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A Portfolio Approach to Estimating the Average Correlation Coefficient for the Constant Correlation Model

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ABSTRACT

This paper presents a portfolio approach to estimating the average correlation coefficient of a group of stocks which are considered for portfolio analysis. The average correlation coefficient has been shown to produce a better estimate of the future correlation matrix than individual pairwise correlations. The advantage of the approach described here is that it does not require the estimation of pairwise correlations for estimating their average.

A MAJOR OBSTACLE TO the implementation of Markowitz's (1952) mean-variance portfolio selection model is the difficulty in estimating the required inputs. Particularly troublesome are the pairwise correlations among all stocks which are being considered for inclusion in a portfolio. The sheer number of required correlation estimates makes it highly impractical to examine a large number of stocks. Sharpe's (1963) Single Index Model (SIM) was developed in response to this problem. It assumes correlations with a common index to be the only source of covariation among stock returns. The necessary number of estimations is much reduced, since only the correlations with the index need be considered.

Accumulated empirical evidence suggests that the SIM is misspecified. Bernard (1987) provides evidence of significant intra-industry cross-sectional correlations in contemporaneous market model residuals. This evidence implies that correlations with a common index will not capture the total covariation among stock returns. An alternative simple structure is the Constant Correlation Model (CCM), which uses the historical average correlation coefficient for the future. In two related articles, Elton and Gruber (1973) and Elton, Gruber, and Urich (1978) have shown that, despite its simplicity, the CCM produces better forecasts of the future correlation matrix than those obtained from the SIM or the full historical correlation matrix.

The superior performance of the CCM should make it an attractive alternative to the SIM. Although simple portfolio selection procedures have been developed for the CCM by Elton, Gruber, and Padberg (1976, 1977a,b), the computational problem will still be large if the full correlation matrix needs to be estimated to compute the average correlation. We present below a simplified (but exact) method of estimating the average correlation coefficient which can be applied

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with equal ease to samples of any size. This method greatly reduces the computational requirements for estimating the average correlation coefficient.

I. The Approach

Consider, first, the case where all N securities are in a single group.[1] Let $\tilde{R}_i$ be a random variable representing the rate of return on security i during a period. Suppose contemporaneous rates of return of T periods on these securities are available to estimate the average correlation. Define

$$r_{it} = \text{the observed value of } \tilde{R}_i \text{ in period } t;$$

$$i = 1, \cdots, N; t = 1, \cdots, T;$$

$$\bar{r}_i = \textstyle\sum_{t=1}^{T} r_i/T = \text{sample mean } \tilde{R}_i; \tag{1}$$

$$s_i^2 = \textstyle\sum_{t=1}^{T} (r_{it} - \bar{r}_i)^2/(T-1) = \text{sample variance of } \tilde{R}_i; \tag{2}$$

$$s_{ij} = \textstyle\sum_{t=1}^{T} (r_{it} - \bar{r}_i)(r_{jt} - \bar{r}_j)/(T-1)$$

$$= \text{sample covariance between } \tilde{R}_i \text{ and } \tilde{R}_j; \tag{3}$$

$$c_{ij} = s_{ij}/(s_i \times s_j) = \text{sample correlation between } \tilde{R}_i \text{ and } \tilde{R}_j; \text{ and} \tag{4}$$

$$\bar{c} = \textstyle\sum_{\substack{i=1 \\ i \neq j}}^{N} \sum_{j=1}^{N} c_{ij}/[N(N-1)]$$

$$= \text{the average correlation coefficient.} \tag{5}$$

The problem is that, to estimate the average correlation coefficient $\bar{c}$, we must estimate N sample variances and $N(N-1)/2$ sample covariances. Notice that most of the effort involves computing sample covariances. We now develop a portfolio approach to estimating $\bar{c}$ that eliminates covariance calculations.

Consider a portfolio of N securities in which the investment in security i is x_i. Let y_t denote the return on this portfolio during period t. Then

$$y_t = \textstyle\sum_{i=1}^{N} x_i r_{it}. \tag{6}$$

The sample variance of this portfolio is

$$s_y^2 = \textstyle\sum_{i=1}^{N} x_i^2 s_i^2 + \sum_{\substack{i=1 \\ i \neq j}}^{N} \sum_{j=1}^{N} x_i x_j s_i s_j c_{ij}. \tag{7}$$

If x_i is set equal to $1/s_i$, then

$$s_y^2 = N + N(N-1)\bar{c}. \tag{8}$$

Thus,

$$\bar{c} = (s_y^2 - N)/[N(N-1)]. \tag{9}$$

[1] Motivation for partitioning the available securities into more than one group may arise if there is significant variation in the average correlation coefficient of different groups, such as industries. The multi-group case is discussed later.

Therefore, the sample variance of the y_t's ($y_t = \sum_{i=1}^{N} r_{it}/s_{it}$; $t = 1, \cdots, T$) is all that we need to determine the average correlation coefficient. This reduces the required computational effort to determine $\bar{c}$, since covariance estimates are not needed.

Extension to the multi-group case is straightforward. For example, consider the three-group construct of Elton and Gruber (1973, p. 1210) with the following matrix:

$$\begin{array}{cccc} & 1 & 2 & 3 \\ 1 & \bar{c}_1 & \bar{c}_{12} & \bar{c}_{13} \\ 2 & & \bar{c}_2 & \bar{c}_{23} \\ 3 & & & \bar{c}_3 \end{array}$$

where $\bar{c}_i$ ($i = 1, 2, 3$) is the average sample correlation coefficient between securities belonging to group i, and $\bar{c}_{ij}$ ($i, j = 1, 2, 3$; $i \neq j$) is the average sample correlation coefficient between securities of group i and group j. Our aim is to estimate the $\bar{c}_i$'s and $\bar{c}_{ij}$'s.

Let N_i be the number of securities in group i. The $\bar{c}_i$'s are determined as before. To illustrate, to estimate $\bar{c}_1$, form a portfolio consisting of all securities belonging to the first group, where the investment in each security is equal to the inverse of its sample standard deviation. Estimate the variance of this portfolio, $s_{y_i}^2$. Then

$$\bar{c}_1 = (s_{y_1}^2 - N_1)/(N_1(N_1 - 1)). \tag{10}$$

In order to estimate $\bar{c}_{ij}$, estimate $s_{y_i}^2$, $s_{y_j}^2$, and $s_{y_{i+j}}^2$, where $s_{y_{i+j}}^2$ is the sample variance of the portfolio of securities in groups i and j; as before, the investment in each security is in inverse proportion to its standard deviation. It can be easily shown that

$$s_{y_{i+j}}^2 = s_{y_i}^2 + s_{y_j}^2 + \textstyle\sum_{p=1}^{N_i} \sum_{q=1}^{N_j} c_{pq} = s_{y_i}^2 + s_{y_j}^2 + 2N_i N_j \bar{c}_{ij}, \tag{11}$$

where p (q) denotes securities belonging to the group i (j). Therefore,

$$\bar{c}_{ij} = [s_{y_{i+j}}^2 - s_{y_i}^2 - s_{y_j}^2]/(2N_i N_j). \tag{12}$$

II. Conclusion

In this paper we have developed a portfolio approach for estimating the average of the pairwise correlations among stock returns. This approach does not require the estimation of pairwise correlations for estimating their average. For N securities in one group, the portfolio approach will estimate only $N + 1$ variances: the variances of N securities and the variance of a portfolio where investment in each security equals the reciprocal of its sample standard deviation. For N securities in k groups, this approach will estimate $N + k + k(k - 1)/2$ variances: the variances of N securities, the variances of k portfolios of securities in the k groups, and $k(k - 1)/2$ portfolios of securities taken two groups at a time. We hope that this paper will generate more interest in the CCM, which, despite its advantages, appears to have been largely ignored.

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