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Equity Issues and Stock Price Dynamics

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ABSTRACT

This paper presents an information-theoretic, infinite horizon model of the equity issue decision. The model predicts that (a) equity issues on average are preceded by an abnormal positive return on the stock, although for some firms the issue is preceded by a loss; (b) equity issues on average are preceded by an abnormal rise in the market; and (c) the stock price drops at the announcement of an issue. The model provides a measure of the welfare cost of asymmetric information; the welfare loss may be small even if the price drop at issue announcement is large.

THE ADVERSE SELECTION PROBLEM when firms issue new securities has been the focus of much recent work in corporate finance. Seasoned equity issues have received particular attention, in part because there are striking stock price patterns around the time of equity issues and considerable variation over time in the number of issues. In this paper we present a dynamic, infinite horizon model of the firm's equity issue decision under adverse selection. The model can simultaneously account for each of the following empirical observations about equity issues:[1]

- Stock prices of issuing firms on average exhibit a large and extended positive abnormal return prior to an equity issue. A sizable fraction of firms, however, have price declines in the period preceding an issue.

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[1] Price behavior around equity issues has been extensively documented by Asquith and Mullins (1986), Masulis and Korwar (1986), Mikkelson and Partch (1986), Barclay and Litzenberger (1988), and Korajczyk, Lucas, and McDonald (1990). A stylized quantitative summary of the results of these studies is as follows: researchers find an abnormal return on the order of 30 percent in the 500 days preceding the issue, but with substantial variation in the price history across firms. Equity issues appear to follow market rises. There is a significant price drop at issue announcement of about 3% for industrial issues. Asquith and Mullins (1986) find that, on average, the drop in the value of a firm's total equity at issue announcement represents approximately 30% of the value of the equity being issued. Similar price behavior is observed for pure secondary issues.

The timing of equity issues is the focus of papers by Korajczyk, Lucas, and McDonald (1988) and Choe, Masulis, and Nanda (1989). The latter find that there are significantly more industrial equity issues in business cycle expansions than in contractions.

- There is substantial variation over time in the volume of equity issues. Issues on average follow increases in the market as a whole.
- The stock price drops significantly upon the announcement of an equity issue. After the issue announcement, returns appear to be normal.

There are, of course, other explanations, not based on adverse selection, for each of these facts. However, the ability of the model to explain all these phenomena in a unified framework adds support to the contention that adverse selection is of major importance in understanding equity issues. The model also sheds light on the relation between stock price behavior and the welfare consequences of adverse selection.

The price drop at issue announcement has received theoretical as well as empirical attention,[2] as has the temporal variation in the volume of issues.[3] The preceding price rise has not been formally modeled, however. Technically, the model differs considerably from earlier models of equity issues due to the assumptions of infinitely lived firms and short-term information asymmetries between managers and outside investors. This allows both the long-term price path and the timing of issues to be characterized.

The idea behind the model is as follows. Suppose that managers have information about the firm today (such as the value of current earnings) that the market will not acquire until next period. The firm is "undervalued" if the revelation of this information will cause the share price to rise and is "overvalued" if revelation will cause the price to drop. Suppose further that managers act in the interest of shareholders and that equity issues are necessary to finance projects. If projects are long-lived and if waiting is not too costly, we would expect undervalued firms to delay issuing until the undervaluation is corrected. On the other hand, overvalued firms issue immediately since waiting may entail the loss of the project and a downward assessment in the valuation of the firm.

This timing behavior by the two types of firms has a clear implication for the price path preceding the announcement of the equity issue. Suppose that project arrival is independent of a firm's price history. Overvalued firms which receive a project will have average performance prior to their equity issue announcement since they issue immediately upon receiving a project. However, undervalued firms which receive a project will wait to issue. They will therefore have above average performance preceding the equity issue since they have waited for the undervaluation to vanish before issuing. Given these two price paths, equity issuers on average have positive abnormal returns preceding the issue.

[2] Scholes (1972) suggested that informational considerations could explain the price drop. In an influential paper, Myers and Majluf (1984) presented a formal model of the price drop based on asymmetric information.

[3] Korajczyk, Lucas, and McDonald (1988) and Choe, Masulis, and Nanda (1989) have proposed theoretical explanations for the clustering of equity issues in certain periods. Both papers emphasize that, if firms can decide when to issue equity, they will do so in periods when asymmetric information is expected to be relatively unimportant. In this paper, by contrast, bunching occurs even though the fundamental firm-level asymmetry between firms and the market is stationary through time.

Since firms announcing equity issues tend to be overvalued according to this explanation, the price may drop at issue announcement.[4] This is a consequence of adverse selection on the part of issuing firms, as in Myers and Majluf (1984).

This story also explains variation over time in the volume of equity issues and the fact that equity issues tend to follow general increases in the market. Suppose that the percentage of firms in the economy which are overvalued or undervalued varies randomly over time, and consider a period in which an above average number of firms have private information that they are undervalued. As information about these firms becomes public, there will be a general market rise. An above average number of firms issue equity following the market rise because firms which anticipated an increase in their price waited to issue equity. Thus, we expect to see bunching of equity issues following a general market rise.

In the next three sections we present the model, define an equilibrium, and study the behavior of the share price around issue announcement.[5] Section IV demonstrates that issues are concentrated after market rises if the percentage of overvalued firms varies over time. In Section V, simulation results are compared to earlier empirical findings. For plausible parameter values, the simulations generate realistic price paths and time variation in issues. Section VI discusses the welfare implications of the analysis. The conventional wisdom that the cost of an equity issue can be measured by the stock price drop at issue is incorrect in this model. We suggest a measure of the true cost.

The ability to issue risk-free debt and the existence of financial slack both lessen the importance of adverse selection. In this paper we focus on the price and timing of equity issues rather than the debt–equity choice, so for simplicity we assume that the firm must use equity financing. Obviously, a complete model of the firm would also account for the choice of security. In Section VII we discuss some of the costs of these alternative financing methods in order to understand why firms might nevertheless issue equity. The possibility that adverse selection could be reduced by signaling is also discussed in Section VII. Section VIII concludes.

I. The Environment

This section describes the environment in which firms and investors operate.

A. The Firm

At time t a firm's market value equals the perceived value of its assets in place A_t and the present value of all future investment opportunities. Each period the market receives news about the value of assets in place, which changes by a

[4] If there is a cost to issuing equity, it is possible for the price to rise at the announcement of the issue. See Section IV.

[5] In practice, the announcement date precedes the actual issue date by several weeks, and the larger price effect is seen at the announcement. In this paper we do not distinguish between the two events, so henceforth we will refer simply to the issue date.

constant proportion each period:

$$A_{t+1} = \begin{cases} uA_t & \text{with probability } p, \\ dA_t & \text{with probability } (1-p), \end{cases}$$

where $u > 1 > d$. (Time variation in p will be introduced in Section IV.) On average, perceived asset value remains unchanged: $up + d(1-p) = 1$. At time t, the manager knows next period's news and hence knows A_{t+1}, the value the market will assign to assets in place next period. Thus, the manager has "one-step-ahead" knowledge of A_t.[6]

Valuable investment opportunities which we call "projects" arrive with constant probability q, independently of whether the value of assets in place increases or decreases. A firm receives at most one project per period. If a firm with a project receives a second project, the second displaces the first so that the firm only has one viable project at a time. (This can be motivated by the assumption that the firm has a scarce resource required to maintain projects which have arrived but not been taken.[7]) We assume that a project can only be undertaken if equity is issued. The market cannot observe whether or not a firm has a project before it is financed.

With probability $1 - \rho$ the firm loses all current projects and future project arrivals and is liquidated for A_t.[8] To maintain a stationary equilibrium, we assume that a new firm with no price history replaces each firm that is liquidated. Once a project arrives, it lasts as long as the firm remains viable.[9]

In order to invest in a project, the firm must raise KA_t from equity issues. The net value of the project (gross of issue costs) is $\beta_H^* A_{t+1}$. Thus, the capital requirement for the project is linked to a publicly known quantity, but the value is linked to next period's asset value and thus is privately known by the manager.

Any firm which issues equity, whether or not it has a project, bears a cost $\beta_L A_{t+1}$, $\beta_L < 0$, which can be thought of as an underwriting fee and associated distribution costs. Thus, a project has a net present value proportional to $\beta_H = \beta_H^* + \beta_L$. Henceforth, we will speak of firms as having a project with value $\beta_H A_{t+1}$, or having no project, in which case the firm can issue equity at a cost $\beta_L A_{t+1}$. The assumption that K, β_H, and β_L are proportional to A_t is made for tractability; similar results will obtain if they are fixed.[10]

[6] Evidence that management can successfully anticipate market price is given by Seyhun (1986), who shows that insiders systematically earn positive abnormal returns on purchases and sales of their stock.

[7] We restrict the number of projects simultaneously available primarily to avoid the complexity of making the amount of capital raised a choice variable.

[8] The possibility of liquidation keeps the value of the firm finite. Otherwise, a non-dividend-paying firm with no chance of liquidation and which is expected to receive an infinite stream of valuable projects would have an infinite value.

[9] It is possible to analyze the case in which projects live only a finite number of periods. This complicates the analysis without changing the qualitative results.

[10] Making the project cost and NPV proportional to A_t generates a homogenous value function, which makes the problem easy to solve. If the project cost and NPV were fixed, the solution would depend upon the size of the firm relative to the project, and there would be regions in which high and low asset value firms separated and regions in which they pooled.

Project value is assumed to be proportional to A_{t+1} so that managers have no incentive to wait to take projects in the absence of asymmetric information.

Since investors cannot observe the arrival of profitable investment opportunities, β_H and β_L firms are ex ante indistinguishable. We assume that investors learn the true value of a project immediately after it is financed.

B. Managers

Managers choose the equity issue policy so as to maximize the value of the firm to current shareholders. As described above, managers know today what the market will learn next period about the value of assets in place. Since the act of issuing equity can lower share price, however, shareholders may disagree about the desired equity issue policy. For example, shareholders who are about to sell shares may prefer that the manager delay or forego a project if issuing equity to finance the project lowers the current stock price.[11] We assume a sufficient number of long-term shareholders so that management acts in their interest.[12]

Our results (and those of Myers and Majluf (1984), for example) depend on the assumption that managers cannot write a contract allowing them to credibly reveal their private information (Dybvig and Zender (1988)). If managers could do so, issuing equity would be costless and all projects would be financed immediately. Although such a contract is easy to envision in this stylized model, there are difficulties with implementing such a contract in practice. First, it may be hard to prove exactly when a manager acquires information. Second, it may be hard to prove that the manager knew how the market would interpret the information. Third, the manager may have quit or retired by the time the information is revealed. Considerations such as these suggest that contracts can reduce but not eliminate problems of asymmetric information.

C. Investors

Investors are competitive and risk-neutral. In buying newly issued equity, they acquire a share of the firm, s, in exchange for providing KA_t. Investors have access to other investments with a zero expected rate of return.

We summarize the assumed timing of events and information within a period in Figure 1. Note that, because of the assumed timing, firms can issue equity to fund a project in the same period in which it arrives.

II. Equilibrium

In any period, firms are assumed to follow a pure strategy of issuing or not issuing equity and are price-takers with respect to the terms of a sale. We define an

[11] We know of no entirely satisfactory solution to this problem. Miller and Rock (1985) and John and Williams (1985) discuss this issue in the context of dividend and investment policy.

[12] For instance, suppose that shareholders are voting on a permanent share-issuance policy. Every shareholder knows that at some time he or she will be a short-term shareholder and at that time would vote for the manager to forego share issuance. However, it will generally still be in the shareholder's interest to vote in favor of taking projects since never taking a project permanently lowers the value of the firm.

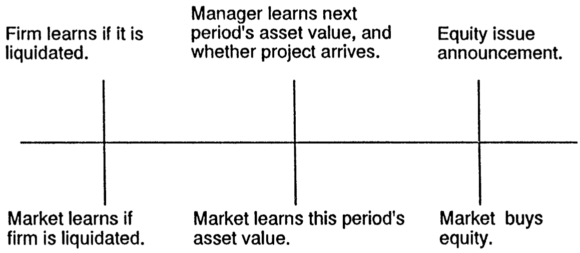

Figure 1. The resolution of uncertainty at time *t*.

equilibrium as follows:

Definition: An equilibrium in the equity issue market is a share s and an issue policy by firms such that

i. investors weakly prefer to pay KA_t for a share s of an issuing firm rather than to invest in an alternative;
ii. firms weakly prefer this issue policy to any other, taking s as given.

The share s may be a function of the commonly observed price history, and the issue policy may be a function of the manager's information.[13]

In this section we posit a time-invariant issue policy and derive sufficient conditions for it to be an equilibrium. Existence is demonstrated by numerical example in Section V.

A. The Value of the Firm from the Manager's Perspective

The manager has private knowledge at time t that the perceived value of assets in place will increase (to uA_t) or decrease (to dA_t) and that the firm either has or does not have a project. Thus, there are four basic types of firms.

Since equity issues will be more costly for undervalued firms than for overvalued firms due to dilution, it is plausible that undervalued firms will defer taking projects, even though they face the prospect that the firm may be liquidated before the project can be taken. If overvalued firms do not take projects, no firm will do so. With this as the intuitive rationale, we will examine an equilibrium with the following properties:

(E1) undervalued firms forego issuing equity even if they have a good project;
(E2) all overvalued firms with a good project issue;
(E3) overvalued firms without a good project do not issue; and
(E4) investors receive a fraction of the firm, s, such that the expected value of their shares is equal to the amount invested, KA_t.

[13] In essence, firms and the market play a game in which firms announce their desire to raise KA_t through an equity issue, and investors accept this offer by demanding a share of the firm s that reflects their beliefs about the value of firms that issue.

The strategy space of the informed players (the firms) is intentionally kept simple (issue or not issue) to avoid the difficult problem of analyzing the response to out-of-equilibrium moves by the informed player in an infinite horizon setting.

Let $V(a, b)A_t$ be the manager's valuation of the firm following the proposed policy (E1)–(E3) at time t in state (a, b), where $a = u, d$ and $b = \beta_L, \beta_H$. This valuation includes the share of the expected value of future new projects accruing to existing shareholders, as well as the value of assets in place. Because the value of assets in place, project value, equity issue costs, and necessary capital are all proportional to A_t, the manager's valuation will also be proportional to A_t. (E1)–(E4) imply that the value of the firm, $V(a, b)A_t$, must satisfy the following set of equations (Appendix A contains a more formal treatment of the maximization problem):

$$V(u, \beta_L) = (1 - \rho)u + \rho(1 - q)u[pV(u, \beta_L) + (1 - p)V(d, \beta_L)] + \rho qu[pV(u, \beta_H) + (1 - p)V(d, \beta_H)], \tag{1}$$

$$V(u, \beta_H) = (1 - \rho)u + \rho u[pV(u, \beta_H) + (1 - p)V(d, \beta_H)], \tag{2}$$

$$\begin{aligned} V(d, \beta_L) &= (1 - \rho)d + \rho(1 - q)d[pV(u, \beta_L) + (1 - p)V(d, \beta_L)] \\ &\quad + \rho qd[pV(u, \beta_H) + (1 - p)V(d, \beta_H)] \\ &= dV(u, \beta_L)/u, \end{aligned} \tag{3}$$

$$V(d, \beta_H) = (1 - s)[d\beta_H + K + V(d, \beta_L)]. \tag{4}$$

Equations (1)–(4) define the value of the firm (per unit of A_t) to existing stockholders, conditional on the manager's information, under the proposed equilibrium issue policy. Equation (1) is the value of a firm in state (u, β_L) that does not issue. With probability $(1 - \rho)$ the firm matures the following period and is liquidated with a terminal payment of u. Otherwise it begins the next period with assets perceived to be worth u times asset value this period and, with the appropriate probability, falls into one of the four states (i.e., the manager will learn that the news two periods hence is u or d and whether a project has arrived). Equation (2) reflects the fact that an undervalued firm postpones or foregoes the project entirely to avoid dilution by pooling with overvalued firms. Equation (3) demonstrates the symmetry between the two types of firms without projects. Finally, (4) indicates that a firm in state (d, β_H) benefits from investment in the project but loses a fraction s of its value to new shareholders.

Equations (1)–(4) treat s as given. The share s is determined by imposing condition (E4) that investors receive a fair value for new shares purchased. Since only overvalued firms with good projects issue in the proposed equilibrium,[14] s is defined by

$$K = s(K + d\beta_H + V(d, \beta_H)). \tag{5}$$

[14] If β_L is not sufficiently negative, then some firms with low asset value may choose to issue equity in order to pool with better firms. It is possible to impose a weaker restriction on β_L so that overvalued firms without a project play a mixed strategy between issuing and not issuing. This results in similar price dynamics but more complicated algebra.

Note that the fair share s under (E1)–(E4) is history-independent because the issue policy is history-independent.

Substituting this into (4) gives

$$V(d, \beta_H) = d\beta_H + V(d, \beta_L). \tag{4'}$$

Note that the system (1), (2), (3), and (4′) is linear in V. The unique solution is given in Appendix A.

B. Sufficient Conditions for Equilibrium

To establish that the proposed policy is an equilibrium, we must ask whether a firm in any given state would do better deviating from the proposed policy, holding the share s fixed. The number of potential policy variations is infinite since a firm can plan to deviate once or multiple times and in any combination of states. It can be shown, however, that, if a one-period deviation decreases firm value in each state individually, so does an arbitrary multiperiod deviation. This result is formalized in Theorem 1, which is stated and proved in Appendix A. Thus, verifying that a proposed strategy is an equilibrium only requires checking that a one-time deviation by a firm in each of the four basic states lowers firm value. The investor's optimization problem clearly is satisfied by the imposition of (E4).

The policy proposed in (E1)–(E4) is not an equilibrium for arbitrary parameter values. As discussed in Section III.*D*, if the asymmetry of information is small, undervalued firms willingly pool with overvalued firms, so all firms with projects issue immediately. In the analysis that follows, we restrict our attention to parameters for which delay is optimal for undervalued firms. The numerical example presented in Section V illustrates that this equilibrium obtains with plausible parameter values.

Note also that multiple equilibria may exist for a given set of parameter values. This occurs when the willingness of a firm to issue depends on whether it believes that other firms of the same type are also issuing. For example, if (u, β_H) firms issue as well as (d, β_H) firms, the equilibrium share s is lower than if only (d, β_H) firms issue because the former are more valuable to investors. Thus, there may be two equilibria: one with a low s and both types issuing and one with a high s and only (d, β_H) firms issuing. Standard refinements of Nash equilibria cannot eliminate this type of multiplicity.

III. Share Price Behavior

A. The Market Price of a Share

Thus far we have concentrated on the manager's valuation of the firm. Investors, however, can only use publicly available information in computing price. Investors know the distribution of firm types in equilibrium, so the market valuation for a firm will be a weighted average of the values of the various types of firms. Interestingly, price is history-dependent because the probability that a firm has an unexploited project increases with the number of periods since the last fall in asset value.

Let q_i denote the probability that a firm with exactly i consecutive asset value increases has a project. Recall that the last time management foresaw an asset value decrease, they adopted any available good project. If the firm has experienced exactly n asset value increases since that time, the probability of having a project is one minus the probability of not having received a project since the initial drop in asset value:

$$q_n \equiv 1 - (1 - q)^{n+1}, \qquad n = 0, 1, 2, \cdots. \tag{6}$$

Let $P(n)A_t$ be the time t stock price immediately prior to announcement of an issue, where n is the number of consecutive price rises for the firm. The price is separable in A_t because the value function is separable. With probability q_n the firm has a project, and with probability p the management privately observes a rise in asset value. Taking these probabilities into account, we have

$$\begin{aligned} P(n) &= p[q_n V(u, \beta_H) + (1 - q_n)V(u, \beta_L)] \\ &\quad + (1 - p)[q_n V(d, \beta_H) + (1 - q_n)V(d, \beta_L)], \qquad n = 0, 1, 2, \cdots. \end{aligned} \tag{7}$$

From equation (7) it is clear that the market price for a firm is increasing in the number of consecutive rises in asset value, as shown in the following lemma:

LEMMA 1: *In an equilibrium characterized by* ($E1$)–($E4$), $P(i)$ *is increasing in* i.

Proof: Lemma A3 in Appendix A shows that $V(\cdot, \beta_H) > V(\cdot, \beta_L)$. Further, q_n is increasing in n. Consequently, the result follows from (7). Q.E.D.

B. Price Behavior at Issue

To measure the short-term effect on stock prices of an equity issue, we compare the market prices immediately before and after issue. Equation (4′) provides the price after issue, with A_t normalized to 1. Subtracting equation (7) gives the price drop for a firm with n consecutive asset value increases:

$$\begin{aligned} d\beta_H + V(d, \beta_L) - \{&p(V(u, \beta_H) + (1 - q)^{n+1}[V(u, \beta_L) - V(u, \beta_H)]) \\ &+ (1 - p)(V(d, \beta_H) + (1 - q)^{n+1}[V(d, \beta_L) - V(d, \beta_H)])\}. \end{aligned} \tag{8}$$

The sign of this expression depends on the degree of information asymmetry and on the number of consecutive asset value increases. As $n \to \infty$, this becomes $p[V(d, \beta_H) - V(u, \beta_H)]$, which is negative. The logic is that a firm with a large number of asset value increases is almost sure to have a project, so the decision to issue equity reveals only that the value of assets in place will fall. For other firms, the decision to issue equity reveals both a negative return on assets in place and the existence of a project, so the price effect can be ambiguous.[15]

[15] Myers and Majluf (1984) note that, if equity issues are costly, the announcement of an equity issue can raise the stock price. The costly equity issue serves as a credible signal about the value of the project.

C. The Equilibrium Price Path Prior to Issue

This section discusses our main result:

THEOREM 2: *In the equilibrium described by* (E1)–(E4), *the average excess returns for a firm issuing equity are positive over the n periods preceding the issue, where n is any positive integer.*

The proof of Theorem 2 is in Appendix B. Here we discuss the logic and robustness of the result.

The idea of the proof is simple. We compute the average return earned over n periods by all firms[16] and compare this to the average return earned over n periods by all firms that issue equity in the nth period. The average return for firms that issue equity exceeds the unconditional return because firms which have had price rises accumulate projects and are therefore overrepresented (relative to all firms) in the sample of firms issuing equity.

Because projects last as long as the firm does, the duration of the price rise is unbounded. However, the incremental price rise from looking back over a longer horizon goes to zero as n becomes large because the probability that a project was postponed for n periods becomes small.[17]

Finally, the model implies that firms that have had a larger price rise prior to the issue will face a larger price drop at announcement of the issue. This occurs because firms with a longer string of price rises on average have a higher probability of having a project and thus have a higher pre-issue price. After the project is taken, all firms have the same price.[18] Empirical evidence on the relation between the price rise and the drop is mixed.[19]

One alternative explanation for the price rise is that the market learns about the arrival of a valuable project, the stock price rises in response to this information, and the firm subsequently issues stock to finance the project.[20] This story, however, leaves unexplained the price drop at issue announcement and also does not account for the observed price rise preceding pure *secondary* issues, in which the firm raises no funds and which therefore are not necessarily linked with project arrival.[21]

D. Other Equilibria

The predicted pattern of price behavior before equity issues depends upon the assumed parameters. There are basically two possible qualitative outcomes in

[16] One might expect a zero average return across all firms, which is the case unconditionally. In this calculation, however, there is a survivorship bias: the selection of firms that were not liquidated in the last n periods induces a positive unconditional return.

[17] It can be shown that, if project life is finite, then the price rise will last no longer than the maximum life of the project.

[18] Unlike the price rise, this result does not appear to be robust with respect to model specification.

[19] Korajczyk, Lucas, and McDonald (1990) show that the price drop is negatively correlated with the price rise over long periods and positively correlated over short periods, a result consistent with apparently contradictory results in Asquith and Mullins (1986) and Masulis and Korwar (1986).

[20] The price rise in our model is not consistent with this story since project arrival is assumed to be unobservable.

[21] Price behavior preceding secondary issues is qualitatively similar to that for primary issues. For a more complete discussion of alternative theories, see Korajczyk, Lucas, and McDonald (1990).

this model: (a) the price path is flat prior to issue, and (b) the price rises prior to issue.

Intuitively, the willingness of undervalued firms to postpone financing a project increases with the degree of information asymmetry (the difference between d and u) and increases with project and firm durability. If the information asymmetry is small, then in equilibrium all firms with projects issue immediately, and on average prices prior to issue are flat. If there is an adverse selection problem (as assumed in the preceding analysis), there is an incentive for good firms to wait to issue, and the issue will therefore be preceded by a rise, the size of which is directly linked to the magnitude of u and d. The empirical implication of this sensitivity to parameters is that firms with different degrees of asymmetric information should have different price behavior preceding and following the issue.[22]

If the issue cost, $|\beta_L|$, is sufficiently small, firms with low quality assets and no project may prefer to issue equity to pool with higher quality firms. In this case overvalued firms without projects enter the issue market to the point where they are indifferent about issuing. Undervalued firms with projects still have an incentive to delay issues, and hence a price rise precedes equity issues.

IV. The Relation Between the Number of Equity Issues and the Return on the Market

In the analysis thus far, the return on the market was assumed to be the same every period. As discussed above, equity issues are not uniform over time; they tend to follow a general rise in the market. A simple modification of the model generates time variation in the number of issues and permits us to analyze the relation between equity issues and the return on the market.

First we calculate the quantity of equity issues in any period when the probability of an increase in asset value is constant. This requires counting the number of issues by firms with different price histories. The unconditional probability of having n consecutive price rises and surviving n periods is $(\rho p)^n$, for $n = 1, 2, \cdots$. Since probabilities add to one, the fraction of firms with a price drop or no price history (i.e., new firms) is $(1 - \rho p)$. Thus, the probability of having exactly n consecutive price increases is $(\rho p)^n(1 - \rho p)$. For a firm to issue in the current period, it must have a project (which it does with probability q_n), and the manager must observe a drop in asset value (which occurs with probability $(1 - p)$). The quantity of issues at any time is therefore

$$(1 - p) \sum_{i=0}^{\infty} q_i(1 - p\rho)(p\rho)^i = (1 - p)q/[1 - \rho p(1 - q)]. \qquad (9)$$

Note that the number of issues in (9) is less than q, the number that would obtain in the absence of asymmetric information.

[22] For instance, asymmetric information may be less of a problem for public utilities because they have regulated rates of return. In fact, utilities appear to depend more on equity issues than other industries, and Masulis and Korwar (1986) find evidence that the price drop for utilities is smaller.

To capture the fact that aggregate returns fluctuate and that returns are correlated across firms, we now suppose that the probability of an increase in the perceived asset value between times t and $t + 1$ for each firm is stochastic:

$$p_t = p + \epsilon_t. \tag{10}$$

ϵ_t has mean zero, support $(-p, 1 - p)$, and is independently and identically distributed over time. Neither the market nor managers observe ϵ_t until time $t + 1$, so at t the expected probability of an increase is p. Equations (1)–(4′) are linear in p; hence, they remain unchanged when the probability of an increase in asset value is given by (10) and when ϵ_t is independent of project arrivals. If p_t exceeds p, the aggregate return will be greater than average. Thus, we can think of a "market rise" as occurring when p_t exceeds p several times in a row.

More precisely, the unconditional market price at time t reflects historical probabilities, the realized values of assets in place, and the relative proportions of firms with different price histories. Let $\bar{A}_t$ denote the average aggregate value of assets in place at time t. Then the unconditional market price at time t is

$$M_t = \bar{A}_t \sum_{i=0}^{\infty} P(i)(1 - \rho p_{t-i-1}) \prod_{k=1}^{i} \rho p_{t-k}, \tag{11}$$

where

$$\bar{A}_t = \bar{A}_{t-1}[u p_{t-1} + d(1 - p_{t-1})].$$

Market returns can be calculated by simply taking ratios of the market price at different points in time.

The quantity of issues at time t is given by equation (9), modified to keep track of the historical fraction of firms with an increase in asset value:

$$(1 - p_t) \sum_{i=0}^{\infty} q_i (1 - \rho p_{t-i-1}) \prod_{k=1}^{i} \rho p_{t-k}. \tag{12}$$

Appendix C establishes that (11) and (12) increase in p_{t-k} for $k > 0$. Thus, following a number of greater than average p's, there tends to be a greater than average number of issues as well as a general rise in the market.[23] We explore the magnitude of this correlation by simulation in the next section.

V. Simulations

This section compares model simulation results with corresponding empirical results from Korajczyk, Lucas, and McDonald (1990).[24] With reasonable param-

[23] Note that the informational asymmetry between managers and outside investors is constant over time, as is the value of a project relative to assets in place. These assumptions rule out the explanations for equity issue timing offered by Choe, Masulis, and Nanda (1989) and Korajczyk, Lucas, and McDonald (1988), which are based on time-varying asymmetry.

[24] The sample contains 549 seasoned equity issues by NYSE/AMEX industrial firms over the period 1974–1983. The abnormal return for each stock was computed relative to an equally weighted portfolio of NYSE/AMEX and OTC stocks, without adjusting for individual firm betas. Stock and index returns are from the Center for Research in Securities Prices tape. The price drop at announcement of the issue was measured over a two-day window including the day before the

eter values, we find that the model reproduces most of the stylized facts discussed in the introduction.

All the simulations employ the parameters listed in Table I. The existence of an equilibrium for these values was verified following Theorem 1. Parameters were chosen to calibrate the model to some general features of the data and to match the empirical distributions. In fact, a large range of parameter values admit qualitatively similar results.[25] The values of p, d, and u imply a quarterly standard deviation for individual firm value of 20%, or an annual standard deviation of 40%. The distribution of ϵ_t leads to a 28% annual, or 14% quarterly, standard deviation of aggregate market returns.[26] The survival probability implies that firms on average last 25 years.

Figure 2 illustrates the simulated average excess returns conditional on issuing over the eight quarters prior to issue and the price drop at issue. The cumulative abnormal return over the entire period is 24.4%, with a drop at issue of 2.8%. This is quite similar to Figure 3, which has a 47% rise and a 3% drop at issue and which was constructed using market data on issuing firms.[27]

The model generates a cross-sectional distribution of price histories for issuing firms as well as the average price rise. The possibility of negative returns prior to an issue arises because some firms which issue will have suffered a string of price decreases followed by a project arrival. The predicted distribution can be derived analytically, following the logic of equation (B1) in Appendix B. The simulated distribution based on this equation is illustrated in Figure 4. Each bar represents the percentage of firms with pre-issue cumulative abnormal returns less than or equal to the number on the x-axis and greater than the previous number on the x-axis. By comparison, Figure 5 shows the empirical distribution of returns in the seven quarters preceding an actual issue. The predicted percentage of firms with a loss in the two years preceding the equity issue is 37%, while the actual percentage is 23%.

Finally, Figure 7 shows that, on average, market rises precede equity issues, while Figure 6 illustrates the same relation for model-generated data. Figure 7 was constructed by averaging the actual market return over the 450 days preceding and the 100 days following each equity issue. To create Figure 6, a sequence of 1000 i.i.d. random numbers was used to generate a probability sequence $\{p_t\}$, as described in Section IV. The number of issues each period was calculated using (12), with the index i truncated at 15. (The approximation error is small since $p^{15} = 10^{-5}$ at $p = 0.5$.) The corresponding unconditional market returns

announcement. Equity issues were obtained from Drexel, Burnham, Lambert (1974–1983), and announcement dates were obtained from the *Wall Street Journal Index*. The data and calculations are described in detail in Korajczyk, Lucas, and McDonald (1990).

[25] All simulations were done in GAUSS. The programs are available upon request.

[26] The total return in a period is $(p + \epsilon)u + (1 - p - \epsilon)d = \epsilon(u - d)$. For the parameters in Table I, $\epsilon(u - d) = 0.14$. Since ϵ is binomially distributed, 0.14 is the quarterly standard deviation.

[27] Korajczyk, Lucas, and McDonald (1990) find that the predicted excess return is very sensitive to how the excess return is estimated. Although all methods yield a positive excess return, estimates using different market indices range from 20% to 80% over the 500-day period. Thus, we put more emphasis on picking "plausible" parameters rather than matching the empirical rise exactly.

Table I

Parameters Used in Model Stimulations (One Period Is One Quarter)

Probability of firm remaining solvent	ρ	0.99
Value of positive NPV project, per dollar of asset value	β_H	0.3
Probability of arrival of positive NPV project	q	0.009
Cost of equity issue per dollar of assets	β_L	0.05
Ratio of future value of assets to current value if asset value increases	u	1.2
Ratio of future value of assets to current value if asset value decreases	d	0.8
Probability that asset value increases	p	0.5
Required equity issue as a fraction of asset value	K	0.2
Magnitude of move in p when market is "high" or "low" (each occurs with probability 0.5)	ϵ_t	0.35

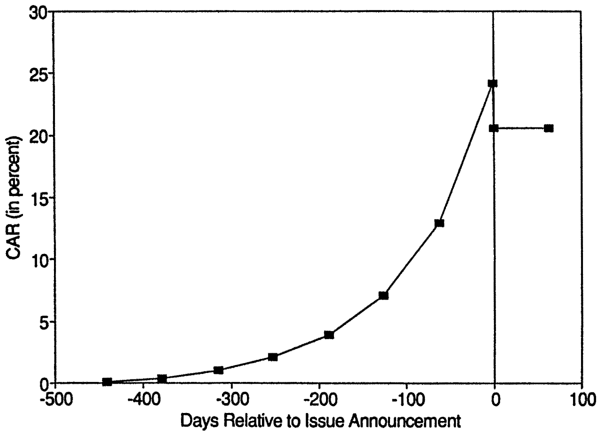

Figure 2. Simulated average cumulative abnormal return over eight quarters preceding an equity issue. Computation described in Apppendix B. Parameters assume a 1% chance of firm liquidation per period; that good projects arrive with probability 0.9% per period, have an NPV of 30% of asset value (net of equity issue costs), and require outside equity financing of 20% of current asset value; that there is a 50% chance that the firm will increase in value by 20% and a 50% chance it will decrease by 20%; and that the cost of an equity issue is 5% of asset value.

history at each date was computed using (11). The price history in the eight quarters preceding each quarter was weighted by the number of issues in that quarter. Summing and dividing by the total number of issues yields the average market price path in Figure 6.[28]

[28] Notice that the market appears to continue rising *after* the issue announcement date in Figure 7. A possible explanation is that firms in our sample both announced an issue and actually issued. If a fall in the market between the announcement day and the issue day makes a firm less likely to proceed with the issue, then we would expect our sample selection procedure to generate abnormal market returns between announcement and issue day.

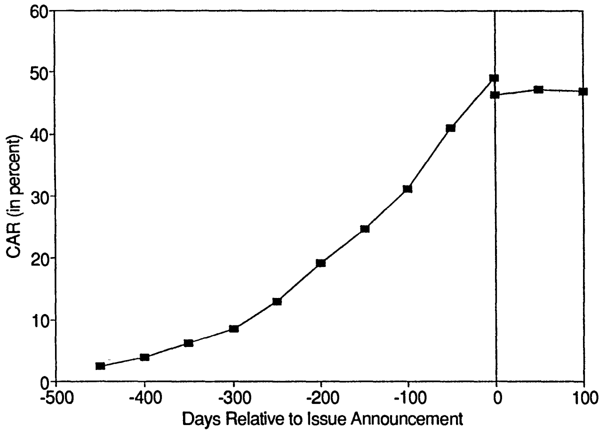

Figure 3. Cumulative abnormal return over 500 trading days preceding an equity issue. Average for 549 seasoned equity issues by NYSE/AMEX industrial firms over the period 1974–1983. Abnormal returns computed relative to an equally weighted portfolio of NYSE/AMEX and OTC stocks, with no adjustment for firm betas.

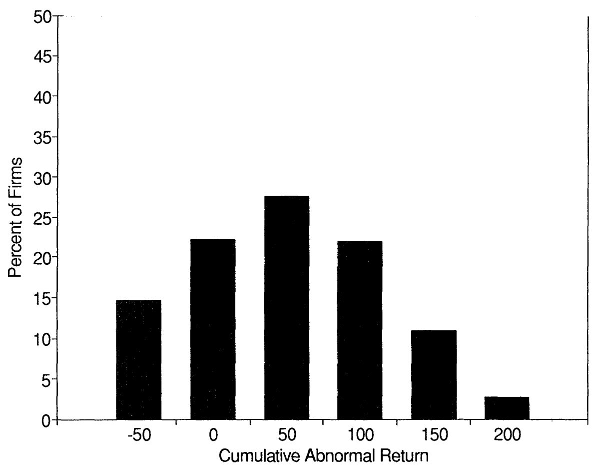

Figure 4. Simulated distribution of cumulative abnormal returns over seven quarters preceding an issue announcement. Parameters assume a 1% chance of firm liquidation per period; that good projects arrive with probability 0.9% per period, have an NPV of 30% of asset value (net of equity issue costs), and require outside equity financing of 20% of current asset value; that there is a 50% chance that the firm will increase in value by 20% and a 50% chance it will decrease by 20%; and that the cost of an equity issue is 5% of asset value.

VI. Welfare Analysis

It is natural to ask whether the magnitude of the price rise or the price drop tells us anything about the welfare costs of asymmetric information. The fact that an equity issue results in a drop in the value of the firm on average equal to 30% of the capital raised (Asquith and Mullins (1986)) has led some observers to conclude

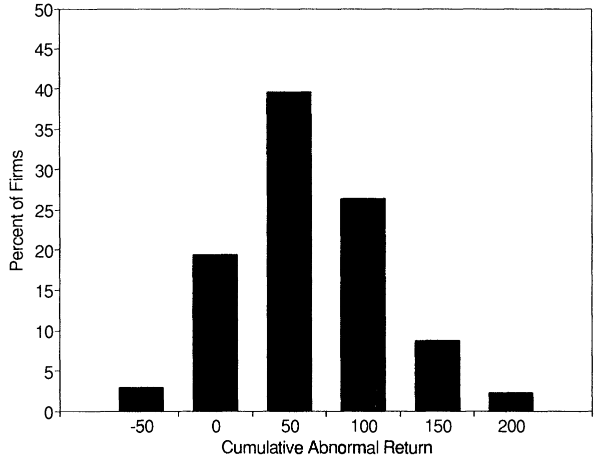

Figure 5. Distribution of cumulative abnormal returns over 500 trading days preceding an issue announcement. Each bar represents the fraction of firms with a CAR less than or equal to the value on the axis below the bar and greater than the value on the axis below the preceding bar, computed for 549 seasoned equity issues by NYSE/AMEX industrial firms over the period 1974–1983. Abnormal returns computed relative to an equally weighted portfolio of NYSE/AMEX and OTC stocks, with no adjustment for firm betas.

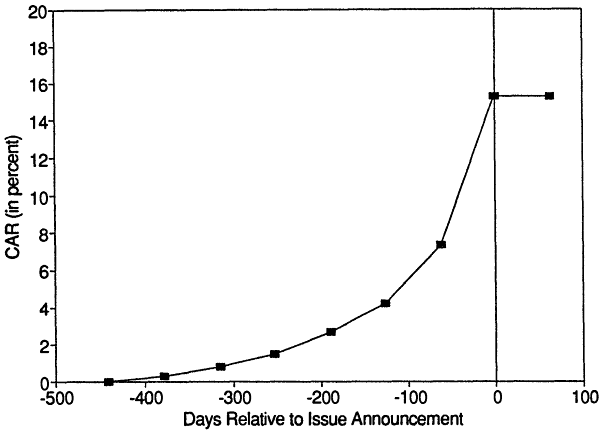

Figure 6. Simulated cumulative abnormal market return over the eight quarters preceding an equity issue. Computed using equations (11) and (12) by creating a simulated market history of 1000 quarters and averaging the market returns over the eight quarters preceding each issue. Parameters assume a 1% chance of firm liquidation per period; that good projects arrive with probability 0.9% per period, have an NPV of 30% of asset value (net of equity issue costs), and require outside equity financing of 20% of current asset value; that there is a 50% chance that the firm will increase in value by 20% and a 50% chance it will decrease by 20%; that the cost of an equity issue is 5% of asset value; and that the percentage of firms going up in value in any period is 85% with probability 0.5 and 15% with probability 0.5.

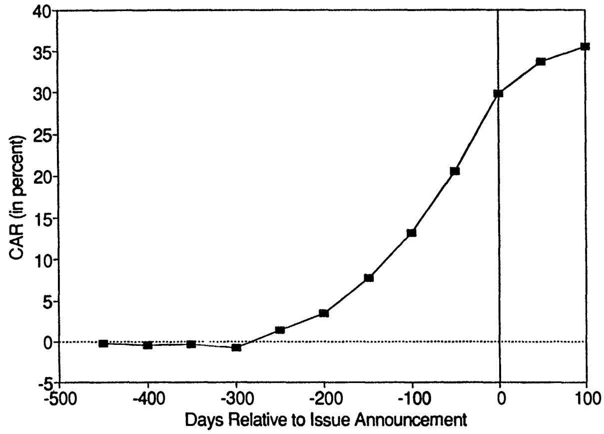

Figure 7. Actual cumulative abnormal market return over the 500 days preceding an equity issue. For each of the 549 seasoned equity issues by NYSE/AMEX industrial firms over the period 1974–1983, we computed the return on an equally weighted portfolio of NYSE/AMEX and OTC stocks. The graph depicts that return, less the then-current risk-free rate, less 8%.

that equity issues are likely to be prohibitively costly. If other forms of financing are unavailable, this suggests potentially large welfare losses. However, this need not be the case. A large price drop at issue can occur even though welfare losses are small.

There are two sources of welfare loss in this model, relative to a full information setting in which all projects would be financed:

- Some projects are lost when firms go out of business while waiting for favorable market conditions to issue. In this model, effective project durability is increasing in ρ, the probability that the firm remains viable for another period.
- A project may be displaced by a new project. The probability of displacement is positively related to the project arrival rate, q.

We have assumed that there is also an explicit equity issue cost β_L, but this cost would be borne even in a full information setting and is not attributable to information problems. The welfare loss due to asymmetric information can be measured using the ratio of projects adopted each period to project arrivals each period. Dividing equation (9) by q gives the steady state fraction of projects accepted:

$$\sigma = \frac{(1 - p)}{1 - (1 - q)\rho p}. \tag{13}$$

Clearly σ increases in ρ and decreases in q. The total loss each period is the fraction of projects foregone times the average value of each project: $(1 - \sigma)\beta_H$. Thus, if projects are fairly durable, and if the frequency with which firms have no alternative but to use equity is low, welfare losses will be minimal.

The reason that the welfare loss need not be related to the price drop at issue (or to the preceding rise) is that the price dynamics depend primarily on the manager's private information about the value of the firm next period. Algebraically, the slope of the price rise and the extent of the drop depend on the ratio u/d, while σ is independent of u/d. By issuing shares, management effectively releases information early. This has no social cost, although it may have a private cost to shareholders who sell immediately following the equity issue announcement. The probability of losing projects is already impounded in the stock price and thus does not affect any of the price reactions. Thus, there is no tight link between price behavior and the social cost of asymmetric information.

VII. Alternative Means of Project Financing

We have focused upon equity issues as a way to finance projects, but there are obviously alternatives, such as debt finance or the accumulation of slack. In addition, there is the question of whether firms can somehow signal to the market their quality to avoid the adverse selection problem. In this section we discuss these issues.

A. Debt

The possibility of debt finance raises two issues: first, why does any firm issue equity instead of debt; and second, what price patterns should be observed around the time of debt issues, and what price patterns are actually observed?

As Myers and Majluf (1984) and others have noted, there is no adverse selection problem associated with issues of riskless debt. There is an adverse selection problem with risky debt, though often it is less than for equity issues (Narayanan (1988)).

Nevertheless, there are reasons why a given firm may prefer an equity issue to a debt issue. Conflicts of interest, restrictive covenants on existing debt, loss of tax shields (which may generate a tax preference for equity issues), and bankruptcy costs all provide reasons why equity issues may be preferred to debt issues, the adverse selection problem notwithstanding.

If firms do issue risky debt, however, the logic of our model should still apply.[29] In general we would expect a price rise preceding a risky debt issue and a price drop at announcement of the debt issue. To the extent that the adverse selection problem with debt is less than with equity, however, these effects should be attenuated.

The evidence on stock price reactions to debt issues suggests that there are small or nonexistent price effects from issuing debt. Mikkelson and Partch (1986) report a small, negative but insignificant price drop at the announcement of a debt issue and a significant −4% stock cumulative abnormal return in the

[29] Consider a world in which debt issues are free of adverse selection problems but in which *existing* debt is more costly than *existing* equity. It seems likely that, in such a model, firms would use debt for "bridge financing" but would issue equity to replace the debt when the firm became overvalued. In such an equilibrium, the predicted price dynamics and timing of equity issues should be qualitatively the same as in the model without debt.

60 days preceding a debt issue. They do not break down the pre-debt issue stock price behavior by debt risk class, and the study does not distinguish between debt issued to retire existing debt and wholly new debt. Eckbo (1986) also reports small insignificant price effects from announcing a straight debt issue, even after controlling for the risk of the debt and the use of funds. To the extent that debt issues are predictable, measured stock price reactions will tend to be zero even if adverse selection is important.

B. *Slack Accumulation*

It has often been suggested that the accumulation of financial slack is a way to avoid the adverse selection problem. In the context of this model, a natural question is whether an overvalued firm without a project would find it worthwhile to issue equity to build slack. The model in this paper does not allow for slack and thus cannot provide a formal answer.[30] However, there appear to be costs as well as benefits to slack in this model.

The benefit of slack is clear: if a project arrives and the firm is undervalued, the project can be immediately financed with slack, avoiding the danger that the project will lose value prior to the firm's becoming overvalued.

The cost is that, by issuing equity to hold slack, the firm pays the equity issue cost β_L before it is necessary to do so. If the firm liquidates prior to receiving a project, this cost would have been borne needlessly. For actual equity issues, Smith (1977) shows that the explicit cost of a large underwritten equity issue is likely to be between 4% and 7% of the value of the issue. If project arrivals are rare and the probability of liquidation (or the interest rate) is high, the future value of this cost will be large compared to the benefit of speeding up project financing once a project arrives. Under these conditions it seems unlikely that equity-financed slack would be optimal.

More generally, however, a profitable firm without a project could accumulate slack without bearing a fixed cost by investing earnings in marketable securities. This also may not be optimal. First, holding marketable securities may be costly for tax reasons: if there is a tax advantage to issuing debt, there is a tax disadvantage to holding marketable debt as slack. Second, slack may be costly for agency reasons since it gives the manager control over extra resources. Shareholders attempting to rein in the manager by limiting free cash flow would not want the firm to hold slack. Finally, even if a firm did optimally accumulate slack, there might still be times when the slack was insufficient and non-slack financing was required.

C. *Signaling*

Rather than delaying an issue, high quality firms with a project could reveal their type by incurring a dissipative signal, in effect "burning money" to prove quality. John and Williams (1985) and Daniel and Titman (1989) give examples of this type of equilibrium.

[30] If a firm without a project were to issue stock, the proceeds after paying β_L would be invested in a zero NPV project with the same risk as the firm.

Depending upon parameters, money-burning may or may not be preferred to delay in our model. For a signal to be effective, the cost must be large enough to prevent low asset quality firms from paying the cost and mimicking high asset quality firms. Deferring a project is costly in that liquidation may occur before the next period, and another project may arrive and be lost. However, with a small enough liquidation probability and project arrival rate, this cost can be quite small. It seems likely, therefore, that delay will often be preferred to money-burning.[31]

One type of signal that has been discussed in the context of equity issues is the choice of project scale. Good quality firms could partially fund a project, while low quality firms could fully fund the project. Firms would thereby reveal their type through the scale of the equity issue. There are at least two reasons for thinking that scale may not be a practical signal. First, it may be costly to reduce scale if, for instance, the firm bears fixed costs of issuing equity (Smith (1977)). Second, the optimal size of a project may not be readily apparent to the market. In order for reduced scale to be a credible signal, a firm must first signal the optimal scale. As long as some firms find delay less costly than signaling, the price dynamics predicted by the model should be observed.

VIII. Concluding Remarks

This paper presents a simple information-based model that accounts for a large number of facts about price behavior around equity issues and the timing of these issues. First, the model replicates the long price rise prior to issue. Interestingly, although the duration of the assumed information asymmetry is short (the manager never has private knowledge of more than next period's valuation), the price rise can be long-lived because firms which by chance have many positive surprises will defer equity issues and appear to have long price run-ups. This policy also generates a realistic cross-sectional distribution of pre-issue price histories. Second, we show that time variation in the aggregate average quality of assets generates the observed positive correlation between the volume of issues and general market returns.

Extensions of the model can potentially explain other phenomena, such as open market share repurchases. Suppose that firms periodically find it optimal to repurchase shares and that management avoids diluting the remaining shareholders. Undervalued firms will repurchase shares immediately, while overvalued firms will wait for the overvaluation to disappear. Equilibrium price behavior should be similar to that in this paper. Repurchases will on average be preceded by an extended price decline, and the announcement will generate a price rise. Vermaelen (1981), in fact, finds that repurchase announcements are preceded by negative cumulative abnormal returns and accompanied by positive abnormal returns.[32]

[31] An example of delay as an effective signal is in Gale (1987), who studies the timing of equity sales by risk-averse entrepreneurs.

[32] Vermaelen (1981) finds that open market repurchases are preceded by a statistically significant CAR of −7% in the 60 days preceding the repurchase announcement. Dann (1981) finds that tendered repurchases are preceded by a mean negative return of about 5% over the same period; Vermaelen, however, finds that the CAR for tendered repurchases is zero.

Appendix A

Sufficient Conditions for Verifying the Existence of Equilibrium

This appendix develops more formally the maximization problem posed in the text and proves Theorem 1, which gives a sufficient condition for a proposed issue policy to be an equilibrium policy. Also, the value function defined by (1)–(4) is solved and characterized.

Let $a_t = u, d$ denote the manager's private knowledge about the perceived value of assets in place at time $t + 1$, $b_t = \beta_H, \beta_L$ denote the presence or absence of a project, and H_t denote all other information in the manager's information set at time t including the firm's history of asset value and project arrivals. Let $\sigma_t = (a_t, b_t, H_t)$ denote the current state. E_t is the conditional expectation operator, where the expectation is conditioned on σ_t. The manager selects an issue policy at time t as a function of his or her information:

$$I_t(\sigma_t) = \begin{cases} 0: & \text{do not issue equity this period,} \\ 1: & \text{issue equity this period.} \end{cases}$$

The manager's problem is to choose I_t each period to solve

$$\begin{aligned} J_t(\sigma_t; I_t) = \max\nolimits_{\{I_t\}}\{&(1 - sI_t) \\ &[(k + a_t b_t)A_t I_t + (1 - \rho)a_t A_t + \rho E_t J_{t+1}(\sigma_{t+1}; I_{t+1})]\}. \end{aligned} \tag{A1}$$

LEMMA A1: *The value of following the optimal strategy is bounded above by*

$$B = u + u\beta_H/(1 - \rho) + K[(1 - \rho) + u\rho]/(1 - \rho).$$

Proof: The value of the firm will obviously be greatest if projects arrive every period ($q = 1$ and $b = \beta_H$ always) and investment capital is free ($s = 0$). Under these assumptions, it is obvious from (A1) that the optimal policy is to issue always ($I_t = 1$). (A1) then becomes (with A_t normalized to 1)

$$\begin{aligned} J(d, \beta_H, \cdot; I) &= k + d\beta_H + (1 - \rho)d \\ &\quad + \rho d(pJ(u, \beta_H, \cdot; I) + (1 - p)J(d, \beta_H, \cdot; I)), \\ J(u, \beta_H, \cdot; I) &= k + u\beta_H + (1 - \rho)u \\ &\quad + \rho u(pJ(u, \beta_H, \cdot; I) + (1 - p)J(d, \beta_H, \cdot; I)). \end{aligned}$$

$J(u, \beta_H, \cdot; I)$ has the solution above. Q.E.D.

We now demonstrate that, if it is not optimal to deviate from a proposed policy for one period, then it is not optimal to deviate for any number of periods. Let I^* denote the proposed optimal policy, $W_t(\sigma_t; I_t^*)$ denote the value of following the proposed policy forever beginning at time t, and $W_t^n(\sigma_t; I_t)$ equal the value of following an alternative policy I for n periods and following I^* thereafter.

LEMMA A2: *If $W_t^1(\cdot; I_t) \leq W_t(\cdot; I_t^*)$, then $W_t^n(\cdot; I_t) \leq W_t(\cdot; I_t^*)$.*

Proof: The proof is by induction. The base case is assumed. We need to show that, if $W_t^{n-1}(\cdot; I_t) \leq W_t(\cdot; I_t^*)$, then $W_t^n(\cdot; I_t) \leq W_t(\cdot; I_t^*)$. By the definition of $W_t^n(\cdot; I_t^*)$, we have

$$\begin{aligned}W_t^n(\sigma_t; I_t) &\equiv (1 - sI_t)[(k + a_t b_t)I_t A_t + (1 - \rho)a_t A_t + \rho E_t W_{t+1}^{n-1}(\sigma_{t+1}; I_{t+1})]\\ &\leq (1 - sI_t)[(k + a_t b_t)I_t A_t + (1 - \rho)a_t A_t + \rho E_t W_{t+1}(\sigma_{t+1}; I_{t+1}^*)]\\ &\equiv W_t^1(\sigma_t; I_t)\\ &\leq W_t(\sigma_t; I_t^*). \quad \text{Q.E.D.}\end{aligned}$$

Although finite deviations from the optimal policy are not optimal, it remains to show that arbitrary *infinite* deviations are not optimal. Suppose that there exists an I' such that $W_t^\infty(\sigma_t, I_t') > W_t(\sigma_t, I_t^*)$ for some σ. Again let W_t^n be the value of following I' for n periods. Solving W_t^n recursively (arguments are suppressed when obvious) gives

$$W_t^n = E_t\left[\left\{\sum_{i=t}^{t+n-1} \rho^{i-t}\left[\prod_{j=0}^{i} (1 - sI_j')\right]((k + a_i b_i)I_i' A_i) + (1 - \rho)a_i A_i\right\} + \left\{\prod_{j=t}^{t+n-1} (1 - sI_j')\right\}\rho^n W_{t+n}\right].$$

Since W_t^∞ and W_t^n are computed using the same issue policy for periods t through $t + n$, we have

$$\begin{aligned}W_t^\infty - W_t^n &= E_t\left[\left\{\prod_{i=t}^{t+n-1} (1 - sI_i')\right\}\rho^n(W_{t+n}^\infty - W_{t+n})\right]\\ &\leq \rho^n B, \quad \text{where } B \text{ is defined in Lemma A1.}\end{aligned}$$

Since $W_t^\infty - W_t \leq W_t^\infty - W_t^n \leq \rho^n \bar{B} \longrightarrow 0$ as $n \longrightarrow \infty$, we have a contradiction, and have proven the following:

THEOREM 1: *If a deviation from the assumed policy is not optimal for one period, it is never optimal.*

Solution to equations (1)–(4′): If (E1)–(E4) are an equilibrium, (A1) reduces to (1)–(4′). The solution is given by

$$V(d, \beta_L) = d[1 + \beta_H \rho q d(1 - p)/(1 - \rho)(1 - \rho u p(1 - q))],$$

$$V(u, \beta_L) = [(1 - \rho)u + \rho u(1 - p)\{V(d, \beta_L) + d\beta_H q/(1 - \rho u p(1 - q))\}]/(1 - \rho u p),$$

$$V(d, \beta_H) = V(d, \beta_L) + d\beta_H,$$

$$V(u, \beta_H) = [(1 - \rho)u + \rho u(1 - p)\{V(d, \beta_L) + d\beta_H\}]/(1 - \rho u p).$$

LEMMA A3: $V(\cdot, \beta_H) > V(\cdot, \beta_L)$.

Proof:

$$V(d, \beta_H) - V(d, \beta_L) = d\beta_H > 0$$

$$V(u, \beta_H) - V(u, \beta_L) = [1 - q/(1 - \rho u p(1 - q))]d\beta_H/(1 - \rho u p) > 0. \quad \text{Q.E.D.}$$

Appendix B

Proof of Theorem 2: We first compute the return for firms which have lived at least n periods, without conditioning upon whether they issue equity in the current period. There are 2^n possible price paths over n periods. 2^{n-1} paths culminate in the sequence $\{d\}$ (a fall in asset value), and all of these result in a terminal price of $P(0)$. Similarly, there are 2^{n-i-1} paths which culminate in the sequence $\{d, u, \cdots, u\}$ (u repeated i times), and all of these result in a terminal price of $P(i)$. There are $\binom{n-i-1}{j}$ ways to arrive at this terminating sequence in which exactly j realizations of a u occurred. (The case with n or more consecutive u's is slightly different and is discussed below.)

To compute returns we also must consider the purchase price n periods ago. Because of independence, the fraction purchased at the price $P(0)$ is $(1 - \rho p)$, at $P(1)$ is $\rho p(1 - \rho p)$, at $P(2)$ is $(\rho p)^2(1 - \rho p)$, and so forth. (This is derived in Section IV.)

Combining terms, the total average return for stocks which reach the terminating price $P(i)$, $i < n$, times the probability of that return, is

$$P(i)du^i\left[\sum_{j=0}^{\infty}\frac{(1-\rho p)(\rho p)^j}{P(j)}\right]\left[\sum_{k=0}^{n-i-1}\binom{n-i-1}{k}((1-p)d)^{n-i-k-1}(up)^k\right];\quad 0 \le i \le n-1. \tag{B1}$$

Since the term in the last set of brackets is the binomial expansion of $[(1-p)d + up]^{n-i-1} = 1$, this can be written

$$P(i)du^i\left[\sum_{j=0}^{\infty}\frac{(1-\rho p)(\rho p)^j}{P(j)}\right];\qquad 0 \le i \le n-1. \tag{B1a}$$

For sequences of consecutive u's extending back further than n periods, the purchase price must reflect exactly n fewer u's than the current price. The combined total average return for stocks which reach the terminating price $P(j)$, $j \ge n$, times the probability of that return, is

$$P(i)u^i\left[\sum_{j=0}^{\infty}\frac{P(i+j)(1-\rho p)(\rho p)^j}{P(i)P(j)}\right];\qquad i = n. \tag{B1b}$$

Weighting by the appropriate probabilities, we get the following expression for the n-period return for all firms which survive n periods:

$$\sum_{i=1}^{n-1}P(i)d(1-p)(up)^i\left(\sum_{k=0}^{\infty}\frac{(1-\rho p)(\rho p)^k}{P(k)}\right) + P(n)(up)^n\left(\sum_{k=0}^{\infty}\frac{P(n+k)(1-\rho p)(\rho p)^k}{P(n)P(k)}\right). \tag{B2}$$

For future reference, (B2) is a weighted average of terms of the form (B1), with weights $\omega_i = (1-p)p^i$ for $i = 0, 1, \cdots, n-1$, and $\omega_n = p^n$. Thus, $\omega_i/\omega_{i-1} = p$ for $i = 0, 1, \cdots, n-1$. Note also that (B1a) and (B1b) are strictly increasing in i.

Now consider the return conditional upon issuing equity. Firms with a price of $P(i)$ have a project with probability q_i. Thus, to compute the return conditional upon an equity issue, we need only weight the terms in the above expression. Then (B1) can be modified:

$$\left\{\sum_{i=0}^{n-1} P(i)q_i d(1-p)(up)^i \left(\sum_{k=0}^{\infty} \frac{(1-\rho p)(\rho p)^k}{P(k)}\right)\right.$$
$$\left. + P(n)(up)^n(1-p) \sum_{i=0}^{\infty} q_{n+i} \frac{P(n+i)(1-\rho p)(\rho p)^i}{P(n)P(i)}\right\} \Big/ [\chi], \quad \text{(B3)}$$

where

$$\chi = \sum_{j=0}^{n-1} q_j(1-p)p^j + \sum_{j=0}^{\infty} q_{j+n}(1-\rho p)\rho^j p^{j+n}.$$

Equation (B3) is again a weighted average of the terms like (B1), with weights given by

$$\Omega_i = [q_i(1-p)p^i]/\chi, \qquad i = 0, 1, \cdots, n-1.$$

Since q_i increases in i, the weight on the last term is greater than

$$\Omega_n = [q_n(1-\rho p)p^n]/\chi.$$

The ratio of successive weights is $q_i p/q_{i-1} > p$ for $i < n$. Also, $\Omega_n/\Omega_{n-1} > \omega_n/\omega_{n-1}$. Thus, greater terms in (B3) have proportionately greater weights than the same terms in (B2), so (B3) > (B2). Q.E.D.

Appendix C

In this appendix, we show that, when the return on the market is time-varying, the number of issues at time t is increasing in p_{t-j}, the fraction of firms with an asset value increase j periods earlier. We do this by showing that (12) is increasing in p_{t-j}. Similar algebra demonstrates that the market price (11) increases in p_{t-j}. To begin, note that, for all j,

$$\sum_{i=j}^{\infty} (1-\rho p_{t-i-1}) \prod_{k=1}^{i} \rho p_{t-k} - \prod_{k=1}^{j} \rho p_{t-k} = 0. \quad \text{(C1)}$$

Setting $j = 0$ demonstrates that the weights in (12) sum to 1, so that (12) is a weighted average of terms of the form $(1-p_t)q_i$.

Differentiating (12) with respect to p_{t-j} gives

$$-q_{j-1}\rho \prod_{k=1}^{j-1} \rho p_{t-k} + \left[\sum_{i=j}^{\infty} q_i(1-\rho p_{t-i-1}) \prod_{k=1}^{i} \rho p_{t-k}\right] \frac{1}{p_{t-j}}. \quad \text{(C2)}$$

This can be rewritten to give

$$\frac{q_{j-1}}{p_{t-j}}\left\{\sum_{i=j}^{\infty} \left(\frac{q_i}{q_{j-1}} - 1\right)(1-\rho p_{t-i-1}) \prod_{k=1}^{i} \rho p_{t-k}\right.$$
$$\left. + \sum_{i=j}^{\infty} (1-\rho p_{t-i-1}) \prod_{k=1}^{i} \rho p_{t-k} - \prod_{k=1}^{j} \rho p_{t-k}\right\} > 0. \quad \text{(C3)}$$

From (C1), the last two terms sum to zero. Since q is increasing, the first term is positive. Q.E.D.

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