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Arbitrage Asset Pricing under Exchange Risk

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ABSTRACT

This paper extends the APT to an international setting. Specifying a linear factor return-generating model in local currency terms, we show that the usual risk-diversification rule in the APT does not yield a riskless portfolio unless currency fluctuations obey the same factor model as asset returns. We then consider an arbitrage portfolio whose exchange risk is hedged by foreign riskless bonds. Under the resulting no-arbitrage conditions, the expected returns are not on the same hyperplane, unlike the closed-economy APT, unless they are adjusted by the cost of exchange risk hedging.

IN ANALYSES OF INTERNATIONAL economic phenomena, exchange risk is one of the most important elements to be considered. This paper studies arbitrage asset pricing in an international setting and shows how the introduction of exchange risk changes the Arbitrage Pricing Theory (APT) formulated in closed-economy models by Ross (1976, 1977), Huberman (1982), Ingersoll (1984), and others.

The extension of the APT to an international framework (IAPT) is successfully undertaken by Solnik (1983). He proves by straightforward application of the APT that, if the asset returns measured in an arbitrarily given numeraire currency jointly follow a linear factor model, then, in the absence of arbitrage, the vector of expected returns in a given currency is spanned by the vector of ones and factor loading vectors.[1] But this direct applicability of the APT to an international setting is somewhat counterintuitive since it is natural to conjecture that exchange risk will introduce a new element into arbitrage activities. The key assumption in this puzzle is that the return-generating process is specified in a numeraire currency. It can easily be seen that under this specification the exchange risk of asset returns is automati-

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[1]The same conclusion is reached by Ross and Walsh (1982). Levine (1989) generalizes Solnik's result in an inflationary model with purchasing power parity deviations. See also Kleidon and Pfleiderer (1983) for a discussion of the Solnik IAPT.

cally diversified when one constructs an arbitrage portfolio according to the rule used in the APT. This enables Solnik to apply the closed-economy APT straightforwardly to international asset pricing.

Instead, we set up a linear factor return-generating process in local currency terms in Section I to emphasize the effect of exchange risk on international arbitrage asset pricing. We first show that the usual risk-diversification rule in the APT does not yield a risk-free portfolio in this setting if exchange rate fluctuations do not have the same factor structure as asset returns. Our main analysis presents a way of constructing an arbitrage portfolio hedged against exchange risk and verifies that expected returns are not on the same hyperplane unless they are adjusted by the cost of exchange risk hedging. Given this result, we also clarify the arbitrage determination of covariances between asset returns and exchange rate fluctuations. The results obtained are compared with Solnik's IAPT in Section II.

I. Arbitrage Asset Pricing with Exchange Risk Hedging

A. A Linear Factor Model with Exchange Risk

We consider a world with N countries indexed 1 to N. In each country, there exist one risky asset and one locally riskless national bond which are freely traded in perfect international capital markets. The risky assets, as well as the national bonds, are assumed to be denominated in their respective local currencies. Given this assumption, we specify the generating process of the risky asset returns as the familiar linear K-factor model in local currency terms:

$$\tilde{r}_i^i = \bar{r}_i^i + b_{i1}\tilde{f}_1 + \cdots + b_{iK}\tilde{f}_K + \tilde{\varepsilon}_i \qquad (i = 1, \ldots, N). \tag{1}$$

In the equation above, $\tilde{r}_i^i$ is the random return on the i-th country risky asset in terms of the local currency, $\bar{r}_i^i$ denotes its expected value, the $\tilde{f}_k$ values ($k = 1, \ldots, K$) represent international common factors with zero means, b_{ik} denotes the sensitivity of return $\tilde{r}_i^i$ to fluctuations in factor k, and $\tilde{\varepsilon}_i$ is a nonsystematic risk component with zero mean, bounded variance, and $E[\tilde{\varepsilon}_i \mid \tilde{f}_k] = 0$ for all i and k.[2] As usual in the APT, the number of risky assets N is assumed to be large enough to permit the law of large numbers to hold.[3]

We assume flexible foreign exchange rates and denote their random processes by the general form:

$$\tilde{s}_i^j = \bar{s}_i^j + \tilde{\delta}_i^j \qquad (i, j = 1, \ldots, N), \tag{2}$$

where $\tilde{s}_i^j$ represents the random rate of appreciation of country i's currency in terms of country j's. Values $\bar{s}_i^j$ and $\tilde{\delta}_i^j$ denote the expected and random

[2] The $\tilde{\varepsilon}_i$ values can be correlated with each other.

[3] Since by assumption there is only one risky asset in each country, this means that the number of countries is large. If there are many risky assets in each country, we can easily consider an alternative world in which there are only a few countries without any change in our main results (below). This point will be discussed at the end of Section II.

parts of currency variation $\tilde{s}_i^j$, respectively. By definition, $E[\tilde{\delta}_i^j] = 0$ for all i and j. Trivially, when $i = j$, $\tilde{s}_i^j$, $\bar{s}_i^j$, and $\tilde{\delta}_i^j$ are all equal to zero.

Here, in contrast to Solnik (1983), we do not assume that currency fluctuations have the same factor structure as equation (1). In order to explicitly demonstrate the implication of not making this assumption, consider risky asset returns from the viewpoint of a given currency, say currency n. If we define P_i^j and S_i^j as the currency j prices of asset i (the risky asset of country i) and currency i, respectively, then the price of asset i measured in terms of currency n is given by the law of one price as $P_i^n = P_i^i S_i^n$. Thus, by application of Ito's lemma, one can compute the currency n return on the asset as:[4]

$$\tilde{r}_i^n = \tilde{r}_i^i + \tilde{s}_i^n + \mathrm{Cov}\left(\tilde{r}_i^i, \tilde{s}_i^n\right) \qquad (i = 1, \ldots, N), \tag{3}$$

where $\mathrm{Cov}(\cdot\,, \cdot)$ denotes covariance. Substitution of equations (1) and (2) (where $j = n$) into (3) yields:

$$\tilde{r}_i^n = \bar{r}_i^n + b_{i1}\tilde{f}_1 + \cdots + b_{iK}\tilde{f}_K + \tilde{\varepsilon}_i + \tilde{\delta}_i^n \qquad (i = 1, \ldots, N), \tag{4}$$

where the expected return $\bar{r}_i^n$ is:

$$\bar{r}_i^n = \bar{r}_i^i + \bar{s}_i^n + \mathrm{Cov}\left(\tilde{r}_i^i, \tilde{s}_i^n\right). \tag{5}$$

The last term on the right hand side of equation (4), $\tilde{\delta}_i^n$, represents the exchange risk for country n's investors. If this term were assumed to be characterized by the same K-factor model as equation (1), then by substituting it for $\tilde{\delta}_i^n$ in (4), the asset returns measured in the numeraire currency n could be rewritten as being generated by the K-factor model as well.[5] Thus, the usual APT (in a closed economy setting) could be applied to that case as in the Solnik model. Empirically, however, it might be difficult to extract international common factors simultaneously, demonstrating both the price variations of risky assets (e.g., stocks) and currency fluctuations while keeping residual or idiosyncratic risk small. This matters because, as shown by Huberman (1982) and Ingersoll (1984), an increase in idiosyncratic risk worsens the fit of the APT equations.[6] Our formulation avoids this difficulty.

In our setting, however, it is impossible to construct riskless portfolio in the same manner as the usual APT because the exchange risk in equation (4), $\tilde{\delta}_i^n$, depends on the asset index i and, at the same time, is undiversifiable, unlike residual risk $\tilde{\varepsilon}_i$. To see this, construct a portfolio from N risky assets denoted by $\omega = (\omega_1, \ldots, \omega_N)'$ with ω_i being the investment proportions,

[4]Following Solnik (1983), we define asset returns over a short interval of time and assume that the conditions required for Ito's calculus are met. See Roll and Ross (1980) and Ikeda (1988) for the continuous time APT.

[5]This case will be discussed explicitly in Section II.

[6]See Cho, Eun, and Senbet (1986) for empirical research of the Solnik IAPT.

according to the usual rule:

$$\left.\begin{aligned} \omega' b_1 &= 0, \\ &\vdots \\ \omega' b_K &= 0, \end{aligned}\right\} \tag{6}$$

$$\omega' \tilde{\varepsilon} \simeq 0, \tag{7}$$

where $b_k = (b_{1k}, \ldots, b_{Nk})'$ and $\tilde{\varepsilon} = (\tilde{\varepsilon}_1, \ldots, \tilde{\varepsilon}_N)'$. Then, the currency n return on the portfolio can be computed from equations (4), (6), and (7) as:

$$\begin{aligned} \omega' \tilde{r}^n &= \omega' \bar{r}^n + \omega' b_1 \tilde{f}_1 + \cdots + \omega' b_K \tilde{f}_K + \omega' \tilde{\varepsilon} + \omega' \tilde{\delta}^n \\ &= \omega' \bar{r}^n + \omega' \tilde{\delta}^n, \end{aligned} \tag{8}$$

where $\tilde{r}^n = (\tilde{r}_1^n, \ldots, \tilde{r}_N^n)'$, $\bar{r}^n = (\bar{r}_1^n, \ldots, \bar{r}_N^n)'$, and $\tilde{\delta}^n = (\tilde{\delta}_1^n, \ldots, \tilde{\delta}_N^n)'$. Equation (8) reveals that the simple hedging rule given by (6) and (7) does not yield a risk-free portfolio since the exchange risk $\omega' \tilde{\delta}^n$ would be left undiversified by this rule. The next subsection confronts this problem by considering an arbitrage portfolio which hedges against exchange risk by means of foreign lending and borrowing at locally riskless rates.

B. Arbitrage Pricing of Hedged Assets

Consider a portfolio of $2N$ assets, $\theta = (\theta_1, \ldots, \theta_{2N})'$, proportion θ_i ($i = 1, \ldots, N$) of which is invested in risky asset i and proportion θ_{N+i} of which is invested in the national bond of country i. Based on the portfolio defined by equations (6) and (7), we now specify portfolio θ as:

$$\theta_i = \omega_i \qquad (i = 1, \ldots, N), \tag{9}$$

$$\theta_{N+i} = -\omega_i \qquad (i = 1, \ldots, N). \tag{10}$$

The resulting bundle is an arbitrage portfolio since equations (9) and (10) imply that investments in the risky assets are fully financed by holding the opposite positions of national bonds.

At the same time the following two facts imply that portfolio θ is riskless. First, as can be seen from equations (6), (7), and (9), the risk from common factors $\tilde{f}_k$ and residual factors $\tilde{\varepsilon}_i$ is diversified, and second, the investment in country i's risky asset is protected against exchange risk by opposite trading in the national bond of the same country. Explicitly, if $\tilde{\rho}_i^j$ denotes the currency j return on the national bond of country i (so that all of the ρ_i^i values are deterministic), return $\tilde{\rho}_i^n$ can be computed in the same way used to derive (3):

$$\tilde{\rho}_i^n = \rho_i^i + \bar{s}_i^n + \tilde{\delta}_i^n \qquad (i = 1, \ldots, N). \tag{11}$$

Thus, setting $\tilde{\rho}^n = (\tilde{\rho}_1^n, \ldots, \tilde{\rho}_N^n)'$, $\rho = (\rho_1^1, \ldots, \rho_N^N)'$, and $\bar{s}^n = (\bar{s}_1^n, \ldots, \bar{s}_N^n)'$, the currency n return on portfolio θ, $\tilde{r}_\theta^n$, reduces to:

$$\begin{aligned}\tilde{r}_\theta^n &= \sum_{i=1}^{N} \theta_i \tilde{r}_i^n + \sum_{i=1}^{N} \theta_{N+i} \tilde{\rho}_i^n \\ &= \omega' \tilde{r}^n - \omega' \tilde{\rho}^n \qquad \text{(by (9) and (10))} \\ &= \omega'(\bar{r}^n - \rho - \bar{s}^n). \qquad \text{(by (8) and (11))} \end{aligned} \tag{12}$$

This verifies that the return measured in currency n is free from any risk.[7]

Now that a riskless arbitrage portfolio has been obtained from the viewpoint of currency n, we can use the same algebraic argument as the APT to determine risk premia for country n's investors. For free lunches to be absent, the currency n return on portfolio θ must be equal to zero. Explicitly, from equation (12), the following equation must be valid:

$$\omega'(\bar{r}^n - \rho - \bar{s}^n) = 0. \tag{13}$$

Given the portfolio formation rule of equation (6), the condition of equation (13) requires that the expected net return vector $(\bar{r}^n - \rho - \bar{s}^n)$ be spanned by factor loading vectors b_k $(k = 1, \dots, K)$ for some parameters $\lambda_1^n, \dots, \lambda_K^n$:

$$\bar{r}^n - \rho - \bar{s}^n = \lambda_1^n b_1 + \cdots + \lambda_K^n b_K.$$

Note here that portfolio θ retains the property of a riskless arbitrage portfolio even if its return is evaluated in any currency other than currency n. It follows that the above discussion is applicable to excluding arbitrage opportunities in terms of any currency. This leads to our main result:

THEOREM: *Suppose that risky asset returns and currency fluctuations are generated by processes* (1) *and* (2), *respectively. Then, if no arbitrage opportunities are left unexploited in terms of any currency, there exist N sets of K scalars,* $\lambda_1^j, \dots, \lambda_K^j$, *such that*:

$$\bar{r}^j - \rho - \bar{s}^j = \lambda_1^j b_1 + \cdots + \lambda_K^j b_K \qquad (j = 1, \dots, N). \tag{14}$$

The left hand side of equation (14) denotes the expected net returns in terms of currency j on risky assets whose exchange risk is hedged by foreign borrowing and lending.[8] On the other hand, given the local currency specification of the return-generating process (1), it is natural to regard b_{ik} as a measure of local factor risk, i.e., factor risk in local currency terms. Thus, the theorem asserts that, in each country or currency, there are K local factor risk prices λ_k^j, which determine the expected net returns on hedged assets.

[7]Exchange risk hedging by holding locally riskless bonds is also proposed in utility-based International Asset Pricing Models (IAPM). See, for example, Solnik (1973, 1974), Stulz (1981), and Adler and Dumas (1983).

[8]To define the concept of exchange risk hedging used here more precisely, it is one in which the resulting excess return on an asset is affected by currency fluctuations only insofar as they affect the local currency return on the asset. Thus, these excess returns do not correspond to returns hedged against currency risk in a minimum-variance sense because the covariance of excess returns with currency fluctuations is not zero.

Put otherwise, the expected returns themselves are not on the same hyperplane, unlike the closed-economy APT, unless they are adjusted by the cost of exchange risk hedging represented by $\rho + \bar{s}^j$.

Formally, if variables $\bar{s}_i^j$, ρ_i^i, and λ_k^j are given, the no-arbitrage conditions of equation (14) yield N^2 expected returns in terms of different currencies $\bar{r}_i^j$. To see the mechanism from another point of view, let us substitute equation (5) (setting $n = j$) for $\bar{r}_i^j$ in the i-th row of (14). We obtain:

$$\bar{r}_i^i + \mathrm{Cov}\left(\tilde{r}_i^i, \tilde{s}_i^j\right) - \rho_i^i = \lambda_1^j b_{i1} + \cdots + \lambda_K^j b_{iK}. \tag{15}$$

This provides an alternative expression showing that N^2 no-arbitrage conditions yield N expected values of the risky assets' returns in terms of respective local currencies and $N\ (N - 1)$ covariances between the risky assets' returns and currency fluctuations.

The arbitrage determination of asset-currency covariances can be shown as follows. First, take the difference between the no-arbitrage conditions given by equation (15) for two different currencies, say currencies j and l. Next, note that $\mathrm{Cov}(\tilde{r}_i^i, \tilde{s}_i^j) - \mathrm{Cov}(\tilde{r}_i^i, \tilde{s}_i^l) = \mathrm{Cov}(\tilde{r}_i^i, \tilde{s}_l^j)$. Then, we obtain our second proposition:

COROLLARY: *Suppose that the conditions in the theorem hold. Then, in the absence of arbitrage, the covariances between local currency returns on risky assets and exchange rate fluctuations are given by*:

$$\mathrm{Cov}\left(\tilde{r}, \tilde{s}_l^j\right) = \lambda_1^{jl} b_1 + \cdots + \lambda_K^{jl} b_K \qquad \left(j, l = 1, \cdots, N\right). \tag{16}$$

where $\mathrm{Cov}(\tilde{r}, \tilde{s}_l^j) = (\mathrm{Cov}(\tilde{r}_1^1, \tilde{s}_l^j), \ldots, \mathrm{Cov}(\tilde{r}_N^N, \tilde{s}_l^j))'$ *and* $\lambda_k^{jl} = \lambda_k^j - \lambda_k^l$.

This shows that covariance vector $\mathrm{Cov}(\tilde{r}, \tilde{s}_l^j)$ is also spanned by factor loadings b_k. But here the weights applied are risk price differences λ_k^{jl}. A close look at the derivation reveals the reason for this. By definition, covariance $\mathrm{Cov}(\tilde{r}_i^i, \tilde{s}_l^j)$ equals the difference between the expected net return on a hedged investment in risky asset i in terms of currency j, $\bar{r}_i^i + \mathrm{Cov}(\tilde{r}_i^i, \tilde{s}_i^j) - \rho_i^i$, and that in terms of currency l, $\bar{r}_i^i + \mathrm{Cov}(\tilde{r}_i^i, \tilde{s}_i^l) - \rho_i^i$. From equation (14), on the other hand, it is international discrepancies in factor risk prices that cause cross-currency differences in expected net returns on hedged investments. It follows that the asset-currency covariance $\mathrm{Cov}(\tilde{r}_i^i, \tilde{s}_l^j)$ must be determined so as to precisely match the effect of cross-currency differences in factor risk prices.

II. A Comparison with Solnik's IAPT

To clarify the implications of our results, we finally compare our discussion with that of Solnik (1983). He specifies the return-generating process in a numeraire currency which is arbitrarily chosen. As pointed out in Section I.A, his analysis can be replicated in our setting by additionally assuming that currency fluctuations have the same factor structure as the asset returns given by equation (1). For example, let currency variation $\tilde{\delta}_i^n$ be generated by:

$$\tilde{\delta}_i^n = c_{i1}^n \tilde{f}_1 + \cdots + c_{iK}^n \tilde{f}_K + \tilde{\mu}_i^n \qquad (i = 1, \ldots, N,\ i \neq n), \tag{17}$$

where c_{ik}^n denotes the factor loading and $\tilde{\mu}_i^n$ represents an idiosyncratic risk component with zero mean, bounded variance, and $E[\tilde{\mu}_i^n \mid \tilde{f}_k] = 0$ $(k = 1, \ldots, K)$. Then, equation (4) reduces to:

$$\tilde{r}_i^n = \bar{r}_i^n + b_{i1}^n \tilde{f}_1 + \cdots + b_{iK}^n \tilde{f}_K + \tilde{\varepsilon}_i^n, \tag{18}$$

where $b_{ik}^n = b_{ik} + c_{ik}^n$ and $\tilde{\varepsilon}_i^n = \tilde{\varepsilon}_i + \tilde{\mu}_i^n$.

This means that the returns in terms of numeraire currency n follow a linear K-factor model, which is the situation Solnik supposes. As he proves, this factor structure is invariant to the currency chosen if the same factor model as equation (17) holds for all currencies. Applying the usual arbitrage argument to the model, we then obtain Solnik's result:

$$\bar{r}^j - \rho_j^j 1 = \pi_1^j b_1^j + \cdots + \pi_K^j b_K^j \qquad (j = 1, \ldots, N), \tag{19}$$

where 1 is an N-dimensional vector of ones, the π_k^j values represent factor risk prices, and the b_k^j values are factor loading vectors. Equation (19) shows that if expected returns are measured in the same currency, all of them are on the same hyperplane as in the case of the closed-economy APT despite the existence of exchange risk.

Equation (18) (and so (17)) plays a key role in producing this result. In this setting, the factors $\tilde{f}_k$ also generate currency fluctuations. Naturally, the factor loadings b_{ik}^j $(= b_{ik} + c_{ik}^j)$ can be regarded as measures of total or composite factor risk, i.e., factor risk in local currency terms plus exchange risk and parameter π_k^j in (19) as the prices of total factor risk including the price of exchange risk. In this sense, Solnik demonstrates international arbitrage pricing in terms of *total* factor risk prices.

In contrast, our model (in the previous section) considers a case in which one cannot successfully extract common factors simultaneously generating asset returns and currency fluctuations. Our theorem avoids this difficulty by explaining arbitrage pricing of hedged securities in terms of *local* factor risk prices.

We can also derive Solnik's result concerning asset-currency covariances from equation (19) in a way similar to that used to obtain (16):[9]

$$\mathrm{Cov}(\tilde{r}^j, \tilde{s}_l^j) = \pi_1^{jl} b_1^j + \cdots + \pi_K^{jl} b_K^j \qquad (j, l = 1, \ldots, N), \tag{20}$$

where $\mathrm{Cov}(\tilde{r}^j, \tilde{s}_l^j) = (\mathrm{Cov}(\tilde{r}_1^j, \tilde{s}_l^j), \ldots, \mathrm{Cov}(\tilde{r}_N^j, \tilde{s}_l^j))'$ and $\pi_k^{jl} = \pi_k^j - \pi_k^l$.

[9]As Solnik (1983) shows, returns on locally riskless bonds are also spanned in the same way as (19):

$$\rho_j^j + \bar{s}_j^l = \rho_l^l + \pi_1^l c_{j1}^l + \cdots + \pi_K^l c_{jK}^l.$$

This relation is used to derive equation (20).

This shows that the covariances between asset returns measured in a given currency, rather than respective local currencies, and currency fluctuations are on the same hyperplane. Again, the result depends crucially on the specification of the return-generating process in terms of a numeraire currency. In this case, factor loadings are measures of *total* factor risk, and therefore the set of K factor loadings measures the risk of return variations (correlated with currency fluctuations) in *numeraire* currency terms. In contrast, according to the result in our corollary, if the return-generating process is specified in local currency terms, arbitrage ensures a similar linear relation between the covariance of the *local* currency return on a risky asset with currency variations and factor loadings b_{ik}, which are measures of *local* factor risk.

Our model can easily be extended to the case in which there is more than one risky asset denominated in each currency. Such an extension incurs the marginal cost of notational complexity (so that we do not treat this case explicitly) but yields the marginal benefit of making it possible, in the absence of a locally riskless bond, to conduct the analysis using a zero beta portfolio constructed from risky assets of the same nationality. Although the results are essentially the same as presented here, one additional result is that, in this case, the returns on risky assets of the same nationality are, if they are large in number, spanned in the same way as in the usual APT. This result corresponds to the aforementioned result of Solnik. This is not surprising because the return process of assets of the same nationality satisfies his assumption concerning asset returns.

III. Conclusions

This paper has pursued an international extension of the APT. In order to focus on the effect of exchange risk on arbitrage asset pricing, we have specified a linear factor model in local currency terms (rather than numeraire currency terms). In this setting, the usual risk-diversification rule in the APT does not yield a risk-free portfolio because the exchange risk of an asset is undiversifiable and at the same time varies depending on the nationality of the asset. Confronted with this problem, we have considered an arbitrage portfolio which hedges against exchange risk by means of foreign lending and borrowing at locally riskless rates. The resulting no-arbitrage conditions require that the expected net returns on risky assets whose exchange risk is covered by foreign borrowing and lending be determined as linear combinations of factor loadings. This means that the expected returns themselves are not on the same hyperplane unless they are adjusted by the cost of exchange risk hedging.

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