline{P}_j + \bar{d}\right) \\ \operatorname{Var}\left(\tilde{W}_{ij}\right) &= X_{ij}^2\left(\sigma_{Pj}^2 + \sigma_d^2\right). \end{aligned} \tag{2}$$

The first-order conditions for investor optimality yield:

$$X_{ij} = \frac{\overline{P}_j + \bar{d} - P_j\left(1 + R_f\right)}{2\Gamma_j\left(\sigma_{Pj}^2 + \sigma_d^2\right)}, \qquad j = 1,2. \tag{3}$$

In equilibrium, risky asset demand per agent must equal one in each state, implying that

$$P_j = \left[\overline{P}_j + \bar{d} - 2\Gamma_j\left(\sigma_{Pj}^2 + \sigma_d^2\right)\right] \Big/ \left(1 + R_f\right), \tag{4}$$

where $\overline{P}_1 = \theta P_1 + (1 - \theta)P_2$ and conversely for $\overline{P}_2$. Owing to symmetry of the state transition matrix, $\sigma_{P1}^2 = \sigma_{P2}^2 = \sigma_P^2 = (\theta - \theta^2)(P_1 - P_2)^2$.

Since dividend realizations contain no information about future returns, the risky asset price fluctuates only because of changes in taste for risk, Γ_j.

[3]If returns were normally distributed, the mean variance utility function is a special case of the Constant Absolute Risk Aversion (CARA) class of utility functions which exhibits declining relative risk tolerance. In the models to follow, individual agents are endowed with one unit of wealth; thus, the risk tolerance measure of an agent is Γ_i^{-1}.

Uncertainty about future cohorts' risk preferences causes the risky asset's return variability to exceed the variability of dividends alone. One might characterize this economy as one in which investor "animal spirits" cause volatility in excess of that justified by the "fundamental" volatility inherent in the dividend process. Market-clearing stock prices are calculated by substituting σ_P^2 into equation (4) (for both $j = 1, 2$) and solving the resulting quadratic equations simultaneously for P_1 and P_2. The unique feasible (nonnegative) pair of equilibrium state prices is

$$P_1 = \frac{\bar{d}}{R_f} + \frac{2[(\Gamma_1 + \Gamma_2)(1 - \theta) + \Gamma_1 R_f][\sqrt{A} - (1 - \theta + R_f/2)]}{4(\theta - \theta^2)R_f(\Gamma_1 - \Gamma_2)^2} \tag{5a}$$

and

$$P_2 = \frac{\bar{d}}{R_f} + \frac{2[(\Gamma_1 + \Gamma_2)(1 - \theta) + \Gamma_2 R_f][\sqrt{A} - (1 - \theta + R_f/2)}{4(\theta - \theta^2)R_f(\Gamma_1 - \Gamma_2)^2}, \tag{5b}$$

where $A \equiv (1 - \theta + R_f/2)^2 - 4\theta(1 - \theta)\sigma_d^2(\Gamma_1 - \Gamma_2)^2$.

Properties of the equilibrium prices are characterized in two propositions. (All proofs are in the Appendix.) Proposition 1 establishes that the risky asset price is higher in state 1 when young investors are less risk averse. Proposition 2 shows that the disparity between state prices widens as the difference between state 1 and state 2 risk tolerance gets larger. Since price variance in proportional to the square of the difference between state prices, an implication of this result is that volatility is increasing in the difference between the state 1 and state 2 risk tolerance measures. Finally, it is intuitively clear that both P_1 and and P_2 are increasing in $\bar{d}$ and decreasing in R_f.

PROPOSITION 1: *The risky asset price is higher in state* 1, *that is, when the young generation has high risk tolerance*: $P_1 > P_2$.

PROPOSITION 2: *Holding all else constant, the difference between the two state prices*, $(P_1 - P_2)$, *is monotonically increasing in the difference between risk aversion in the two states*, $(\Gamma_1 - \Gamma_2)$. *As* $(\Gamma_1 - \Gamma_2) \to 0$, $P_1 \to P_2$ *and* $\sigma_P^2 \to 0$.

C. Stochastic Preferences

In a model where investors of any given generation have homogenous preferences or beliefs, margin requirements affect all investors identically, and, consequently, the scope for investigating the effect of margin policy is quite limited. In this section we disaggregate; specifically, each cohort is assumed to be composed of two types of agents differentiated by their risk tolerances. There are low risk tolerance agent comprising some fraction ω, $(0 < \omega < 1)$, of each cohort of N agents, all of whom have a risk tolerance coefficient of γ_H. It is assumed that the remainder of the young population, the high risk-tolerance agents, are born with one of two possible levels of taste for risk-bearing. We let γ_{L1} (γ_{L2}) be the risk tolerance of the high risk-tolerance agents in state 1 (state 2), where $\gamma_{L1} < \gamma_{L2} < \gamma_H$. State transitions are governed by a symmetric two-state Markov process where the probability of a state change is $(1 - \theta)$.

The share demands are given by substituting the appropriate risk tolerance coefficients in equation (3). Market clearing requires all N shares to be purchased: $N = X_{Lj}\omega N + X_{Hj}(1 - \omega)N$ for $j = 1\,2$. Substitution and simplification yields the following expressions for the state prices:

$$P_j = \left[\bar{P}_j + \bar{d} - \frac{2\gamma_H\gamma_{Lj}}{\omega\gamma_H + (1-\omega)\gamma_{Lj}}\left(\sigma_{Pj}^2 + \sigma_d^2\right)\right] \Big/ \left(1 + R_f\right), \qquad j \in \{1,2\}. \quad (6)$$

Defining $\Gamma_j = \gamma_H\gamma_{Lj}/(\omega\gamma_H + (1-\omega)\gamma_{Lj})$ to be the average, or "representative", risk aversion coefficient in state j highlights the isomorphism between this model and that of the previous section. Equilibrium price expressions are thus identical to those given by equation (5). It is easily shown that $\gamma_{L1} < \gamma_{L2}$ implies $\Gamma_1 < \Gamma_2$, and, consequently, by Proposition 1, that $P_1 > P_2$.[4]

D. Stochastic Proportions of Risk-Tolerant Investors

Now consider an alternative model of stochastic risk tolerance, where each generation again includes investors with high and low levels of risk tolerance, γ_L and γ_H, $\gamma_L < \gamma_H$, but where the *proportion* of agents with high risk tolerance is a random variable characterized by a two-state Markov process. Let the two states of nature be indexed by the proportion of high risk-tolerance (γ_L) agents born in each state, ω_j, where $\omega_1 > \omega_2$. Symmetric transition probabilities are again characterized by θ.

The share demand functions are again given by equation (3), and market clearing here requires that $N = X_{Lj}\omega_j N + X_{Hj}(1 - \omega_j)N$ for $j = 1, 2$. Again, substituting the individual demand functions into the market clearing equations and manipulating yields

$$P_j = \left[\bar{P}_j + \bar{d} - \frac{2\gamma_H\gamma_L}{\omega_j\gamma_H + (1-\omega_j)\gamma_L}\left(\sigma_{Pj}^2 + \sigma_d^2\right)\right] \Big/ \left(1 + R_f\right). \quad (7)$$

By defining $\Gamma_j \equiv \gamma_H\gamma_L/(\omega_j\gamma_H + (1-\omega_j)\gamma_L)$ to be the average, or "representative", risk tolerance coefficient in state j, we again drawn upon the isomorphism with the original model. The representative agent characterization of risk tolerance yields equilibrium price expressions identical to those in (6). In equation (7), $\Gamma_1 < \Gamma_2$ and thus $P_1 > P_2$ both follow from $\omega_1 > \omega_2$ and Proposition 1.

[4]The result that the harmonic mean of investor risk aversion coefficients is a sufficient statistic for market risk aversion is a consequence of mean-variance expected utility (or constant absolute risk aversion with normal returns) as first shown by Lintner (1969) in the CAPM framework.

II. Margin Requirements

A. Initial Margins and Individual Investor Demands

Initial margin requirements for common stock determine the maximum legal collateral value of a marginable security. For example, a margin requirement of 40 percent restricts brokers, banks, and other regulated lenders from lending in excess of 60 percent of the market value of a marginable equity security pledged as collateral. Initial margin requirements thereby limit the extent to which investors can legally leverage their equity portfolios.[5] Since 1974, initial margin requirements for common stock, set by the Board of Governors of the Federal Reserve System, have remained at 50 percent.

When margin requirements bind, it is clearly the more risk-tolerant investors who are most constrained.[6] If margins bind in only one of the two states, then their direct effect will be to lower the effective demand of the constrained agent and thereby reduce the price of the risky asset in that state. Indirectly, margins will also affect the price of the risky asset in the nonbinding state, in part through their effect on price variance and the equilibrium required risk discount. Intuitively, whether margins cause an increase or decrease in volatility depends upon whether they bind upon the risk-tolerant agents in the high- or low-price state. If margins only bind in the high-price state, they might be expected to lower volatility since their direct effect will be lower effective demand, and thus a lower price, in the high-price state. Similarly, if margins bind in the low-price state, they might be expected to raise volatility.

In the following two propositions, we establish predictions concerning the state in which a constant margin requirement is more likely to constrain the type γ_L agents, the agents that take the largest position in each of our heterogeneous-agent models.

PROPOSITION 3: *In the stochastic preferences economy* (*model* 1), *it is always the case that margins are more binding on the type* γ_L *agent in state* 1, *the higher priced state.*

PROPOSITION 4: *In the stochastic proportions economy* (*model* 2), *there is a nontrivial subspace of parameters for which margins are more binding on the type* γ_L *agent in state* 2, *the lower priced state.*

The proof to Proposition 3 amounts to showing that both the risky asset's price and the quantity demanded by a type γ_L investor are higher in state 1. The intuition behind Proposition 4 is that margins are more binding in the

[5]This ignores, of course, futures and options markets, which could be used to change the leverage characteristics of an equity portfolio, markets where initial margins have quite different definitions and functions. Also, because of the single holding period in our formal model, the distinction between maintenance margins are irrelevant.

[6]The ratio of the shares demanded by the less risk-tolerant agent to that of the more risk-tolerant agent is $\gamma_L/\gamma_H < 1$.

low-price state if the risk-tolerant investor's demand is elastic over the range of the equilibrium state prices. Since demand elasticity is increasing in price, the condition is satisfied by parameters yielding equilibrium prices that are not too low, for example, by a dividend variance or an average level of risk aversion that is not too large. The remaining discussion refers to the case where such conditions are satisfied.

Propositions 3 and 4 highlight a critical difference between two alternative economies that display identical representatives agent characterizations and equilibrium prices when unconstrained. In model 1, margins are always more binding in the high-price state, whereas, in model 2, margins may be more binding in the low-price state. Below, this microeconomic asymmetry is shown to give rise to opposing stock price volatility responses when binding margin requirements are imposed on the alternative models.

B. Equilibrium When Margins Bind

Consider a constant margin requirement M large enough to bind (on risk-tolerant agents) in only one sate. In the *stochastic preference economy*, this occurs in state 1, the high-price state, whereas in the *stochastic proportions economy*, this occurs in state 2, the low-price state. In either case, the new margin-constrained state prices are computed by replacing the market-clearing condition in the state when margins bind with an appropriate condition that incorporates the constraint on the less risk-averse agent. For example, in the stochastic preference economy, the quantity of the risky asset bought by a type γ_L agent in state 1 is $1/MP_1$; the new "market-clearing" condition for state 1 is thus

$$1 = \frac{\omega}{MP_1} + (1-\omega)\frac{\overline{P}_1 + d - (1+R_f)P_1}{2\gamma_H(\sigma_P^2 + \sigma_d^2)}. \tag{8}$$

Although no reduced form solution exists (in either economy) when margins bind, a comparison can still be drawn between the margin-constrained equilibrium and equilibrium in the absence of margin requirements.[7] Consider a binding margin in the economy with stochastic tastes (model 1). Construct a hypothetical investor whose demand would just equal that of the constrained (high risk-tolerance) agent's demand at the margin-constrained equilibrium prices. The risk-tolerance coefficient of such an agent in labeled $\hat{\gamma}_{L1}$ and is defined by

$$\hat{X}_1 = \frac{1}{MP_1^*} = \frac{\overline{P}_1^* + \bar{d} - P_1^*(1+R_f)}{2\hat{\gamma}_{L1}(\sigma_P^2 + \sigma_d^2)}, \tag{9}$$

[7]For a wide range of real parameter values there are four numeric solutions to either system—one solution with positive prices, one pair of solutions that are complex conjugates, and a fourth solution that includes a negative price. In such cases, the latter three solutions are inadmissible, leaving a unique equilibrium with positive prices.

where * indicates the margin-constrained equilibrium prices. Clearly, since margins were binding in the original economy, the hypothetical agent must be more risk averse than the true type γ_L investor: $\hat{\gamma}_{L1} > \gamma_{L1}$. If we construct a hypothetical economy which only differs in that the state 1 risk-tolerant agent (type γ_{L1}) is replaced by this hypothetical agent, then equilibrium prices in such an economy must be identical to those of the margin-constrained economy.

The aggregate risk-tolerance coefficient of the hypothetical economy in state 1, $\hat{\Gamma}_1$, is computed as:

$$\hat{\Gamma}_1 = \frac{\gamma_H \hat{\gamma}_{L1}}{\omega \gamma_h + (1 - \omega) \hat{\gamma}_{L1}}, \tag{10}$$

where $\hat{\gamma}_{L1} > \gamma_{L1}$ implies $\hat{\Gamma}_1 > \Gamma_1$. Since margins do not actually bind in the hypothetical formulation, the unconstrained equilibrium price formulas of equations (5a) and (5b) apply, with $(\hat{\Gamma}_1, \Gamma_2)$ being the state 1 and state 2 aggregate risk-tolerance parameters. Thus, a comparison of the (nonconstrained) economies characterized by state risk tolerances (Γ_1, Γ_2) and $(\hat{\Gamma}_1, \Gamma_2)$ yields a comparison of the original economy with and without the binding margin requirements. Since $\Gamma_2 > \hat{\Gamma}_1 > \Gamma_1$, by invoking Proposition 2, it can be concluded that a binding margin requirement in the stochastic preference economy results in lower stock price volatility.

In the stochastic proportions economy, we can analogously define a hypothetical state 2 agent ($\hat{\gamma}_L > \gamma_L$) who demands precisely the quantity purchased by the constrained agent (γ_L) in state 2. The hypothetical economy is constructed with Γ_2 replaced by $\hat{\Gamma}_2$, where

$$\hat{\Gamma}_2 = \frac{2\gamma_H \hat{\gamma}_L}{\omega_2 \gamma_H + (1 - \omega_2) \hat{\gamma}_L}. \tag{11}$$

It is easily shown that $\hat{\gamma}_L > \gamma_L$ implies $\hat{\Gamma}_2 > \Gamma_2 > \Gamma_1$. An argument analogous to that made above can be used to establish that the imposition of a binding margin requirement in model 2 raises stock price volatility.

C. Three-Period Extension

Because investors in our model have only one-period investment horizons, we cannot explicitly examine the role of margin calls on investors who, in a declining market, desire to hold on to previously purchased risky shares. Although our model does not directly examine this so-called "de-pyramiding" effect described, for example, by Kindleberger (1978), it does capture its essence—a constraint on the effective demand for the risky asset.

In order to make the margin call analogy more concrete, imagine a three-cohort, three-period horizon version of our OLG economy in which investors rebalance their portfolios in their second period and consume in the third. Consider a three-period version of the stochastic proportion economy in which, for example, either 0.10 or 0.50 of each generation is of the risk-tolerant type. In such a case, 0.1 or 0.3 or 0.5 of the total investing population—the young and middle-aged—is of the risk-tolerant type.

Suppose this economy was in the state where the young generation has 0.1 and the middle-aged generation has 0.5 risk-tolerant investors, implying that a total of 0.3 of the two actively investing generations are risk-tolerant. This state would be characterized by a price somewhere between the other two state prices (where both generations either have 0.1 or 0.5 of the risk-tolerant type). In this state, it may be the case that the risk-tolerant investors are leveraged, investing more than one dollar in the risky asset.

If, in the following period, the new young generation had only 0.1 of the risk-tolerant investors, then the aggregate proportion of risk-tolerant investors would fall to 0.1; the risky asset price would fall as well. In this world, not only would margins likely bind on the risk-tolerant young (if Proposition 4 conditions are met), but the middle-aged risk tolerant investors might be forced to liquidate some of their original position due to the fall in the value of their collateral—the risky asset. Instead of increasing their position and stabilizing prices in the "bad" state, their forced liquidation would depress stock prices further.

Finally, it should be pointed out that, once multiperiod investment horizons are allowed for, wealth effects potentially become a natural endogenous force behind portfolio rebalancing activities on which margin requirements impinge. For example, if it was assumed that (i) risk aversion of some investors was a function of their wealth, and (ii) current dividends contained information about future dividends, then portfolio rebalancing would arise because of wealth effects associated with information on fundamentals. Intuitively, one might associate the portfolio behavior of wealth-sensitive investors with that of "portfolio insurers".[8]

In this world, again, one might expect that margins could enhance or reduce volatility. Here, it would depend upon whether they were more binding on constant-risk-aversion investors in the low-price (low-dividend) state, or portfolio insurers in the high-price (high-dividend) state. In the former case margins would raise volatility, whereas in the latter they would lower it.

III. A Noise Trader Interpretation

An alternative approach to modeling heterogeneity and the effects of margin requirements focuses on agents' subjective beliefs.[9] Similar to the previous analysis, we construct two alternative models of time-variation in heterogeneous beliefs. In each model, we assume the existence of both "rational" agents, who know the (constant) mathematical expectation of

[8]This example was suggested by our referee. The behavior of equilibrium asset prices in an economy where investors have a utility function that gives rise to portfolio insurance-like behavior is studied by Marcus (1989). The utility function adopted there is characterized by a subsistence level of consumption; as the subsistence level approaches zero, the utility function approaches the constant relative risk aversion (CRRA) case.

[9]This type of heterogeneity underlies not only the "noise trader" analysis of De Long, Shleifer, Summers, and Waldman (1989), it is also the focus of a quite general analysis of the equilibrium asset pricing implications of heterogeneity by Abel (1989).

dividends, and "noise traders", who have either "optimistic' or "pessimistic" expectations. First, we propose a "stochastic beliefs" economy, where all noise traders of a given cohort have identical beliefs that evolve stochastically across generations. We then analyze a "stochastic proportions" economy, where each cohort includes rational investors as well as both kinds of noise traders but where the proportion of those that are optimistic changes over time. These characterizations are used to show that, as in the original framework, binding margin requirements can either lower or raise volatility.

A. Stochastic but Homogenous Noise Trader Beliefs

As before, we assume there exists 1 unit of the risky asset with a dividend characterized by constant mean and variance $(\bar{d}, \sigma_d^2)$. There are now assumed to be ω rational agents (R) and $(1 - \omega)$ noise traders (N). Noise traders in a given generation believe that the dividend process is characterized by mean and variance $(\bar{d} + \varepsilon_j, \sigma_d^2)$, $j \in \{1, 2\}$, $\varepsilon_1 > 0 > \varepsilon_2$. In state 1, noise traders are all optimists (ε_1), whereas, in state 2, they are all pessimists (ε_2). Similar to the original model, transition probabilities between states (of noise trader beliefs) are characterized by θ.

The demands of rational and noise traders, each endowed with a unit of wealth, are calculated as before and denoted by X_{Rj} and X_{Nj}, where

$$X_{Rj} = \frac{\bar{P}_j + \bar{d} - P_j(1 + R_f)}{2\Gamma(\sigma_P^2 + \sigma_d^2)} \tag{12a}$$

and

$$X_{Nj} = \frac{\bar{P}_j + \bar{d} + \varepsilon_j - P_j(1 + R_f)}{2\Gamma(\sigma_P^2 + \sigma_d^2)} \tag{12b}$$

for $j = (1, 2)$. Again, price variance is identical across states.

Imposing the market clearing conditions, the state contingent equilibrium prices are:

$$P_j = \frac{\bar{d} - 2\Gamma(\sigma_P^2 + \sigma_d^2)}{R_f} + \frac{(1 - \omega)(1 - \theta)(\varepsilon_1 + \varepsilon_2)}{R_f[2(1 - \theta) + R_f]} + \frac{(1 - \omega)\varepsilon_j R_f}{R_f[2(1 - \theta) + R_f]} \tag{13}$$

for $j = (1, 2)$. The first term is common to both prices and is equal to the price in a world with no noise traders (with $\sigma_P^2 = 0$). The second term, also identical across states, in a function of the bias in noise traders' unconditional mean beliefs; this term is zero if $\varepsilon_1 = -\varepsilon_2$. The third term is positive in state 1 and negative in state 2. The difference between state prices is:

$$P_1 - P_2 = \frac{(1 - \omega)(\varepsilon_1 - \varepsilon_2)}{2(1 - \theta) + R_f} > 0. \tag{14}$$

Clearly, the larger the percentage of investors that are noise traders (the smaller is ω), the greater is the difference between state prices.

B. Stochastic Proportions of Optimistic Noise Traders

This economy is similar to the one above except that there exist both optimists and pessimists in each state. Noise trader dividend expectations are characterized by

$$\text{optimists:}\ \tilde{d} \sim \left(\bar{d} + \varepsilon_O, \sigma_d^2\right) \tag{15a}$$

and

$$\text{pessimists:}\ \tilde{d} \sim \left(\bar{d} + \varepsilon_P, \sigma_d^2\right), \tag{15b}$$

where $\varepsilon_O > 0 > \varepsilon_P$. For simplicity, assume that the total proportion of noise traders per generation is $1/2$. Let ω_1 and ω_2, where $\omega_1 > \omega_2$ be the fraction of noise traders that are optimists in states 1 and 2, respectively. Once the correspondence between ε_O (ε_P) and ε_1 (ε_2) in equation (12b) is noted, it should be clear that investor demand functions again are given by (12a) and (12b). Here (12b) represents individual demands of optimists (X_{Oj}) and pessimists (X_{Pj}) populating the young cohort in state j.

Again, the market-clearing conditions can be used to calculate equilibrium prices, the difference between which can be written as

$$P_1 - P_2 = \frac{(\omega_1 - \omega_2)(\varepsilon_O - \varepsilon_P)}{2(1-\theta) + R_f} > 0. \tag{16}$$

Clearly, the greater the discrepancy between the proportion of optimists (ω_j) in the two states, the greater is the difference between state prices.

C. The Impact of Margin Requirements

It is straightforward to show and intuitively clear that, in the stochastic beliefs economy, margins are more binding on noise traders when they are optimistic (state 1) that when they are pessimistic (state 2). This is a consequence of the fact that both the equilibrium price and the individual noise trader's quantity demanded are higher in state 1 than state 2.

As will be stated and proved in Proposition 5, as long as there are not "too many" noise traders in a given generation, the nominal demand of a noise trader in state 1 is greater than that of a rational traders in state 2. Intuitively, if there are enough rational traders to take up the slack in the pessimistic state, then margins will constrain optimistic noise traders before they constrain rational traders. Consequently, under such conditions, margin requirements will be more binding overall in state 1 than in state 2. When binding, margin requirements lower volatility in this economy.

PROPOSITION 5: *In the stochastic beliefs economy, margins are more binding on noise traders in state* 1, *the high prices state, if* $P_1 X_{N1} > P_2 X_{N2}$. *A condition sufficient to guarantee satisfaction of this inequality is* $\varepsilon_1/(-\varepsilon_2) > (1-\omega)/\omega$. *If noise trader beliefs are symmetric,* $\varepsilon_1 = -\varepsilon_2$, *this reduces to* $\omega > 1/2$.

In contrast, in the "stochastic proportions" version of the noise trader model it can easily be shown that, under fairly weak conditions on the parameters, margins are more binding on optimistic noise traders (and rational traders) in state 2, the low-price state.

PROPOSITION 6: *In the stochastic proportions model, margins are more binding on optimistic noise trader investors in state 2 if* $P_1 X_{O1} < P_2 X_{O2}$; *that is, if demand is elastic over the range of the equilibrium prices. This condition is satisfied if and only if* $(P_1 + P_2)/2 < (\bar{d} + \varepsilon_O)/2(1 - \theta + R_f)$.

The condition under which margins are also more binding on rational agents in the low-price state is derived by setting $\varepsilon_O = 0$ in the condition above. As in Proposition 4, such conditions are satisfied when the variance of returns is not too larger relative to investor risk aversion. This will be the case, in particular, as long as noise trader influence, characterized by $(1 - \omega)$ and $(\varepsilon_O - \varepsilon_P)$, is not too large relative to risk aversion levels.

When such conditions hold, margins, when binding, raise volatility. By constraining optimistic, and possibly rational, traders from buying up the risky assets when excessive pessimism abounds in the aggregate, binding margin requirements further depress stock prices in the low-price state, thereby increasing volatility.

IV. Conclusion

In this paper we have established within an equilibrium framework that the imposition of a binding margin requirement can either increase or decrease the price volatility of the risky asset to which the requirement applies. The realized effects of a binding margin requirement depend critically upon the microstructure of investor heterogeneity in the economy, and cannot be predicted *ex ante* from the economy's representative agent characterization. This implication, that the impact of margin requirements cannot be predicted based upon the observation of aggregate variables prior to imposition, is suggestive of the dangers implicit in using representative agent models. Uncovering the "deep parameters" of a representative agent model may be insufficient or even useless for macrofinancial policy analysis.

The results highlight the difficulties inherent in formulating a real-world margin policy designed with the intent of reducing stock-price volatility. Even in our comparatively simple models, slight variations in the source "excess" volatility in the risky asset's returns generates uncertainty concerning the basic qualitative impact of margin requirements. In the real world, the sources of volatility are not only likely to be more complex, but also different at different times. If variations in investors' preferences and expectations were identifiable on a timely basis, these data could, perhaps, be used to guide the selective use of margin requirements for moderating volatility. Although our results may be suggestive in this regard, it would, of course, be unwise to formulate policy based upon the implications of a model that abstracts from complexities of real-world financial markets such as limited liability and bankruptcy.

Appendix

Proof, Proposition 1: From equations (5a) and (5b), compute

$$P_1 - P_2 = \frac{\left[\sqrt{A} - \left(1 - \theta + R_f/2\right)\right]}{2(\theta - \theta^2)(\Gamma_1 - \Gamma_2)}.$$

Since $\Gamma_1 < \Gamma_2$, and $\theta \in (0,1)$, $P_1 > P_2$ if $[\sqrt{A} - (1 - \theta) + R_f/2)] < 0$. Let $[\sqrt{A} - (1 - \theta + R_f/2)] \equiv G(\sigma_d^2)$. Notice that $\lim_{\sigma_d^2 \to 0} G(\sigma_d^2) = 0$, $G(\sigma_d^2)$ is continuous in σ_d^2. Finally, $\delta G(\sigma_d^2)/\delta\sigma_d^2 < 0$ for allowable σ_d^2, that is for σ_d^2 small enough such that a real solution exists ($A \geq 0$). Thus $G(\sigma_d^2) < 0$ for all σ_d^2 in the relevant range.[10] Q.E.D.

Proof, Proposition 2: The first part is verified by computing $\delta(P_1 - P_2)/\delta(\Gamma_1 - \Gamma_2)$; the second is shown by application of L'Hopital's rule and the formula for price variance.

Proof, Proposition 3: Since $P_1 > P_2$, it is sufficient to show that $X_{L1} > X_{L2}$ or that the risk-tolerant agent purchases more shares in state 1 than state 2. Because the proportion of agents of type γ_L is identical in the two states, this is equivalent to demonstrating that each of the type γ_H agents purchase fewer shares in state 1. Using the equation for demand (3), it is easy to show that $X_{H1} < X_{H2}$ for $R_f > 0$.

Proof, Proposition 4: The proof amounts to establishing a feasible restriction on the parameters under which $P_2 X_{L2} > P_1 X_{L1}$—the type L agent desires to invest more (in nominal terms) in states 2 than in state 1. Substitution of the expressions for demand into the inequality and algebraic manipulation yields a restatement of this inequality: $P_1 + P_2 > \bar{d}/(1 - \theta + R_f)$.

Summing the equilibrium price expressions (5a and 5b) yields:

$$P_1 + P_2 = \frac{2\bar{d}}{R_f} + \frac{(\Gamma_1 + \Gamma_2)\left[4(1 - \theta) + 2R_f\right]}{4(\theta - \theta^2)(\Gamma_1 - \Gamma_2)} \cdot G(\sigma_d^2),$$

where (as shown in Proposition 1) $G(\sigma_d^2) < 0$ and $G(\sigma_d^2) \to 0$ as $\sigma_d^2 \to 0$. Clearly, for σ_d^2 close to zero, $P_1 + P_2 \approx 2\bar{d}/R_f > \bar{d}/(1 - \theta + R_f)$ for all $0 < \theta < 1$, $R_f > 0$, and any $\{\Gamma_1, \Gamma_2\}$ in the relevant range. To satisfy the constraint, the larger σ_d^2, the smaller must be the economy's unconditional risk-tolerance coefficient $(\Gamma_1 + \Gamma_2)/2$, holding other parameters constant.[11]

[10] Other parameters given, for σ_d^2 too large, 'A' becomes negative and the equilibrium price solutions implied by equation (6) are no longer real-valued.

[11] In model simulation experiments the inequality was not violated for any of the parameters models we tested. This includes the examples constructed in the working paper, where σ_d^d appears quite large relative to equilibrium prices.

Proof, Proposition 5: Rearranging the inequality and subtracting unity from both sides yields:

$$\frac{P_1 - P_2}{P_2} > \frac{X_{R2} - X_{N1}}{X_{N1}}.$$

Prices, demand, and $P_1 - P_2$ are positive, so a sufficient condition for the desired inequality is that $X_{R2} - X_{N1}$ be less than or equal to 0. Substituting the equilibrium prices P_1 and P_2 give by equation (13) into the demand functions, it is easily shown that

$$X_{R2} - X_{N1} < 0 \qquad \text{if } \varepsilon_1 > (1 - \omega)(-\varepsilon_2)/\omega.$$

If the noise traders' expectation errors are symmetric ($\varepsilon_1 = -\varepsilon_2$), then the condition simplifies to $\omega > 1/2$.

Proof, Proposition 6: The condition is equivalent to:

$$P_1 X_{O1} - P_2 X_{O2} = (P_1 - P_2)\{(\theta - 1 - R_f)(P_1 + P_2) + \bar{d} + \varepsilon_o\} \times \{2\Gamma(\sigma_P^2 + \sigma_d^2)\}^{-1} < 0.$$

$P_1 > P_2$ implies $P_1 X_{O1} < P_2 X_{O2}$ if $P_1 + P_2 > (\bar{d} + \varepsilon_O)/(1 - \theta + R_f)$.

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