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Corporate Risk Management and the Incentive Effects of Debt

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ABSTRACT

This paper demonstrates how the incentive of manager-equityholders to substitute toward riskier assets, commonly referred to as the "asset substitution problem," is related to the level of observable risk in the firm. When observable and unobservable risks are sufficiently positively correlated, increases (decreases) in observable risk generate the incentive for manager-equityholders to increase (decrease) unobservable risk. Thus, credible commitments to hedge observable risk can benefit the firm's manager-equityholders by reducing the incentive to shift risk and the associated agency cost of debt. This provides a positive rationale for hedging diversifiable risk at the firm level.

IN THE LAST FEW years, a sizeable literature has emerged which is devoted to explaining how shareholders can benefit from hedging undertaken by the firm. Such hedging may include the purchase of insurance, the use of financial futures and options, and even diversification at the level of the firm. Four lines of reasoning have been pursued in the literature. Smith and Stulz (1985) and Campbell and Kracaw (1987) rely on the fact that under optimal contracting between managers and shareholders managers are less diversified than are shareholders. Hedging by the firm which reduces the manager's exposure to risk can improve on the optimal compensation contract between the manager and equityholders. This argument may be applied to a firm with any capital structure or any degree of leverage. The other arguments apply to firms which are financed with risky debt. Smith and Stulz show that hedging may be useful in reducing expected bankruptcy costs. They also show, as does MacMinn (1985), that hedging by the firm may reduce the probability that depreciation tax shields may be lost due to financial distress. Finally, Diamond (1984) has shown that monitoring costs may be reduced through diversification.

MacMinn's result is of particular interest. Part of his analysis is based on an earlier paper by Green and Talmor (1985) which demonstrates that the equity-holders of a levered firm will have an incentive to underinvest in risky assets if the firm cannot capture its interest and depreciation tax shields with certainty. In Green and Talmor's analysis, the firm has no mechanism to reduce or hedge the uncertainty surrounding its tax shields other than to alter its investment decisions. However, as MacMinn argues, if the firm is allowed to hedge its risks,

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say in the futures markets, then it can reduce the incentive to choose suboptimal investments. Therefore, hedging can reduce the agency cost of debt arising from the existence of uncertain tax shields.

The argument developed in our paper is similar to that offered by MacMinn. The incentive effect of debt we are concerned with is the incentive of manager-equityholders to increase the riskiness of the firm's investments after the terms of the firm's debt financing have been determined—commonly referred to in the literature as the "asset substitution problem." This aspect of the incentive effect of debt has been the focus of attention in Jensen and Meckling (1976), Green (1984), Green and Talmor (1986), and Gavish and Kalay (1983). In this paper, the model developed by Green and Talmor (1986) and originated by Gavish and Kalay (1983) is extended to show how the level of observable risk within the firm influences the incentive to engage in (unobservable) asset substitution.

A perspective on the problem is provided by the recent example of a covenant included in a loan agreement between the R. H. Macy Corporation (with Macy Credit Corporation) and a syndicate of commercial banks—a covenant which is now common in much debt (particularly LBO) financing.[1] The loan agreement for a floating-rate loan of $500 million reportedly included a clause stipulating that within 60 days from the extension of the loan Macy's would arrange for a hedge which would offset, to the degree possible, the interest rate risk involved in that loan. Why would the borrower prefer a contract which required the interest rate risk in this loan agreement to be hedged, and why would the lender prefer such a contract over a fixed-rate loan which included no hedging requirement?

One explanation is that the covenant may simply be a device employed by the lender to insure that if terms, i.e., a spread over a reference interest rate, are quoted assuming that a portion of the interest rate risk is going to be hedged, then that risk actually *is* hedged. For any given set of terms, if hedging were unrestricted, the borrower would have an incentive to increase risk by avoiding hedging and thereby shift wealth from the lender to the borrower. Hence, the lender needs a contract that compels the borrower to choose a level of hedging that allows the lender to at least break even ex ante. This explanation implies that there should be available loan terms corresponding to various degrees of hedging, among which the borrower should be indifferent. But this provides no reason why loans which include such covenants would be preferred to ones which do not.

An alternative explanation of why the hedging covenant may be used is that it is difficult or costly for the bank group to directly observe the operating risk inherent in Macy's business. Moreover, both the borrower and the lenders understand that as financing risk is added to operating risk, Macy's incentive to increase its operating risk is enhanced. Therefore, the equityholders of Macy's find it advantageous to agree to limit their financial risk so as to change their incentive to increase operating risk. One way to accomplish this might be for the bank group to absorb the risk of changing interest rates. However, the banks in

[1] The bank credit was arranged by Goldman Sachs & Co. in conjunction with the leveraged buy-out of the company by a Macy management group. See Gilman (1985) for additional information.

the group face virtually the same incentive problems in funding their operations. Hence, it is optimal for both Macy's and the banks to shift this risk to some other market participant whose incentive to increase an unobservable risk is less than their own. As a result, it is beneficial for each party to agree to a covenant which requires that the interest rate risk be hedged in the futures market. As our formal analysis will show, such covenants will be most likely to diminish the incentive problem for Macy's if the risks to which those covenants pertain are highly correlated with Macy's operating risk. Since the company in question is Macy's credit subsidiary, it seems highly likely in this instance that operating and financial risks are highly correlated.

In the remainder of this paper we develop a model which permits us to formally demonstrate how and why controlling the level of an observable risk influences the incentive to increase an unobservable risk. In so doing, our analysis provides a new explanation of why corporate hedging is beneficial to the firm.

The paper is organized as follows: Section I specifies the model; Section II derives our result pertaining to the incentive effects of hedging observable risks; and Section III contains a short discussion and concluding comments.

I. Risk and the Asset Substitution Problem

A. The Model

Consider a firm with end-of-period cash flow $\tilde{X}$ where

$$\tilde{X} = \mu(\alpha) + \sigma(\alpha)\tilde{e}_1 + k\tilde{e}_2. \tag{1}$$

The random variables $\tilde{e}_1$ and $\tilde{e}_2$ have a continuous joint density function $g(\tilde{e}_1, \tilde{e}_2)$. Each random variable has a mean of zero, so that the firm's expected cash flow is equal to $\mu(\alpha)$. Both equityholders and debtholders are risk neutral which implies that the total value of the firm is determined entirely by $\mu(\alpha)$. The variable α is a scale parameter which lies in the set [0, 1]. The firm's equityholders control α and their choice of α is not observable by debtholders. Increases in α will decrease the expected cash flows of the firm, i.e., $\mu'(\alpha) < 0$, as well as increase the riskiness of those cash flows, i.e., $\sigma'(\alpha) > 0$. We will refer to $\sigma(\alpha)$ as the unobservable risk of the firm—that which can be altered by asset substitution. The risk of cash flows depends on k, which we take to be directly observable by debtholders; k reflects risk that cannot be altered by substituting assets but may be reduced by other actions such as hedging. We also assume that the correlation between $\tilde{e}_1$ and $\tilde{e}_2$ is non-negative and $k \geq 0$ so that $\tilde{e}_2$ is not a "natural hedge" of $\tilde{e}_1$. The firm is financed by equity and by single-period debt in the amount D, bearing a promised payment F. Further, the amount of financing D is assumed to be given. Finally, we assume that the riskless interest rate is zero and that shareholders select the unobservable risk after the firm's debt is issued.

The sequence of decisions facing manager-equityholders and debtholders is described in the time-line shown in Figure 1. At time 1, manager-equityholders announce their decision on k, the observable level of risk exposure. At time 2, debtholders choose the promised payment for debt F, knowing the revealed value

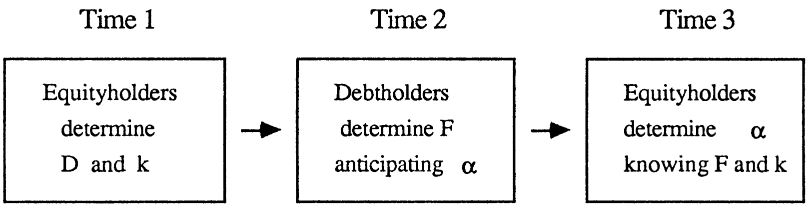

Figure 1. Decision sequence in the three-date model. At time 1, manager-equityholders determine the value of debt D and the level of observable risk k. At time 2, debtholders price debt by determining the face value F, anticipating the level of unobservable risk α. At time 3, manager-equityholders determine the optimal level of unobservable risk α, given F and k are fixed.

of k and anticipating how manager-equityholders will choose α, once F and k are fixed. Finally at time 3, manager-equityholders choose the level of unobservable risk via α, given the values of k and F determined at time 2. The recursive nature of this problem is crucial. The ultimate question we want to answer is: What is the total impact of a change in k on unobservable risk? The answer must take into account both the direct effect of k on α (or $\partial\alpha/\partial k$) and the indirect effect of k changing F (or dF/dk) and in turn F changing α (or $\partial\alpha/\partial F$). However, dF/dk is determined at time 2 as debtholders anticipate the direct effect of the change in k on α and the indirect effect of their change in F on α.

If we assume that k is given exogenously, then the decision problem facing equityholders who have obtained debt financing in the amount D, is to choose α, for a given F and k, in order to maximize the value of their claim:

$$\max_{\alpha} W = \int_{-\infty}^{\infty}\int_{a}^{\infty} \{\mu(\alpha) + \sigma(\alpha)\tilde{e}_1 + k\tilde{e}_2 - F\} g(\tilde{e}_1, \tilde{e}_2)\, d\tilde{e}_2\, d\tilde{e}_1 \tag{2}$$

where

$$a = \frac{F - \mu(\alpha) - \sigma(\alpha)\tilde{e}_1}{k}.$$

Equation (2) is similar to that derived in Green and Talmor (1986). Here there is an additional risk $\tilde{e}_2$ which is scaled by k appearing in the integrand and in the limit of integration a. The first-order condition for the optimal value of α is:

$$\int_{-\infty}^{\infty}\int_{a}^{\infty} \{\mu'(\alpha) + \sigma'(\alpha)\tilde{e}_1\} g(\tilde{e}_1, \tilde{e}_2)\, d\tilde{e}_2\, d\tilde{e}_1 = 0. \tag{3}$$

For any value of F and k, equation (3) defines the following implicit function for the optimal level of α which we label $\alpha^*(F, k)$:

$$-\frac{\mu'(\alpha^*(F, k))}{\sigma'(\alpha^*(F, k))} = \frac{\int_{-\infty}^{\infty}\int_{a}^{\infty} \tilde{e}_1 g(\tilde{e}_1, \tilde{e}_2)\, d\tilde{e}_2\, d\tilde{e}_1}{\int_{-\infty}^{\infty}\int_{a}^{\infty} g(\tilde{e}_1, \tilde{e}_2)\, d\tilde{e}_2\, d\tilde{e}_1} = E\{\tilde{e}_1 \mid \tilde{e}_2 > a\}. \tag{4}$$

This is similar to the expression derived by Green and Talmor (1986, equation (8)). Their interpretation is that the optimal α is chosen by "setting the tradeoff between mean and increased risk equal to the benefit to the equityholders from the extra risk *scaled up* by the probability that this benefit will be realized."

B. Competitive Pricing of Debt

If we assume that debt is priced competitively, then lenders determine the promised payment F to break even, anticipating that α will be chosen by the shareholders to maximize their wealth. We define $\alpha^* = \alpha^*(F, k)$ to be the value that maximizes the wealth of manager-equityholders. Hence, debt will be priced as:

$$D = \int_{-\infty}^{\infty} \int_{\infty}^{a} \{\mu(\alpha^*(F, k)) + \sigma(\alpha^*(F, k))\tilde{e}_1 + k\tilde{e}_2\} g(\tilde{e}_1, \tilde{e}_2)\, d\tilde{e}_2\, d\tilde{e}_1 + \int_{-\infty}^{\infty} \int_{a}^{\infty} F g(\tilde{e}_1, \tilde{e}_2)\, d\tilde{e}_2\, d\tilde{e}_1 \tag{5}$$

Next, we define a necessary condition for the existence of an equilibrium debt contract F. This condition will be useful in our subsequent analysis. The condition is that increases in F do not decrease the value of debt D:

$$\frac{\partial D}{\partial F} = \int_{-\infty}^{\infty} \int_{\infty}^{a} \{\mu'(\alpha) + \sigma'(\alpha)\tilde{e}_1\} \frac{\partial \alpha^*}{\partial F} g(\tilde{e}_1, \tilde{e}_2)\, d\tilde{e}_2\, d\tilde{e}_1 + \int_{-\infty}^{\infty} \int_{a}^{\infty} g(\tilde{e}_1, \tilde{e}_2)\, d\tilde{e}_2\, d\tilde{e}_1 > 0. \tag{6}$$

Since it plays a crucial role in the analysis which follows we will analyze dF/dk. Recall that the sequence of decisions is that k is selected first, then F, and finally α. While debtholders cannot observe α, they can anticipate how α will be influenced by their choice of both F and k. Hence, we must treat α as endogenous when we evaluate dF/dk. To evaluate dF/dk, we set the total differential of (5) equal to zero:

$$dD = \frac{\partial D}{\partial k} dk + \frac{\partial D}{\partial F} dF = 0 \tag{7}$$

where $\partial D/\partial F$ is defined in (6), and

$$\frac{\partial D}{\partial k} = \int_{-\infty}^{\infty} \int_{\infty}^{a} \{\mu'(\alpha) + \sigma'(\alpha)\tilde{e}_1\} \frac{\partial \alpha^*}{\partial k} g(\tilde{e}_1, \tilde{e}_2)\, d\tilde{e}_2\, d\tilde{e}_1 + \int_{-\infty}^{\infty} \int_{\infty}^{a} \tilde{e}_2 g(\tilde{e}_1, \tilde{e}_2)\, d\tilde{e}_2\, d\tilde{e}_1. \tag{8}$$

In order to simplify notation, we define:

$$V \equiv -\frac{\partial^2 W/\partial\alpha\partial k}{\partial^2 W/\partial\alpha\partial F} = -\frac{\partial\alpha^*/\partial k}{\partial\alpha^*/\partial F}, \tag{9}$$

where W is the value of equity defined by (2). V is the ratio of the partial effects of k and F on the optimal level of unobservable risk α^*. We then label the first term in (6):[2]

$$q \equiv \int_{-\infty}^{\infty} \int_{\infty}^{\infty} \{\mu'(\alpha) + \sigma'(\alpha)\tilde{e}_1\} \frac{\partial \alpha^*}{\partial F} g(\tilde{e}_1, \tilde{e}_2)\, d\tilde{e}_2\, d\tilde{e}_1 = \frac{\partial \alpha^*}{\partial F} \mu'(\alpha), \quad (10)$$

or the marginal effect on the value of the firm caused by a change in the level of unobservable risk which is induced by a change in F. Also, we define the probability of solvency as:

$$z \equiv \int_{-\infty}^{\infty} \int_{a}^{\infty} g(\tilde{e}_1, \tilde{e}_2)\, d\tilde{e}_2\, d\tilde{e}_1. \quad (11)$$

Since the mean of $\tilde{e}_2$ is zero:

$$\int_{-\infty}^{\infty} \int_{a}^{\infty} \tilde{e}_2 g(\tilde{e}_1, \tilde{e}_2)\, d\tilde{e}_2\, d\tilde{e}_1 = -\int_{-\infty}^{\infty} \int_{\infty}^{a} \tilde{e}_2 g(\tilde{e}_1, \tilde{e}_2)\, d\tilde{e}_2\, d\tilde{e}_1$$

and we can define:

$$E_2^* \equiv \frac{\int_{-\infty}^{\infty} \int_{a}^{\infty} \tilde{e}_2 g(\tilde{e}_1, \tilde{e}_2)\, d\tilde{e}_2\, d\tilde{e}_1}{z}. \quad (12)$$

E_2^* can be interpreted as the expected value of $\tilde{e}_2$ conditional on the firm being solvent. Since the unconditional expectation of $\tilde{e}_2$ is zero, this conditional expectation must be positive. Notice also that the partial derivative of E_2^* with respect to F is positive. This means that E_2^* increases with leverage. Then, by substituting into (7) we can write:

$$\frac{dF}{dk} = \frac{qV + zE_2^*}{q + z}. \quad (13)$$

Finally, (13) can be rearranged to give:

$$\frac{dF}{dk} - V = \frac{z}{q + z} [E_2^* - V]. \quad (14)$$

To interpret (14), it is useful to first consider the case where α is determined exogenously (i.e., $\partial\alpha/\partial F = \partial\alpha/\partial k = 0$) so that debtholders take α as given when pricing debt. Under these circumstances, we can see from (6) and (8) that dF/dk

[2] It follows from (3) that the expression for q can be rewritten as:

$$q \equiv \frac{\partial \alpha^*}{\partial F} \int_{-\infty}^{\infty} \int_{\infty}^{\infty} \{\mu'(\alpha) + \sigma'(\alpha)\tilde{e}_1\} g(\tilde{e}_1, \tilde{e}_2)\, d\tilde{e}_2\, d\tilde{e}_1$$

which can be rearranged to give:

$$q \equiv \frac{\partial \alpha^*}{\partial F} \mu'(\alpha) \int_{-\infty}^{\infty} \int_{\infty}^{\infty} g(\tilde{e}_1, \tilde{e}_2)\, d\tilde{e}_2\, d\tilde{e}_1 + \frac{\partial \alpha^*}{\partial F} \sigma'(\alpha) \int_{-\infty}^{\infty} \int_{\infty}^{\infty} \tilde{e}_1 g(\tilde{e}_1, \tilde{e}_2)\, d\tilde{e}_2\, d\tilde{e}_1.$$

By definition, the integral in the first term is equal to one and the integral in the second term is equal to zero so that:

$$q = \frac{\partial \alpha^*}{\partial F} \mu'(\alpha).$$

would reduce to E_2^*. Note that with α fixed, E_2^* is the rate at which marginal changes in k would affect W, holding F constant. Since the *total* value of the firm is being held constant (i.e., α is constant), this is the same rate at which debtholders must increase the promised debt payment F to maintain the value of debt D. Thus, E_2^* represents the tradeoff between F and k necessary to maintain a given value of debt D *without incentives for manager-equityholders to shift risk.*

Now consider the more general case where there is an incentive problem, and α is endogenously determined. Under what circumstances would dF/dk again be equal to E_2^*? From the necessary condition for an equilibrium debt contract (6), $q + z > 0$, and by definition $0 < z < 1$. Therefore, from (14) the necessary and sufficient condition for $dF/dk = E_2^*$ is that $V = E_2^*$. In this case, the partial effects on α^* resulting from an increase in k, and from the corresponding increase in F, offset each other such that dF/dk is again equal to E_2^*—i.e., the lender requires the same tradeoff between F and k as when no incentive problem exists.

II. The Impact of Observable Risk on Asset Substitution

In this section we determine how the level of observable risk will affect the incentive of equityholders to alter the level of unobservable risk. In general, we cannot restrict the sign of $d\alpha^*/dk$. However, we will show that when the observable and the unobservable risks are perfectly correlated, or when there is one underlying source of risk with observable and unobservable exposures, then $d\alpha^*/dk$ is positive. We will also clarify the determinants of $d\alpha^*/dk$ in the general case and relate it to the competitive pricing of debt.

A. The Determinants of the Incentive Effect of Observable Risk

We begin by specifying the determinants of $d\alpha^*/dk$. Our analysis is similar in some ways to that of Green and Talmor. They analyzed how increases in the amount of leverage influence the optimal value of risk α. Hence, they evaluated $d\alpha^*/dF$. By contrast, we are interested in how increases in the level of observable risk k, or the risk which is not subject to moral hazard or the asset substitution problem, influences the optimal level of α.

Our basic argument is that the impact of k on α can be decomposed into two terms. One is the direct effect: the impact of k on α, holding F constant. The other is an indirect effect: the impact of k on α which occurs from the effect of k on F and the resulting effect of F on α. The sign of $d\alpha^*/dk$ will depend on the relative magnitude and sign of the two effects. The propositions in this section deal with this issue.

Differentiating totally the first-order condition (equation (3)) and rearranging we obtain:

$$\frac{d\alpha^*}{dk} = -\frac{(\partial^2 W/\partial\alpha\partial F)(dF/dk) + \partial^2 W/\partial\alpha\partial k}{\partial^2 W/\partial\alpha^2} = \frac{\partial\alpha^*}{\partial F}\frac{dF}{dk} + \frac{\partial\alpha^*}{\partial k} \tag{15}$$

In order to assess the magnitude and sign of the two effects we note that, at an interior optimum, $\partial^2 W/\partial\alpha^2 < 0$, and therefore:

$$\operatorname{sign}\left[\frac{d\alpha^*}{dk}\right] = \operatorname{sign}\left[\frac{\partial^2 W}{\partial\alpha\partial F}\frac{dF}{dk} + \frac{\partial^2 W}{\partial\alpha\partial k}\right]. \tag{16}$$

If we assume that:

$$\frac{\partial^2 W}{\partial\alpha\partial F} > 0,$$

equation (16) can be rewritten as:

$$\operatorname{sign}\left[\frac{d\alpha^*}{dk}\right] = \operatorname{sign}\left[\frac{dF}{dk} - V\right], \tag{17}$$

where V is defined in (9) above. Using (14) this implies:

$$\operatorname{sign}\left[\frac{d\alpha^*}{dk}\right] = \operatorname{sign}\left[\frac{z}{q+z}\,(E_2^* - V)\right]. \tag{18}$$

Since $\mu'(\alpha) < 0$, it follows from (10) that $q < 0$ if $\partial\alpha^*/\partial F > 0$.[3] As a result, $z > q + z$. Combined with the previous result that $q + z > 0$:

$$\frac{z}{q+z} > 1. \tag{19}$$

It follows directly that:

$$\frac{d\alpha^*}{dk} \gtreqless 0 \quad \text{as} \quad E_2^* \gtreqless V, \tag{20}$$

and from (14) that:

$$\frac{dF}{dk} \gtreqless E_2^* \quad \text{as} \quad E_2^* \gtreqless V. \tag{21}$$

Comparing (20) and (21) gives the result summarized in the following proposition.

PROPOSITION 1:

$$\frac{d\alpha^*}{dk} \gtreqless 0 \quad as \quad \frac{dF}{dk} \gtreqless E_2^*. \tag{22}$$

To interpret the proposition, recall that E_2^* is the effect of k on F if α is exogenous, or if $\partial\alpha^*/\partial k$ and $\partial\alpha^*/\partial F$ are offsetting (in the sense that $E_2^* = V$).

[3] From the total differential of the first-order condition, holding k constant:

$$\frac{\partial\alpha^*}{\partial F} = -\frac{\partial^2 W/\partial\alpha\partial F}{\partial^2 W/\partial\alpha^2}.$$

Therefore if the second-order condition is satisfied:

$$\operatorname{sign}\left[\frac{\partial\alpha^*}{\partial F}\right] = \operatorname{sign}\left[\frac{\partial^2 W}{\partial\alpha\partial F}\right].$$

Equation (21) asserts that if V is less than (greater than, if $\partial\alpha/\partial F < 0$) E_2^*, $dF/dk > E_2^*$—i.e., increases in k cause debtholders to charge a premium over E_2^*. Proposition 1 states that whenever increases in k cause α^* to increase, or when $d\alpha^*/dk$ is positive, dF/dk will exceed E_2^* by a positive premium. Otherwise the firm's debt would not be competitively priced. Notice that Proposition 1 draws no conclusion about the sign of $d\alpha^*/dk$. Rather, it establishes how debt will be priced in equilibrium, given the sign of $d\alpha^*/dk$. For instance, when observable risk does contribute to the incentive for manager-equityholders to shift unobservable risk—or, when $d\alpha^*/dk$ is positive—debtholders will price that prospect into the equilibrium yield. The yield is increased by requiring the return F per unit of observable risk k to be commensurately higher than E_2^*.

Finally, recall that (20) and (21) are based on the temporary assumption that:

$$\frac{\partial^2 W}{\partial\alpha\partial F} > 0.$$[4]

The assumption that the sign of this derivative is negative implies that $\partial\alpha/\partial F < 0$ (see footnote 3) and that $z/(q + z) < 1$. These changes cause the comparisons made in (20) and (21) to be inverted. But since *both* relationships are inverted at the same time, the general result (22) remains intact.

With this result in hand, we now analyze how the degree of correlation between observable and unobservable risk influences the direct and indirect effects of k on α.

B. The Impact of the Degree of Correlation Between Risks

We can generate additional insight by considering the special case where $\tilde{e}_1$ and $\tilde{e}_2$ are perfectly correlated, or equivalently where there is a single source of risk $\tilde{e}$, but there are observable and unobservable exposures to this risk. Under these circumstances, the equityholder's maximization problem given by (2) simplifies to:

$$\max_{\alpha} W = \int_a^{\infty} \{\mu(\alpha) + [\sigma(\alpha) + k]\tilde{e} - F\} g(\tilde{e})\, d(\tilde{e}), \tag{2'}$$

where

$$a = \frac{F - \mu(\alpha)}{\sigma(\alpha) + k}.$$

The necessary first-order condition (3) now becomes:

$$\frac{\partial W}{\partial \alpha} = \int_a^{\infty} \{\mu'(\alpha) + \sigma'(\alpha)\tilde{e}\} g(\tilde{e})\, d\tilde{e} = 0. \tag{3'}$$

[4] Green and Talmor (1986) have already shown that in the single risk case:

$$\frac{\partial^2 W}{\partial\alpha\partial F} > 0.$$

By introducing a second exposure of risk our analysis allows the possibility that the sign may be negative.

Differentiating (3′) we can write:

$$\frac{\partial \alpha^*}{\partial k} = -\frac{1}{\sigma(\alpha^*) + k}\left[\frac{[\mu'(\alpha^*) + \sigma'(\alpha^*)a]g(a)}{\partial^2 W/\partial \alpha^2}\right]a. \tag{23}$$

And since:

$$\frac{\partial \alpha^*}{\partial F} = \frac{1}{\sigma(\alpha^*) + k}\left[\frac{[\mu'(\alpha^*) + \sigma'(\alpha^*)a]g(a)}{\partial^2 W/\partial \alpha^2}\right] > 0, \tag{24}$$

it follows that:

$$\operatorname{sign}\left[\frac{\partial \alpha^*}{\partial k}\right] = \operatorname{sign}[-a] = \operatorname{sign}[\mu - F]. \tag{25}$$

Hence, the sign of direct effect of k on α depends on the degree of leverage chosen by the firm, or F. Note that this effect works entirely through the impact of k on a and that increases in k always drive a closer to zero. When the firm has relatively little leverage, or when $\mu - F$ is positive and a is negative, then increases in k lead to increases in a, which decrease the incentive to engage in asset substitution. But when the firm has sufficient leverage that $\mu - F$ is negative and a is positive, then increases in k reduce a (i.e., make a closer to zero) which exacerbates the asset substitution problem.

Also note that from the definition of V in (9), $V = a$. In the case of a single source of risk, the counterpart of E_2^* is:

$$E^* = \frac{\int_a^\infty \tilde{e} g(\tilde{e})\, d\tilde{e}}{\int_a^\infty g(\tilde{e})\, d\tilde{e}}.$$

It follows directly that $E^* > V$ since $E^* > a$ by definition. Then from (20) it follows that $d\alpha^*/dk > 0$.

The intuition behind this result is that $d\alpha^*/dk$ can be negative only if the direct effect $(\partial\alpha^*/\partial k)$ is negative and of sufficient magnitude to overpower the positive indirect effect. Since the sign of the direct effect is the same as that of $[\mu - F]$, where μ is fixed, the direct effect can be negative only if the firm is sufficiently levered. But the condition that $E^* > a$ implies that the indirect effect cannot be swamped by the direct effect, regardless of how much debt the firm employs. The reason is that a change in leverage affects *not only* the direct effect of k on α *but also* E^* the incremental F lenders would demand for increases in k if $d\alpha^*/dk = 0$. Hence, increases in the firm's leverage can cause the direct effect of k on α to become negative but never sufficiently negative to overpower the positive indirect effect contributed by dF/dk.

We summarize this result in the following proposition.

PROPOSITION 2: *When the observable and unobservable sources of risk are perfectly correlated, an increase (decrease) in the observable level of risk will always generate an increase (decrease) in the unobservable level of risk. Or,* $d\alpha^*/dk > 0$.

Now, return to the general case where there is less-than-perfect positive correlation between the observable and unobservable sources of risk. In this case, a is stochastic, as it depends on $\tilde{e}_1$ so that $\partial\alpha^*/\partial k$ and $\partial\alpha^*/\partial F$ now involve conditional expected values across $\tilde{e}_1$ of $[\mu'(\alpha) + \sigma'(\alpha)\tilde{e}_1]\partial a/\partial k$ and $[\mu'(\alpha) + \sigma'(\alpha)\tilde{e}_1]\partial a/\partial F$, respectively:

$$\frac{\partial \alpha^*}{\partial k} = \frac{1/k \int_{-\infty}^{\infty} [\mu'(\alpha) + \sigma'(\alpha)\tilde{e}_1](-a)g(\tilde{e}_1, a)\, d\tilde{e}_1}{\partial^2 W/\partial \alpha^2}, \tag{26}$$

and:

$$\frac{\partial \alpha^*}{\partial F} = \frac{1/k \int_{-\infty}^{\infty} [\mu'(\alpha^*) + \sigma'(\alpha^*)\tilde{e}_1]g(\tilde{e}_1, a)\, d\tilde{e}_1}{\partial^2 W/\partial \alpha^2}. \tag{27}$$

Therefore, we can no longer rely on a direct comparison of E_2^* with a as we did in the single-risk case to establish restrictions on the direct effect of k on α, nor can we claim that $\partial\alpha^*/\partial F$ is greater than zero.[5]

However, we can make the following general comparison of the direct and indirect effects using V. Note that:

$$V \equiv \frac{\partial \alpha^*/\partial k}{\partial \alpha^*/\partial F} = \frac{-\int_{-\infty}^{\infty} \{\mu'(\alpha) + \sigma'(\alpha)\tilde{e}_1\}ag(\tilde{e}_1, a)\, d\tilde{e}_1}{-\int_{-\infty}^{\infty} \{\mu'(\alpha) + \sigma'(\alpha)\tilde{e}_1\}g(\tilde{e}_1, a)\, d\tilde{e}_1}. \tag{28}$$

Dividing numerator and denominator by $\sigma'(\alpha)$ yields:

$$V = \frac{\int_{-\infty}^{\infty} \{\tilde{e}_1 - E_1^*\}ag(\tilde{e}_1, a)\, d\tilde{e}_1}{\int_{-\infty}^{\infty} \{\tilde{e}_1 - E_1^*\}g(\tilde{e}_1, a)\, d\tilde{e}_1}, \tag{29}$$

where, from equation (4), $-\mu'(\alpha)/\sigma'(\alpha) = E\{\tilde{e}_1 \mid \tilde{e}_2 > a\}$ is labeled E_1^*.

From (29), V can now be interpreted as the weighted expectation of a, across $\tilde{e}_1$, where the weights are the deviations of $\tilde{e}_1$ from E_1^* relative to the *expected* deviation of $\tilde{e}_1$ from E_1^*. This expression approaches the value of a in the case of a single source of risk since, in that case, a may be factored from the numerator of (29) and $V = a$. This leads to Proposition 2.

In the general case, for any degree of correlation and any value of $F - \mu(\alpha)$, since the sign of $d\alpha^*/dk$ depends on the relative magnitude of V and E_2^*, we can state the following general result.

PROPOSITION 3:

$$\frac{d\alpha^*}{dk} \gtreqqless 0 \quad as \quad E_2^* \gtreqqless \frac{\int_{-\infty}^{\infty} \{\tilde{e}_1 - E_1^*\}ag(\tilde{e}_1, a)\, d\tilde{e}_1}{\int_{-\infty}^{\infty} \{\tilde{e}_1 - E_1^*\}g(\tilde{e}_1, a)\, d\tilde{e}_1} \qquad if\ \frac{\partial \alpha^*}{\partial F} > 0,$$

and

$$\frac{d\alpha^*}{dk} \gtreqqless 0 \quad as \quad E_2^* \lesseqqgtr \frac{\int_{-\infty}^{\infty} \{\tilde{e}_1 - E_1^*\}ag(\tilde{e}_1, a)\, d\tilde{e}_1}{\int_{-\infty}^{\infty} \{\tilde{e}_1 - E_1^*\}g(\tilde{e}_1, a)\, d\tilde{e}_1} \qquad if\ \frac{\partial \alpha^*}{\partial F} < 0.$$

[5] This result diverges from that of Green and Talmor (1986) by allowing the possibility that $\partial\alpha^*/\partial F < 0$. The reason becomes clearer by comparing (24) with (27). In the case of a single source of risk, (24) is consistent with their result that increases in risky debt are accompanied by increases in the level of (unobservable) risk. This result obtains because both the numerator and denominator of (24) are negative. In the case where two imperfectly correlated risks are involved, the numerator of (27) can be either positive or negative depending on the distribution of $\tilde{e}_1$.

From the previous analysis, it is apparent that for sufficiently high degrees of correlation between $\tilde{e}_1$ and $\tilde{e}_2$, $d\alpha^*/dk$ will be positive. However, as the degree of correlation falls, then $d\alpha^*/dk$ will decline and may become negative.[6]

Proposition 3 is best understood by comparing it to Proposition 2. Both propositions involve an analysis of the effect of a change in the level of observable risk on the first-order condition. When there is a single source of risk, increases in the observable exposure to that risk cause the probability of solvency to decline. Since there is only one random variable that determines solvency, there is a unique value of that variable at which the firm is just solvent. Hence, it is then necessary to determine whether that single value increases or decreases in order to ascertain the total effect of a change in k. However, when there are two sources of risk, the effect of a change in k on the first-order condition involves the expected value of the product of the marginal value of a change in α (or $\mu'(\alpha) + \sigma'(\alpha)\tilde{e}_1$) and a, across all possible values of the other random variable $\tilde{e}_1$. Since this expectation is nonlinear, it is not possible to rule out the prospect that the direct effect of a change in k on α will be negative and larger in absolute value than the indirect effect of k on F and F on α, nor is it possible to completely rule out the prospect that $\partial\alpha^*/\partial F$ is negative.

It is important to note that nothing in the basic economics of the direct and indirect effects of k on α changes when there are imperfectly correlated risks. Increases in leverage still increase a for any given $\tilde{e}_1$ and therefore alter $\partial\alpha^*/\partial k$ and E_2^* as in the case with a single source (or perfectly correlated sources) of risk. However, when there is a single source of risk there is a unique value of $\tilde{e}$ such that the firm just breaks even. This makes it possible to place a lower bound on the direct effect of k on α while such a lower bound is not determinate when there are less than perfectly correlated sources of risk.

This means that equityholders would have an incentive to avoid observable risks that are not highly correlated with unobservable risks only when the net effect of increasing such risks increases incentive problems. If the total effect of increases in the observable risk is to diminish the incentive problem pertaining to the unobservable risk, then it is valuable to hedge this risk. Hence, we expect to see efforts to reduce observable risks when their total effect is to increase incentive problems. In the example pertaining to Macy's discussed earlier we would expect this to be the case as long as Macy's credit subsidiary's operating risk is highly correlated with interest rates, which seems likely.

III. Conclusions

Previous research (Gavish and Kalay 1983; Green and Talmor 1986) has formally demonstrated how the existence of risky debt generates an incentive for manager-equityholders to substitute from low- to high-risk assets. This research has also

[6] Even in the case of independently distributed sources of risk, the sign of $d\alpha^*/dk$ is still ambiguous. Because of the dependence of the limit of integration a on $\tilde{e}_1$, equations (28) and (29) cannot be simplified as in the case of perfectly positively correlated risks. Thus, Proposition 3 also describes the implications of independent risks.

shown that the incentive to shift risk—and the associated agency costs of debt—increase monotonically with the use of risky debt. Similar results are shown in this paper regarding the level of observable risk in the firm's cash flows.

Our paper demonstrates how the incentive to substitute toward riskier assets is related to the observable risk of the firm. When observable and unobservable risks are perfectly positively correlated, increases (decreases) in the level of observable risk generate incentives which *always* lead to increases (decreases) in the level of unobservable risk. Since observable risks are ones on which hedging contracts can be written—e.g., in the form of forward or futures contracts—hedging against observable risks can benefit the firm's equityholders by reducing the agency cost associated with risky debt. Thus, the analysis provides a positive rationale for manager-equityholders to hedge diversifiable risk. The analysis also shows that when observable and unobservable sources of risk are not sufficiently positively correlated, the opposite result cannot be precluded. Our results imply that manager-equityholders should benefit from hedging when observable and unobservable risks are sufficiently positively correlated.

Our results suggest an additional reason, compared to those already found in the literature, why hedging may be beneficial. They also explain why it may be optimal to include covenants which require that the borrower hedge an observable risk in debt contracts—e.g., interest rate risk, or commodity price risk—when a lending institution or intermediary is involved in the credit decision. Finally, our results suggest that investment projects which increase the risk of the firm's cash flows will generate an externality. Therefore, it is possible that as investment in risky new projects occurs, the incentive costs of debt are increased—equityholders have a greater incentive to increase the risk of existing assets—causing the value of the firm to decrease. So new capital budgeting alternatives should be evaluated to include the marginal incentive effects associated with the firm's other assets. This constitutes one example of the problems which result whenever the separation between investment and financing decisions breaks down.

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