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Performance Measurement under Asymmetric Information and Investment Constraints

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ABSTRACT

The fact that investment policies are often restricted appears to have been neglected in the performance measurement literature. This paper, using a standard information model, shows how the introduction of constraints on the proportion of assets to be invested in the market affects the expected portfolio returns and the value of a portfolio manager's performance. The results are related to the classical Treynor and Mazuy (1966) conjectures about characteristic lines.

IN A RATIONAL MARKET, a portfolio manager's ability to produce a superior returns distribution should be associated with possession of superior information or greater ability to process information. Thus, Merton (1981) proposes an equilibrium theory for the value of a manager's timing skills that gives rise to a kinked relation between the expected portfolio return and the market return. Other information models, such as those of Pfleiderer and Bhattacharya (1983) or Admati and Ross (1985), rather suggest a smooth, quadratic relation between the two. In practice, however, one is unlikely to observe high expected portfolio returns for large negative market returns, and so a quadratic characteristic curve appears unrealistic.

The purpose of this paper is to propose an information model with investment constraints that confirms an intuition of Treynor and Mazuy (1966) about the relation between a mutual fund and the market rate of return. These authors conjecture the existence of a curved relation over a range of market values whose degree of curvature depends on how heavily management bets on its expectations. They describe the characteristic line for a fund that outguesses the market with better than average success as "a gradual transition of fund volatility from a flat slope at the extreme left ... to a steep slope at the extreme right, with the slope varying more or less continuously in between, producing a smoothly curved characteristic line pattern ... rather than the kinked pattern ...." It is precisely this type of relation that our model achieves. It will be shown, in a market timing context, how the introduction of constraints on investment strategies affects a manager's expected portfolio returns and the value of his or her performance.

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This will be accomplished using a standard partial equilibrium information model for portfolio selection.

Section I describes the basic information model. In Section II, it is shown how this model can be adjusted to account for constraints on the proportion of the assets to be invested in the market. Finally, the impact of investment constraints on performance measurement is discussed in Section III.

I. A Basic Information Model

The model described below is adapted from Gendron (1988) but is similar to many others in the information theory literature. It rests on the Bayesian paradigm, which involves an updating of beliefs and the maximization of expected utility. In this context, the portfolio returns distribution, conditional upon the potential realization of the market return, is of particular interest. Its expectation may be regarded as reflecting the manager's ability to manage funds and can be used to value his or her performance.

More specifically, consider a manager with an exponential utility function[1] defined over returns, r, with absolute risk aversion parameter, $\theta > 0$, viz.

$$U(r) = -(1/\theta)\exp(-\theta r), \quad r \in (-\infty, +\infty),$$

and suppose that the market return, in excess of the risk-free rate,[2] follows a normal distribution with mean μ and variance σ^2, denoted by

$$R_m \sim N(\mu, \sigma^2). \tag{1}$$

Now assume that the manager has the special ability to observe a signal, Y, which provides information on the future value of the market. To be specific, let us suppose that, conditional on the true value of the market return, $R_m = r_m$, the manager's signal is normally distributed around r_m with variance σ_y^2, viz.

$$Y \mid r_m \sim N(r_m, \sigma_y^2). \tag{2}$$

Equations (1) and (2) define a joint distribution for the variables R_m and Y. Once the value of the latter is observed, the manager uses Bayes' theorem to update his or her subjective distribution for R_m. A standard calculation[3] yields

$$R_m \mid y \sim N(\nu, \tau^2), \tag{3}$$

where $\nu = \tau^2(\mu/\sigma^2 + y/\sigma_y^2)$ and $1/\tau^2 = 1/\sigma^2 + 1/\sigma_y^2$.

In this market timing situation, the manager chooses his or her portfolio, i.e., the proportion X_m to be invested in the market, in such a way as to maximize the conditional expected utility of his or her portfolio return, $R_p = X_m R_m$, upon observing $Y = y$. In other words, $X_m = X_m(y)$ must be chosen so that

$$E\{U(R_p \mid y)\} = \max E[-(1/\theta)\exp\{-\theta(R_p)\} \mid y]. \tag{4}$$

[1] This convenient choice of utility function is common in the performance measurement literature.

[2] All returns will be in excess of the risk-free rate.

[3] See, for instance, DeGroot (1970), p. 167.

It is easy to see that

$$X_m = \mathrm{E}(R_m \mid y)/\theta \ \mathrm{var}(R_m \mid y), \tag{5}$$

which, in view of (3), implies that X_m is a linear function of y, viz.

$$X_m = a + by, \tag{6}$$

where $a = \mu/\theta\sigma^2$ and $b = 1/\theta\sigma_y^2$.

This prescription appears reasonable, provided that the manager has a fixed risk aversion parameter. Indeed, a favorable market (i.e., with large mean) or stable market conditions (as evidenced by a small variance) should induce the manager to invest a larger proportion of his or her portfolio in the market, as conveyed by equation (6). Note, however, that X_m can take arbitrarily large, positive or negative values. The joint distribution of X_m and R_m is seen to be bivariate normal with parameters

$$\begin{pmatrix} R_m \\ X_m \end{pmatrix} \sim \mathrm{MVN}_2\left[\begin{pmatrix} \mu \\ a + b\mu \end{pmatrix}, \begin{pmatrix} \sigma^2 & b\sigma^2 \\ b\sigma^2 & b^2(\sigma^2 + \sigma_y^2) \end{pmatrix}\right]. \tag{7}$$

The expected return on the portfolio given a certain realized market return, $\mathrm{E}(R_p \mid r_m)$, which we will refer to as the *payoff function*, is of special interest. It represents the manager's product, or the portfolio return we can expect, given a realized market return, if we follow his or her advice.

From (7) and the definition of R_p, it is immediate that the conditional distribution of R_p given $R_m = r_m$ is itself normal with mean

$$\mathrm{E}(R_p \mid r_m) = ar_m + br_m^2 = (\mu/\theta\sigma^2)r_m + (1/\theta\sigma_y^2)r_m^2 \tag{8}$$

and variance

$$\mathrm{var}(R_p \mid r_m) = b^2 r_m^2 \sigma_y^2 = r_m^2/(\theta^2\sigma_y^2). \tag{9}$$

Looking first at expression (9), note that $\mathrm{var}(R_p \mid r_m)$ is inversely proportional to the variance of the signal, σ_y^2, and the risk aversion parameter, θ. This could have been anticipated since a risk-averse or poorly informed manager should invest a smaller proportion, X_m, of his or her portfolio in the market. The quadratic dependence of $\mathrm{var}(R_p \mid r_m)$ on r_m could also be expected since one is likely to have a more volatile portfolio in bull and bear markets. Turning to the conditional mean of R_p displayed in (8), note that the expression is linear in μ, the mean of the market, and that high values of θ and σ_y^2 will have the same effect as on the variance. This again is in accordance with intuition. Thus, the model developed in this section shares with those of Jensen (1972) and Pfleiderer and Bhattacharya (1983), as well as Admati and Ross (1985), a quadratic functional dependence of R_p on R_m. Intuitively, it might be reasonable to expect the payoff function to be positive and increasing in $\mid r_m \mid$, at least in a neighborhood of the origin. In the present case, however, this behavior is predicted by the model on the *entire* range of values of r_m, implying that a manager with superior information should have been able to foresee poor market conditions, short sell the market, and make profits. This is unrealistic. A more sophisticated information model which addresses this problem will be described in the following section.

II. A Model with Investment Constraints

It is common for managers to face constraints or limitations on their investment strategies. In Canada, for example, Section 2.05 of National Policy Statement No. C-39 on investment practices in mutual funds effectively prohibits borrowing to invest or short selling securities. Such constraints can be suitably modeled by restricting the proportion, X_m, of assets invested in the market to the interval (0, 1). The more general case where X_m is restricted to an interval (k_1, k_2) will now be considered.

Assume, as before, that R_m is distributed according to (1) and that the manager has the ability to observe a signal, Y, whose conditional distribution given $R_m = r_m$ is displayed in equation (2). Upon observing $Y = y$, he or she would then proceed as before to select the optimal proportion, X_m, to be invested in the market. Now suppose that restrictions k_1 and k_2 exist on his or her portfolio investment strategy. Because of these external constraints, the *actual* proportion invested by the manager would then be given by

$$\begin{aligned} X_m^* &= k_1 \quad \text{for } X_m \le k_1, \\ &= X_m \quad \text{for } X_m \in (k_1, k_2), \\ &= k_2 \quad \text{for } X_m \ge k_2, \end{aligned} \tag{10}$$

where X_m is defined as in (6).

Obviously, the conditional distribution of X_m^* given $R_m = r_m$ is not normal since non-zero probability masses are accumulated at $X_m^* = k_1$ and k_2. Note, however, that its density remains proportional to

$$N(a + br_m,\ b^2\sigma_y^2) \tag{11}$$

on the open interval (k_1, k_2).

The corresponding "restricted" payoff function is then given by $\mathrm{E}(R_p \mid r_m) = r_m \mathrm{E}(X_m^* \mid r_m)$. Substituting the conditional distribution of X_m^* and integrating, we obtain the following explicit form for the payoff function when the investment policy is restricted to the range (k_1, k_2):

$$\begin{aligned} \mathrm{E}(R_p \mid r_m) = {} & r_m k_1 \mathrm{pr}(Z \le k_1^*) + r_m k_2 \mathrm{pr}(Z \ge k_2^*) \\ & + (ar_m + br_m^2)\mathrm{pr}(k_1^* \le Z \le k_2^*) + b\sigma_y r_m \int_{k_1^*}^{k_2^*} zf(z)\, dz, \end{aligned} \tag{12}$$

where $k_i^* = (k_i - a - br_m)/b\sigma_y$, $i = 1, 2$, and $Z \sim N(0, 1)$.

Figure 1 compares an unrestricted payoff function (dotted line) and the restricted payoff function (dashed line) obtained when $k_1 = 0$ and $k_2 = 1$ for fictitious but realistic parameter values $a = 0$, $b = 10$, and $\sigma_y = 0.05$. It illustrates what happens to the payoff curve when the manager is instructed not to short sell or borrow to invest in the market. As we can see, the expected portfolio

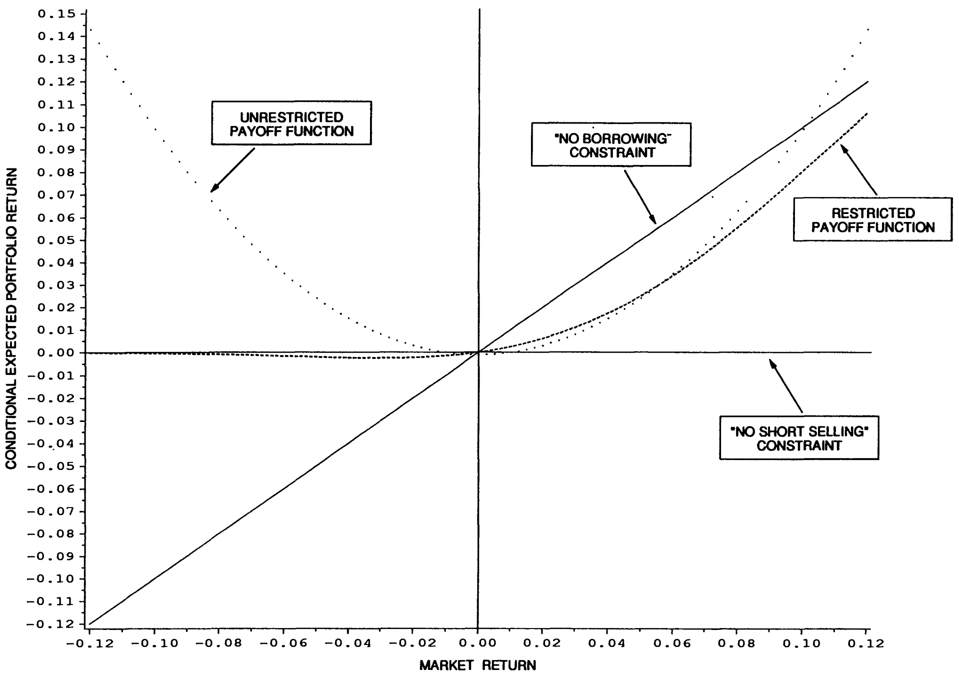

Figure 1. A restricted and an unrestricted payoff function. The dotted line represents the conditional expected return $E(R_p \mid r_m) = ar_m + br_m^2$ of an unrestricted portfolio under the basic information model of Section I, with parameter values $a = 0$ and $b = 10$. The dashed line shows how this payoff function is altered if the manager is instructed not to short sell or borrow to invest in the market. Observe that, in that case, $E(R_p \mid r_m)$ is comprised between the x-axis and the solid line y = x.

return is always comprised between the (solid) lines $r_p = 0$ and $r_p = r_m$ and so is negative for negative realizations of the market. This is exactly the type of behavior predicted by Treynor and Mazuy (1966) for the characteristic line of a fund that outguesses the market with better than average success. More generally, the introduction of constraints of the form (10) on a manager's investment policy implies that his or her payoff function will always lie between $k_1 r_m$ and $k_2 r_m$, the value of R_p approaching either $k_1 r_m$ or $k_2 r_m$ under extreme market conditions. Note also that, when k_1 and k_2 approach a common value, k, the manager has less and less room to maneuver and his or her portfolio return becomes approximately equal to kr_m.

III. Performance Valuation

In order for fund management to translate into benefit for a client, Treynor and Mazuy (1966) indicate that a manager's ability to use private information to outguess the market must enable him or her "to vary the fund volatility systematically in such a fashion that the resulting characteristic line is concave upward. . . ." Where constraints are being imposed on a portfolio investment strategy, the performance of a manager who has timing ability should thus be characterized by the degree of curvature of his or her payoff function in the restricted range of market values where the constraints have little or no effect on the investment policy.

One convenient way of summarizing the curvature of a manager's payoff function would be to regard the latter as a contingent claim, assume continuous trading, and compute its value using a risk-neutral valuation relationship (Brennan (1979), p. 57). For the basic information model of Section I, based on Gendron (1988), the performance value would then be obtained by solving

$$V = (1 + r_f)^{-1} \int_{-\infty}^{\infty} \mathrm{E}(R_p \mid r_m)\, d\bar{F}(r_m), \qquad \bar{F}(r_m) \sim N(0, \sigma^2), \tag{13}$$

which yields

$$V = (1 + r_f)^{-1} \sigma^2 / \theta \sigma_y^2 \tag{14}$$

for the value of the manager's performance, where r_f is the risk-free rate.

This constant is representative of the price that a client would be willing to pay for a manager's services. For realistic (positive) values of the risk aversion and risk-free rate parameters, V will be positive and larger for "better" managers, inducing a ranking among them. From formula (14), we see that the value of a manager's performance upon whom no constraints are imposed will increase with the precision of his or her signal, the variability of the market return, and, in accordance with Treynor and Mazuy (1966), the manager's willingness to bet on his or her own forecast as quantified by θ, the risk aversion parameter.[4] Alternatively, V could be viewed as a measure of the curvature of a manager's quadratic payoff function. If the latter were simply linear, V would be 0 from the symmetry of the normal distribution. This coincides with our intuition that a manager is not worth consulting if his or her advice is to invest a fixed proportion of the client's portfolio in the market, irrespective of the anticipated market levels.

A similar valuation procedure can be followed for the restricted model, where the payoff function is given by equation (12). A straightforward but tedious calculation shows that

$$V_r = (1 + r_f)^{-1}(V_1 + V_2 + V_3), \tag{15}$$

where

$$V_1 = k_1 \frac{\sigma^2}{\sigma^2 + \sigma_y^2} \int_{-\infty}^{c} y\, dF(y),$$

$$V_2 = k_2 \frac{\sigma^2}{\sigma^2 + \sigma_y^2} \int_{d}^{\infty} y\, dF(y),$$

$$V_3 = \frac{a\sigma^2}{\sigma^2 + \sigma_y^2} \int_{c}^{d} y\, dF(y) + \frac{b\sigma^2}{\sigma^2 + \sigma_y^2} \int_{c}^{d} y^2\, dF(y),$$

$$c = \frac{k_1 - a}{b}, \quad d = \frac{k_2 - a}{b}, \quad \text{and} \quad F(y) \sim N(0, \sigma^2 + \sigma_y^2).$$

[4] Preference free measures could also be devised. For instance, Professor Tinic suggested, in a private discussion, a conditional reward to variability ratio which, from (8) and (9), would be preference free in the unrestricted case.

Although this expression is more intricate than (14), its interpretation remains essentially unaltered. Note, in particular, that, when $k_1 \to -\infty$ and $k_2 \to \infty$, V_1, V_2, and the first summand in V_3 become negligible, so that (15) reduces to (14).

For illustration purposes, take $\sigma^2 = 0.01$ and consider once again the fictitious parameter values used in Figure 1, with $k_1 = 0$ and $k_2 = 1$ as in the Canadian mutual fund example. In that situation, the value of the manager's performance, as calculated from (15), would reduce to 31.45% of its unrestricted level. This illustrates the considerable impact that imposing constraints on the investment policy may have on the value of a well informed manager. This suggests also that ignoring constraints when they are present may lead to incorrect conclusions concerning the performance of a portfolio manager.

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