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Source: *The Journal of Finance*, Vol. 45, No. 2 (Jun., 1990), pp. 397-429
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2328663
Accessed: 26/11/2009 07:17

THE JOURNAL OF FINANCE • VOL. XLV, NO. 2 • JUNE 1990

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ABSTRACT

Multibeta asset pricing models are examined using proxies for economic state variables in a framework which exploits time-varying expected returns to estimate conditional betas. Examples include multiple consumption-beta models and models where asset returns proxy for the state variables. When the state variables are not specified, the tests indicate two or three time-varying expected risk premiums in the sample of quarterly asset returns. Conditional betas relative to consumption generate less striking evidence against the model than betas relative to asset returns, but both the consumption and the market variables fail to proxy for the state variables.

MODELS IN WHICH EXPECTED returns are linear in their betas have long been important in finance. This paper studies multibeta models with time-varying expected returns. The approach integrates two strands in the existing literature. The first group of studies specifies proxies for the economic "state variables" in multibeta models.[1] These studies traditionally have focused on unconditional or "average" expected returns. The second group of studies examines time-varying, conditional expected returns as "latent variables." In these studies, the underlying state variables are not specified. This paper extends the latent variables approach of Gibbons and Ferson (1985) by specifying proxies for the state variables.[2] There are several potential advantages to such an approach. One is to control for the "systematic" variance of unanticipated returns. Precision and power should thereby be enhanced. Another is to provide a framework for exploring the links between time-varying expected returns and conditional betas.

This paper therefore looks at two related types of models. The first type

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[1] Examples include Chan, Chen, and Hsieh (1985), Chen, Roll, and Ross (1986), Sweeney and Warga (1986), Huberman and Kandel (1987), Shanken and Weinstein (1987), and others.

[2] Beta pricing models with time-varying expected returns are also examined, using different approaches, by Campbell (1987), Bollerslev, Engle, and Wooldridge (1988), Ferson, Kandel, and Stambaugh (1987), Bodurtha and Mark (1989), Shanken (1990), and others.

assumes that expected returns for a set of N assets are linear in the betas relative to K observable state variables. Expected returns and betas are conditional on a set of L predetermined instruments, and the conditional betas are assumed to be constants. The second type, the latent variables model, is the same except that the state variables are not observed, so weaker restrictions can be tested. Quarterly returns for a sample of Treasury bills, long-term government and corporate bonds, and common stock "size" portfolios are examined. The tests indicate that there are two or three latent state variables in the time-varying expected returns. Various observable variables are used to proxy for the state variables. A vector of consumption measures is one example.[3] Models where asset returns proxy for the state variables are also examined.

This paper is organized as follows. Section I reviews the asset pricing models, states the statistical assumptions, develops the econometric model, and describes the estimation and test procedures. Section II describes the data. Section III presents the empirical results. Section IV discusses some diagnostics and extensions. Section V summarizes and concludes.

I. Methodology

A. Beta Pricing Models

Models consistent with state-dependent expected returns of securities follow from the analysis of optimal portfolio choice. The optimality conditions imply that expected returns are related to the conditional covariances with a measure of marginal utility. In some theoretical models, the covariances of returns with marginal utility are proportional to "consumption betas" (e.g., Breeden (1979)). However, few empirical studies have examined consumption beta models for conditional expected returns.[4] Breeden showed that the returns of portfolios can proxy for marginal utility. When a linear function of a number of state variables can proxy for marginal utility, a "multibeta" model describes expected returns.[5]

Let R_{it} be the rate of return of asset i between times $t-1$ and t. Beta pricing models posit the existence of "premiums" $\lambda_j(Z_{t-1})$, $j = 0, \cdots, K$, such that expected returns, conditional on the information Z_{t-1}, can be written in the familiar form[6]:

$$E(R_{it} \mid Z_{t-1}) = \lambda_0(Z_{t-1}) + \textstyle\sum_{j=1}^{K} b_{ij}\lambda_j(Z_{t-1}), \qquad i = 0, \cdots, N. \tag{1}$$

[3] Multiple consumption betas can be motivated as a proxy for marginal utility using "noisy" consumption data. The hypothesis is that differences in assets' conditional covariances with the "true" marginal utility can be captured using these variables.

[4] Consumption beta models were examined using unconditional moments by Hazuka (1984), Mankiw and Shapiro (1986), and Breeden, Gibbons, and Litzenberger (1989). Studies that used aggregate consumption data in other formulations have not been very successful in fitting expected returns across assets. (Singleton (1988) provides a review of this literature.)

[5] See, for example, Merton (1973), Sharpe (1977), Cragg and Malkiel (1982), Grinblatt and Titman (1983), and Shanken (1987).

[6] Examples of asset pricing models similar to equation (1) include the models of Sharpe (1964), Black (1972), Merton (1973), Long (1974), Ross (1976), Breeden (1979), and Cox, Ingersoll, and Ross (1985).

Z_{t-1} is assumed to be public information when prices are set at time $t-1$. The information variables Z_{t-1} may include, but are not limited to, past values of the state variables, f_{jt}. The b_{ij}'s are multiple regression coefficients ("betas"), conditional on Z_{t-1}, of the returns R_{it} on the K state variables. The b_{ij} could in general depend on Z_{t-1}. The state variables are specified as part of the model. As referred to here, the unanticipated components of the state variables play much the same role that the so-called "factors" play in much of the asset pricing literature. There will be no special assumptions about the form of the residual covariance matrix, however, in this paper.

If there is a portfolio with betas equal to zero on all K state variables, then $\lambda_0(Z_{t-1})$ is the expected return of this "zero beta" portfolio. If portfolios of traded assets can be used as the state variables, then the model is equivalent to the statement that a combination of the portfolios is conditionally mean-variance efficient. The simplest example is the Capital Asset Pricing Model (CAPM), where $K = 1$ and the "market portfolio" is a state variable.

For notational convenience, drop the time subscript and suppress the dependence of the premiums on Z. Letting b stand for the $K \times (N+1)$ matrix of the b_{ij}'s, equation (1) is restated as follows: $E(R \mid Z) = \lambda_0 \underline{1} + \lambda b$, where R is the $(N+1)$-vector of returns, $\underline{1}$ is a row vector of ones, and λ is the K-vector of the $\lambda_j, j = 1, \cdots, K$. The reader should keep in mind that Z is predetermined relative to R. Define the N vector of "excess returns," $r = \{r_i\}$; $i = 1, \cdots, N$, relative to the arbitrarily chosen asset zero, where $r_i = R_i - R_0$. Equation (1) implies that the following expression holds for the expected excess returns:

$$E(r \mid Z) = \lambda\beta, \tag{2}$$

where β is the $K \times N$ matrix of regression betas for the excess returns ($\beta_{ij} = b_{ij} - b_{0j}$). It will be assumed that the excess return betas are constant parameters. This is an important assumption in the context of models with conditional expectations.[7] Using measures of the state variables, the constant beta assumption can be tested. Such tests are conducted in a later section.

It simplifies the exposition, the econometric models, and the empirical analysis to work with excess returns. Of course, such tests will not exploit all of the restrictions of the asset pricing theory. For example, equation (2) does not distinguish between a "zero beta" model and a model which assumes the existence of a riskless asset.

Partition the N excess returns as $r = (r_1, r_2)$, where r_2 is a K-vector and r_2 contains the remaining $N - K$ excess returns. Partition of the matrix of betas conformably: $\beta = (\beta_1, \beta_2)$. The K assets in r_1 will be called the "REFERENCE ASSETS." These are assumed to be chosen so that the $K \times K$ matrix β_1 is nonsingular. The $N - K$ assets in r_2 will be called the "TEST ASSETS." If the expected excess returns of N assets are spanned by K time-varying risk premiums

[7] Since a conditionally mean-variance efficient portfolio will virtually always exist, there will be a single-beta model of expected returns. The single-beta model will typically have a time-varying beta. This makes the constant beta assumption an important part of the model specification. Wheatley (1989) criticizes latent variable tests along these lines, noting that the constant beta part of the joint hypothesis cannot be examined when the state variables are not specified.

according to equation (2), then K reference assets must provide a basis for the N expected excess returns. Using the paritioned equation (2), solve for the K risk premiums, λ, in terms of the expected excess returns of the K reference assets: $\lambda = E(r_1 \mid Z)\beta_1^{-1}$. Substitute this expression for λ to obtain a restriction on the $N - K$ test assets:

$$E(r_2 \mid Z) = E(r_1 \mid Z)\beta_1^{-1}\beta_2. \tag{3}$$

The restrictions of the asset pricing models for time-varying expected returns are based on equation (3).

B. The Statistical Assumptions

The statistical framework is described as follows:

$$\begin{aligned} f &= E(f \mid Z) + \eta, \\ r &= E(r \mid Z) + \eta\beta + \varepsilon, \\ E(f \mid Z) &= Z\gamma. \end{aligned} \tag{4}$$

Given constant conditional betas, the first two equations of (4) have little content. The first equation defines the unanticipated part of the state variables, η. The second equation defines the conditional betas, β, as the regression coefficients of the unexpected returns on η. The ε is the nonsystematic part of the unexpected returns. It is not necessary to assume that the covariance matrix of ε is diagonal, is constant over time, for has bounded eigenvalues as N increases. The third equation is a linear regression model for the conditional means of the state variables, given the L-vector of instruments Z, which includes a constant term. γ is assumed to be an $L \times K$ matrix of fixed regression coefficients. Linear regression models for the state variables are examined in this paper, but the approach can accommodate other (i.e., nonlinear) models. The conditional expected excess returns of the reference assets are assumed to be linear in the instruments:

$$E(r_1 \mid Z) = Z\delta_1, \tag{5}$$

where δ_1 is an $L \times K$ matrix of regression coefficients.

C. The Conditional Beta Pricing Model

The asset pricing restriction (3) and the regression model (5) imply that a regression like (5) holds for the test assets: $E(r_2 \mid Z) = E(r_1 \mid Z)\beta_1^{-1}\beta_2 = Z\delta_1\beta_1^{-1}\beta_2$. Combining the above with (4) and (5) produces the econometric model:

$$\begin{aligned} f &= Z\gamma + \eta, \\ r_1 &= Z\delta_1 + (f - Z\gamma)\beta_1 + \varepsilon_1, \\ r_2 &= Z\delta_1(\beta_1^{-1}\beta_2) + (f - Z\gamma)\beta_2 + \varepsilon_2. \end{aligned} \tag{6}$$

The first equation of system (6) is the model for the conditional means of the state variables. Asset pricing models imply that the innovations in the state variables represent the relevant risks. The first equation is used to identify the innovations η. Any predictable behavior of the state variable proxies that is captured by Z will not affect the model, because γ is a free parameter and only the η affect the other equations. The second equation incorporates the regression model for the expected excess returns of the K reference assets and identifies β_1 as their conditional regression coefficients. The third equation defines ε_2, the "nonsystematic" portion of the unanticipated excess returns of the $N - K$ test assets, r_2. The β_2 are the conditional betas of the test assets. The unanticipated returns and state variables provide direct estimates of the conditional betas. The comovement of expected returns through time brings additional cross-sectional information to bear on the estimates of the betas, via the restriction $\delta_2 = \delta_1[\beta_1^{-1}\beta_2]$, where δ_2 is the regression coefficient of r_2 on Z. This says that the conditional means of the test assets are related to those of the reference assets by their conditional betas. The model implies that the error term $u = (\eta, \varepsilon_1, \varepsilon_2)$ has conditional mean zero given the instruments Z.

The conditional beta pricing model summarized in system (6) generalizes the models examined by Hansen and Hodrick (1983) and Gibbons and Ferson (1985). Those studies did not specify the state variables. When the state variables are not specified, it is not possible to identify both β_1 and β_2. However, there are still testable restrictions when $[\beta_1^{-1}\beta_2]$ is replaced by a $K \times (N - K)$ matrix, C. The error terms of this model combine the systematic with the nonsystematic unanticipated returns. The second and third equations of (6), with this substitution, form a "LATENT VARIABLES MODEL" for the time-varying expected returns. Such models serve as baseline models to help interpret the empirical results in this paper. While Gibbons and Ferson work with raw returns, this paper examines excess returns.[8]

D. Econometric Method

The models are estimated using the Generalized Method of Moments (GMM). Each model defines an error term u, which should satisfy the orthogonality condition $E(u \mid Z) = 0$. Iterated expectations imply $E(u'Z) = 0$. Define a vector of sample orthogonality conditions: $g = \text{vec}(\hat{u}'\hat{Z}/T)$, where $\hat{u}$ is the $T \times \dim(u)$ matrix of sample residuals and $\hat{Z}$ is the $T \times L$ instrument matrix. Dim(u) is $N + K$ in equation (6), and $\dim(u)$ is N is the latent variable model. An algorithm searches for parameter values to minimize a quadratic form $g'Wg$. Hansen (1982) discusses the choice of weighting matrix W and provides general conditions under which the parameter estimates are consistent and asymptotically normal and the minimized value of the quadratic form is asymptotically chi-square. The quadratic form provides a goodness-of-fit statistic for the model. The degrees of freedom

[8] With raw returns, a zero-beta model implies that the elements of the matrix C sum to 1.0 across the state variables for each asset. Measuring excess returns simplifies the implementation of the tests since the restriction may always be satisfied by choosing the implicit weight on the 0th asset. The problem of specifying an appropriate price deflator for real returns is also avoided since excess real returns and excess nominal returns are empirically very similar.

are the difference between the number of orthogonality conditions and the number of parameters.[9]

Gibbons and Ferson (1985) use a likelihood ratio (LR) statistic to test the restrictions of latent variable models. Foerster (1989) examines the LR and also the Lagrange multiplier statistic (LM). Such approaches make stronger assumptions about the data than GMM. Most studies using the LR and the LM assume that the residuals in the regression system have a fixed covariance matrix, while GMM allows general forms of conditional heteroskedasticity and can be adjusted for autocorrelation. Campbell (1987), Stambaugh (1988), and others use the GMM approach in tests of latent variable models.

Given N excess returns, L instruments, and K state variables, there are $(N + K)L$ orthogonality conditions $[E(g) = 0]$ implied by system (6).[10] The number of parameters $(\delta_1, \gamma, \beta_1, \beta_2)$ is $K(2L + N)$. The model is overidentified provided that the number of orthogonality conditions exceeds the number of parameters. Overidentification constrains the sample design: $N > [KL/(L - K)]$, $L > K$, $N > K$.

II. The Data

Quarterly rates of return are measured for five value-weighted portfolios of New York Stock Exchange common stocks selected from "size" deciles. The deciles are based on the market value of equity outstanding at the beginning of each year. Three fixed-income returns are included: a long-term government bond, a long-term corporate bond, and the three-month return to a strategy of rolling over one-month Treasury bills. The returns of the eight portfolios are in excess of a three-month Treasury bill return. The data are from the Center for Research in Security Prices at the University of Chicago (CRSP). The sample period is 1947:2–1985:4.[11]

The consumption measures used for the state variable proxies are real, per capita, aggregate consumption expenditures. Many studies of consumption-based asset pricing use seasonally adjusted consumption. On one hand, Miron (1986) and Ferson and Harvey (1990) argue that there are reasons to avoid seasonally adjusted data. On the other hand, seasonal adjustment might proxy in some

[9] GMM may be implemented in a two-stage procedure (discussed in Hansen and Singleton (1982)), where the weighting matrix W is the identity matrix in the first stage. The initial parameter estimates are then used to construct the weighting matrix for the second stage. In this paper, a multi-stage procedure is used, where the parameter estimates from the nth stage are used to update the weighting matrix for stage $n + 1$ until convergence or a minimum is obtained. Ferson and Foerster (1990) find that such an approach has superior finite sample properties in tests of latent variable models. Typically, 3 to 20 stages are sufficient.

[10] The model also implies that the ε are uncorrelated with η. These orthogonality conditions can be imposed by including f, along with Z, in the set of instrumental variables used for the second and third equations of system (6). The examples in this paper do not impose these orthogonality conditions explicitly, because of the increased size and complexity of the econometric system. Including the additional restrictions should increase the power of the tests.

[11] For 1947:2–1959:2, the three-month Treasury bill rate is from Salomon Brothers' Analytical Record of Yields and Yield Spreads. The return is based on the three-month yield observed in the last month of the prior quarter.

crude way for seasonal tastes or technology. The adjustment could transform expenditure data into a better measure of marginal utility for asset pricing. Since the choice is not obvious, both seasonally adjusted and unadjusted data are examined. The unadjusted series are constructed from nominal, unadjusted personal consumption expenditures provided by the Commerce Department and are deflated by the not seasonally adjusted consumer price index.

Table I presents summary statistics for the quarterly data. The average excess returns and their standard deviations decrease, moving from the small stock portfolio (Dec1) to the large stock portfolio (Dec10)—reflecting well known "size effects" on stock returns—and the pattern continues with the corporate and government bonds. One-month Treasury bills and the long-term bonds returned slightly less than three-month bills over this sample. The autocorrelations of the excess returns are typically small, relative to their standard errors.

The sample standard deviations of the not seasonally adjusted consumption growth rates, relative to the seasonally adjusted (SA) consumption growth rates, show that the X-11 seasonal-adjustment program employed by the Commerce Department removes a substantial fraction of the variability. (It also affects the means of the series.) Miron (1986) and Ferson and Harvey (1990) note that the more volatile unadjusted consumption series lead to smaller estimates of risk

Table I

Summary Statistics: Quarterly Data, 1947:2–1985:4 (155 Observations)

Real growth of consumption of nondurables, durables, and services at time t is the logarithm of period t consumption per capita divided by period $t-1$ consumption per capita. (SA) denotes seasonally adjusted. All returns are arithmetic rates of return per quarter, in decimal fractions, and are measured in excess of the return of a three-month Treasury bill. Dec$N$ refers to a value-weighted portfolio of common stocks of the Nth size decile. "Treasury bills" measures the return to rolling over one-month bills each month. ρ_j is the jth order sample autocorrelation.

Variable	Mean (Quarterly Decimal)	Std. Dev. (Quarterly Decimal)	Autocorrelations							
			ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	ρ_8
Consumption Growth:										
Nondurables	0.00248	0.11657	−.66	.33	−.65	.96	−.64	.32	−.62	.93
Nondurables (SA)	0.00272	0.00860	.06	.04	.14	−.04	−.08	−.06	.02	−.18
Durables	0.00657	0.15487	−.75	.60	−.74	.86	−.71	.54	−.70	.82
Durables (SA)	0.00862	0.04181	−.09	.16	−.13	−.03	.01	−.06	−.03	−.19
Services	0.00742	0.01428	−.27	.01	−.17	.61	−.32	−.02	−.31	.54
Services (SA)	0.00589	0.00582	.14	−.02	.03	−.03	−.16	.11	.14	.09
Excess Return:										
Treasury bills	−0.00058	0.00160	.10	−.17	.27	−.14	−.15	.19	.03	.09
Government bond	−0.00164	0.04602	−.14	.09	.07	.04	−.16	.01	.00	−.08
Corporate bond	−0.00011	0.04689	−.05	.06	.09	.03	−.17	−.02	−.02	−.04
Stocks: Dec1	0.03573	0.13348	.01	−.10	−.08	.13	−.09	−.08	−.15	−.04
Stocks: Dec3	0.03002	0.11437	.04	−.09	−.04	.04	−.05	−.07	−.11	−.05
Stocks: Dec5	0.02587	0.10053	.10	−.09	−.06	−.03	−.03	−.09	−.15	−.07
Stocks: Dec7	0.02462	0.09575	.10	−.11	−.06	−.08	−.02	−.09	−.12	−.12
Stocks: Dec10	0.01645	0.07297	.15	−.02	−.02	−.02	−.03	−.11	−.14	−.04

aversion in representative-agent consumption models. For a given correlation with returns, greater volatility should also attenuate estimates of consumption betas.

The sample autocorrelations of seasonally adjusted consumption growth rates are fairly small, but the unadjusted data are highly autocorrelated. Nondurable and durable goods show high autocorrelation at the seasonal lags. Such evidence of seasonality is less pronounced for services but remains statistically significant.[12]

Table II reports correlations among the variables. The upper panel contains sample correlations of the excess returns and consumption growth measures. The lower panel reports correlations of the residuals from these variables, regressed on the predetermined instruments described below. These provide a feel for the conditional correlations. In the model of equation (6), predictable seasonal variation of expected consumption growth does not affect the conditional betas and need not affect the conditional expected risk premiums of assets.

The upper panel of Table II indicates that, unconditionally, unadjusted nondurable and durable goods expenditures are highly correlated and negatively correlated with the growth rate of services. Much of this correlation is due to predictable seasonal behavior. With seasonally adjusted data, the three components of consumption expenditures are weakly positively correlated. The residuals of the not seasonally adjusted measures are also weakly positively correlated. However, the correlations between the seasonally adjusted and unadjusted versions of the consumption measures typically are not so large that including both versions would be redundant.

III. Empirical Results

A. Selection of Instrumental Variables

Instrumental variables, Z, are used to model the conditional expectations. The instruments should be public information and correlated with market expectations. Assuming rational expectations, previous research suggests instruments by documenting their ability to forecast future stock and bond returns. This study chooses a common set of instruments for both the assets and the state variable proxies.

Table III summarizes regression models for the expected excess returns and consumption growth measures. The instruments include 1) a nominal Treasury bill rate, 2) a Treasury bill return spread, 3) a long-term real stock return, and 4) consumption expenditure growth rates. A total of seven instruments plus a constant are included. The table shows t-ratios and summary statistics for the regressions.

Nominal bill returns are found by Fama and Schwert (1977) and others to be negatively related to future asset returns. Table III includes the past quarter's nominal return to the one-month bill strategy (denoted "RFNOM") as a predictor

[12] Ferson and Harvey (1990) observe that the unadjusted consumption growth rates are highly seasonal but appear to be stationary time series.

Table II

Correlation Matrices of Assets' Excess Returns and Real, per Capita Consumption Growth Rates Quarterly Data, 1947:2–1985:4

The real growth rates of consumption expenditures for time t are the logarithm of period t consumption per capita divided by period $t-1$ consumption per capita. Non denotes nondurable, Serv is services, and Dur is durable goods expenditures. (SA) denotes seasonally adjusted. All returns are arithmetic rates of return per quarter, in decimal fractions, and are measured in excess of the return of a three-month Treasury bill. Dec$N$ refers to a value-weighted portfolio of common stocks of the Nth size decile. TB is the return to rolling over one-month Treasury bills each month. GB is the long-term government bond return, and CB is the long-term, high-grade corporate bond return. The regression residuals are from the regressions summarized in Table III, where each variable is regressed on predetermined values of a nominal Treasury bill rate, a Treasury bill return spread, a long-term real stock return, and four lagged nondurables consumption expenditure growth rates.

Sample Correlations 1947:2–1985:4 (155 Observations)

	Non	Serv	Dur	Non(SA)	Serv(SA)	Dur(SA)	TB	GB	CB
Non	1.00								
Serv	−0.61	1.00							
Dur	0.91	−0.53	1.00						
Non(SA)	0.02	0.19	0.10	1.00					
Serv(SA)	0.05	0.25	0.13	0.39	1.00				
Dur(SA)	−0.05	0.15	0.18	0.37	0.15	1.00			
TB	0.11	−0.06	0.11	0.13	0.22	0.11	1.00		
GB	0.08	0.00	0.02	−0.07	−0.11	−0.16	−0.47	1.00	
CB	0.06	0.05	−0.01	−0.05	−0.09	−0.16	−0.50	0.95	1.00
Dec1	−0.26	0.27	−0.25	0.14	0.14	0.12	−0.20	0.16	0.23
Dec3	−0.17	0.23	−0.17	0.16	0.12	0.07	−0.22	0.21	0.29
Dec5	−0.10	0.18	−0.10	0.14	0.12	0.06	−0.22	0.27	0.34
Dec7	−0.04	0.14	−0.05	0.12	0.12	0.02	−0.20	0.29	0.36
Dec10	0.11	0.09	0.11	0.15	0.13	0.01	−0.15	0.31	0.38

Correlations of Regression Residuals 1948:2–1985:4 (151 Observations)

	Non	Serv	Dur	Non(SA)	Serv(SA)	Dur(SA)	TB	GB	CB
Non	1.00								
Serv	0.09	1.00							
Dur	0.39	0.08	1.00						
Non(SA)	0.41	0.13	0.33	1.00					
Serv(SA)	0.24	0.30	0.24	0.36	1.00				
Dur(SA)	0.31	0.11	0.71	0.39	0.21	1.00			
TB	0.02	−0.03	0.11	0.12	0.20	0.13	1.00		
GB	0.02	0.02	−0.15	−0.05	−0.12	−0.05	−0.49	1.00	
CB	0.04	0.04	−0.16	−0.03	−0.09	−0.17	−0.53	0.95	1.00
Dec1	0.11	0.09	−0.02	0.20	0.19	0.07	−0.18	0.13	0.21
Dec3	0.07	0.09	−0.05	0.20	0.17	0.02	−0.22	0.17	0.26
Dec5	0.05	0.06	−0.03	0.18	0.16	0.02	−0.22	0.23	0.30
Dec7	0.04	0.05	−0.05	0.16	0.14	−0.02	−0.22	0.24	0.32
Dec10	0.11	0.09	0.11	0.15	0.13	0.01	−0.15	0.31	0.38

Table III
Regressions of Excess Returns and Growth Rates of Consumption on Predetermined Instruments
Quarterly Data, 1948:2–1985:4 (151 Observations)

Non, Serv, and Dur are the growth rates of real nondurables, services, and consumer durables expenditures per capita. (SA) indicates seasonally adjusted. GB is the long-term government bond return, and CB is the long-term, high-grade corporate bond return. Dec*n* is the return of the size-based common stock portfolio from decile *n*. All excess returns are nominal returns less the three-month Treasury bill rate. TB is the excess return to rolling over one-month Treasury bills each month, and TB1 is its one-quarter lagged value. 2r1 is the two-year real return of the CRSP equally weighted stock index for the period ending one quarter before a dependent variable is measured. The real return is the nominal return deflated by the consumer price index. RFNOM is the lagged, one-quarter nominal return to rolling over one-month Treasury bills. Non*J* is the growth of nondurables consumption expenditures at lag *J*. *t*-Ratios are formed using heteroskedasticity- and autocorrelation-consistent standard errors. *R*sq1 is the regression *R*-square with all seven predictors plus a constant included in the regression. *R*sq2 is the *R*-square excluding Non1 through Non4. *p*val is the right-tail *p*-value of an autocorrelation- and heteroskedasticity-consistent chi-square test for the hypothesis that the coefficients of Non1 through Non4 are equal to zero.

	Dependent Variables (columns):													
	Non	Dur	Serv	Non(SA)	Serv(SA)	Dur(SA)	TB	GB	CB	Dec1	Dec3	Dec5	Dec7	Dec10
Independent Variables (rows):														
	t-Ratios													
2r1	2.65	1.69	1.12	3.32	0.31	1.22	−0.47	−2.10	−1.83	−2.01	−1.70	−1.66	−2.03	−1.16
TB1	0.36	−1.40	−6.96	−0.95	−3.50	−1.77	−0.20	−0.71	−0.72	−4.02	−5.17	−5.87	−5.13	−5.00
RFNOM	−0.94	−2.22	−2.73	−1.80	−2.56	−1.63	−3.31	−0.30	−0.44	−2.07	−2.36	−2.52	−2.74	−3.87
Non1	−5.57	−3.02	1.13	−0.23	2.13	−1.04	2.53	−1.45	−1.54	−0.75	−0.90	−1.09	−1.04	−1.51
Non2	−5.29	−1.86	0.26	−0.22	2.89	−1.15	2.43	−1.50	−1.75	−0.87	−1.12	−1.27	−1.25	−1.65
Non3	−5.28	−3.11	−0.93	−0.27	2.62	−1.14	2.76	−1.85	−2.03	−1.27	−1.55	−1.66	−1.65	−2.09
Non4	6.48	1.00	−2.29	−0.31	2.20	−1.24	2.88	−1.57	−1.74	−1.62	−1.60	−1.63	−1.46	−1.62
	Coefficients of Determination													
*R*sq1	0.98	0.85	0.59	0.08	0.12	0.06	0.19	0.07	0.07	0.16	0.14	0.13	0.13	0.17
*R*sq2	0.01	0.01	0.04	0.07	0.05	0.03	0.09	0.04	0.04	0.06	0.06	0.07	0.09	0.11
*p*val	.000	.000	.000	.977	.004	.538	.001	.189	.234	.004	.004	.066	.053	.001
	Residual Autocorrelations													
lag1	−0.09	−0.27	−0.18	0.01	0.12	−0.06	0.04	−0.18	−0.10	0.06	0.04	0.09	0.07	0.10
lag4	−0.25	0.21	0.36	−0.07	−0.11	−0.02	−0.26	0.05	0.04	0.13	0.06	0.00	−0.04	−0.03
lag8	−0.09	0.09	0.18	−0.12	−0.03	−0.19	0.05	−0.08	−0.05	−0.08	−0.06	−0.06	−0.12	−0.10

for the excess returns.[13] Its relation to the long-term bonds is weak, but large negative t-ratios are observed in regressions for the short-term bills and common stock returns. A negative relation to future consumption growth is also observed.

Fama (1984) and Campbell (1987) find that differences in Treasury bill rates, measuring the slope of the term structure in the short maturities, can be useful predictors of bond and stock returns. Table III includes the lagged return to rolling over one-month bills minus the three-month bill rate (denoted "TB1"). Since the excess return is approximately a "real" quantity, including it together with the nominal return may allow for real and nominal effects that differ across assets.[14] TB1 appears to contribute additional explanatory power. It is strongly negative related to the future excess returns of the common stocks and to some of the consumption measures.[15]

A two-year, real return to the CRSP equally weighted stock index, lagged one quarter (denoted "2r1"), is included to capture potential reversion of expected returns to their long-run means. Mean reversion is suggested by the evidence of Keim and Stambaugh (1986), Fama and French (1988a, b, 1989), and others. The implication is that, if stock returns have been lower than average, then conditional expected returns may be higher than average. Table III indicates that the lagged two-year return is negatively related to all of the excess returns, with t-ratios near 2.0. 2r1 also has some ability to forecast the future growth of nondurables consumption. Its coefficient is positive in all of the consumption growth regressions. This is consistent with the view that stock returns anticipate future real growth (e.g., Fama (1981)).

Lagged values of the growth of real, not seasonally adjusted, nondurables consumption are included to capture information in past consumption expenditures. Previous studies indicate that past expenditure growth rates can predict excess returns.[16] The measures are not seasonally adjusted, to avoid the forward filtering by X-11 and to parsimoniously capture the predictable seasonal patterns in consumption expenditures. Four quarterly lags include the seasonal lag and avoid the multicollinearity that additional lagged values would generate.[17]

The lagged consumption variables jointly contribute significant marginal explanatory power for the bill and stock return regressions. Past consumption growth rates are negatively related to most of the excess returns. The one-month bill premium is the exception, where the relation is strongly positive. The one-month bill premium is negatively correlated with the other assets. Since the covariance of two returns can be decomposed into the covariance of the expected part plus the covariance of the unexpected part, Table III suggests that the

[13] Using the nominal, three-month bill rate instead of the lagged return to the one-month bill strategy produced similar regression results.

[14] Real bill returns are found to predict future bond and stock returns by Fama and Gibbons (1982), Hansen and Singleton (1983), Ferson and Harvey (1990), and others.

[15] The sample correlation between TB1 and RFNOM in these data is −0.179.

[16] For example, see Hansen and Singleton (1983). Regressions of Ferson and Harvey (1990) suggest that unadjusted nondurables may be a better predictor than the other consumption expenditures.

[17] Tauchen's (1986) simulation evidence suggests that the small sample properties of GMM estimators may suffer if excessive numbers of lagged values are used as instruments.

negative covariances of the bill strategy reflect in part negative covariances of the expected returns.

It should be possible to obtain more precise forecasts by selecting instruments on an equation-by-equation basis. Such an approach is cumbersome and is more susceptible to overfitting. The common set of instruments in Table III seems to produce fairly well-specified regressions. The proportion of variance explained is similar to other studies. The independent variables do a reasonable job of controlling autocorrelation, and there seem to be some interesting relations among the variables.[18]

In some experiments, subsets of the instruments are used. The "CONSUMPTION INSTRUMENT SUBSET" includes only the constant and the four lagged values of nondurables consumption growth. The "FINANCIAL INSTRUMENT SUBSET" comprises the constant, the lagged nominal bill return, the bill spread, and the lagged two-year real stock return.

B. Models with Unspecified State Variables

This section tests for the number of latent variables in the expected returns. The latent variables model imposes a cross-equation restriction on the regressions in Table III which must hold if expected excess returns can be formed as fixed-weight linear combinations of the regressions for K reference assets. This is required if there are K portfolios, where the excess returns have constant conditional betas relative to these portfolios, and a combination (which may depend on Z) of the portfolios is conditionally mean-variance efficient. The results of the latent variable tests are summarized in Table IV.

If a single latent variable model ($K = 1$) is accepted, the c_i's can be interpreted as ratios of the beta for asset i to the beta of the reference asset. The reference asset is the smallest decile of common stocks, so the beta of portfolio i is c_i times as large as the beta of the small stock portfolio. The estimates $\hat{c}_i$ for $K = 1$ decline with firm size in Table IV, ranging from 0.67 for the third decile to -0.02 for the largest decile. The t-ratios of the coefficients exceed 4.5 for all stock portfolios except the largest decile. The $\hat{c}_i$'s of the government and corporate bonds are positive but numerically small, with small t-ratios. The negative c_i estimate of the Treasury bill return (t ratio: -5.6) is consistent with a negative correlation between the expected excess returns of the bill and the small stocks. However, the interpretation of the coefficients as relative betas assumes a single-premium

[18] It should be noted that heteroskedasticity is not a misspecification in this context. Residual autocorrelation can in general indicate that the instruments omit some of the "market's" information (see, e.g., Huizinga and Mishkin (1986)) but does not imply a misspecification of the conditional means given the instruments. Because quarterly observations of 2r1 imply overlapping data, autocorrelation of the residuals is expected to at least lag 7. The standard errors in Table III are adjusted for conditional heteroskedasticity and autocorrelation using the Neway and West (1987) estimator, with nine moving-average terms. OLS standard errors and heteroskedasticity-consistent standard errors assuming no autocorrelation tended to produce slightly larger t-ratios for the lagged consumption variables in the asset return regressions. Otherwise, the results were similar.

model, and the asymptotic distributions of the "t-statistics" are conditional on the null hypothesis being true.

Table IV shows that the restrictions of a single-latent-variable model can be rejected. The test statistic is significant at conventional levels, using either the full instrument set or the subsets of the instruments. This extends to quarterly data the evidence against single-latent-variable models for daily and monthly returns.[19] The statistics provide little evidence against $K = 2$ or $K = 3$ latent variables (the right-tail p-values are 0.29 or larger).

If a two-beta model is accepted, the estimates of C can be interpreted as follows. The betas of the test assets on each latent state variable are linear combinations of the reference asset betas: $\beta_2 = \beta_1 C$, where β_1 is the 2×2 matrix of the conditional betas of the reference assets. Each column of the $2 \times (N - 2)$ matrix C gives the linear combination for a test asset. The same linear combination is used for the betas on both state variables. The estimate of a column of C is shown as a row in Table IV. The reference assets for $K = 2$ are the small-firm and the large-firm stock portfolios.[20] The estimates reveal some intuitively reasonable patterns. The implicit betas of the common stock portfolios are approximately formed as positive, convex combinations of the small- and large-firm portfolios. The negative weights of the Treasury bill on the stock portfolios indicate negative correlation of the expected excess returns.

Figure 1 examines the fit of the models in more detail. A "pricing error" is defined for each portfolio as the difference between the actual excess return and the fitted value of a given model. The pricing error is therefore equal to the sum of the model error in fitting the expected return, plus the deviation of the return from the "true" expected return. To the extent that the sample mean return approximates the average expected return, the mean pricing error will approximate the average model error.

Figure 1 shows the mean pricing errors and the mean absolute errors for the latent variable models with $K = 2$ and $K = 3$ factors. The largest mean pricing error is less than 5% per year. This compares favorably with the average mispricing of the smallest of the size portfolios using a five-factor APT model, as reported by Connor and Korajczyck (1988). However, the mean pricing errors for the other common stock portfolios are not much smaller. (All of the mean errors for the size portfolio are between 0.85% and 1.19% per quarter.) The mean pricing errors for the Treasury bill and the bonds are negative and smaller than 1.2% per year in absolute value. The mean absolute errors are smaller for bonds than for stocks. This is expected if the fluctuation of fixed-income returns around their expected returns is small compared with stocks. The mean errors are not systematically smaller in the three-latent-variable model than in the two-variable

[19] See Ferson, Foerster, and Keim (1988) for daily stock return data, Foerster (1989) for monthly stock returns, Stambaugh (1988) for monthly bond returns, and Campbell (1987) and Ferson (1989) for monthly bond and stock data.

[20] I am grateful to the referee for suggesting that the sample values of the test statistics may be invariant to the choice of reference assets. Assuming that the conditional means of the reference assets are rank K, the invarance holds both for the latent variable models and for the model of system (6). A proof is available by request.

Table IV
Asset Pricing Models with Unspecified State Variables

Results of estimating models with K = 1, 2, and 3 latent variables, using quarterly data for 1948:2–1985:4 (151 observations). The model is

$$r_1 = Z\delta_1 + e_1,$$

$$r_2 = Z\delta_1 C + e_2,$$

where r_1 represents the returns of K reference assets in excess of the three-month Treasury bill rate. r_2 is the excess returns of $N - K$ test assets. Z is the set of predetermined instruments. δ_1 and $C = \{c_{ij}\}$ are coefficients. c_{ij} is the relative beta of asset i with respect to the jth reference asset. $t(\cdot)$ is the t-ratio of the c_{ij} coefficient estimate. Estimates of the coefficients using the full instrument set are reported. The latent variable model implies that $E(e_1, e_2|Z) = 0$. χ^2 is the sample value of the test statistic for the null hypothesis that the orthogonality conditions implied by a K-latent-variable model are satisfied. df is the degrees of freedom, and p-value is the probability that a chi-square variable exceeds the sample value of the statistic.

	$K = 1$		$K = 2$				$K = 3$					
Reference Asset:	Dec1		Dec10		Dec1		Dec10		Dec1		TB	
Excess Return	$\hat{c}_i$	t	$\hat{c}_{i1}$	$t(\cdot)$	$\hat{c}_{i2}$	$t(\cdot)$	$\hat{c}_{i1}$	$t(\cdot)$	$\hat{c}_{i2}$	$t(\cdot)$	$\hat{c}_{i3}$	$t(\cdot)$
GB	.006	0.18	0.26	2.74	−0.06	−1.29	0.51	3.90	−0.11	−1.48	−0.93	−0.15
CB	.020	0.62	0.34	3.43	−0.02	−0.48	0.53	3.58	−0.08	−1.09	−1.47	−0.25
TB	−.009	−5.61	−0.01	−0.76	−0.01	−4.32	—[a]	—	—	—	—	—
Dec3	.667	15.80	0.41	7.29	0.68	21.49	0.44	6.96	0.69	16.19	−1.05	−0.46
Dec5	.466	8.38	0.60	8.38	0.48	13.06	0.71	8.02	0.44	9.31	−4.37	−1.39
Dec7	.323	4.65	0.76	12.21	0.31	8.04	0.95	9.73	0.27	5.39	−5.30	−1.61
Dec10	−.020	−0.23	—	—	—	—	—	—	—	—	—	—

Table IV—*Continued*

	$K = 1$	$K = 2$	$K = 3$
		Full Instrument Set	
χ^2 Statistic	69.51	39.78	24.49
df	49	36	25
p-Value	0.029	0.305	0.491
		Consumption Instrument Subset	
χ^2 Statistic	44.34	20.72	7.75
df	28	18	10
p-Value	0.026	0.294	0.653
		Financial Instrument Subset	
χ^2 Statistic	32.72	11.56	3.04
df	21	12	5
p-Value	0.049	0.482	0.693

[a] Dash indicates that the asset is used as a reference asset.

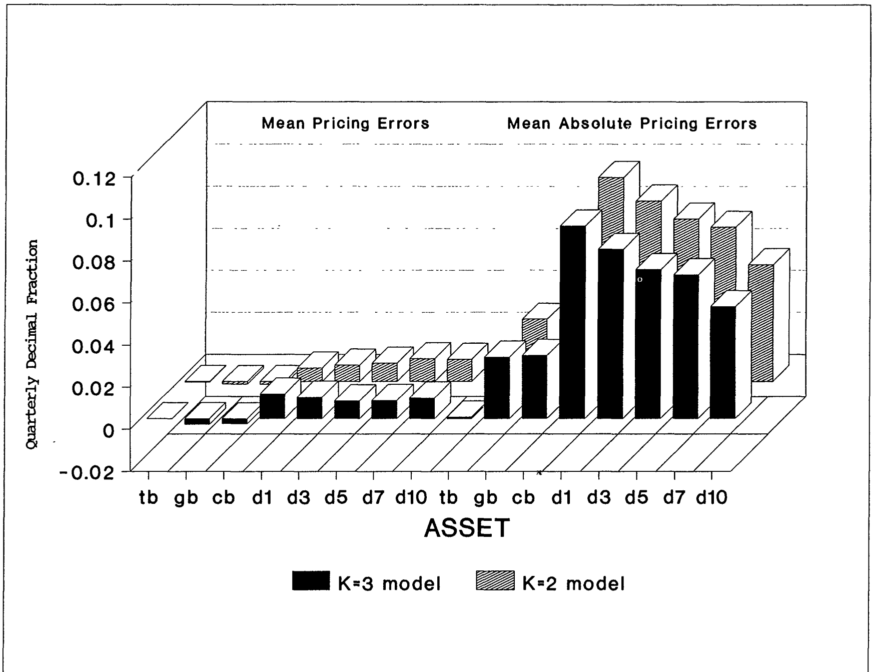

Figure 1. Pricing errors of the latent variable models, 1948:2–1985:4. The latent variable models are

$$r_1 = Z\delta_1 + e_1,$$

$$r_2 = Z\delta_1 C + e_2,$$

where r_1 represents the quarterly returns of K reference assets in excess of the three-month Treasury bill rate. r_2 represents the excess returns of $N - K$ test assets. Z is the set of predetermined instruments. δ_1 and $C = \{c_{ij}\}$ are coefficients. c_{ij} is the relative beta of asset i with respect to the jth reference asset. The pricing errors are the sample values of the fitted residuals e_1 and e_2. The mean pricing errors are the sample average of the residuals, and the mean absolute pricing errors are the sample average of the absolute values of the residuals. $K = 2(3)$ indicates a model with 2(3) latent variables. The assets are denoted as follows:

tb = Treasury bill,

gb = government bond,

cb = corporate bond,

dn = common stock portfolio from size decile n.

model, and there is only subtle visual evidence that a third latent variable reduces the mean absolute errors.[21]

C. Asset Pricing Models with Multiple Consumption Betas

Table V summarizes the results of estimating models with $K = 1$ state variable. Five consumption measures serve as alternative proxies for the state variable in

[21] It follows from the invariance of the model error terms to the choice of reference assets that the pricing errors are the same when different assets are the reference assets. An additional experiment

system (6). The estimates of the conditional consumption betas of the common stock portfolios are positive, and they are typically smaller for the larger firm portfolios. The betas of the Treasury bill and the government bond are each negative in four out of five cases. The conditional beta estimates, relative to not seasonally adjusted consumption, are typically closer to zero than the betas relative to seasonally adjusted consumption. Of course, the interpretation and the asymptotic properties of the estimates rely on the validity of the single-consumption-beta models. The test statistics for the models are displayed at the bottom of the table. Each of the right-tail p-values is smaller than the value obtained for $K = 1$ latent variable in Table IV, which rejects the models.

Table VI summarizes the results for models with $K = 2$ and $K = 3$ consumption betas. Several versions of each model are examined, using different combinations of the consumption variables. Unlike Table IV, which did not reject $K = 2$ latent variables, the test statistics in Table VI imply small p-values. Even when $K = 3$ the χ^2 statistic is very large.[22]

There are several potential explanations for these results. The consumption models impose more structure than the latent variable model, by relating expected returns to the conditional betas. The models control the systematic variance of unanticipated returns using the state variables. The consumption beta model might fit the data less closely because of these additional restrictions. Possibly, the tests can detect a smaller departure from the restrictions because of the reduced error variance.

It is likely that the consumption measures are "noisy." However, if measurement error in the consumption variables is uncorrelated with asset returns, then under reasonable assumptions the null hypothesis remains valid.[23]

Even accurately measured consumption expenditures might be a poor proxy for marginal utility. Recent papers present arguments to this effect. Dunn and

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confirmed the robustness of the latent variables model results. The latent variables model implies that $E(u_2 \mid Z) = 0$, where $u_2 = r_2 - r_1 C$. The model was estimated by GMM. Unlike the tests in Table IV, this formulation does not assume that a linear regression describes the conditional means given Z. Different size-decile portfolios (deciles 2, 4, 6, and 8) were used as the reference assets in combination with the eight test assets. The results are similar to Table IV. Models with $K = 1$ latent variable are rejected at standard significance levels (the p-values range from 0.021 to 0.044), but the $K = 2$ and $K = 3$ models are not rejected.

[22] Given that a number of cases are shown, the evidence may be interpreted in terms of multiple comparisons. It is clear that such an approach would not change the inferences. Additional experiments (not shown) examined the $K = 2$ and $K = 3$ models using subsets of the test assets. The results were similar, providing no indication that particular assets are influential in these rejections. A few cases were examined using the consumption instrument subset, and again the results were similar.

[23] If a regression of the consumption measures on the instruments yields an error term equal to the true innovation in the state variables, given the instruments, plus measurement error that is uncorrelated with returns, then the conditional betas will be biased because the conditional variance of the state variables is overstated. However, the bias will cancel out of $\beta_1^{-1}\beta_2$, and the relative betas can be consistently estimated. If the first equation of (6) has conditional mean zero, then $f - Z\gamma$ multiplied by any constant matrix has conditional mean zero. Therefore, the second and third equations of (6) can have conditional mean zero in large samples even if the conditional betas themselves are inconsistent. Under the null hypothesis, the asymptotic distribution of the test statistic can be invariant to this type of measurement error.

Table V

Estimates and Tests of Asset Pricing Models with a Single Consumption Beta

Results of estimating system (6), with $K = 1$, for alternative consumption variables, using quarterly data for 1948:2–1985:4 (151 observations). The model is

$$f = Z\gamma + \eta,$$

$$r_1 = Z\delta_1 + (f - Z\gamma)\beta_1 + \varepsilon_1,$$

$$r_2 = Z\delta_1(\beta_1^{-1}\beta_2) + (f - Z\gamma)\beta_2 + \varepsilon_2,$$

where $r = (r_1, r_2)$ is a vector of quarterly returns in excess of the three-month Treasury bill rate, f is the proxy for a single state variable, and Z is a vector of predetermined instrumental variables. γ, δ_1, and the β's are parameters. β_1 is the conditional beta coefficient of the excess return of the reference asset r_1 on the state variable. The β_2 are the conditional betas of the test assets, r_2. $t(\hat{\beta}_i)$ is the t-ratio. The beta coefficient estimates in the table are based on the full instrument set. This includes a constant, four lagged values of real nondurables consumption growth, a lagged nominal and a lagged excess Treasury bill return, and a lagged two-year real return on the CRSP equally weighted stock index. The consumption instrument subset comprises only the first five instruments. The model implies that the combined error term $u = (\eta, \varepsilon_1, \varepsilon_2)$ has conditional mean zero given the instruments Z. The system is estimated by generalized method of moments, and the null hypothesis is examined with the chi-square goodness of fit statistic. χ^2 is the test statistic, *df* is the degrees of freedom, and p-value is the right-tail probability value. Non, Dur, and Serv are real, per capita growth rates of nondurables, durables, and services expenditures, respectively. (SA) denotes seasonally adjusted. Dec*n* is the common stock portfolio from the *n*th size decile. TB is the excess return to a strategy of rolling over one-month Treasury bills. GB is the long-term government bond, and CB is the long-term, high grade corporate bond. All returns are in excess of the return of a three-month Treasury bill.

State Variable:	Non		Non(SA)		Serv(SA)		Dur		Dur(SA)	
Test Assets (rows):	$\hat{\beta}_i$	$t(\hat{\beta}_i)$	$\hat{\beta}_i$	$t(\hat{\beta}_i)$	$\hat{\beta}_i$	$t(\hat{\beta}_i)$	$\hat{\beta}_i$	$t(\hat{\beta}_i)$	$\hat{\beta}_i$	$t(\hat{\beta}_i)$
GB	0.99	7.50	−0.04	−0.03	−0.02	−0.02	−0.52	−5.30	−0.03	−0.44
CB	0.17	11.35	0.16	0.12	0.16	0.13	−0.77	−7.45	0.05	0.78
TB	0.01	5.76	−0.12	−0.43	−0.05	−0.84	−0.02	−6.54	−0.01	−1.21
Dec1	0.58	23.11	3.00	28.04	4.68	21.81	6.15	8.72	0.44	21.26
Dec3	0.47	39.54	2.56	1.35	4.09	2.23	0.30	1.07	0.35	4.98
Dec5	0.35	24.42	2.20	1.05	3.52	1.82	1.16	5.67	0.29	3.51
Dec7	0.34	21.72	2.07	0.90	3.81	1.69	1.49	8.46	0.26	2.86
Dec10	0.20	10.43	1.55	0.55	2.55	1.11	2.11	13.60	0.13	1.15
Full Instrument Set										
χ^2 Statistic	91.14		79.97		88.42		101.62		75.50	
df	48		48		48		48		48	
p-Value	0.000		0.003		0.000		0.000		0.007	
Consumption Instrument Subset										
χ^2 Statistic	69.13		66.15		68.32		83.84		51.69	
df	27		27		27		27		27	
p-Value	0.000		0.000		0.000		0.000		0.003	

Table VI

Tests of Asset Pricing Models with $K = 2$ and $K = 3$ Consumption Betas

The tests are based on quarterly data for 1948:2–1985:4 (151 observations). The model is

$$\begin{aligned} f &= Z\gamma + \eta, \\ r_1 &= Z\delta_1 + (f - Z\gamma)\beta_1 + \varepsilon_1, \\ r_2 &= Z\delta_1(\beta_1^{-1}\beta_2) + (f - Z\gamma)\beta_2 + \varepsilon_2, \end{aligned}$$

where $r = (r_1, r_2)$ is a vector of returns in excess of the Treasury bill rate, f is a vector of K (= 2 or 3) state variables, and Z is a vector of predetermined instrumental variables. γ, δ_1, and the β's are parameters. β_1 is the matrix of conditional betas of the excess returns of the reference assets r_1 on the state variables. The ε_2 are the "nonsystematic" portion of the unanticipated excess returns on the 8-K test assets, r_2. The β_2 are the conditional betas of the test assets. The model implies that the error term $u = (\eta, \varepsilon_1, \varepsilon_2)$ has conditional mean zero given the instruments, Z. The system is estimated by generalized method of moments, and the null hypothesis is examined with the chi-square goodness of fit statistic. χ^2 is the test statistic, df is the degrees of freedom, and p-value is the right-tail probability value. The eight instruments Z are a constant, four lagged values of real nondurables consumption growth, a lagged nominal and a lagged excess Treasury bill return, and a lagged two-year real return on the CRSP equally weighted stock index. Non, Dur, and Serv are real, per capita growth of nondurables, durables, and services expenditures, respectively. (SA) denotes seasonally adjusted. The test assets are the quarterly returns, in excess of a three-month Treasury bill, of portfolios of common stocks from size deciles 1, 3, 5, 7, and 10, a long-term government bond, a long-term, high-grade corporate bond, and a strategy of rolling over one-month Treasury bills.

Number of State Variables, K	State Variables	χ^2 Statistic (df)	p-Value
2	Non Non(SA)	117.7 (32)	0.000
2	Non Dur	115.4 (32)	0.000
2	Dur Dur(SA)	147.5 (32)	0.000
2	Non(SA) Serv(SA)	100.3 (32)	0.000
2	Non Serv(SA)	111.2 (32)	0.000
2	Dur(SA) Serv(SA)	151.0 (32)	0.000
3	Non Serv(SA) Dur	113.2 (16)	0.000
3	Non(SA) Serv(SA) Dur(SA)	113.8 (16)	0.000

Singleton (1986) emphasize the durability of consumer goods, which implies that expenditures differ from "true" consumption. Constantinides (1990) studies habit formation, which has a similar effect, and Bergman (1985) studies nonseparability in preferences over time. Brown (1988), Sheinkman and Weiss (1986), and Grossman and Laroque (1990) emphasize market imperfections which prevent consumers from equating marginal rates of substitution in consumption with security returns. In such models, security market betas can capture expected returns while a consumption beta model does not.

D. Models with Asset Market State Variables

Table VII uses asset returns as proxies for the state variables. Theory and previous empirical work suggest a role for a "market portfolio" of aggregate wealth as a state variable. The first asset market proxy is the return of the CRSP value-weighted stock index (VW). The second is the real return of a three-month Treasury bill (RR). This variable follows Merton (1973), Cox, Ingersoll, and Ross (1985), and others, who show that a short-term interest rate can proxy for the state of investment opportunities. The real return is measured as the nominal bill rate less the (CPI) rate of inflation. Additional state variable proxies include the CRSP equally weighted stock index (EW) and size portfolios from deciles 2 and 8. Table VII shows that the test results using these proxies are similar to the results for the consumption variables. Although such comparisons can be hazardous, the statistics for the $K = 3$ models, with the same degrees of freedom, are even larger than the ones in Table VI.

Figure 2 depicts the pricing errors of models with three consumption betas and models with three asset market betas. The pricing errors are defined as before, so they include the model error in fitting the expected returns, plus the systematic and the nonsystematic unanticipated returns. The mean pricing errors and the mean absolute errors of the consumption models are generally larger than the errors of the latent variable model, which are included in the figure for comparison.[24] However, the visual evidence of worsened fit is not dramatic, because the scale of the figure is chosen to accommodate the pricing errors of a model with $K = 3$ asset returns (VW, EW, R) as state variables. The fit of this model appears worse than the fit of the consumption beta models. Both the mean errors and the mean absolute errors are much larger.

IV. Some Diagnostics and Extensions

One interpretation of the results in Tables IV–VII and Figure 2 is that the conditional consumption betas capture the time-varying expected returns at least as well as the asset market betas but that neither type of variable is an adequate proxy for the state variables. However, the model assumes that conditional expectations are linear and that conditional betas are constant over time. Such a specification has limited theoretical justification, so an examination of these

[24] Figure 2 depicts the consumption model with seasonally adjusted data. The pricing errors using not seasonally adjusted consumption appear more similar to the latent variable model.

Table VII

Tests of Asset Pricing Models with K = 1, 2, and 3 Asset Market State Variables

Quarterly data for 1948:2–1985:4 (151 observations) are used. The model is

$$f = Z\gamma + \eta,$$

$$r_1 = Z\delta_1 + (f - Z\gamma)\beta_1 + \varepsilon_1,$$

$$r_2 = Z\delta_1(\beta_1^{-1}\beta_2) + (f - Z\gamma)\beta_2 + \varepsilon_2,$$

where $r = (r_1, r_2)$ is a vector of returns in excess of the Treasury bill rate, f is a vector of K (=1, 2, or 3) state variables, and Z is a vector of predetermined instrumental variables. γ, δ_1, and the β's are parameters. β_1 is the matrix of conditional betas of the excess returns of the reference assets r_1 on the state variables. The β_2 are the conditional betas of 8-K test assets, r_2. The model implies that the error term $u = (\eta, \varepsilon_1, \varepsilon_2)$ has conditional mean zero given the instruments, Z. The eight instruments are a constant, four lagged values of real nondurables consumption growth, a lagged nominal and a lagged excess Treasury bill return, and a lagged two-year real return on the CRSP equally weighted stock index. The consumption instrument subset comprises only the first five instruments; the financial subset includes only the constant and the last three. The system is estimated by generalized method of moments, and the null hypothesis is examined with the chi-square goodness of fit statistic. χ^2 is the test statistic, *df* is the degrees of freedom, and *p*-value is the right-tail probability value. Dec*n* is the common stock portfolio from the *n*th size decile. RR is the real return of a three-month treasury bill. VW is the return of CRSP value-weighted common stock index. EW is the return of the CRSP equally weighted stock index.

No. State Variables	State Variables	Instruments	χ^2 Statistic (*df*)	*p*-Value
1	RR	All Eight	92.5 (48)	0.000
1	VW		94.9	0.000
1	RR	Consumption	82.8 (27)	0.000
1	VW		94.5	0.000
1	RR	Financial	58.0 (20)	0.000
1	VW		35.1	0.020
2	RR, Dec2	All Eight	144.0 (32)	0.000
2	RR, VW		150.9	0.000
2	RR, Dec2	Consumption	151.0 (14)	0.000
2	RR, VW		142.6	0.000
2	RR, Dec2	Financial	124.7 (8)	0.000
2	RR, VW		125.4	0.000
3	RR, Dec2 Dec8	All Eight	127.4 (16)	0.000
3	RR, VW, EW		150.9	0.000

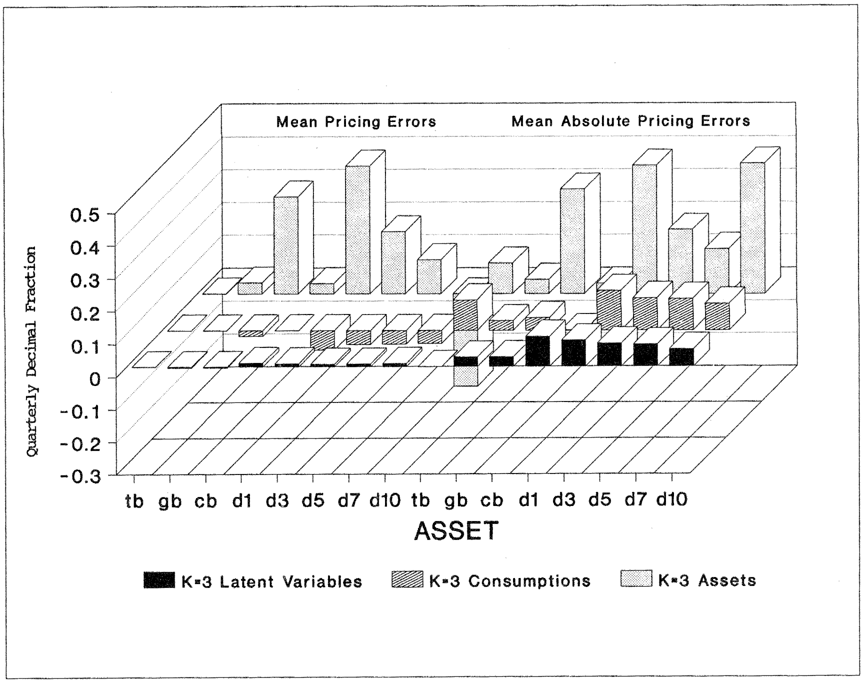

Figure 2. Pricing errors of models with 3 state variables, quarterly data for 1948:2–1985:4. The model is

$$f = Z\gamma + \eta,$$

$$r_1 = Z\delta_1 + (f - Z\gamma)\beta_1 + \varepsilon_1,$$

$$r_2 = Z\delta_1(\beta_1^{-1}\beta_2) + (f - Z\gamma)\beta_2 + \varepsilon_2,$$

where $r = (r_1, r_2)$ is a vector of returns in excess of the Treasury bill rate, f is a vector of 3 state variables, and Z is a vector of predetermined instrumental variables. γ, δ_1, and the β's are parameters. β_1 is the matrix of conditional betas of the excess returns of the reference assets r_1 on the state variables. The β_2 are the conditional betas of the five test assets, r_2. The eight instruments, Z, are a constant, four lagged values of real nondurables consumption growth, a lagged nominal and a lagged excess Treasury bill return, and a lagged two-year real return on the CRSP equally weighted stock index. The mean pricing errors are the sample means of $r_1 - Z\delta_1$ and $r_2 - Z\delta_1(\beta_1^{-1}\beta_2)$. The mean absolute pricing errors are the sample means of the absolute values of these terms. The three measures of consumption which serve as the state variables are per-capita expenditures for nondurables, services, and durable goods (seasonally adjusted). The assets used as state variable proxies are an equally weighted stock index, a value-weighted stock index, and a real interest rate. The test assets are denoted as follows:

tb = Treasury bill,

gb = government bond,

cb = corporate bond,

dn = common stock portfolio from size decile n.

assumptions is important.[25] The objective is to get some feel for the sensitivity of the results to departures from these assumptions. For example, the models with the asset market state variables might have more important departures from the statistical assumptions than the latent variable or the consumption beta models.

Previous empirical evidence suggests that conditional variances and covariances of asset returns vary with predetermined information,[26] and tests using the instruments in this study reach a similar conclusion for most of the returns.[27] A constant conditional beta models allows variation in the conditional variances and covariances but restricts the conditional covariances to be proportional to the conditional variances of state variables.

There is also evidence that conditional betas vary with predetermined variables.[28] Ferson and Harvey (1989) find that such variation contributes only a small amount to the predictable variation of the excess returns of monthly size and industry portfolios, while time-varying expected risk premiums per unit of beta are a most important source of predictability.

Of course, there are alternatives to a constant-beta model. One approach is to assume that the ratio of the expected risk premium to the conditional variance for a state variable is a constant parameter.[29] Another approach specifies a functional form for the conditional beta.[30] It is possible to extend the approach of this paper along these lines by appending additional moment conditions for time-varying betas.

The statistical specification of this paper is examined in Tables VIII and IX using an approach that is similar to the tests of the asset pricing models. However, the tests do not impose the asset pricing restrictions on the expected returns. Consider the following system of equations, estimated by GMM for asset i and

[25] Some theoretical models are consistent with these assumptions. See, for example, a version of the Cox-Ingersoll-Ross (1985) model described in Stambaugh (1988), or see Connor and Korajczyk (1989) or Donaldson and Mehra (1984). Assuming stationary joint normality of the returns, state variables and information implies that conditional expectations are linear and that conditional betas are constant over time.

[26] See, for example, Hasbrouck (1986), Campbell (1987), French, Schwert, and Stambaugh (1987), Bollerslev, Engle, and Wooldridge (1988), and Ferson (1989).

[27] These tests are conducted by forming products of the OLS residuals, from regressions of the asset returns and the state variables on the instruments, and regressing the product on the instruments. Evidence of conditional covariances that are correlated with the instruments is found in the form of significant slope coefficients.

[28] See, for example, Keim and Stambaugh (1986), Campbell (1987), Ferson (1989), Bodurtha and Mark (1989), and Shanken (1990).

[29] Constant ratios of expected excess return to conditional covariances with marginal utility are implied by some consumption models (e.g., Grossman and Shiller (1982), Hansen and Singleton (1983), and Ferson (1983)). In these models the ratio is a coefficient of risk aversion. Ferson and Harvey (1990) find evidence against a constant ratio in quarterly and annual consumption data. Campbell (1987) models monthly conditional variances of portfolios as linear in predetermined variables and rejects the hypothesis of constant expected excess return to variance ratios. Harvey (1989) rejects constant ratios for stock market indexes against a general alternative.

[30] Examples of such an approach include Campbell (1987), Ferson, Kandel, and Stambaugh (1987), and Shanken (1989a). ARCH-type models (see, e.g., Engle (1982)) are used for conditional betas in Bollerslev, Engle, and Wooldridge (1988), Bodurtha and Mark (1989), and others.

Table VIII

Estimates and Tests of Constant Conditional Consumption Betas Quarterly Data, 1948:2–1985:4 (151 Observations)

The model is

$$f_k = Z\gamma_k + \eta_k,$$

$$\omega_i = (f_k - Z\gamma_k)r_i - (f_k - Z\gamma_k)^2\Theta_{ik},$$

where Θ is a constant parameter which is equal to the conditional beta if the beta is constant. Z is a vector of predetermined instrumental variables. γ is a parameter. (t) is the t-ratio of the Θ coefficient estimate for a single-asset, single state variable model. The system is estimated by generalized method of moments. pv is the right-tail probability value of the asymptotic χ^2 statistic testing the orthogonality conditions. The asset system tests stack the equations for ω_i together across assets. The three-beta models include equations for each of the three state variable proxies, f_k, and generalize the system to multiple regression betas. Non, Dur, and Serv stand for real, per capita growth of nondurables, durables, and services expenditures, respectively. (SA) denotes seasonally adjusted. Dec$n$ is the common stock portfolio from size decile n. TB is the excess return to rolling over one-month Treasury bills. GB is the long-term government bond, and CB is the long-term, high-grade corporate bond return. All returns are in excess of a three-month Treasury bill return.

Table VIII—*Continued*

Test Asset Excess Return (rows):	Non $\hat{\Theta}$ (t)	Non χ^2 pv	Serv $\hat{\Theta}$ (t)	Serv χ^2 pv	Dur $\hat{\Theta}$ (t)	Dur χ^2 pv	Three-Beta χ^2 pv	Non(SA) $\hat{\Theta}$ (t)	Non(SA) χ^2 pv	Serv(SA) $\hat{\Theta}$ (t)	Serv(SA) χ^2 pv	Dur(SA) $\hat{\Theta}$ (t)	Dur(SA) χ^2 pv	Three-Beta χ^2 pv
	State Variable Proxies (columns):													
TB	0.01	5.80	−0.00	9.84	0.00	5.22	21.4	nc[a]		nc		0.00	5.89	nc
	(0.68)	0.56	(−0.77)	0.20	(1.01)	0.63	0.44					(1.03)	0.55	
GB	−0.10	3.42	0.19	7.86	−0.03	2.61	21.7	−0.29	4.75	−0.24	6.60	−0.03	5.90	18.18
	(−0.90)	0.84	(0.82)	0.35	(−1.21)	0.92	0.42	(−1.16)	0.69	(−0.89)	0.47	(−0.80)	0.55	0.64
CB	−0.06	2.8	0.37	10.6	−0.35	3.16	26.9	−0.16	6.23	0.08	7.66	−0.04	6.43	20.53
	(−0.54)	0.91	(1.56)	0.16	(−1.30)	0.87	0.17	(−0.71)	0.51	(0.32)	0.36	(−1.21)	0.49	0.49
Dec1	0.89	7.63	0.08	12.8	−0.00	2.59	23.8	2.77	2.97	3.77	7.87	0.12	5.73	18.06
	(2.01)	0.37	(0.10)	0.08	(−0.01)	0.90	0.30	(2.45)	0.89	(2.66)	0.34	(0.53)	0.57	0.65
Dec3	0.39	7.70	0.23	12.2	−0.03	2.98	25.4	2.38	3.30	2.21	7.66	0.51	7.39	29.10
	(1.05)	0.36	(0.29)	0.10	(−0.23)	0.89	0.23	(2.48)	0.86	(1.95)	0.37	(0.27)	0.39	0.11
Dec5	0.34	7.50	0.21	10.0	0.03	4.60	22.9	1.81	2.50	2.20	7.29	0.11	7.91	29.16
	(1.08)	0.38	(0.29)	0.19	(0.39)	0.71	0.35	(2.22)	0.93	(2.11)	0.40	(0.77)	0.34	0.11
Dec7	0.22	9.56	0.10	10.0	−0.04	4.65	26.5	1.45	3.95	1.50	8.31	−0.03	6.55	26.60
	(0.80)	0.22	(0.16)	0.19	(−0.42)	0.71	0.19	(1.88)	0.79	(1.56)	0.31	(−0.17)	0.48	0.18
Dec10	0.06	7.48	0.50	8.29	−0.28	4.62	18.8	1.38	3.64	1.14	14.4	−0.03	6.90	24.65
	(0.24)	0.38	(0.93)	0.31	(−0.45)	0.71	0.60	(2.45)	0.82	(1.51)	0.04	(−0.21)	0.44	0.26
Asset System Test	49.7		62.8		47.8							58.9		
	0.71		0.25		0.77			nc		nc		0.37		

[a] nc indicates that the algorithm encountered a singular covariance matrix of the sample orthogonality conditions.

Table IX

Estimates of Tests of Constant Conditional Betas

Quarterly data for 1948:2–1985:4 (151 observations) are used. The model is

$$f_k = Z\gamma_k + \eta_k$$

$$\omega_i = (f_k - Z\gamma_k)r_i - (f_k - Z\gamma_k)^2\Theta_{ik},$$

where Θ is a constant parameter which is equal to the conditional beta if the beta is constant. Z is a vector of predetermined instrumental variables, and γ_k is a vector of parameters for the conditional mean of the state variable, f_k. The system is estimated by generalized method of moments. (t) is the t-ratio of the Θ coefficient estimate for a single-asset, single state variable model. pv is the right-tail probability value of the χ^2 statistic for the hypothesis that $E(\eta_k, \omega_i|Z) = 0$. The asset system tests stack the equations for ω_i together across assets. The multibeta models use a five-dimensional vector for the state variables and the Θ parameter, and generalize the system for multiple regression betas. The state variable proxies are denoted as follows:

Dec$n$ is the common stock portfolio from size decile n,
VW is the value-weighted common stock index,
EW is the equally weighted common stock index, and
RR is the real return of a three-month Treasury bill.

The test assets are measured in excess of a three-month Treasury bill return and are denoted as follows:

Dec$n$ is the common stock portfolio from size decile n,
TB is the strategy of rolling over one-month Treasury bills,
GB is the long-term government bond return, and
CB is the long-term corporate bond return.

Table IX—*Continued*

Test Asset Excess Return (rows):	State Variable Proxies (columns): Dec2		Dec8		VW		RR		EW		Multi-Beta
	$\hat{\Theta}$ (t)	χ^2 pv	$\hat{\Theta}$ (t)	χ^2 pv	$\hat{\Theta}$ (t)	χ^2 pv	$\hat{\Theta}$ (t)	χ^2 pv	$\hat{\Theta}$ (t)	χ^2 pv	χ^2 pv
TB	−0.00	6.07	−0.00	6.56	0.00	7.77	−0.02	9.03	−0.00	7.06	nc[a]
	(−0.60)	0.53	(−0.57)	0.48	(0.16)	0.35	−3.64	0.25	(−0.53)	0.42	
GB	0.10	6.73	0.47	10.0	0.06	13.3	0.99	4.71	0.03	8.46	48.4
	(0.46)	0.79	(1.74)	0.19	(1.87)	0.07	(3.71)	0.69	(1.10)	0.29	0.07
CB	0.06	6.67	0.08	10.1	0.16	18.0	1.03	5.76	0.67	8.55	47.3
	(2.91)	0.46	(3.11)	0.19	(5.44)	0.01	(3.96)	0.57	(2.82)	0.29	0.08
Dec1	1.05	8.80	1.18	4.82	1.27	4.12	0.50	10.6	1.15	5.47	35.3
	(37.4)	0.27	(21.3)	0.68	(18.6)	0.76	(0.40)	0.16	(28.7)	0.60	0.45
Dec3	0.94	8.96	1.12	4.99	1.22	3.60	−0.08	9.49	1.08	8.10	47.9
	(70.0)	0.26	(35.2)	0.66	(27.3)	0.82	(−0.08)	0.22	(60.3)	0.32	0.07
Dec5	0.81	12.9	1.05	5.47	1.17	5.28	0.16	9.84	0.98	4.62	59.8
	(32.4)	0.07	(45.9)	0.60	(35.3)	0.63	(0.19)	0.20	(66.3)	0.71	0.01
Dec7	0.74	10.8	1.01	13.1	1.12	6.30	0.35	9.73	0.92	10.1	35.5
	(32.3)	0.15	(53.2)	0.07	(38.5)	0.51	(0.44)	0.29	(64.4)	0.18	0.04
Dec10	0.41	10.6	0.70	5.96	0.91	3.00	0.51	5.66	0.56	10.9	45.7
	(12.7)	0.16	(19.7)	0.54	(49.0)	0.88	(0.80)	0.58	(17.3)	0.15	0.11
Asset System Test	63.7		56.6		56.6		63.8		56.1		
	0.22		0.45		0.45		0.22		0.47		

[a] nc indicates that the program encountered a singular covariance matrix of the sample orthogonality conditions.

state variable k:

$$f_k = Z\gamma_k + \eta_k,$$
$$\omega_i = (f_k - Z\gamma_k)r_i - (f_k - Z\gamma_k)^2\Theta_{ik}. \tag{7}$$

The first equation of (7) models the conditional expected value of the kth state variable as a linear regression on Z. When Z is the instrument, this equation does not generate overidentifying restrictions because the numbers of parameters and orthogonality conditions are equal. (The first equation alone produces the same estimate of γ_k as equation-by-equation ordinary least squares, summarized in Table III.) In the second equation, Θ_{ik} will equal the conditional beta β_{ik} if the latter is constant. However, the conditional expectation of ω_i will not be identically zero for any fixed value of Θ_{ik} if β_{ik} is not a constant. The equation for ω_i implies L orthogonality conditions but only one additional parameter, so the degrees of freedom of the χ^2 statistic are $L - 1$.

Table VIII summarizes tests of the constant beta assumption with the consumption growth measures as the state variables. Each asset and consumption measure is examined separately to see whether particular variables or assets are important sources of misspecification. Joint tests across the assets, for a single state variable, are shown in the bottom row. The columns labeled "Three-Beta" report results for a vector of three state variables and a single asset.

The goodness-of-fit statistics produce little evidence against the statistical specification. The smallest p-value in the table is 0.04, and only three are less than 10 percent. Two of these are for not seasonally adjusted services expenditures. The estimates of the conditional consumption betas for the stocks relative to both seasonally adjusted nondurables and services are close to the estimates of the c_i in Table IV, with $K = 1$ latent variable. The t-ratios are near 2.0. Most of the other t-ratios and test statistics are small.[31]

Equation (7) assumes a linear model for the expected values of the state variables, but it does not assume that the expected returns of the assets are linear. The conditional beta pricing model does assume that the expected excess returns are linear. A modification of system (7) is examined, imposing the additional linearity assumption.[32] The goodness-of-fit tests produce only weak evidence against the specification.[33]

Table IX summarizes the tests of equation (7) when asset returns proxy for the state variables. The magnitudes of the conditional beta estimates are similar to the unconditional beta coefficients reported in the literature for "market model" type regressions. The t-statistics show that some of the estimates are quite precise. It should not be surprising that betas relative to asset returns are

[31] When the model is estimated jointly across the assets, the t-ratios of the betas are frequently much larger than in the single-asset models. These t-ratios are not shown in the table.

[32] A third equation is appended for each asset: $r_i = Z\delta_i + e_i$; and the expression $(r_i - Z\delta_i)$ replaces r_i in the second equation of (7).

[33] The largest test statistics are obtained for not seasonally adjusted services, where the p-values are between 0.01 and 0.11 for individual assets. Because of the additional orthogonality conditions, joint tests could only be conducted across subsets of the assets. The p-value of the multiple-asset test is slightly below 0.05 using seasonally adjusted nondurables and slightly larger than 0.05 for unadjusted nondurables.

estimated with greater precision than are consumption betas. There is virtually no evidence against the constant beta assumption in the single-beta models (4 of the 40 p-values are below 0.10).[34]

Even if the simple conditional betas are constant, it is possible that the multiple betas are time-varying if the covariances among the state variables are time-varying. The right-hand column of Table IX provides some evidence. In multiple-beta models with all five state variables, five of the eight p-values are below 0.10 and two are below 0.05. When three-beta versions corresponding to the models that were rejected in Table VII are examined, three cases produce p-values below 0.10 and one is below 0.05.

In summary, there is some weak evidence against the statistical specification, at least for the asset market state variables. However, it seems unlikely that the results are fully explained by nonconstant conditional betas or nonlinearities in the conditional means. Of course, the tests of the pricing models could be more sensitive to nonlinearities and to nonconstant betas than are the tests in Tables VIII and IX.[35]

Some extensions and variations on the model formulation provide further insights about the sensitivity of the results. (Further details and tables of these results are available by request.) When subsets of the asset returns are maximally correlated with the state variables, the risk premiums can be identified as their expected excess returns.[36] Shanken and Weinstein (1987) and Shanken (1989b) use this observation in multibeta models for unconditional mean returns. The extension to time-varying conditional expected returns is straightforward. One such model consists of two equations which are similar to the latent variables formulation and a third equation which identifies the conditional betas of the test assets. The third equation can be formulated in various ways. Three experiments were conducted using different formulations.

In the first experiment, the equation which specifies the conditional betas does not control for the variance of the systematic unanticipated returns. The conditional betas are identified using the expected values of the products of asset return residuals. The larger error variance of this model, compared with system (6), could reduce the power of the tests even through the model incorporates the restriction that the expected risk premium is equal to the expected excess return of a proxy portfolio. The tests appear to reject the $K = 1$ and $K = 2$ models at standard significance levels, but there is mixed evidence against the $K = 3$ models. Three of six ($K = 3$) models produce p-values below 0.07, and three of the p-values are larger than 0.20. The average pricing errors are slightly larger

[34] The results are similar when the linear equations for the expected excess returns of the test assets are included in the system (7).

[35] Alternatively, there could be small-sample problems which are more severe in the more complex system (6). However, it is not clear whether this would bias the statistic in favor of accepting or rejecting the model, and the pattern of the pricing errors is consistent with the goodness-of-fit tests.

[36] Different authors characterize this situation in slightly different ways. Breeden (1979) discusses maximum correlation portfolios. Shanken (1987, 1989b) analyzes assets which span a mean-variance efficient benchmark. Huberman and Kandel (1987) discuss conditional "mean-variance spanning" and mean-variance intersection. Huberman, Kandel, and Stambaugh (1987) discuss "mimicking portfolios." Grinblatt and Titman (1987) describe portfolios which are "locally efficient."

than the errors of the latent variable models, but they are smaller than the errors of the conditional beta pricing models. The mean absolute errors are similar to the latent variable models.

A second experiment modifies the previous model to partially control the systematic variance of unexpected returns. This is accomplished by making the assumption that the proxy portfolio returns are perfectly correlated with the state variables. This model is soundly rejected by the tests.

A third experiment examines consumption models without controlling the systematic error variance of the assets. In this formulation the consumption betas are not uniquely identified, but the relative betas are identified using the state variable innovations. Matrix size limitations precluded versions of this model with more than a single state variable. Replicating the ten cases examined in Table V produced p-values for the goodness-of-fit statistics between 0.009 and 0.163. Four of the ten were larger than 0.05.

Of course, the diagnostics are not conclusive, and comparisons of the p-values and pricing errors across the models can be misleading. However, the experiments suggest a pattern of sensitivity in the results. They suggest that identifying the coefficients in latent variables models as constant conditional betas may not be the only factor driving the rejections. The ability to control systematic unanticipated return variance is also important. Controlling the variance of unexpected returns should be important if the variation of expected returns is low relative to the total variance, implying a low "signal to noise ratio." This is one aspect in which the consumption models and the models with the asset market state variables differ. The portion of the unanticipated return variance that is controlled with the three consumption variables used in Figure 2 is between 2.0 and 4.2 percent of the total variance of an excess return. When the asset market variables are used, the fraction is much higher. (It ranges from 11.2 to 84.2 percent across the test assets.)

V. Conclusions

This paper examines multibeta pricing models with time variation in expected returns. The method extends the approach of Gibbons and Ferson (1985) by specifying proxies for the state variables. This allows a model to incorporate state-dependent expected returns and conditional betas in a unified framework. The approach is illustrated using quarterly bond and stock returns, with aggregate consumption measures and asset returns as the state variable proxies. The evidence suggests that the approach derives some power from using the state variables to control the systematic variance of unexpected returns. The conditional consumption betas seem to fit the model better than the asset market betas. However, consumption measures do not control variance as effectively, and the hypothesis that the consumption or the market variables can proxy for the state variables is rejected.

Several extensions of the investigation seem worthwhile. It should be interesting to examine other economic state variables (as in Chen, Roll, and Ross (1986), for example), factor-mimicking portfolios (e.g., Lehman and Modest (1988)), and other information variables. The methodology can be refined to examine raw returns instead of excess returns. The distinction between the zero-beta and the risk-free rate forms of the models can then be drawn. The assumptions of linear expectations and constant betas can be replaced with alternative functional forms. Measuring returns at multiple frequencies could produce insights about the relation of conditional expected returns and risk over different investment horizons.

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