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Equilibrium Block Trading and Asymmetric Information

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ABSTRACT

This paper investigates the existence of equilibria with information-based block trading in a multiperiod market when no investor is constrained to block trade. Attention is restricted to equilibria in which a strategic uninformed institution (i.e., one which is forced to rebalance its portfolio but is free to choose an optimal rebalancing strategy) is willing to trade a block rather than "break up" the block into a series of smaller trades. Examples of such equilibria are found and analyzed.

A STRIKING FACT ABOUT the New York Stock Exchange is that roughly half of the volume is traded in blocks of over 10,000 shares.[1] However, despite the obvious importance of block trading, the types of market microstructures which generate block trades are not well understood. This paper provides a theoretical rationale for block trades by modeling an equilibrium in which blocks are endogenously traded. In particular, we show that, even when a block can be "broken up" into a sequence of small trades, blocks may still be traded as part of both informed and uninformed investors' optimal trading strategies.

The analysis is conducted in a simple market in which there are competitive dealers and specialists, a group of small "noise" traders, and a strategic institution which trades either to exploit private information or because it is constrained to rebalance its portfolio. The main results about block trading in this setting are as follows:

(1) For any size block there is a market parameterization with a "separating" equilibrium in which the institution optimally trades a block to rebalance and trades dynamically using market orders when informed.
(2) Increasing the amount to be rebalanced (holding fixed the rest of the "separating" parameterization) leads to a "partial-pooling" equilibrium in which the institution still trades blocks to rebalance but randomizes between block trades and dynamic strategies when informed. In this second equilibrium, block trades are "large enough" to have the sort of "perma-

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[1] See the *New York Stock Exchange Fact Book 1989*, p. 73. Also, 43 percent of the 1988 NASDAQ volume consists of block trades.

nent" price effect found in Kraus and Stoll (1972) and Holthausen, Leftwich, and Mayers (1987).[2]

(3) The model is consistent with the observation that, in addition to being "large", blocks are typically executed in "lumpy" trading strategies which also include periods of no trade.

These results extend recent work on block trading by Easley and O'Hara (1987) and Gammill (1985) in three directions. First, the set of admissible trading strategies is expanded to include dynamic strategies. This is unavoidable if the model is to explain why blocks are not "broken up" and why "lumpy" strategies are used.[3] Second, the uninformed institution is not forced to trade a block but is instead allowed to follow an optimal trading strategy for executing its portfolio rebalancing.[4] In particular, the timing of its rebalancing is made flexible enough to permit dynamic strategies as alternatives to block trading. Third, dealers and specialists provide competing market-making mechanisms.[5] This feature allows the model to capture the fact that block trades are typically executed via off-exchange "block positioning" through a network of dealers which coexists alongside the specialist system on the exchange floor. From our perspective, the most significant difference between these two market-making mechanisms is that dealers negotiate block trades with investors whose identities are known. This is in sharp contrast to the comparatively anonymous execution of orders by specialists.

Optimal rebalancing strategies receive particular attention in our analysis. Given the adverse selection problem in trading with a potentially informed institution, dealers set block prices so that probabilistically their expected losses on trades with an informed institution are offset by expected gains on trades with an uninformed one. However, for the uninformed institution to be willing to trade a block, its expected loss on the block must be less than the expected loss on any available alternative trading strategy that also satisfies its rebalancing constraint.

[2] Seppi (1988) provides evidence that permanent price changes are partially due to information revelation by documenting a positive correlation between earnings forecast errors and block trade price changes during the week before quarterly earnings announcements. This correlation is interpreted as a specific example of the type of information blocks reveal.

[3] Two alternative explanations are transaction costs and risk aversion. Constantinides (1986) shows that transaction costs can lead to "lumpy" trading. The argument for risk aversion is less clear. If the probability of *exogenous* resolution of uncertainty (leading to price changes) increases with time, then dynamic strategies over very short intervals should dominate block trades. The question of whether price volatility due to the *endogenous* revelation of information through trading can induce block trading is the central question in this paper.

[4] Admati and Pfleiderer (1987) and Foster and Viswanathan (1987) also examine the equilibrium implications of allowing uninformed investors to trade optimally. Their intent, however, is to explain pricing and volume patterns in daily and weekly data.

[5] Gammill (1985) considers a market with a monopolistic market maker and two potentially informed investors and shows that there are noncooperative equilibria with block trading. In these equilibria the market maker executes the first block at a loss as a way of inducing the first investor to become informed to reveal its information. The market maker can then (because of its monopoly power) set subsequent prices so as to recoup this loss and earn positive profits on later trades by liquidity traders. With competitive market makers as in this paper (and Easley and O'Hara (1987)), this market-making strategy is not feasible.

Block trades in this paper are intended to resemble actual block trades in which a brokerage house acts as a principal (i.e., takes at least part of the other side on its own account rather than "shopping the block" to other investors). Such block trades account for a significant proportion of observed block trades. In 1988, total off-exchange member trading (of which block positioning was the most significant component) equalled half of the total block trade volume.[6] Such trades typically involve implicit commitments between dealers and investors. One common understanding is that the investor initiating the block should not trade the stock again soon (i.e., while the block is still on the dealer's inventory). Failure to keep this commitment is called "bagging the street" and can lead to a refusal by brokerage houses to use their own capital in executing future trades for a frequent offender. (See Glynn (1983) and Smith (1985).)

Such commitments provide a natural way to endogenize "lumpy" trading strategies. The economic interpretation of a binding "no bagging" commitment is that it allows the dealer to condition his or her price quote both on the institution's current block order and on knowledge of the institution's future market orders (i.e., given a binding "no bagging" commitment, the dealer knows that for the relevant future the institution will make no additional trades). In contrast, the specialist can condition quotes only on past and current aggregate orders.

This paper is organized as follows: Section I describes the basic market structure used in the block trading model, defines a "fair price" equilibrium, and proves that one always exists for this model. Section II finds sufficient conditions under which a market has an equilibrium with block trades and binding "no bagging" commitments. Section III discusses some empirical implications of the model. Section IV contains a summary.

I. Basic Market Structure

This section presents a simple model of a multiperiod asset market in which optimal trading strategies and pricing rules can be calculated.[7] This model is then used in Section II to solve for an equilibrium with block trading.

The model represents a security market as a finite horizon game. The structure of the game is presented in the following order: first, the traded assets are described; second, the market participants are introduced; third, the information sets available to the market at each trading date are described; and fourth, a "fair price" equilibrium is formally defined for this structure, and optimal trading strategies, equilibrium beliefs, and prices are characterized. The section ends with an existence result.

Traded assets—There is a risky stock and a numeraire in terms of which the stock price is quoted (i.e., cash). The terminal value of the risky stock $\tilde{v}$ is a

[6] In 1988, 22.3 billion shares were traded on the NYSE in blocks of 10,000 shares or more. Total off-floor member trading was 11.2 billion shares. *NYSE Fact Book 1989*, pp. 73, 75.

[7] The model is most closely related to Kyle (1985). Easley and O'Hara (1987) use an alternate modeling strategy based on Glosten and Milgrom (1985). The advantage of Kyle's approach is that it allows for dynamic strategies of trade through the specialist. A discrete-space specification is used here because mixed strategies are more tractable in this setting.

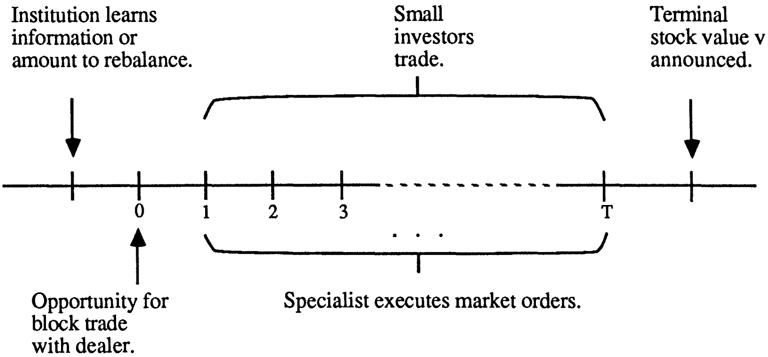

Figure 1. Timing of events in a simple multiperiod market.

random variable with unconditional expected value μ. To abstract from issues of time preference, the risk-free interest rate is assumed to be zero.

Market participants—There are four risk-neutral market participants: 1) a group of N small (price-taking) uninformed investors with random exogenous trading needs, 2) a single large (non-price-taking) institutional investor with either an exogenous portfolio rebalancing shock or private information, 3) a group of competitive specialists on an organized exchange, and 4) a group of competitive off-exchange dealers. Knowledge of the game structure and of the parameters of the joint distribution of the investor state variables (i.e., trading shocks and private information) is referred to as the market parameterization M and is common to all market participants. Knowledge of the realized values of the investor state variables is private to each individual investor and cannot be communicated in a verifiable manner to other market participants except to the extent that it is revealed in equilibrium by trading actions. Figure 1 illustrates the timing of events in the market.

Market makers (i.e., dealers and specialists) facilitate trading by small investors and the institution by receiving orders, clearing markets, and setting prices. At time 0, dealers stand ready to trade a block on their own account.[8] At times $1, \cdots, T$, specialists execute any market orders sent to the floor of the exchange. Imbalances between buy and sell orders are offset by specialists on their own account, and all market orders are executed at the price at which the specialists clear the market. Market orders are submitted anonymously to the exchange. In a block trade, however, the dealer knows the identity of the institution. This allows the dealer and the institution to enter into additional commitments beyond simply agreeing to the terms of the trade (i.e., the price and quantity). The particular commitment examined in Section II is a "no bagging" commitment prohibiting subsequent trading by the institution.

Trading by small investors—Small investors have no discretion over the timing or size of their trades. At dates $t = 1, \cdots, T$, each individual small trader $i = 1, \cdots, N$ submits a market order to trade a random number of shares $\tilde{e}_{it}$ ($e_{it} > 0$ indicates a buy order) for reasons that are exogenous.

[8] By restricting block trades to time 0, the timing of block trading is potentially suboptimal. This prejudices the model against the existence of an equilibrium with block trading. However, in the context of a later example, this timing is shown to be optimal.

The process generating small investors' orders is assumed to satisfy three conditions. First, the individual orders $\tilde{e}_{it}$ are jointly distributed such that the net order e_t has a binomial distribution:

$$e_t = \sum_{i=1}^{N} e_{it} = \begin{cases} e_b & \text{with probability } \pi(e_b), \\ e_s & \text{with probability } \pi(e_s) = 1 - \pi(e_b), \end{cases} \tag{1}$$

where $e_b > 0 > e_s$. The net order $\tilde{e}_t$ is identically distributed at each trading date t and is independent of all other variables in the model. Second, there is always one individual (without loss of generality $i = 1$) who sells $e_{1t} = e_s$ shares. The remaining $N - 1$ individual orders take one of only three possible values $e_b - e_s$, $e_s - e_b$, and 0. Trades of these sizes are referred to as "round lots". Third, the number of small investor crosses $\tilde{n}_t$ (i.e., pairs of offsetting buy and sell orders for nonzero round lots submitted by small investors at date t) is assumed to be independently and identically distributed with an arbitrary probability distribution over the integers 0, $\cdots$, $(N - 2)/2$. For convenience,[9] the number of small traders N is assumed to be an even integer ≥ 4.

Detailed assumptions about the joint distribution of individual orders are necessary[10] if, as in this model, the specialist is allowed to observe individual orders in addition to the aggregate order flow.[11] These particular assumptions greatly improve the mathematical tractability of the model by simplifying the optimal trading strategy for the institutional investor. For example, an immediate consequence of these assumptions about small investors' orders is that any institutional order x_t other than a round lot is immediately apparent to market makers. As a result, the institution will (endogenously) submit only round lot market orders in the equilibria examined below. This fact also suggests the quantity $|e_b - e_s|$ as a metric of block size. If $|B| > |e_b - e_s|$, then a block B is economically "large" in the sense that it could have alternatively been broken up into a sequence of buy (or sell) round lot market orders.

Trading by the institution—The institution has two possible reasons for trading: either it must rebalance its portfolio due to an exogenous noninformative shock or it wants to exploit private information about the terminal stock value v. Let $\tilde{g}$ denote a random portfolio rebalancing shock taking values g_b, g_s, or g_0. A realization g_b indicates a constraint to buy $g_b > 0$ shares of stock by a deadline at date T.[12] A realization g_s indicates an analogous sell constraint. For example, the

[9] The assumption that the number of small trades N is an even integer greater than or equal to four is motivated by the following considerations. First, $N > 2$ means that at least one cross is possible. Second, if N is not even, then the number of small investor crosses n_t and the net small investor order e_t may not be independent. For example, if N is odd and $n_t = (N\text{-}1)/2$ (i.e., the maximum number of small investor crosses), then the net small investor order must be $e_t = e_s$.

[10] These assumptions provide a minimum level of buying and selling "noise" for the institution. In defense of the fact that these (and subsequent) assumptions lead to only a limited number of order sizes being traded in equilibrium, we note that actual transactions are far more discrete than would follow from a simple integer constraint (e.g., trades of 50 round lots appear to be much more common than trades of 49).

[11] Kyle (1985) assumes that the specialist observes only the net order flows but not the crosses. However, although brokers may "bunch" orders of the same sign for execution on the NYSE, all offsetting buys and sells must be crossed with the specialist.

[12] A deadline on (or alternatively before) date T precludes trivial equilibria in which the uninformed institution avoids any costs of being confused with an informed institution by deferring trade until after the resolution of uncertainty.

institution might be an insurance company which needs to invest incoming premiums or liquidate part of its portfolio to pay a claim on a known future date. A realization g_0 indicates that there are no such constraints. Let θ denote a private signal about the terminal stock value v taking values θ_u, θ_d, or θ_0 indicating good news, bad news, and no news in the sense that the institution's resulting conditional expectations are ranked $\mu_u > \mu_0 = \mu > \mu_d$.

The institution learns its state (g, θ) before the start of trade. The possible combinations of (g, θ) are distributed [13]

$$(g,\theta) = \begin{cases} (g_b,\theta_0) & \text{must buy } g_b \text{ with probability } \pi(g_b), \\ (g_s,\theta_0) & \text{must sell } g_s \text{ with probability } \pi(g_s), \\ (g_0,\theta_0) & \text{no news } \theta_0 \text{ with probability } \pi(\theta_0), \\ (g_0,\theta_u) & \text{good news } \theta_u \text{ with probability } \pi(\theta_u), \\ (g_0,\theta_d) & \text{bad news } \theta_d \text{ with probability } \pi(\theta_d). \end{cases} \tag{2}$$

The institution can trade by submitting a block order b to a dealer at time 0 and a sequence of anonymous market orders $\underset{\sim}{x} = (x_1, \cdots, x_T)$ to the specialists on the exchange (where $b > 0$ and $x_t > 0$ indicate buy orders). If informed, the institution is free to exploit any pricing discrepancies until the terminal stock value becomes public after time T. If forced to rebalance its portfolio, the institution is still free to follow any trading strategy which satisfies its deadline T.

Price rules—Prices depend on the orders market makers receive because orders reveal information about possible information-based trading and thus indirectly about the terminal value v. The raw "trading data" can be compactly described by the following variables. A block trade is simply described by its size b. Sufficient statistics for a set of market orders observed on a trading date t are $h_t = (y_t, \nu_t, o(x_t))$, where y_t is the order flow

$$y_t = e_t + x_t, \tag{3}$$

defined as the sum of the N small investors' orders and the institution's order, ν_t is the total number of "crosses" or pairs of exactly offsetting buy and sell orders,

$$\nu_t = \begin{cases} n_t + 1 & \text{if} \quad x_t = e_s - e_b \quad \text{and} \quad e_t = e_b, \\ n_t & \text{otherwise,} \end{cases} \tag{4}$$

and $o(x_t)$ is an indicator variable for non-round lot institutional orders x_t, where

$$o(x_t) = \begin{cases} 0 & \text{if} \quad x_t \in \{e_b - e_s,\ 0,\ e_s - e_b\} \quad \text{(i.e., } x_t \text{ is a round lot),} \\ x_t & \text{otherwise.} \end{cases} \tag{5}$$

All trades are assumed to be publicly observable so that at each date t the information set on which prices can be conditioned is the trading history

$$\underset{\sim}{h}_t = (b,h_1,\cdots,h_t), \qquad t = 0,\cdots,T. \tag{6}$$

[13] Cross events are not likely to be very interesting or common in practice. For example, an institution in a "sell with bad news" state (g_s, θ_d) simply has a lower bound on the number of shares it must sell. Also, as an empirical matter, one wonders how frequently "sell with good news" or "buy with bad news" states occur given that real world investors actually have a choice of assets to buy or sell.

Formally, price rules are functions mapping trading histories $\underset{\sim}{h}_t$ into transaction prices.[14] Let $q(b)$ be the price at which a dealer will execute a block trade of b shares. Let $p = \{p_1, \cdots, p_T\}$ be a series of functions p_t where $p_t(\underset{\sim}{h}_t)$ is the price at which market orders are executed at time t given the current and past orders.

Commitment rules—Commitments entered into by trading a block are represented as a penalty function $c(\underset{\sim}{h}_T)$ specifying additional terminal payments to the dealer from the institution as a function of the final observed trading history $\underset{\sim}{h}_T$.[15] One particular commitment, the "no bagging" commitment, is discussed further in Section II.

Equilibrium—A "fair price" equilibrium (FPE) consists of an optimal trading strategy for the institution and competitive price rules and consistent beliefs for market makers.

The institution's trading strategy is a comprehensive description of how the institution would trade at each date given any previous "play of the game" when facing fixed price and commitment rules q, p, and c. Formally, a strategy is a series of probability distributions $\underset{\sim}{\psi} = \{\psi_0, \cdots, \psi_T\}$ over possible trade quantities. $\psi_0(b{:}g,\theta)$ is the probability with which an institution in state (g,θ) trades a block b. Similarly, given any mathematically possible series of previous trades $b, x_1, \cdots, x_{t-1}$ and history $\underset{\sim}{h}_{t-1}$, $\psi_t(x_t{:}\ g, \theta, b, x_1, \cdots, x_{t-1}, \underset{\sim}{h}_{t-1})$ is the probability with which the institution submits a market order for x_t.[16]

At each trading date an optimal strategy assigns positive probability only to blocks or market orders which maximize the institution's expected profit over the remaining rounds of trade (subject, of course, to any unsatisfied rebalancing constraint). These "optimal" orders are defined as follows:

Definition 1: A block trade b is optimal for an institution which must rebalance by g_b or g_s if it begins a sequence of orders b, $\underset{\sim}{x}$ which solves the dynamic programming problem:

$$\max_{b,\underset{\sim}{x}} \; [\mu - q(b)]b + \textstyle\sum_{t=1}^{T}\{\mu - E[p(\tilde{\underset{\sim}{h}}_t)\,|\,x_t,\underset{\sim}{h}_{t-1}]\}x_t - E[c(\tilde{\underset{\sim}{h}}_T)] \tag{7a}$$

subject to a portfolio rebalancing constraint

$$b + \textstyle\sum_{t=1}^{T} x_t = g \qquad (\text{where } g = g_b \textit{ or } g_s). \tag{7b}$$

Similarly, a market order x_t is optimal given previous trades $b, x_1, \cdots, x_{t-1}$ and a history $\underset{\sim}{h}_{t-1}$ if it begins a series of trades $x_t, \cdots, x_T$ which solves a truncated version of problem (7) starting at date t.

Definition 2: A block trade b is optimal for an unconstrained institution with

[14] Past prices contain no additional information beyond that in the past trading history. For empirical evidence on the relation between transactions and prices, see Glosten and Harris (1988).

[15] An alternative type of commitment specifies future payments as a function of the realized terminal stock value $c(v)$ rather than the subsequent trading history. Essentially, this involves the institution giving the dealer a "warranty". (See Grossman (1981).) There is some anecdotal evidence of implicit warranties in the form of poorer trade execution following perceived abuses of a broker-customer relationship (e.g., selling the broker a large block of stock the day before a disastrous earnings report).

[16] Although in equilibrium only sequences of optimal orders will be traded (and hence only certain histories will be observed), the hypothetical probability ψ_t is well defined for all orders and histories.

information θ_u, θ_0, or θ_d if it begins a sequence of orders b, $\underline{x}$ which solves the unconstrained problem:

$$\max_{b,\underline{x}} \; [m - q(b)]b + \textstyle\sum_{t=1}^{T}\{m - E[p(\tilde{\underline{h}}_t)\,|\,x_t,\underline{h}_{t-1}]\}x_t - E[c(\tilde{\underline{h}}_T)] \tag{8}$$

$$\text{(where } m = \mu_u,\mu,\mu_d).$$

A market order x_t is again optimal given previous trades b, x_1, $\cdots$, x_{t-1} and a history $\underline{h}_{t-1}$ if it begins a series of trades x_t, $\cdots$, x_T which solves a truncated version of (8) beginning at date t.

Although the institution is sometimes constrained to rebalance its portfolio, it trades optimally to accomplish this. By providing the rebalancing institution with alternatives to block trading in the form of dynamic market order strategies, the results here extend those in Easley and O'Hara (1987).[17]

If more than one block quantity b is optimal at date 0 or if more than one market order x_t is optimal at a subsequent date t, then the institution will be indifferent between any distributions ψ_0 or ψ_t randomizing over these orders. Put differently, programs (7) and (8) characterize *sets of optimal trades* at each date but impose no restrictions on the *probabilities* with which different optimal orders are submitted. Note that this implies that any strategy $\underline{\psi}$ is optimal provided that the sets of orders which are assigned positive probability are subsets of the sets of optimal trades.

Having described optimal trading strategies, the next step is to define market makers' beliefs and price rules. Let $\underline{\psi}^b = \{\psi_0^b, \cdots, \psi_T^b\}$ be the trading strategy market makers believe the institution will follow and let $\beta(g,\theta{:}\underline{h}_t)$ denote their assessment of the probability that the institution is in state (g,θ) conditional on a trading history h_t. Using the strategy beliefs $\underline{\psi}^b$ and Bayes' Theorem, the conditional probability $\beta(g,\theta{:}\underline{h}_t)$ is well defined for any current trading data h_t which occurs with positive probability under ψ_t^b given a previous history $\underline{h}_{t-1}$. However, market makers' beliefs must be complete in the sense that they are defined for *any* hypothetical h_t.[18] Thus, a "sequentially rational" extension of β must be found for all h_t's which do not occur under ψ_t^b. (See Kreps and Wilson (1982).)

Given their beliefs, market makers quote prices at which they are willing to clear the market. As in Kyle (1985), competition between market makers is assumed to force the price to equal the expected terminal stock value conditional on the observed trading history at each trading date:

$$\begin{aligned} q(b) &= [\beta(g_b{:}b) + \beta(g_s{:}b) + \beta(\theta_0{:}b)]\mu + \beta(\theta_u{:}b)\mu_u + \beta(\theta_d{:}b)\mu_d, \\ p(\underline{h}_t) &= [\beta(g_b{:}\underline{h}_t) + \beta(g_s{:}\underline{h}_t) + \beta(\theta_0{:}\underline{h}_t)]\mu + \beta(\theta_u{:}\underline{h}_t)\mu_u + \beta(\theta_d{:}\underline{h}_t)\mu_d. \end{aligned} \tag{9}$$

[17] Easley and O'Hara (1987) examines optimal trading strategies for informed investors and the resulting equilibria given that some uninformed investors are constrained to trade in blocks.

[18] Suppose that using "backwards induction" to solve the institution's problem, some order sequence b, $\underline{x}$, is optimal. This means that no alternative orders lead to a higher expected profit. To make such a statement, however, we need to be able to say at what prices suboptimal orders would be executed. Prices must, however, be "fair" with respect to some set of beliefs. Hence, there is a requirement that beliefs β be defined for all h_t.

Prices which satisfy (9) are called "fair prices" given the market makers' beliefs. At "fair prices" a competitive, risk-neutral market maker is just compensated for clearing the market.

One immediate implication of the "fair price" requirement is that for *any* beliefs β the range of each of the price functions $q(b)$ and $p(\underline{h}_t)$ is restricted to the interval $[\mu_d, \mu_u]$. The worst inference a market maker can draw from a trading history is that the institution has bad news θ_d, and the best inference is that it has good news θ_u.

The following definition summarizes the requirements for equilibrium:

Definition 3: A "fair price" equilibrium is a quintuple $\{\underline{\psi}, \beta, q, p, c\}$, where $\underline{\psi}$ is an optimal strategy given the price and commitment rules, β are "sequentially rational" conditional probabilities based on consistent beliefs $\underline{\psi}^b = \underline{\psi}$, and the price rules q and p are fair given the beliefs β.

Although in equilibrium $\underline{\psi}$ must be an optimal strategy when facing fair prices based on beliefs $\underline{\psi}^b = \underline{\psi}$, the institution will be indifferent between this strategy and any alternative strategy which assigns positive probability only to optimal orders. However, given that the institution is willing to use $\underline{\psi}$, the model is closed by assuming that it does so.

Having defined a "fair price" equilibrium FPE for this market, we have the following existence result for the specialist market:

PROPOSITION 1: *For any market parameterization M there is a "fair price" equilibrium in which (except possibly at date T) the institution trades only round lots with the specialist.*

Proof: See Appendix A.[19]

With a functioning multiperiod specialist market in hand, we turn to the main question of whether there are particular parameterizations which support block trading through a coexisting network of broker/dealers.

II. Block Trading with "No Bagging" Commitments

Block trades are executed by brokers in one of two ways. The broker can try to "shop the block" to potential buyers/sellers, or he or she may take part or all of the other side of the block on his or her own account and then subsequently work off the trade at a later date.[20] In the latter case, as a service to his or her client, the broker is a principal in the trade and bears the subsequent price risk. The block trades modeled here are intended to resemble blocks in which a brokerage house acts as a principal.[21]

[19] Since the set of possible prices is not a finite set, it is not sufficient to appeal to Proposition 1 of Kreps and Wilson (1982) for existence. Also, market makers are not really "players" since prices are constrained to be fair. It might appear that this could be fixed by assuming that market makers' payoffs are a large negative number if they set "unfair" prices. However, such payoff rules seem to be outside of the scope of those considered in Kreps and Wilson.

[20] Schwartz (1988) provides a good description of the mechanics of block trading.

[21] Burdett and O'Hara (1987) model the process of shopping a block.

A common institutional practice in such trades is for the client who initiates the block to commit implicitly not to trade again in the stock until the brokerage house has traded the acquired position off of its inventory. From conversations with block positioners it appears that such a commitment might last for one day after a block which is large relative to normal market volume and for up to two days after a very large block.[22] Failure to keep this commitment, commonly called "bagging the street", can lead to a refusal by brokerage houses to act as principal in future block trades with an offending client. (See Glynn (1983) and Smith (1985).) Thus, in contrast to the relatively anonymous and "commitment free" trading through NYSE specialists, the lack of anonymity in block trades allows investors to make multiperiod commitments which are then monitored and enforced by brokers.

In this section the simple market in Section I is used to show that with binding "no bagging" commitments there is always a parameterization M which supports "large" (i.e., $|B| > |e_b - e_s|$) and sometimes informationally motivated block trades in a "fair price" equilibrium.

Because of the one-shot nature of this model (e.g., the institution only rebalances once), the analysis abstracts from additional constraints which arise in models with multiple rounds of block trading where long-term relationships between brokers and investors are possible.[23] In addition, rather than explicitly modeling the cost of being shut out of a multi-round block trade market, it is simply assumed that brokers have access to a penalty technology through which prohibitive ex post penalties $c(\underline{h}_T)$ can be imposed if the final trading history $\underline{h}_T$ reveals that the institution has "bagged" its dealer.[24]

The first result of this section finds a "separating" equilibrium with arbitrarily large block trades. If forced to rebalance, the institution trades a block with the dealer at a competitive price equal to the unconditional expected value μ. If informed, it trades only with the specialist. For an uninformed institution, the opportunity cost of the "no bagging" commitment is zero. However, for an informed institution, there is a tradeoff between trading a limited number of shares as a block at a good price and potentially trading many round lots but at less favorable prices. If the block, although possibly "large" relative to a single market order, is not "large enough" relative to the total number of possible market orders, then block trades are suboptimal for the informed institution.

[22] Although comparatively short, one or two days may be the relevant time horizon over which it is possible to trade on private information obtained in advance of many types of public announcements. Seppi (1988) shows that price changes on block trades executed during the last five-day trading week before a quarterly earnings announcement are correlated with earnings forecast errors but that price changes on block trades executed two weeks before the announcement are not correlated. In addition, anecdotal evidence that occasionally institutions do in fact "bag" their broker suggests that this constraint, although short, is nontrivial.

[23] Reputation acquisition is one possible example.

[24] A "forcing contract" is possible because aggregate order flow realizations other than e_b or e_s unambiguously reveal the presence of an institutional order. In practice, the monitoring of "no bagging" commitments is more subtle. Smith (1985) describes an informal network of connections between market makers through which brokers ferret out information about suspicious trading activity. In addition, periodic public disclosures by institutions with fiduciary responsibilities also allow (imperfect) ex post monitoring of portfolio holdings.

PROPOSITION 2: *For any block size B there is a market with a separating "fair price" equilibrium in which only the uninformed institution trades a block B and makes a binding "no bagging" commitment.*

Proof: Suppose $B > 0$. The proof consists of showing that for some parameterization M there is a FPE in which the institution uses the following pure strategies: if constrained to rebalance $g_b = B$, it buys a block B (or sells a block g_s if constrained to sell); if uninformed and unconstrained, it does not trade; and, if informed, it buys round lots every period given good news and sells round lots given bad news:

$$\psi_0(b = B{:}g_b) = \psi_0(b = g_s{:}g_s) = \psi_0(b = 0{:}\theta_0)$$
$$= \psi_0(b = 0{:}\theta_u) = \psi_0(b = 0{:}\theta_d) = 1, \quad \text{(10a)}$$

$$\psi_t(x_t = 0{:}g_b) = \psi_t(x_t = 0{:}g_s) = \psi_t(x_t = 0{:}\theta_0) = \psi_t(x_t = e_b - e_s{:}\theta_u)$$
$$= \psi_t(x_t = e_s - e_b{:}\theta_d) = 1,\ t = 1,\cdots,T. \quad \text{(10b)}$$

In particular, the informed institution never trades blocks. Market makers have consistent beliefs $\underline{\psi}^b = \underline{\psi}$ and sequentially rational out-of-equilibrium beliefs as given in Step 1 of the proof of Proposition 1. The corresponding fair price rules are

$$q(b) = \begin{cases} \mu_u & \text{if } b > 0 \text{ and } b \neq B, \\ \mu & \text{if } b = B \text{ or } g_s, \\ \mu_d & \text{if } b < 0 \text{ and } b \neq g_s, \end{cases} \quad \text{(10c)}$$

$$p(\underline{h}_t) = \begin{cases} \mu_u & \text{if } o(x_t) > 0 \quad \text{or if } y_t = 2e_b - e_s \\ & \text{or if } y_t = e_b \text{ and } \beta(\theta_u{:}\underline{h}_{t-1}) + \beta(\theta_d{:}\underline{h}_{t-1}) = 1, \\ \dfrac{\pi(e_s)^t\pi(\theta_u)\mu_u + \pi(e_b)^t\pi(\theta_0)\mu}{\pi(e_s)^t\pi(\theta_u) + \pi(e_b)^t\pi(\theta_0)} & \text{if } \underline{y}_t = (e_b,\cdots,e_b) \\ \mu & \text{if } b = g_s \text{ or } B \text{ or if } b = 0 \text{ and } \underline{y}_t \text{ includes} \\ & \text{both an } e_b \text{ and an } e_s \text{ realization}, \\ \dfrac{\pi(e_b)^t\pi(\underline{n}_t = \underline{\nu}_t - \underline{1}_t)\pi(\theta_d)\mu_d + \pi(e_s)^t\pi(\underline{n}_t = \underline{\nu}_t)\pi(\theta_0)\mu}{\pi(e_b)^t\pi(\underline{n}_t = \underline{\nu}_t - \underline{1}_t)\pi(\theta_d) + \pi(e_s)^t\pi(\underline{n}_t = \underline{\nu}_t)\pi(\theta_0)} & \\ & \text{if } \underline{y}_t = (e_s,\cdots,e_s), \\ \mu_d & \text{if } o(x_t) < 0 \quad \text{or if } y_t = 2e_s - e_b \\ & \text{or if } y_t = e_s \text{ and } \beta(\theta_u{:}\underline{h}_{t-1}) + \beta(\theta_d{:}\underline{h}_{t-1}) = 1, \end{cases} \quad \text{(10d)}$$

where $\underline{y}_t = (y_1,\cdots,y_t)$, $\underline{\nu}_t = (\nu_1,\cdots,\nu_t)$, and $\underline{1}_t$ is a t-long series $(1,\cdots,1)$.

By construction the price rules are fair given the market makers' beliefs. The proof that the trading strategies in (10a) and (10b) are optimal given these price rules is divided into three steps. First, sufficient conditions are found on M such that an informed institution does not trade blocks B or g_s with binding "no bagging" commitments. Because this is immediate for all other block sizes given the block price rule, forgoing block trades is optimal for an informed institution. Second, conditions are found such that (10b) is the optimal trading strategy for an informed institution in the specialist market. Third, it is verified that under these conditions a block trade is the optimal way to rebalance and that not trading is optimal for the institution when it is uninformed and is not forced to rebalance.

STEP 1: *For an informed institution, the expected gain from buying a block B and making a binding "no bagging" commitment is less than the expected gain from buying through the specialist if the block size B is "small" in that*

$$\frac{B}{e_b - e_s} < k = \sum_{t=1}^{T} \frac{[\pi(e_s)\pi(e_b)]^t \pi(\theta_0)}{\pi(e_s)^t \pi(\theta_u) + \pi(e_b)^t \pi(\theta_0)}. \tag{11}$$

Proof of Step 1: The proof is immediate for an institution with bad news θ_d.[25] Consider an institution with good news θ_u which deviates from the proposed strategy. Given the price rule (10c) it could buy a block B at the competitive price μ. However, because the "no bagging" commitment precludes post-block trading in the specialist market, the informed institution's total expected profit under this strategy would be limited to

$$G(B|\theta_u) = [\mu_u - \mu]B. \tag{12}$$

Next, consider the proposed strategy (denoted $\underline{x}_b$) of buying round lots from the specialist on dates $1, \cdots, T$. On each buy $e_b - e_s$ there is a positive (expected) profit until the first date small investors buy e_b causing an order flow $y_t = 2e_b - e_s$, thereby unambiguously revealing that the institution is buying. The informed institution's total expected profit is

$$G(\underline{x}_b|\theta_u) = \sum_{t=1}^{T} \pi(e_s)^t [\mu_u - p(\underline{h}_{t,b})](e_b - e_s), \tag{13}$$

where $p(\underline{h}_{t,b})$ is the price given any trading history consisting of an unbroken t-long sequence of net buy realizations $\underline{y}_t = (e_b, \cdots, e_b)$.[26] Substituting from (10d) into (13) and rearranging gives

$$G(\underline{x}_b|\theta_u) = \left(\sum_{t=1}^{T} \frac{[\pi(e_s)\pi(e_b)]^t \pi(\theta_0)}{\pi(e_s)^t \pi(\theta_u) + \pi(e_b)^t \pi(\theta_0)} \right) (\mu_u - \mu)(e_b - e_s). \tag{14}$$

Thus, (11) is sufficient for $G(B|\theta_u) < G(\underline{x}_b|\theta_u)$ and insures that a block trade with a binding "no bagging" commitment is not optimal for an informed institution with good news θ_u.

The same logic can be used to show that, if g_s is small, then only the rebalancing institution will sell blocks. This also takes care of the case where $B < 0$. Q.E.D.

[25] If $B > e_b - e_s$, then the "no bagging" commitment can be relaxed to prohibit only subsequent buying (rather than all trading) after a block buy, and an institution with bad news will still not try to manipulate by buying a block.

[26] Notice that, given the assumption that the number of small traders N is even, the number of crosses ν_t contains no additional information when the order flow y_t is positive.

STEP 2: *If the probability of being informed is sufficiently small relative to $\pi(\theta_0) > 0$, then the optimal strategy for the informed institution in the specialist market is (10b).*

Proof of Step 2: See Appendix B.

STEP 3: *The optimal strategy for an uninformed institution when rebalancing is to use blocks with binding "no bagging" commitments. When not constrained to rebalance, it is optimal for the uninformed institution not to trade.*[27]

Proof of Step 3: Consider the expected profit for an uninformed institution which deviates from the proposed strategy by trading market orders. Under the price rule (10d), any buy market order $x_t > 0$ is executed at an expected price greater than the unconditional expected asset value μ regardless of the prior trading history h_{t-1}. Similarly, any sell order $x_t < 0$ is executed at an expected price less than μ. Thus, the expected gain on any trading strategy involving market orders is strictly negative for an uninformed institution. Because a gain of *zero* can be obtained by trading the necessary shares as a block, the proposed block trade strategy is the optimal way to rebalance. Similarly, not trading is an optimal strategy when the uninformed institution is not constrained to trade. Q.E.D.

Proposition 2 illustrates that simply because a block B is "large" relative to single market orders does not, in a multiperiod market, mean that it has a "permanent" price effect. Unlike the Easley and O'Hara (1987) model, blocks are traded here only for uninformative rebalancing reasons. The next result, however, shows that, if the amount to be rebalanced g_b exceeds the critical level $(e_b - e_s)k$, then a partial-pooling equilibrium is obtained in which information-based blocks are also traded.

PROPOSITION 3: *For any block size B there is a market with a partial-pooling mixed-strategy "fair price" equilibrium in which the institution still trades blocks to rebalance but randomizes between market orders and a block B if informed.*

Proof: Again suppose $B > 0$. Fix a "separating" parameterization from Proposition 2 with $B > (e_b - e_s)k$, and then increase the rebalance amount until $g_b = B$. The resulting market has a partial-pooling FPE in which (10) is modified by setting $\psi(b = 0{:}\theta_u) = 1 - \psi(B{:}\theta_u)$ in (10a), replacing $\pi(\theta_u)$ in (10d) with $\pi(\theta_u)[1 - \psi(B{:}\theta_u)]$, and setting

$$q(B) = \frac{\pi(\theta_u)\psi(B{:}\theta_u)\mu_u + \pi(g_b)\mu}{\pi(\theta_u)\psi(B{:}\theta_u) + \pi(g_b)}, \tag{10c$'$}$$

[27] This step is harder than it appears. To show that given θ_0 the institution prefers an expected profit of zero requires computing prices for each possible deviation involving market orders. If the informed institution randomizes over multiple market orders in both the (g_0, θ_u) and (g_0, θ_d) states, it is hard in general to show that there are not profitable strategies for an uninformed institution in states (g_0, θ_0), (g_b, θ_0), or (g_s, θ_0). The reason for this difficulty is that the uninformed institution (unlike the specialist) knows that there is no informed institution trading.

where the probability of a block B given good news $\psi(B{:}\theta_u)$ either is the solution to $G(B|\theta_u) = G(\underset{\sim}{x}_b|\theta_u)$ (i.e., B and $\underset{\sim}{x}_b$ are equally profitable given good news) or is one if $G(B|\theta_u) > G(\underset{\sim}{x}_b|\theta_u)$ for all $\psi(B{:}\theta_u)$.

In verifying that this is a FPE, there are two differences from the proof of Proposition 2. First, an informed institution now optimally trades blocks given good news. This follows from (12) and (14) (which imply that the informed institution is indifferent between B and $\underset{\sim}{x}_b$ when $B = g_b = (e_b - e_s)k$ and $\psi(B{:}\theta_u) = 0$) and the fact that $G(B|\theta_u)$ is increasing in B and decreasing in $\psi(B{:}\theta_u)$ and that $G(\underset{\sim}{x}_b|\theta_u)$ is increasing in $\psi(B{:}\theta_u)$.

Second, although a rebalancing institution now pays more than μ for a block $B = g_b$ (i.e., blocks are costly), it still does not deviate and "break up" g_b into market orders. For deviating to be optimal, the share-weighted average expected price paid to buy g_b using some strategy $\underset{\sim}{x}'$ must be less than $q(B)$ from (10c′). However, since $G(B|\theta_u) \geq G(\underset{\sim}{x}_b|\theta_u)$ when $B > (e_b - e_s)k$ and since the strategy $\underset{\sim}{x}_b$ is still optimal whenever the *informed* institution trades market orders (the proof of Proposition 2 Step 2 is unchanged), the existence of a strategy $\underset{\sim}{x}'$ with lower average expected prices contradicts the optimality of block trades for the informed institution. Q.E.D.

In particular, the critical quantity k in Propositions 2 and 3 can be made arbitrarily larger than one by setting the probability $\pi(e_s)$ close to one (i.e., round lots can be bought with low probability of certain detection), the probability $\pi(\theta_u)$ of good news close to zero (i.e., buy orders which do not reveal that the institution bought trade at prices close to μ), and the number of rounds of trade T to be large. If g_b is less than $(e_b - e_s)k$, a separating equilibrium results. Only if more than $(e_b - e_s)k$ must be bought to rebalance is a partial-pooling equilibrium obtained in which blocks are "large enough" to have "permanent" price effects $q(B) = p_1 = \cdots = p_T > \mu$.

Three other points can be made. First, the probabilities of uninformed blocks $\pi(g_b)$ and $\pi(g_s)$ constrain the (endogenous) probabilities of informed blocks and hence the total frequency of all blocks. However, the requirement that $G(B|\theta_u) \gtrsim G(\underset{\sim}{x}_b|\theta_u)$ (and the corresponding condition for informed block sells) implies that a simple disparity in the probabilities of uninformed blocks $\pi(g_b = B) < \pi(g_s = -B)$ alone is not sufficient to account for the observation that large block buys are less frequent than block sells. (See Holthausen, Leftwich, and Mayers (1987, p. 246).) In particular, the probability of uninformed sell *market* orders $\pi(e_s)$ cannot be too large relative to $\pi(e_b)$.

Second, the previously cited empirical evidence indicates that the information effect of block trades is small. This suggests that the empirically relevant part of the parameter space is likely to be the region close to $g_b = (e_b - e_s)k$ (and the corresponding region for sells) where informed blocks trade with low probability.

Third, if g_b is either less (or not too much larger) than $(e_b - e_s)k$, then the timing of blocks at date 0 is in fact optimal for the rebalancing institution. This follows because the opportunity cost of the "no bagging" commitment to the informed institution is increasing in the number of rounds of forgone trade. Thus, for example, it is possible that a block which is suboptimal for the informed institution at date 0 could become optimal if traded at some later date. Since the uninformed institution bears the cost of informed block trades, it clearly prefers to trade such blocks at date 0.[28]

Other equilibria—We are also interested in the existence of equilibria other than the separating and partial-pooling varieties considered thus far. If a slightly stronger constraint is imposed on the uninformed institution requiring its portfolio rebalancing to be completed by some date T_g strictly before date T, then it is easy to show that no "full" pooling equilibria exist in which the institution randomizes between block trades and market orders both when informed and when rebalancing.

PROPOSITION 4: *If $T_g < T$, then "full" pooling mixed-strategy equilibria are impossible.*

Proof: Suppose the proposition does not hold; then, for the rebalancing institution to randomize between buying a block B and following some dynamic strategy $\underset{\sim}{x}'$, the share-weighted average expected price paid must be the same. However, since the strategy $\underset{\sim}{x}'$ satisfies the constraint T_g, the dynamic strategy dominates blocks for the informed institution since with $\underset{\sim}{x}'$ the informed institution retains the option of trading after date T_g. Thus, the informed institution will not trade blocks, but this is a contradiction. Q.E.D.

The absence of a "full" pooling equilibrium is a very general result in this model. It holds not only for "round lot" equilibria but also for any alternative equilibrium in which non-round lot market orders are traded. Because the notion of a final resolution of uncertainty is simply a modeling fiction, rebalancing must always be completed before all uncertainty is fully resolved. Thus, $T_g < T$ is arguably the empirically relevant case.

III. Other Empirical Implications

Although our primary focus has been simply to motivate block trading as equilibrium behavior, the model can also illustrate other testable properties of block trades. First, however, the model needs to be enriched to allow for multiple block sizes. This is done by replacing assumption (2) with an assumption that there are different amounts which an uninformed institution may be constrained to trade. The same logic as in Section II can then be used to show that a block trading equilibrium exists for the modified market.[29]

In any equilibrium in which different size blocks $b_1, \cdots, b_m$ trade, the block

[28] The issue of optimal timing becomes more complicated as blocks become larger because of a possible tradeoff between 1) the fact that there are fewer forgone rounds of trade on blocks executed after $t = 0$ (which increases the probability of informed block trades and hence worsens the terms of trade for rebalancing) and 2) the fact that the informed institution may increase the probability that it trades market orders before trading blocks and decrease the probability that it trades just a block. Since the pre-block trading history is informative about pre-block institutional trading, this can improve the ex ante terms of trade for the uninformed institution.

[29] For example, any portfolio rebalancing in which fewer than $(e_b - e_s)k$ shares must be bought will also be executed by a block trade. States with rebalancing quantities greater than $(e_b - e_s)k$ can also be introduced.

prices $q(b_1), \cdots, q(b_m)$ have, as in Easley and O'Hara (1987), the following intuitive property:

PROPOSITION 5: *Block prices q(b) are increasing in block size b for quantities which are traded with positive probability in equilibrium.*

Proof: With a binding "no bagging" commitment, the institution's expected trading profit on a block trade b is

$$G(b|\theta) = \begin{cases} [\mu_u - q(b)]b & \text{given good news } \theta_u, \\ [\mu_d - q(b)]b & \text{given bad news } \theta_d. \end{cases} \tag{15}$$

Let b_1 and b_2 be two blocks which are traded with positive probability in equilibrium. Suppose that $b_1 > b_2$ and that $q(b_1) \leq q(b_2)$; then from (15) an institution will never optimally trade b_2 given good news θ_u and will never trade b_1 given bad news θ_d. This, however, contradicts the assumption that the prices $q(b_1)$ and $q(b_2)$ are "fair". Thus, in equilibrium block prices must be increasing in block size $q(b_1) > q(b_2)$. Q.E.D.

It is important to recognize, however, that most presently available transactions data sets (e.g., ISSM or Fitch) pool trades executed through different market-making mechanisms. The present model illustrates the important fact that the price-trade quantity relation can be nonmonotone in pooled data sets of market orders and block trades. For example, principal-executed "large" block buys of fewer than $(e_b - e_s)k$ shares will have no price impact, while individual "small" market orders can have permanent price effects. Even for blocks larger than $(e_b - e_s)k$, it is possible for the total price impact to be less than that of the average market order. Further, even when the total block price effect exceeds that of a single market order, this "market impact" cost must be less than on dynamic alternatives which also meet the typical rebalancing time constraint. This suggests that, in regressions estimating the per-share price impact of NYSE trades (e.g., Glosten and Harris (1988)), one might want to use specifications which allow for a differential "per share" price impact on large and small trades.

Another implication of Proposition 5 is that, even if the *strategy* probabilities $\psi(b{:}\theta_u)$ and $\psi(b{:}\theta_d)$ are nonmonotonic in block size, the *conditional* probability $\beta(\theta_u{:}b)$ is nondecreasing and $\beta(\theta_d{:}b)$ is nonincreasing for traded blocks. Thus, if a sample of block trade data is generated by multiple repetitions of a market like the one modeled here, then the frequency of information-based block trades should be increasing in samples sorted by unsigned trade quantities. Seppi (1988) uses this idea to develop a stronger test for the presence of information-based block trades around quarterly earnings announcements.

IV. Summary

This paper provides simple examples of multiperiod markets in which block trades are not "broken up" into sequences of smaller market orders. Parameterizations of the market are found such that 1) a strategic uninformed institution trades "large" blocks in a separating equilibrium and 2) both strategic informed and uninformed institutions trade "large" (sometimes information-based) blocks in a partial-pooling equilibrium.

These results are of interest for several reasons. First, the partial-pooling equilibrium provides a theoretical basis for the empirical hypothesis that block trades reveal private information. Second, they show that from a modeling perspective it is not necessary to treat uninformed institutions like "noise" traders. Blocks can be an optimal way for an institution to rebalance its portfolio. Third, the ability of dealers and institutions to make commitments beyond simply agreeing on the terms of the trade is shown to allow an institution to credibly signal (at least imperfectly) that it is uninformed. The particular commitment examined is a "no bagging" commitment restricting subsequent trading by investors after block trades in which the dealer takes the other side of the trade. Other implicit commitments accompanying block trades (e.g., reputation acquisition, warranties) can be viewed as alternative signaling mechanisms.

The model also suggests topics where further work is needed. These include the following.

Robustness—Given the scarcity of tractable market microstructure models, it was necessary to work in a very stylized market setting. A natural first step to enrich the model would be to replace the binomial distribution for small investors' orders with a multinomial distribution. Because this adds more "noise" in the specialist market, the relative profitability of block trades by an informed institution will decrease (i.e., Proposition 2, Step 1 becomes easier to satisfy). However, multinomial distributions complicate the proof that a strategic uninformed institution cannot also do better using market orders (e.g., Proposition 2, Step 3).

Refinements—It is easy to show that there are multiple equilibria in this market. What equilibrium refinements lead to block trading? Related issues are: who benefits from block trading (e.g., how do the small "noise" traders fare with and without blocks trading)?; and does block trading increase or decrease the specialist's spreads for market orders?

Role of extra-market commitments—The "no bagging" commitment, by essentially reducing block trade strategies to single-period strategies, plays a central role in this analysis. Are such commitment devices necessary for the viability of "lumpy" block trade strategies in multiperiod markets, or do they simply improve the terms of trade for uninformed institutions?

Appendix A

PROPOSITION 1: *For any market parameterization M there is a "fair price" equilibrium in which (except possibly at date T) the institution trades only round lots with the specialist.*

Proof: The proof is divided into three steps. First, fair prices are found which cause the institution to trade only round lots (provided that it is feasible to rebalance using round lots). Second, it is shown that a FPE exists for the specialist market if only round lots are traded. Third, the proof is modified to allow for non-round lot trades on date T when it is not feasible to rebalance using only market orders.

STEP 1: *If* $|g_b/(e_b - e_s)|$ *and* $|g_s/(e_b - e_s)|$ *are integers* $\leq T$ *(i.e., round lot rebalancing is feasible), then there are sequentially rational beliefs and fair price rules which deter non-round lot market orders.*

Proof of Step 1: Suppose at each date t market makers 1) impute non-round lot buys to an informed institution with good news and non-round lot sells to one with bad news:

$$\begin{aligned} \beta(\theta_u{:}\underline{h}_t) &= 1 \text{ if } o(x_t) > 0, \\ \beta(\theta_d{:}\underline{h}_t) &= 1 \text{ if } o(x_t) < 0, \end{aligned} \tag{A1}$$

2) conjecture that at subsequent dates $t' > t$ the institution will use pure strategies:

$$\psi^b(x_{t'} = e_b - e_s{:}\theta_u) = \psi^b(x_{t'} = e_s - e_b{:}\theta_d) = 1, \tag{A2}$$

and 3) interprets any subsequent histories inconsistent with (A2) using beliefs

$$(\beta(\theta_u{:}\underline{h}_{t'}),\beta(\theta_d{:}\underline{h}_{t'})) = \begin{cases} (1,0) & \text{if } \ o(x_{t'}) > 0 \ \text{ or if } \ o(x_{t'}) = 0 \text{ and } y_t \in \{2e_b - e_s, e_b\}, \\ (0,1) & \text{if } \ o(x_{t'}) < 0 \ \text{ or if } \ o(x_{t'}) = 0 \text{ and } y_t \in \{2e_s - e_b, e_s\}, \end{cases} \tag{A3}$$

and then resume assuming that (A2) is the institution's strategy.

Given these beliefs, the price rule

$$\begin{aligned} p(\underline{h}_t) &= \begin{cases} \mu_u & \text{if } \ o(x_t) > 0, \\ \mu_d & \text{if } \ o(x_t) < 0, \end{cases} \\ p(\underline{h}_{t'}) &= \begin{cases} \mu_u & \text{if } \ o(x_{t'}) > 0 \ \text{ or if } \ o(x_{t'}) = 0 \\ & \qquad\qquad\qquad \text{ and } \ y_t \in \{2e_b - e_s, e_b\}, \\ \mu_d & \text{if } \ o(x_{t'}) < 0 \ \text{ or if } \ o(x_{t'}) = 0 \\ & \qquad\qquad\qquad \text{ and } \ y_t \in \{2e_s - e_b, e_s\}, \end{cases} \end{aligned} \tag{A4}$$

is fair. In addition, given prices (A4), buying $e_b - e_s$ at each subsequent date t' is optimal given good news θ_u (i.e., no other order makes the institution strictly better off), and selling $e_s - e_b$ shares is optimal given bad news θ_d. Thus, the beliefs (A2) and (A3) are sequentially rational.

Clearly an unconstrained institution will not submit non-round lot orders so as to avoid having to trade at prices from (A4). In addition, if (as was assumed) it is feasible for the constrained institution to rebalance using an integer number of round lots, it will also choose to do so. Thus, non-round lot orders are deterred. Q.E.D.

STEP 2: *If* $|g_b/(e_b - e_s)|$ *and* $|g_s/(e_b - e_s)|$ *are integers* $\leq T$, *then there is a FPE for the specialist market in which the institution trades only round lots.*

Proof of Step 2: Given Step 1, the general market (in which non-round lots are hypothetically possible) can be treated as a simpler market in which all trades are round lots. In this market the specialist's beliefs about the institution's strategy is a vector $\underline{\psi}^b$ consisting of a triplet of probabilities $(\psi_t^b(e_b - e_s{:}\theta,g,\underline{h}_{t-1})$, $\psi_t^b\,(0{:}\theta,\,g,\,\underline{h}_{t-1})$, $\psi_t^b(e_s - e_b{:}\theta,g,\underline{h}_{t-1}))$ for each type of institution (θ,g) and for each possible "round lot" history $\underline{h}_{t-1}$ at each date $t = 1, \cdots, T$. Thus, $\underline{\psi}^b$ lies in a compact space U formed by the Cartesian product of a finite number of unit simplexes.

Given any strategy beliefs $\underline{\psi}^b$, prices along histories $\underline{h}_t$ that occur with positive probability under $\underline{\psi}^b$ can be calculated by using Bayes' Law to calculate conditional beliefs $\beta(g,\theta{:}\underline{h}_t)$ and then imposing the "fair price" condition. Prices must be set using some alternate beliefs for histories that do not occur under $\underline{\psi}^b$.

Given prices based on some $\underline{\psi}^b$, truncated versions of programs (7) and (8) can be solved to obtain the institution's "best response" correspondence $R(\underline{\psi}^b)$ (i.e., the set of all strategies $\underline{\psi}$ which it would be willing to use when trading at prices based on beliefs $\underline{\psi}^b$). A "fair price" equilibrium in the simplified round-lots-only market is a fixed point of the best response correspondence $\underline{\psi} \in R(\underline{\psi})$. However, in applying the Kakutani Fixed Point Theorem (see Hildenbrand and Kirman (1976, p. 201)) to show that such a fixed point exists, there is a complication: although convex-valued, the correspondence $R(\underline{\psi}^b)$ may not be closed on the subset of U where prices are set for trading histories using the specialists' alternate beliefs.

Consider, however, a related game in which the institution is exogenously constrained to use a totally mixed strategy satisfying

$$\min\{\psi_t^b(e_b - e_s{:}\theta,g,\underline{h}_{t-1}),\ \psi_t^b(0{:}\theta,g,\underline{h}_{t-1}),\ \psi_t^b(e_s - e_b{:}\theta,g,\underline{h}_{t-1})\} \geq \epsilon \qquad \forall t, \forall \underline{h}_{t-1} \forall (g,\theta). \quad \text{(A5)}$$

The specialist's beliefs $\underline{\psi}^b$ can also be restricted to the closed subspace of U satisfying an analogous constraint. Because all "round lot" histories are possible in the ϵ constrained game, no alternate beliefs are needed to set prices. Thus, the conditions of the fixed point theorem are met and there is a solution $\underline{\psi}^\epsilon$.

If constraint (A5) is not binding at the solution, then $\underline{\psi}^\epsilon$ is also a solution to the unconstrained game. If the constraint is binding, then consider the sequence of solutions $\{\underline{\psi}^\epsilon\}$ to a sequence of constrained games in which $\epsilon \to 0$. Since the unconstrained space U is compact, there is a subsequence $\{\underline{\psi}^{\epsilon_j}\}$ which converges to some $\underline{\psi}^*$.

Suppose that the limiting vector $\underline{\psi}^*$ is not an equilibrium in the unconstrained game. This occurs only if, for some history $\underline{h}_{t-1}$ at some date t for some institutional state (θ,g), an order that is optimal along the subsequence is not optimal in the limit. However, because prices lie in the (compact) interval $[\mu_d, \mu_u]$, they also converge to limiting prices. This implies in turn that the sequences of differences in expected total profits over the remaining rounds of trade given different orders at date t,

$$G_t^\epsilon(x_t = e_b - e_s|\theta,g,\underline{h}_{t-1}) - G_t^\epsilon(x_t = 0|\theta,g,\underline{h}_{t-1}),$$

$$G_t^\epsilon(x_t = e_b - e_s|\theta,g,\underline{h}_{t-1}) - G_t^\epsilon(x_t = e_s - e_b|\theta,g,\underline{h}_{t-1}), \quad \text{(A6)}$$

also converge. Since these differences characterize the set of optimal orders at date t, this contradicts the hypothesis that the set of optimal orders in the limit is smaller than the set of orders to which $\underline{\psi}^*$ assigns positive probability. Thus, the orders to which $\underline{\psi}^*$ assigns positive probability are optimal. Since the institution is indifferent between probability distributions over optimal orders, the vector $\underline{\psi}^*$ is an equilibrium of the unconstrained game. Q.E.D.

STEP 3: *A modification of Steps 1 and 2 is described here to handle cases where round lot rebalancing is not feasible (i.e., $|g_b/(e_b - e_s)|$ or $|g_s/(e_b - e_s)|$ are either non-integer or are $>T$). One solution is to change beliefs and strategies to allow for a (finite) number of possible non-round lot orders on date T so that the constrained institution can make up any shortfall after trading in round lots at all previous dates. Because the proofs of Steps 1 and 2 depend only on there being a finite number of possible orders, they are easily adapted to this case.* Q.E.D.

Appendix B

STEP 2 OF PROPOSITION 2: *If the probability of being informed is sufficiently small relative to $\pi(\theta_0) > 0$, then the optimal strategy for the informed institution in the specialist market is (10b).*

Proof: At the last trading date T, conditional on good news θ_u, it is always optimal to buy $e_b - e_s$ shares. Conditional on bad news θ_d, it is always optimal to sell $e_s - e_b$ shares. This is because fair prices lie in the interval $[\mu_d,\mu_u]$ regardless of the prior trading history $\underline{h}_{T-1}$.

A backwards induction argument can then be used to show that, if at date $t' + 1$ there exist positive bounds $a_{u,t'+1}$ and $a_{d,t'+1}$ on the probabilities of being informed $\pi(\theta_u)$ and $\pi(\theta_d)$ such that (10b) is the optimal strategy for the informed institution at dates $t' + 1, \cdots, T$, then at date t' there exist bounds $a_{u,t'}$ and $a_{d,t'}$ such that (10b) is optimal for the informed institution at dates $t', \cdots, T$. This is done by showing that for sufficiently small $\pi(\theta_u)$ and $\pi(\theta_d)$ the expected profit for an institution with good news θ_u under the proposed strategy

$$G(x_{t'} = e_b - e_s|\theta_u,\underline{h}_{t'-1}) = \pi(e_b)\textstyle\sum_{t=t'}^{T}[\mu_u - \mu_u](e_b - e_s) + \pi(e_s)\sum_{t=t'}^{T}\{\mu_u - E[p(\tilde{\underline{h}}_t)|y_{t'} = e_b]\}(e_b - e_s) \tag{B1}$$

is greater than under any deviation. (The proof for an institution with bad news θ_d is identical.)

First, given the proposed price rule (10d), any deviation in which the institution submits a non-round lot order $x_{t'} \notin \{e_s - e_b,0,e_b - e_s\}$ is clearly suboptimal.

Second, consider deviations in which the institution does not trade at date t' and then follows the optimal strategy of buying round lots at dates $t' + 1, \cdots, T$. Given $x_{t'} = 0$, the expected profit for the institution is

$$G(x_{t'} = 0\,|\,\theta_u,\underline{h}_{t'-1}) = \pi(e_b)\textstyle\sum_{t=t'+1}^{T}\{\mu_u - E[p(\tilde{\underline{h}}_t)\,|\,y_{t'} = e_b]\}(e_b - e_s) + \pi(e_s)\sum_{t=t'+1}^{T}\{\mu_u - E[p(\tilde{\underline{h}}_t)\,|\,y_{t'} = e_s]\}(e_b - e_s). \tag{B2}$$

As the probability of the institution receiving private information becomes small, the expected profit under the deviation $x_{t'} = 0$ drops below that of the proposed strategy $x_{t'} = e_b - e_s$ because 1) if the limiting values of the proposed price rule

$$\lim_{\substack{\pi(\theta u)\to 0\\ \pi(\theta d)\to 0}} p(\underline{h}_t) = \begin{cases} \mu_u \text{ if } o(x_t) > 0 \text{ or if } y_t = 2e_b - e_s \\ \quad \text{or if } y_t = e_b \text{ and } \beta(\theta_u:\underline{h}_{t-1}) + \beta(\theta_d:\underline{h}_{t-1}) = 1, \\ \mu \text{ if } \underline{y}_t \text{ includes only } e_b \text{ and } e_s \text{ realizations} \\ \quad \text{and the number of crosses } \nu_t \neq N/2, \\ \mu_d \text{ if } o(x_t) < 0 \text{ or if } y_t = 2e_s - e_b \\ \quad \text{or if } y_t = e_s \text{ and } \nu_t = N/2 \\ \quad \text{or if } y_t = e_s \text{ and } \beta(\theta_u:h_{\underline{t}-1}) + \beta(\theta_d:h_{\underline{t}-1}) = 1, \end{cases} \tag{B3}$$

are substituted into (B1) and (B2), then the difference between expected profits under the two strategies is strictly positive in the limit

$$\lim_{\substack{\pi(\theta u)\to 0\\ \pi(\theta d)\to 0}} G(x_{t'} = e_b - e_s \mid \theta_u, \underline{h}_{t'-1}) - G(x_{t'} = 0 \mid \theta_u, \underline{h}_{t'-1}) \tag{B4}$$

$$= \pi(e_s)^T(\mu_u - \mu)(e_b - e_s)$$

$$\geq 0$$

and 2) the prices from (10d) and thus the expected profit functions are continuous in the probabilities $\pi(\theta_u)$ and $\pi(\theta_d)$. Thus, there are positive bounds $a_{u,t'}$ and $a_{d,t'}$ such that, if $a_{u,t'} \geq \pi(\theta_u) > 0$ and $a_{d,t'} \geq \pi(\theta_d) > 0$, then the difference in the expected gains is still positive.

Third, consider a deviation in which the institution sells $e_s - e_b$ shares at date t' and then follows the optimal strategy of buying round lots at dates $t' + 1, \cdots, T$. Given $x_{t'} = e_s - e_b$, the expected profit given good news is

$$\begin{aligned} &G(x_{t'} = e_s - e_b \mid \theta_u, \underline{h}_{t'-1}) = \\ &\quad \pi(e_b)[1 - \pi(n_{t'} = (N-2)/2)] \left(\{\mu_u - E[p(\underline{\tilde{h}}_{t'}) \mid y_{t'} = e_s, \nu_{t'} < N/2]\} (e_s - e_b)\right. \\ &\qquad + \textstyle\sum_{t=t'+1}^{T} \{\mu_u - E[p(\underline{\tilde{h}}_t) \mid y_{t'} = e_s, \nu_{t'} < N/2]\} (e_b - e_s)) \\ &\qquad + [\pi(e_s) + \pi(e_b)\pi(n_{t'} = (N-2)/2)] [(\mu_u - \mu_d)(e_s - e_b) \\ &\qquad + (T - t')(\mu_u - \mu_u)(e_b - e_s)]. \end{aligned} \tag{B5}$$

Comparing (B2) and (B5) and using the fact that by (B3) the future expected price levels $E[p(\underline{h}_t)|y_{t'} = e_b]$, $E[p(\underline{h}_t)|y_{t'} = e_s]$, and $E[p(\underline{h}_t)|y_{t'} = e_s, \nu_{t'} < N/2]$ are equal in the limit, the limiting expected profit under the deviation $x_{t'} = e_s - e_b$ can be shown to be strictly less than under the deviation $x_{t'} = 0$ and, thus by (B4), strictly less than under the proposed strategy $x_{t'} = e_b - e_s$. As above, continuity of the gain functions implies that, for some positive probabilities of receiving private information, these differences are still negative. Q.E.D.

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