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ABSTRACT

In a dual-currency, flexible exchange rate model, both nominal and real foreign exchange premia depend on investor risk attitudes, consumption parameters, and the stochastic structure of currency and commodity supplies. When supplies are random, their joint correlation structure determines the sign of the premia. If the money supplies are identically distributed, then all foreign exchange premia, regardless of the currency of denomination, are zero. A positive correlation between the value of a country's currency and its nominal interest rate need not indicate real interest rate movements. Relative bond prices can be negatively correlated with the terms of trade.

RECENTLY, THERE HAS BEEN a great deal of interest in the construction and use of general equilibrium models of an international economy subject to random disturbances.[1] However, with the exceptions of Stulz (1984), Domowitz and Hakkio (1985), Hodrick and Srivastava (1986), and Stockman and Svensson (1987), little effort has been devoted to expressing the relationships examined in these models in terms of parameters describing the underlying stochastic processes of the exogenous variables.[2] Stockman and Svensson incorporated international capital flows into the model of Svensson (1985) and calculated covariances between investment, the current account, the exchange rate, and the terms of trade in terms of exogenous money and commodity supplies. Focusing more on financial market issues, Domowitz and Hakkio and Hodrick and Srivastava simplified the case-in-advance model of Lucas (1982) by assuming that the exogenous variables are independently lognormal and that investors have loglinear utility functions, a procedure which allowed them to relate the premium required on forward exchange contracts to money supply variances. Stulz obtained similar results in a continuous time model where real money balances enter the utility function.

This paper employs a model similar to that developed by Domowitz and Hakkio (1985) and Hodrick and Srivastava (1986), but its emphasis and objectives are quite different. Those authors were concerned only with demonstrating that a time-varying risk premium is a theoretical possibility prior to conducting a series of empirical tests only loosely related to the model itself. Consequently, they did

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[1] See Stockman (1980), Lucas (1982), Domowitz and Hakkio (1985), Svensson (1985), Hodrick and Srivastava (1986), Stockman and Svensson (1987), and Sibert (1989) for discrete time examples. Models with continuous trading have been developed by Stulz (1984, 1987).

[2] Many authors have noted the desirability of achieving such a goal. See, for instance, Hansen and Hodrick (1980, 1983), Adler and Dumas (1983), Giovannini and Jorion (1987), and Huang (1989).

not investigate or exploit the full theoretical implications of their models. This is the objective of this paper. It first considers the difference in nominal expected returns required on bonds denominated in different currencies—the foreign exchange premium. This is determined by investor risk attitudes, consumption parameters, and the stochastic structure of money and commodity supplies. If monetary policies are identical, the premium is zero, regardless of the currency of denomination, a feature which helps explain the Siegel (1972) Paradox.

Unlike the nominal premium, the real premium—the difference in expected real returns—is zero when investors are risk neutral. This is because only fluctuations in real consumption constitute real risk, and risk neutrality implies that investors are indifferent to these fluctuations. However, nominal risk also comprises price level fluctuations, and standard risk neutrality does not imply that investors are indifferent to these fluctuations. As with the nominal premium, the effect of changes in preference parameters on the real premium depends on the stochastic structure of supply.

The model can also be used to address other issues. Using a macroeconomic model, Engel (1986) demonstrated that a positive correlation between a country's currency value and its nominal interest rate need not indicate real interest rate movements. The asset pricing model of this paper confirms this finding and provides additional intuition. If outputs are deterministic, then real interest rates are constant but monetary fluctuations can cause consumption to be positively correlated with the exchange rate. Fluctuations in consumption cause inverse fluctuations in the marginal utility of consumption, which in turn is positively related to the nominal rate of interest. Thus, a country's exchange rate can be negatively correlated with its nominal interest rate even when real interest rates are constant.

A final discussion relates relative bond prices to the terms of trade. Although this relationship does not appear to have been specifically addressed in the literature, the conventional view of balance-of-payments adjustment (as expressed by Turnovsky (1977, Chapter 9)) suggests that relative interest rates should be negatively correlated with the terms of trade: a rise in the terms of trade improves the current account, thereby necessitating a fall in relative interest rates so as to induce a deterioration in the capital account. However, this paper demonstrates that there can in fact be a negative correlation between the terms of trade and relative bond prices. This outcome occurs if each country's money supply is more positively correlated with its own output than with the other country's output.

The results of this paper highlight the importance of money and monetary uncertainty for the pricing of internationally traded assets. Fluctuations in the money supplies generate exchange rate movements which, in turn, affect the home currency returns on foreign assets. Pure monetary uncertainty (monetary fluctuations which occur independently of real output) has no effect on international differences in real returns due to purchasing power parity but constitutes a systematic risk in evaluating nominal (i.e., currency-denominated) returns. The interaction of money and output uncertainty constitutes a systematic risk in evaluating both nominal and real returns.

The paper is organized as follows. Section I reviews the Lucas (1982) model of international consumption and asset choice and derives nominal interest rates and the foreign exchange premium as functions of endogenous variables. Section II outlines the equilibrium implications of the assumed set of monetary arrangements. Section III introduces some additional assumptions which enable the foreign exchange premium to be expressed as a function of the stochastic structure of supply, a task which is carried out in Section IV. Section V derives some comparative static results for this premium. Section VI examines the real premium, while Section VII considers other relationships between endogenous variables in the international economy. Section VIII contains concluding remarks.

I. The Model

The model has two goods and an infinite time horizon. There are two countries $k = 1,2$, each issuing its own currency and containing a representative investor whose initial wealth and underlying preferences are independent of national origin. Thus, each representative investor has the same utility function with the time-separable form:

$$\sum_{t=1}^{\infty} \delta^{t-1} U(X_{1t}^{k}, X_{2t}^{k}),$$

where X_{jt}^{k} is time t consumption of commodity j by the country k investor, δ is the subjective one-period discount factor, and $U(\cdot)$ is the single-period utility function which exhibits concavity and nonsatiation. In period t, the supply of good $j = 1,2$ is Y_{jt} and the supply of currency $i = 1,2$ is M_{it}. Country 1 is completely specialized in the production of good 1, while country 2 is completely specialized in the production of good 2. The government of each country acts as a supplier of its own currency via transfers but plays no other role, prices being determined by the private sector. There are no impediments to international trade or financial flows so that the law of one price holds for all goods and assets.

All prices and payoffs are expressed in monetary terms. As in Lucas (1982), money is introduced into the model by requiring that, in any period, goods purchases be made with money obtained before the goods markets open.[3] Each period, investors trade in goods, currencies, and securities according to the following sequence of events. Each investor enters period t with a portfolio of previously accumulated asset holdings and receives money from payoffs on these portfolios as the state of the world is revealed. Financial markets then open and each investor chooses his or her holdings of money and securities, including claims to future monetary authority transfers.[4] Since investors are assumed to make their cash choices *after* the state of the world is known, positive nominal interest rates ensure that investors desire money balances solely for consumption

[3] This method of introducing money, originally due to Clower (1967), has been incorporated into asset pricing models by Stockman (1980), Lucas (1982), Svensson (1985), Domowitz and Hakkio (1985), and Hodrick and Srivastava (1986). See Lucas (1980) and Kohn (1981) for in-depth discussions of the plausibility of such a device.

[4] Trade in such claims is necessary for the existence of a stationary equilibrium. See Lucas (1982) and Svensson (1985) for further discussion.

purposes.[5] The securities markets then close and commodity markets open, inaugurating the consumption stage of the period t decision, during which the investors purchase goods with the money chosen in the earlier investment stage. Period $t + 1$ then opens with the receipt of payoffs on period t securities as the above process is iterated. Since preferences and initial wealth are assumed to be the same in both countries, there is a perfectly pooled equilibrium in which the supplies of commodities and securities are shared equally between the representative investors. As money is the required medium of exchange, period t nominal prices and exchange rates are determined by market equilibrium conditions and the period t money supplies.

Equilibrium interest rates can be derived by specifying the state-contingent payoffs on bonds.[6] Let r_{it} denote the period t nominal rate of interest on a single-period currency i denominated bond. If an investor purchases a currency 1 bond in period t at currency 1 price $b_{1t} \equiv \frac{1}{1 + r_{1t}}$, he or she receives one unit of currency 1 in period $t + 1$. Given nonsatiation, a standard application of the Bellman Equation to either investor's period t expected utility maximization problem yields the requirement that the discounted marginal utility of the period $t + 1$ payoff expected in period t equal the marginal utility of consumption foregone in period t by purchasing the bond.[7] This marginal utility foregone equals the currency 1 bond price in period t, b_{1t}, times the period t marginal utility of consumption denominated in currency 1.[8] The period $t + 1$ marginal utility equals the period $t + 1$ bond payoff times the marginal utility of period $t + 1$ wealth denominated in currency 1 which, by the Envelope Theorem, equals the period $t + 1$ marginal utility of consumption denominated in currency 1. Equilibrium interest rates are those that satisfy these requirements when all supplies are willingly held. If, in period t, Λ_t is the equilibrium marginal utility of consumption denominated in currency 1, then

$$1 + r_{1t} = \frac{\Lambda_t}{\delta E_t[\Lambda_{t+1}]}, \tag{1}$$

where $E_t[\cdot]$ denotes the period t expectations operator.

Let e_t denote the period t spot exchange rate—the currency 1 price of one unit of currency 2. If either investor purchases a currency 2 denominated bond in period t at currency 2 price $b_{2t} \equiv \frac{1}{1 + r_{2t}}$, then he or she receives one unit of currency 2 in period $t + 1$. In period t, $e_t\Lambda_t$ is the marginal utility of consumption denominated in currency 2. Hence, an identical argument to that given above for

[5] An alternative assumption is that investors obtain cash balances in order to finance future consumption and hence make their cash choices before the state of the world is known. See Svensson (1985) and Stockman and Svensson (1987).

[6] Bonds are in zero net supply in the representative investor model. Such a framework is commonly employed to highlight the essential forces determining equilibrium security prices. However, it bypasses the role of financial securities in shifting risk between heterogeneous economic agents.

[7] See Lucas (1982) for a formal statement of the dynamic programming maximization problem.

[8] Since the law of one price holds in the absence of hindrances to trade, investors can measure and evaluate asset payoffs and prices in terms of either currency.

the currency 1 bond implies

$$1 + r_{2t} = \frac{e_t\Lambda_t}{\delta E_t[e_{t+1}\Lambda_{t+1}]} = \frac{\Lambda_t}{\delta E_t[D_{t+1}\Lambda_{t+1}]}, \tag{2}$$

where $D_{t+1} \equiv \dfrac{e_{t+1}}{e_t}$ is one plus the rate of currency 1 depreciation. Both interest rates are strictly positive as long as monetary growth is sufficiently high relative to the subjective discount factor δ. Henceforth, it is assumed that this requirement is satisfied.

The foreign exchange premium expressed in terms of currency i is the additional expected currency i return required by investors on the currency h denominated bond, $h \neq i$. In period t, the expected currency 1 return on a currency 1 unit investment in the currency 1 bond is equal to $(1 + r_{1t})$, while the same investment in the currency 2 bond yields an expected currency 1 return of $(1 + r_{2t})E_t[D_{t+1}]$. Hence, $F_{1t} \equiv \ln E_t[(1 + r_{2t})D_{t+1}] - \ln(1 + r_{1t})$ is the log-difference in expected currency 1 returns, i.e., the foreign exchange premium in terms of currency 1. Dividing (2) by (1) yields

$$\begin{aligned} F_{1t} &= -\ln\left\{1 + \frac{\operatorname{cov}_t(e_{t+1},\ \Lambda_{t+1})}{E_t[e_{t+1}]E_t[\Lambda_{t+1}]}\right\} \\ &= -\ln\{1 + \operatorname{Cov}_t(e_{t+1},\ \Lambda_{t+1})\}, \end{aligned} \tag{3}$$

where $\operatorname{Cov}_t(e_{t+1},\ \Lambda_{t+1}) \equiv \dfrac{\operatorname{cov}_t(e_{t+1}\ ,\Lambda_{t+1})}{E_t[e_{t+1}]E_t[\Lambda_{t+1}]}$ is the normalized period t covariance between e_{t+1} and Λ_{t+1}, i.e., the regular covariance $\operatorname{cov}_t(e_{t+1}\ ,\Lambda_{t+1})$ divided by the mean of each argument. Equation (3) implies that $F_{1t} >(<)(=)\ 0$ as $\operatorname{Cov}_t(e_{t+1},\Lambda_{t+1}) <(>)(=)\ 0$. The intuition for this is straightforward: if e is negatively correlated with Λ, then the currency 2 bond has a high (low) currency 1 payoff when the payoff has a low (high) marginal value to investors. Hence, investors require a higher expected return on that bond than on the currency 1 bond, whose payoff is independent of consumption opportunities.

The expected currency 2 return on a currency 2 unit investment in the currency 1 bond is equal to $E_t\left[\dfrac{1 + r_{1t}}{D_{t+1}}\right]$, while the same investment in the currency 2 bond yields an expected currency 2 return of $(1 + r_{2t})$. Hence, $F_{2t} \equiv \ln E_t\left[\dfrac{1 + r_{1t}}{D_{t+1}}\right] - \ln(1 + r_{2t})$ is the log-difference in expected currency 2 returns, i.e., the foreign exchange premium in terms of currency 2. Noting that $E_t[\Lambda_{t+1}] = E_t\left[D_{t+1}\Lambda_{t+1}\left(\dfrac{1}{D_{t+1}}\right)\right]$, dividing (1) by (2) yields

$$F_{2t} = -\ \ln\left\{1 + \operatorname{Cov}_t\left(\frac{1}{e_{t+1}}, e_{t+1}\Lambda_{t+1}\right)\right\}. \tag{4}$$

Equation (4) states that $F_{1t} >(<)(=)\ 0$ as $\operatorname{Cov}_t\left(\dfrac{1}{e_{t+1}},\ e_{t+1}\Lambda_{t+1}\right) <(>)(=)\ 0$. The intuition for this finding is analogous to that for equation (3).

Equations (3) and (4) express the foreign exchange premia in terms of covariance between variables which are themselves determined endogneously as functions of the commodity and currency supplies. To express the premia in terms of these exogenous supplies, the equilibrium values of the endogenous variables need to be identified. This task is performed in the next section.

II. Equilibrium Prices and Exchange Rates

Each investor is assumed to have rational period t expectations about the probability distributions of prices and the exchange rate in period $t + 1$. Since the utility function is time-additive, the behavior of commodity prices and the exchange rate in any period can be inferred from the equilibrium requirements in the commodity and money markets for that period. The money demand functions appearing in these market-clearing conditions reflect the uses of money in this two-currency world. If the investor of a given country can purchase imports with his or her own currency, then he or she has no need to hold foreign currency. If, on the other hand, imports can only be purchased with foreign currency, then the investor will demand both currencies. The latter mechanism is the one employed by Stockman (1980), Lucas (1982), Domowitz and Hakkio (1985), Svensson (1985), Hodrick and Srivastava (1986), and Stockman and Svensson (1987) and is assumed to operate here also.[9]

In this model, consumers are required to purchase goods with money obtained before the goods markets open. Let N_{it}^k denote the country k investor's demand for currency i in period t and P_{1t} (P_{2t}) be the currency 1 (2) price of commodity 1 (2) in period t. If nominal interest rates are strictly positive, then the country k investor faces the finance constraints:

$$P_{1t}X_{1t}^k = N_{1t}^k, \qquad k = 1,2, \tag{5}$$

$$P_{2t}X_{2t}^k = N_{2t}^k, \qquad k = 1,2. \tag{6}$$

Equilibrium in the representative investor model is perfectly pooled so that

$$X_{jt}^1 = X_{jt}^2 = \frac{Y_{jt}}{2}, \qquad j = 1,2, \tag{7}$$

$$N_{it}^1 = N_{it}^2 = \frac{M_{it}}{2}, \qquad i = 1,2. \tag{8}$$

Substituting the market-clearing conditions (7) and (8) into the finance constraints (5) and (6) together with the law of one price implies that, in equilibrium,

$$e_t = \alpha(Y_{1t}, Y_{2t})\frac{M_{1t}}{M_{2t}}, \tag{9}$$

where $\alpha(\cdot)$ is the nominal expenditure share ratio: nominal expenditure on good 2 divided by nominal expenditure on good 1. In equilibrium, the finance con-

[9] A discussion of the implications of alternative assumptions about the payments system for asset pricing in a stochastic environment is contained in Boyle (1989). See also Helpman and Razin (1984) for an analysis of alternative payments systems in a deterministic setting.

straints imply that the velocities of circulation both equal one so that money is neutral and $\alpha(\cdot)$ depends only on Y_1 and Y_2.

Let $U^e_{jt} \equiv \dfrac{\partial U(Y_{1t}/2, Y_{2t}/2)}{\partial X_j}$ denote the equilibrium marginal utility of good j in period t. The first-order conditions for optimal consumption choice in period t require that the marginal utility of consumption denominated in currency i equal the marginal utility of good j divided by the currency i price of good j. When commodity markets are in equilibrium (i.e., (7) holds), this can be expressed as

$$\Lambda_t = \frac{U^e_{1t}}{P_{1t}} \quad \text{and} \quad e_t\Lambda_t = \frac{U^e_{2t}}{P_{2t}}. \tag{10}$$

It follows from (5), (7), and (8) that $P_{1t} = \dfrac{M_{1t}}{Y_{1t}}$, while (6), (7), and (8) imply that $P_{2t} = \dfrac{M_{2t}}{Y_{2t}}$. Hence,

$$\Lambda_t = \frac{Y_{1t}U^e_{1t}}{M_{1t}}, \tag{11}$$

$$e_t\Lambda_t = \frac{Y_{2t}U^e_{2t}}{M_{2t}}. \tag{12}$$

III. Investor Preferences and the Stochastic Structure of Supply

Relationships between the endogenous variables in this dual-currency economy can be explicitly linked to the stochastic structure of supply by making two simplifying assumptions:

(i) Commodity and money supplies are lognormal in all periods;
(ii) Each investor's one-period utility function has the loglinear, or Cobb-Douglas, form:

$$U(X_{1t}, X_{2t}) = H(X^S_{1t}X^{1-S}_{2t}), \tag{13}$$

where $H(X^S_{1t}X^{1-S}_{2t}) \equiv H(\mathbf{X}_t)$ is a function exhibiting constant relative risk aversion ρ; i.e.,

$$H(\mathbf{X}_t) = \begin{cases} \dfrac{1}{1-\rho}\mathbf{X}_t^{1-\rho} & \text{if} \quad \rho \neq 1, \\ \ln\mathbf{X}_t & \text{if} \quad \rho = 1. \end{cases}$$

Assumption (i) ensures that all supplies are non-negative and could be justified by the Central Limit Theorem if the supplies were subject to a large number of independent, multiplicatively interacting shocks with finite variances. For any lognormal variable Z,

$$\ln E[Z] = E[\ln Z] + (1/2)\text{var}(\ln Z). \tag{14}$$

As demonstrated in the Appendix, (14) implies:

LEMMA 1: *If the random variables Z and W are lognormal, then*

$$\ln\{1 + \mathrm{Cov}(W,Z)\} = \mathrm{cov}(\ln W,\ln Z).$$

Thus, $\mathrm{Cov}(W,Z)$ has the same sign as $\mathrm{cov}(\ln W,\ln Z)$, the "logcovariance" of W and Z. Given two pairs of lognormal variables (W,Z), the pair with the higher logcovariance also has the higher value of $\mathrm{Cov}(W,Z)$.

Assumption (ii) ensures that each investor spends a fixed share S $(1 - S)$ of his or her nominal consumption on commodity 1 (2) in period t. The exchange rate equation (9) becomes

$$e_t = \frac{(1 - S)M_{1t}}{SM_{2t}}, \tag{15}$$

while the marginal utilities appearing in (11) and (12) are given by

$$\Lambda_t = \frac{SR_t^{1-\rho}}{M_{1t}}, \tag{16}$$

$$e_t\Lambda_t = \frac{(1 - S)R_t^{1-\rho}}{M_{2t}}, \tag{17}$$

where $R_t = Y_{1t}^S Y_{2t}^{1-S}$ measures aggregate real output in period t.[10]

IV. Nominal Expected Returns

Let $m_{it} \equiv \dfrac{M_{it}}{M_{ht}}$, $h \neq i$, be the ratio of the period t supply of currency i to that of currency h. Substitution of (15) and (16) into the currency 1 premium equation (3) and (15) and (17) into the currency 2 premium equation (4) yields

$$F_{it} = -\ln\left\{1 + \mathrm{Cov}_t\left(m_{it+1}, \frac{R_{t+1}^{1-\rho}}{M_{it+1}}\right)\right\}, \quad i = 1,2. \tag{18}$$

Both arguments of the normalized covariance term in (18) are loglinear functions of the lognormal supplies; hence, they are themselves lognormal. Applying Lemma 1 to (18) then yields:

PROPOSITION 1: *The equilibrium currency i-denominated premium in period t is given by*

$$F_{it} = \mathrm{cov}_t(\ln m_{it+1}, \ln M_{it+1}) - (1 - \rho)\mathrm{cov}_t(\ln m_{it+1}, \ln R_{t+1}), \quad i = 1,2. \tag{19}$$

Using the definition of m_i to expand (19), Proposition 1 states that the additional expected currency i return required in period t on a currency h ($h \neq i$) denominated bond over and above that required on a currency i denominated

[10] That R is indeed real output can be seen by calculating nominal output and then deflating by the exact price index induced by the homothetic utility function (13). As discussed by Grauer and Litzenberger (1979), this index is given by (in currency 1 units)

$$\pi_1 = \frac{(P_1)^S(eP_2)^{1-S}}{S^S(1 - S)^{1-S}}.$$

bond is increasing in the logvariance of currency i, $\mathrm{var}_t(\ln M_{it+1})$, and decreasing in the logcovariance between currency i and currency h, $\mathrm{cov}_t(\ln M_{it+1}, \ln M_{ht+1})$. Moreover, if $\rho >(<) 1$, then this premium is also increasing (decreasing) in the logcovariance between currency i and aggregate output, $\mathrm{cov}_t(\ln M_{it+1}, \ln R_{t+1})$, but decreasing (increasing) in the logcovariance between currency h and aggregate output, $\mathrm{cov}_t(\ln M_{ht+1}, \ln R_{t+1})$. Each of these conclusions has a straightforward intuitive explanation, presented in turn below.

An increase in the supply of currency i causes a depreciation of that currency and hence an increase in the currency i payoff from a currency h bond, but also decreases the marginal utility of consumption denominated in currency i (see (16) and (17)). Hence, stochastic fluctuations in the supply of currency i induce a negative correlation between the currency i payoff from a currency h bond and the marginal utility of consumption denominated in currency i. The currency h bond therefore has greater risk than the currency i bond and so requires a higher expected return.

An increase in the supply of currency h causes that currency to depreciate and hence to decrease the currency i payoff from a currency h bond. Thus, a positive correlation between the supplies of currencies i and h induces a positive correlation between the currency i payoff from a currency h bond and the marginal utility of consumption denominated in currency i. The currency h bond therefore requires a lower expected return.

The intuition for the significance of the terms in (19) involving correlation between the real and monetary sectors can be clarified by interpreting equation (16) ((17)) for the equilibrium marginal utility of consumption denominated in currency 1 (2) as the equilibrium marginal utility of real consumption $R^{-\rho}$ divided by the equilibrium currency 1 (2) price level $\frac{M_1}{SR}\left(\frac{M_2}{(1-S)R}\right)$.[11] An increase in aggregate output R therefore decreases the marginal utility of nominal consumption by decreasing the marginal utility of real consumption, but also increases the marginal utility of nominal consumption by decreasing commodity prices. Under high (low) risk aversion ρ, the equilibrium marginal utility of real consumption is sensitive (insensitive) to changes in aggregate output, so the first (second) effect dominates. Hence, a positive correlation between the supply of currency i and aggregate output induces a negative (positive) correlation between the currency i payoff on a currency h bond and the marginal utility of consumption denominated in currency i if relative risk aversion is high (low), i.e., if $\rho >(<) 1$. A similar argument applies to the correlation between the supply of currency h and aggregate output.

A. *The Siegel Paradox*

Note that, since $\ln m_1 = -\ln m_2$, (19) implies that $\{F_{1t} + F_{2t}\} = \mathrm{var}(\ln m_{1t+1}) \geq 0$. Hence, both premia can be positive or one premium can be positive and the other negative. However, they cannot both be negative. This is the essential point

[11] That $\frac{M_1}{SR}$ is indeed a price index can be established formally by substituting the equilibrium prices into the exact price index π. See footnote 10.

of the "Siegel paradox". Siegel (1972) observed that, if one premium was zero, then the other had to be positive due to Jensen's inequality. Subsequently, McCulloch (1975, p. 170) argued that this result arises "...from...probability theory and has no apparent economic cause." Most other authors have also dismissed the paradox as a mathematical inconvenience having no economic significance. The linking of foreign exchange premia to the underlying stochastic processes of currencies and outputs in the current model demonstrates that the paradox has, in fact, a very simple economic explanation. Given stochastic money and commodity supplies, (19) implies that, if one premium is zero, then the other premium is also zero *if and only if* var $(\ln m_{1t+1}) = 0$, i.e., the money supplies are perfectly positively correlated. Thus, the Siegel paradox is simply a manifestation of inter-country differences in monetary policy. It is these differences that give rise to exchange rate fluctuations and hence to the paradox. Intuitively, since each premium is denominated in terms of a different currency, the result that differences in the stochastic processes governing the supplies of these currencies cause the premia to differ should come as no surprise.

B. Risk Neutrality

Equation (19) implies that the two bonds are imperfect substitutes even when investors are risk-neutral ($\rho = 0$). This point has also been made by Frenkel and Razin (1980), Hodrick and Srivastava (1986), and Sibert (1989), although the present model expresses the divergence in terms of exogenous, rather than endogenous, variables. Those authors argued that this divergence cannot be a risk premium since it exists when $\rho = 0$. However, the model employed here makes it clear that such a view is strictly valid only for a narrow interpretation of risk. Given stochastic asset payoffs, risk arises because the marginal utility of nominal consumption is state-dependent. As discussed earlier, this latter term is equal to the marginal utility of real consumption $R^{-\rho}$ divided by the price level. Thus, risk can arise because of fluctuations in real consumption, or in the price level, or both. Standard risk-neutrality ($\rho = 0$) implies only that investors are indifferent to fluctuations in real consumption; it does not imply that they are indifferent to price fluctuations. Hence, $\mathrm{cov}_t\left(\ln m_{it+1}, \ln\left(\frac{M_{it+1}}{R_{t+1}}\right)\right)$, the term remaining in (19) when $\rho = 0$, can indeed be interpreted as a risk premium; it is the currency i premium required for bearing the price level risk of the bond denominated in currency h.

V. Comparative Statics

Equation (19) indicates that the relationship between nominal foreign exchange premia and the stochastic structure of supply depends on the value of two preference parameters: the share of consumption expenditure S devoted to good 1 and the coefficient of relative risk aversion ρ. Substituting the definition of R_{t+1} into (19) and then differentiating with respect to S yields:

Proposition 2: *Suppose $\rho >(<)1$. Then the period t currency i premium is an* *increasing function of S if and only if*

$$\mathrm{cov}_t\left(\ln m_{it+1},\ \ln\left\{\frac{Y_{1t+1}}{Y_{2t+1}}\right\}\right) >(<)\ 0, \quad i = 1,2. \tag{20}$$

Proposition 2 can be explained as follows. Following the rise in S, aggregate real output R is higher (lower) than previously in states where $\frac{Y_1}{Y_2}$ is high (low). Hence, if $\frac{Y_1}{Y_2}$ is positively correlated with m_i, then high (low) values of m_i are associated with higher (lower) values of R following the rise in S and hence with lower (higher) values of the marginal utility of real consumption $R^{-\rho}$. Thus, the covariance between the currency h bond payoff and the marginal utility of real consumption falls. If $\rho >(<) 1$, then, as explained earlier, the marginal utility of nominal consumption moves in the same (opposite) direction as the marginal utility of real consumption so that the nominal premium increases (decreases).

Differentiating (19) with respect to ρ yields:

PROPOSITION 3: *The currency i premium is an increasing function of ρ if and only if*

$$\mathrm{cov}_t(\ln m_{it+1}, \ln R_{t+1}) > 0, \quad i = 1,2. \tag{21}$$

For given money and output distributions, a rise in ρ causes fluctuations in the marginal utility of real consumption, $R^{-\rho}$, to become more acute. Hence, the absolute value of the correlation between the currency h bond payoff and the marginal utility of real consumption rises. If $\mathrm{cov}_t(\ln m_{it+1}, \ln R_{t+1}) >(<) 0$, then that correlation is initially negative (positive). The rise in ρ therefore reduces (increases) it. Since changes in ρ do not affect the stochastic behavior of the price level, the correlation between the currency h bond payoff and the marginal utility of nominal consumption also falls (rises), thereby increasing (decreasing) the nominal premium.

VI. Real Expected Returns

Using the exact price index corresponding to the utility function (13) permits the calculation of the "real" foreign exchange premium—the inter-country difference in expected real returns. Recalling that b_{1t} (b_{2t}) is the currency 1 (2) price of the currency 1 (2) denominated bond in period t, and letting π_{1t} denote the currency 1 price level in period t, the expected real returns $\hat{r}_{it}$ on the two bonds are defined by

$$1 + \hat{r}_{1t} = \frac{E_t[1/\pi_{1t+1}]\pi_{1t}}{b_{1t}} = \frac{E_t[1/\pi_{1t+1}]\pi_{1t}\Lambda_t}{\delta E_t[\Lambda_{t+1}]} \qquad \text{by (1)},$$

$$1 + \hat{r}_{2t} = \frac{E_t[e_{t+1}/\pi_{1t+1}]\pi_{1t}}{e_t b_{2t}} = \frac{E_t[e_{t+1}/\pi_{1t+1}]\pi_{1t}\Lambda_t}{\delta E_t[e_{t+1}\Lambda_{t+1}]} \qquad \text{by (2)}.$$

Therefore,

$$\frac{1+\hat{r}_{2t}}{1+\hat{r}_{1t}}=\frac{E_t[e_{t+1}/\pi_{1t+1}]}{E_t[e_{t+1}]E_t[1/\pi_{1t+1}]}\Bigg/\frac{E_t[e_{t+1}\Lambda_{t+1}]}{E_t[e_{t+1}]E_t]\Lambda_{t+1}]}=\frac{1+\mathrm{Cov}_t(e_{t+1},1/\pi_{1t+1})}{1+\mathrm{Cov}_t(e_{t+1},\Lambda_{t+1})}$$

and

$$\hat{F}_t\equiv\ln\left\{\frac{1+\hat{r}_{2t}}{1+\hat{r}_{1t}}\right\}=\ln\{1+\mathrm{Cov}_t(e_{t+1},1/\pi_{1t+1})\}-\ln\{1+\mathrm{Cov}_t(e_{t+1},\Lambda_{t+1})\}. \quad (22)$$

Thus, the "real" premium $\hat{F}_t$ is equal to the nominal (currency 1) premium less an adjustment for correlation between the exchange rate and the price index. As noted in Section IV, the utility function (13) induces an exact currency 1 price index π_1 which is proportional to $\frac{M_1}{R}$, the level of currency 1 expenditure per unit of aggregate output. Since supplies are lognormal so is π_1. Applying Lemma 1 to (22), using the exchange rate equation (15) and the nominal premium equation (19), and simplifying yield;[12]

PROPOSITION 4: *The equilibrium real premium in period t is given by*

$$\hat{F}_t=\rho\mathrm{cov}_t(\ln m_{1t+1},\ln R_{t+1}). \quad (23)$$

In contrast to the nominal premium, $\rho = 0$ does imply a zero real premium. In a "real" context, risk is due solely to fluctuations in real consumption. Standard risk neutrality ($\rho = 0$) implies that investors are indifferent to these fluctuations and thus no premium is required. Moreover, since (23) also holds when the currency 2 price level is used to deflate nominal returns, the Siegel paradox disappears when the premia are expressed in real terms. This conclusion was also reached by Adler and Dumas (1983), but the present formulation provides additional economic insight. As shown above, the Siegel paradox arises because of imperfectly correlated money supplies. Real premia, however, do not depend on pure monetary fluctuations so that the phenomenon inducing the paradox does not operate when calculating these real premia. Consequently, no paradox occurs.

Comparative static results similar to Propositions 2 and 3 can also be obtained for the real premium. Indeed, the same procedure used to derive those results indicates that $\hat{F}_t$ is an increasing function of S (ρ) if and only if (20) ((21)) holds. The intuitive explanations are implicit in those provided for Propositions 2 and 3.

VII. Interest Rates, Exchange Rates, and the Terms of Trade

A. Correlation between Interest Rates and Exchange Rates

While there has been considerable debate in the empirical literature as to whether there exists a time-varying risk premium, most authors have concluded

[12] In a continuous time model where $\rho = 1$ by assumption, Stulz (1984) found that $\hat{F}_t = \mathrm{cov}_t(\ln m_{1t+1},\ln R_{t+1})$. Proposition 4 therefore demonstrates that his result can be extended in a straightforward way to cases where $\rho \neq 1$.

that at least some of the variation in the forward factor $\frac{f_t}{e_t}$ is due to fluctuations in the expected spot factor $\frac{E_t[e_{t+1}]}{e_t}$. Since $\frac{f_t}{e_t} = \frac{1 + r_{1t}}{1 + r_{2t}}$ by interest parity, the positive relationship between $\frac{f_t}{e_t}$ and $\frac{E_t[e_{t+1}]}{e_t}$ can be seen as indicating a positive correlation between the value of a country's currency and its nominal rate of interest. This has been interpreted by some authors (see, for instance, Frenkel and Mussa (1981) and Frenkel (1983)) as providing evidence for the view that changes in the real interest rate are the primary cause of changes in the nominal interest rate, in contrast to Fama's (1975) finding that changes in nominal interest rates mirror changes in expected inflation. Subsequently, Engel (1986) has shown that such an interpretation may be unwarranted, that a negative correlation between the exchange rate and the nominal rate of interest is not necessarily an indication of real interest rate movements. Engel employs a monetarist macro-model in which nominal foreign prices and interest rates are exogenously fixed. In the microeconomic model of this paper, neither assumption is made, but, as this section demonstrates, Engel's conclusion remains valid.

If real output R is constant, then real interest rates are also constant. Moreover, let the stochastic money supplies be given by the simple loglinear autoregressive scheme:

$$\ln M_{it} = \lambda_i \ln M_{it-1} + \ln \epsilon_{it}, \tag{24}$$

where λ_i is a constant and $\ln \epsilon_{it}$ is a random variable representing the residual uncertainty regarding the period t supply of currency i after period $t - 1$ supplies have been revealed. The vector $(\epsilon_{1t}, \epsilon_{2t})$ is assumed to be multivariate lognormal and independent of supplies in periods before t so that, for instance, $\text{var}_{t-1}(\ln M_{it}) = \text{var}_{t-1}(\ln \epsilon_{it})$ is a constant, independent of $\ln M_{it-1}$.

With real output fixed—so that real interest rates are also fixed—and money supplies given by (24), the period t currency 1 rate of interest satisfies, by (1), (16), and (14),

$$\ln(1 + r_{1t}) = (\lambda_1 - 1)\ln M_{1t} - \ln\delta - (\tfrac{1}{2})\text{var}_t(\ln \epsilon_{1t+1}) + E_t(\ln \epsilon_{1t+1}).$$

The exchange rate equation (15) therefore implies

$$\text{cov}_{t-1}(\ln e_t, \ln(1 + r_{1t})) = (\lambda_1 - 1)\text{cov}_{t-1}(\ln m_{1t}, \ln M_{1t}). \tag{25}$$

Examination of the right-hand side of (25) indicates that the value of a country's currency is positively correlated with its nominal interest rate if, for instance, the serial correlation in its money supply is not too positive ($\lambda_1 < 1$) and the period $t - 1$ foreign exchange premium is positive ($\text{cov}_{t-1}(\ln m_{1t}, \ln M_{1t}) > 0$).[13] This is easily explained. A positive foreign exchange premium occurs in period $t - 1$ if and only if e_t is perceived to be negatively correlated with Λ_t (see (3)). However, with $\lambda_1 < 1$, (1) and (16) imply that r_{1t} is positively correlated with Λ_t. Hence, e_t is negatively correlated with r_{1t}, even though real interest rates are constant. Thus, a negative correlation between exchange rates and nominal

[13] Equations (2), (17), and (24) imply that $\text{cov}_{t-1}(\ln(1 + r_{2t}), \ln(1/e_t)) = (\lambda_2 - 1)\text{cov}_{t-1}(\ln m_{2t}, \ln M_{2t})$.

interest rates need not indicate movements in real interest rates. In fact, $\lambda_1 < 1$ implies that both the country 1 interest rate and the expected rate of country 1 inflation are negatively correlated with $\ln M_1$ so that nominal interest rate movements are associated with changes in expected inflation.

B. Correlation between Relative Bond Prices and the Terms of Trade

A great deal of research effort has been devoted to examining the effect of terms-of-trade changes on other real sector variables such as saving and the current account.[14] By contrast, there has been (to my knowledge) no systematic study of the relationship between the terms of trade and financial market prices, even though terms-of-trade changes presumably alter investor wealth and therefore asset prices. In general equilibrium, of course, it is incorrect to talk about terms-of-trade changes *causing* asset price changes since both are endogenous. The stochastic relationship between them therefore depends on the exogenous supply processes. The model of this paper permits the derivation of such relationships.

In this model, the terms of trade are given by the relative price of goods 1 and 2, $p \equiv \frac{P_1}{eP_2}$. By (1) and (2), the logcovariance between the terms of trade and the relative price of bonds, $b \equiv \frac{b_1}{eb_2}$, is

$$\mathrm{cov}_{t-1}(\ln p_t, \ln b_t) = \mathrm{cov}_{t-1}\left(\ln p_t, \ln\left\{\frac{E_t[e_{t+1}A_{t+1}]}{E_t[A_{t+1}]}\right\}\right). \tag{26}$$

Since $\frac{S}{1-S} = \frac{pY_1}{Y_2}$, it follows that $p = \frac{SY_2}{(1-S)Y_1}$. Moreover, as (16) and (17) imply that Λ and $e\Lambda$ are both lognormal, $\ln\left\{\frac{E_t[e_{t+1}A_{t+1}]}{E_t[A_{t+1}]}\right\}$ can be obtained by applying (14) to those equations and simplifying. Assuming that the money stocks evolve according to (24) then yields

$$\mathrm{cov}_{t-1}(\ln p_t, \ln b_t) = \mathrm{cov}_{t-1}\left(\ln\left\{\frac{Y_{2t}}{Y_{1t}}\right\}, \ln\left\{\frac{M_{1t}^{\lambda_1}}{M_{2t}^{\lambda_2}}\right\}\right). \tag{27}$$

Equation (27) indicates that the relationship between the terms of trade and relative bond prices can be positive or negative. However, given positive serial correlation in the money stocks, the relationship will be negative as long as the supply of currency 1 (2) is more positively correlated with good 1 (2) output than it is with good 2 (1) output. This can be compared with the standard view of balance-of-payments adjustment, which holds that a rise in the terms of trade improves the curenty account, thereby necessitating a fall in relative interest rates so as to induce a deterioration in the capital account. This contrast highlights the dangers involved in imputing causality to relationships between endogenous variables.

[14] For a summary of this literature, see Svensson and Razin (1983) and the references therein.

VIII. Concluding Remarks

This paper has expressed a number of international financial sector relationships in terms of parameters describing the underlying stochastic processes of the exogenous variables—the stochastic structure of supply. For instance, if all currency and commodity supplies are random, then their joint correlation structure determines whether the foreign exchange premium is positive or negative. This contrasts with most earlier treatments of these issues where the relationships were expressed in terms of marginal utilities and other endogenous variables.

The analysis and results of this paper highlight the importance of monetary uncertainty for nominal assets denominated in different currencies. By altering the exchange rate, money supply fluctuations affect nominal returns; by altering nominal commodity prices, they affect real returns. Thus, for example, even real premia depend not only on the stochastic processes of commodity outputs, but also on the joint distribution of these with the money supplies. This suggests that some caution needs to be exercised in interpreting results generated by nonmonetary models of international asset pricing.

In this context, further research in this area should concentrate on expanding the economic role of money. In this model, the holding of money is motivated by the use of a cash-in-advance constraint. This generates a simple equilibrium solution but results in a constant velocity of circulation. Svensson (1985) has developed a general equilibrium model of asset pricing in which velocity is endogenous, but, as demonstrated in Stockman and Svensson (1987), results of the type generated here can be readily obtained only by using Taylor's Series approximations and by assuming that all supplies have zero serial correlation. Other discrete-time models which generate a variable money velocity include Helpman and Razin (1985), Danthine and Donaldson (1986), and Boyle and Young (1988). It may be possible to adapt these models so as to address the types of issues raised in this paper, but this is left for future research.

Appendix

Proof of Lemma 1: If W and Z are lognormal, then (14) implies

$$\ln E[W] = E[\ln W] + \frac{1}{2}\text{var}(\ln W),$$

$$\ln E[Z] = E[\ln Z] + \frac{1}{2}\text{var}(\ln Z),$$

$$\begin{aligned}\ln E[WZ] &= E[\ln W + \ln Z] + \frac{1}{2}\text{var}(\ln W + \ln Z)\\ &= E[\ln W] + E[\ln Z] + \frac{1}{2}\{\text{var}(\ln W)\\ &\quad + 2\text{cov}(\ln W, \ln Z) + \text{var}(\ln Z)\}.\end{aligned}$$

Therefore,

$$\ln\{1 + \mathrm{cov}(W,Z)/E[Z]E[W]\} = \ln\frac{E[WZ]}{E[W]E[Z]}$$

$$= \ln E[WZ] - \ln E[W] - \ln E[Z] = \mathrm{cov}(\ln W, \ln Z).$$

Q.E.D.

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