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The Effect of Temporal Risk Aversion on Optimal Consumption, the Equity Premium, and the Equilibrium Interest Rate

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ABSTRACT

This paper demonstrates that temporal risk aversion makes smoothing consumption over time less attractive, while the usual risk aversion makes it more attractive. As temporal risk aversion increases, the equilibrium interest rate decreases and the equity premium increases. This paper also shows a striking and novel result that an increase in time impatience can lead to either a decrease or an increase in the interest rate, depending on the nature of the nonseparability.

TEMPORAL RISK AVERSION (TRA) REFERS to a risk attitude toward an uncertain consumption stream. It is of importance to practical investors. Even though the concept of TRA is easy to rationalize and is accepted, most models use time-additive preferences and do not exhibit TRA. Consequently, most of the literature on portfolio selection fails to show the effect of TRA on optimal consumption and investment choices. (See Samuelson (1969), Merton (1969), and Hakansson (1970).)

The effects of TRA on lifetime consumption, the equity premium, and the equilibrium interest rate are examined by employing utility functions exhibiting TRA.[1] The utility functions employed have time-additive preference as a special case, so comparison with the traditional models is easily accomplished.

There is a growing body of literature which shows that a standard representative consumer model with time-additive preference cannot explain some impor-

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[1] Karady (1982) has shown that TRA lowers the excess required rate of return for an irreversible project over a risk-free interest rate. This study did not, however, provide an extensive study of TRA on portfolio selection. Selden (1978) introduces the "ordinal certainty equivalence" (OCE) utility hypothesis, which permits one to prescribe risk and time preference separately. Selden (1979) demonstrates by employing an OCE utility function that the effect of a mean-preserving increase in risk on savings depends on the elasticity of substitution, which is interpreted as a measure of intertemporal complementality. His analysis is different from ours in important respects. First, OCE utility is not von Neumann-Morgenstern (vNM), whereas preference exhibiting TRA studied in this paper is vNM. Second, he uses a two-period framework, whereas ours has an infinite horizon. Third, the elasticity of substitution used in his paper does not have a homeomorphic relationship with TRA since the elasticity of substitution can be greater (or less) than unity without TRA. Finally, he does not examine TRA and the issues on this risk aversion.

tant empirical regularities. For example, Deaton (1986) has noted that consumption grows even during periods of negative interest rates. This result implies that, with time-additive preferences, the subjective discount factor must be greater than unity, which is economically implausible. Hansen and Singleton (1982) show that the data reject the overidentifying restrictions from the model with time-additive preference. Recently, Ahn (1988) shows that problems with time-additive preference can be resolved by employing preference exhibiting TRA. He obtains favorable empirical evidence for a model with preference exhibiting TRA. The model is not rejected by the data. These results suggest that it is useful to study TRA.

Section I defines TRA and introduces two plausible utility functions which exhibit TRA. We find a measure of TRA for each utility function. Section II derives the optimal consumption rule, the equity premium, and the equilibrium interest rate under each utility function. The effect of TRA on them is analyzed. Section III provides a discussion of the results and conclusions.

I. Preferences and Temporal Risk Aversion

Let x and y be two attributes of the utility function of a consumer. The consumer is said to be multivariate risk averse if he or she prefers a lottery which has equal probability of $(\underline{x}, \bar{y})$ and $(\bar{x}, \underline{y})$ to one which has equal probability of $(\underline{x}, \underline{y})$ and $(\bar{x}, \bar{y})$, where $\underline{x} < \bar{x}$ and $\underline{y} < \bar{y}$. In other words, negatively correlated gambles are valued more than positively correlated gambles. This is because the consumer is afraid of getting $(\underline{x}, \underline{y})$, the worst outcome for x and y. An equal chance of obtaining $(\underline{x}, \underline{y})$, the worst outcome, outweighs an equal chance of obtaining $(\bar{x}, \bar{y})$, the best outcome, for the multivariate risk-averse consumer. Richard (1975) shows that a necessary and sufficient condition for multivariate risk aversion is the negativity of the cross partial derivative of the utility function with respect to two arguments (i.e., $\partial^2 U/\partial x \partial y < 0$, where U denotes the utility function). If this partial derviative is zero or positive, the consumer is said to be multivariate risk neutral or risk seeking.

The arguments of utility functions in this paper are consumption at different periods. Multivariate risk aversion, risk neutrality, or risk preference over consumption at different periods is defined as temporal risk aversion, risk neutrality, or risk preference, respectively. Define U as the von Neumann-Morgenstern (vNM) utility index for multiperiod consumption. We denote $\partial U/\partial C_t$ as U_{C_t}, $\partial^2 U/\partial C_s \partial C_t$ as $U_{C_s C_t}$, etc., where C_t and C_s are consumption at periods s and t, respectively. It follows that $U_{C_s C_t}$ is negative if the consumer is temporally risk averse. It is easy to see that time-additive preference exhibits temporal risk neutrality. Thus, time-additive preference is not adequate to examine the effect of TRA.

For the later analysis, we introduce two vNM expected utility indices.

Utility Index A:

$$U = \int_0^\infty u(C(t)) \exp\left[-\int_0^t v(C(s))ds\right] dt, \qquad (1)$$

where $u(C(t)) = -(1/\eta)\exp[-\eta C(t)]$ and $v(C(s)) = aC(s) + b$ with positive constants η, a, and b.[2] Utility Index A has been developed by Koopmans, Diamond, and Williamson (1964) and Uzawa (1968) and used by Nairay (1981), Epstein (1983, 1987), and Bergman (1985). None of this literature, however, explores the concept of TRA. Utility Index A has been called variable time preference since the subjective discount rate varies over time. Note that the derivative of the discount function is positive, i.e., $v_C = a > 0$. This is referred to as "increasing marginal impatience" advocated by Uzawa (1968), Epstein (1983, 1987), and Lucas and Stokey (1984). The existence and stability of a steady state under a positive derivative are demonstrated by Epstein (1983). However, if the derivative is negative, local stability of steady states may fail in standard enviroments, even in single agent models. (See Epstein (1987) for arguments supporting a positive derivative.) Furthermore, a positive derivative is assumed here to focus the analysis on TRA. If the derivative is negative, (1) exhibits temporal risk preference.

Utility Index B:

$$U = (1/\gamma)\exp[\int_0^\infty \gamma\beta^t u(C(t))dt], \tag{2}$$

where $u(C(t)) = \ln C(t)$, γ is a parameter related to TRA, and β is a subjective constant discount factor satisfying $0 < \beta < 1$. Utility Index B, called multiplicatively separable preference, has been used by Pye (1973). He does not, however, examine TRA. In order to focus on TRA, we assume throughout the paper that γ is negative.[3]

TRA can be easily established for (1) and (2) since it is sufficient to show that cross partial derivatives of (1) and (2) are negative. This can be done by straightfoward differentation.[4] In order to perform comparative statics in the later analysis, we need a measure for a degree of TRA. We will use $-U_{C_sC_t}/U_{C_t}$ or $-U_{C_sC_t}C_t/U_{C_t}$ as local TRA measures, where C_t denotes consumption at a later date than C_S.[5] For Utility Index A, the measure of TRA, $-U_{C_sC_t}U_{C_t}$, is a marginal

[2] Concavity and monotonicity of (1) have been shown in Epstein (1983). Even though a positive concave discount function, $v(C)$, is permissible, we assume that $v(C)$ has an affine form in order to obtain closed-form solutions.

[3] If γ is zero, then (2) becomes time-additive preference. If γ is positive, then (2) exhibits temporal risk preference. If γ is not zero, the marginal rates of substitution between time t and time $t + 1$ depend on time zero consumption. Thus, an empirical study should seriously consider when the world begins. This preference is nonstationary. Consequently, multiplicatively separable preference is problematic in its empirical implementation. We thank an anonymous referee for bringing up this point.

[4] Since utility indices are defined in the continuous time framework, some technical conditions are needed to insure valid differentiation of utility indexes with respect to consumption. See Ash (1972, p. 52) for these conditions.

[5] As the value of the measure of TRA, $-U_{C_sC_t}/U_{C_t}$, increases, the magnitude of the risk premium which a consumer would pay to avoid an arbitrarily small gamble will increase, ceteris paribus, as shown in Ronn (1988). These measures are analogous to the Arrow-Pratt measures of absolute and relative risk aversion. Since they are normalized by marginal utility of consumption, which is assumed to be positive, they are invariant to positive affine transformations of U.

discount rate function, i.e., $v_C = a$, which is a positive constant. Further, it is readily seen that TRA is interrelated with time impatience, which means time preferences for current consumption over future consumption. The higher the marginal time impatience, the higher the TRA. For Utility Index B, the measure of TRA is $-U_{C_sC_t}C_t/U_{C_t} = -\gamma\beta^t$. This measure is critically affected by the parameter γ and the "discount factor" β. TRA increases with a decrease in γ and a decrease in time impatience. Thus, the higher the time impatience, the lower the TRA. This is, however, exactly opposite to the case of Utility Index A.

II. The Effect of Temporal Risk Aversion

In this section, we derive the optimal consumption rule and the equilibrium interest rate under both utility indices in a general equilibrium setting. We examine the effect of TRA on optimal consumption, the equity premium, and the interest rate as well as the effect of time preference on the interest rate.

We consider an economy with a large number of indefinitely lived consumers, identical in their endowments and preferences. Assume that each consumer maximizes his or her lifetime vNM expected utility index,

$$E_0[U], \tag{3}$$

where U denotes Utility Index A or B and E_0 is a conditional expectation operator. We assume that the investment opportunity set consists of one risky production technology and one risk-free asset. Let w denote the proportion of wealth invested in the production technology. At each instant, the consumer's wealth is allocated between consumption, the production technology, and one risk-free asset. Each consumer maximizes (3) subject to his or her budget equation. Since no trading is Pareto-optimal in an identical consumer economy, we have the equilibrium condition that $w = 1$. We now solve the consumer's problem under two utility assumptions.

Utility Index A: The output of the production technology, dq, is assumed to be governed by a "square root" process,

$$dq = q\alpha dt + \sqrt{q}\sigma dz, \tag{4}$$

where α and σ are constants and dz is a standard Wiener process. Thus, the consumer has the budget equation given by

$$dW = [w(\alpha - r)W + rW - C]dt + w\sqrt{W}\sigma dz, \tag{5}$$

where W denotes wealth, C means consumption, and r denotes the interest rate.

It can be shown that the optimal consumption rule is

$$C^* = \frac{\psi}{\eta}W(t) - \frac{1}{\eta} + \frac{b}{\psi - a}, \tag{6}$$

where $\psi = (2\eta\alpha + 2a)/(2 + \eta\alpha^2)$. Substituting optimal consumption (6) into the budget equation (5) delivers the dynamics of wealth as

$$dW = \left[\left(\alpha - \frac{\psi}{\eta}\right)W + \frac{1}{\eta} - \frac{b}{\psi - a}\right]dt + \sqrt{W}\sigma dz. \tag{7}$$

Straightforward calculations give that $\partial\psi/\partial a > 0$ and $\partial(\psi - a)/\partial a < 0$. The expected growth of wealth (i.e., $E[dW]$) is negative with a sufficiently large value for "a", TRA. Further, as TRA increases, the expected growth of wealth decreases (i.e., $\partial E[dW]/\partial a < 0$). This relationship makes economic sense because, as TRA (i.e., marginal time impatience) increases, current consumption increases and future consumption decreases. Equation (6) implies that $dC^* = (\psi/\eta)dW$. Since ψ/η is positive, the expected level of optimal consumption is decreasing over time with a sufficient level of TRA.

Using (6) and (7), we can find the effect of TRA on the expected growth of consumption by

$$\frac{\partial E[dC^*]}{\partial a} = \frac{E[dW]}{\eta}\left(\frac{\partial\psi}{\partial a}\right) + \frac{\psi}{\eta}\left[-\frac{W}{\eta}\left(\frac{\partial\psi}{\partial a}\right) + \frac{b}{(\psi-a)^2}\left(\frac{\partial(\psi-a)}{\partial a}\right)\right]. \tag{8}$$

If the consumer is very temporally risk averse (i.e., with a large value of "a"), the sign of (8) will clearly be negative. Consequently, the expected growth of consumption, which is negative, decreases as TRA increases. Thus, expected differences between optimal current consumption and optimal future consumption will increase as TRA increases.

It can be shown that the equilibrium interest rate is

$$r = \frac{\alpha(2 - \eta\sigma^2) - 2a\sigma^2}{2 + \eta\sigma^2}. \tag{9}$$

It follows that $\partial r/\partial a < 0$. The equilibrium interest rate decreases with increases in TRA. Since the discount function, $v(C)$, is a linear function of consumption, an increase in marginal time preference implies an increase in time impatience given a level of consumption. Consequently, we have the proposition that the higher the time impatience, the lower the equilibrium interest rate. The equity premium, $(\alpha - r)$, is $(2\eta\alpha + 2a)\sigma^2/(2 + \eta\sigma^2)$. Thus, as TRA increases, the equity premium increases.

Utility Index B: The output of the production technology, dq, is assumed to be governed by a lognormal process,[6]

$$dq = q\alpha dt + q\sigma dz, \tag{10}$$

where α and σ are constants and dz is a standard Wiener process. Thus, the consumer has a budget equation given by

$$dW = [w(\alpha - r)W + rW - C]dt + wW\sigma dz, \tag{11}$$

where W denotes wealth, C means consumption, and r denotes the interest rate.

It can be shown that the optimal consumption rule is

$$C^* = -(\ln\beta)W. \tag{12}$$

[6] We use different processes in (4) and (10) since they are required to obtain closed-form solutions for the interest rates under different utility indices.

Substituting optimal consumption (12) into the budget equation (11) yields the following wealth dynamics:

$$dW = [\alpha + \ln\beta]Wdt + \sigma W dz. \tag{13}$$

The expected growth rate of wealth (i.e., $E[dW]/W$) is positive with a sufficient level of TRA (i.e., with a large value of "β"). Equation (12) implies that $dC^*/C^* = dW/W$. Thus, the expected growth rate of consumption is positive if the consumer is very temporally risk averse.

Further, from (12) and (13), the effect of TRA on the expected growth rate of consumption is given by

$$\frac{\partial(E[dC^*]/C^*)}{\partial\beta} = \frac{1}{\beta}. \tag{14}$$

If the consumer is very temporally risk averse (i.e., with a large value of β), the sign of (14) will clearly be positive. Consequently, the expected growth rate of consumption, which is positive, increases as TRA increases. Thus, expected differences between optimal current consumption and optimal future consumption will increase as TRA increases.

It can be shown that the equilibrium interest rate is

$$r = \alpha - (1 - \lambda)\sigma^2 = \alpha - \left(1 + \frac{\gamma\beta^t}{\ln\beta}\right)\sigma^2. \tag{15}$$

It follows that $\partial r/\partial\beta < 0$ and $\partial r/\partial\gamma > 0$. The equilibrium interest rate increases as time impatience increases.[7] Since time impatience and the parameter γ are negatively correlated with TRA for Utility Index B, we obtain the implication that the higher the TRA, the lower the equilibrium interest rate. The equity premium, $(\alpha - r)$, is $\{1 + (\gamma\beta^t/\ln\beta)\}\sigma^2$. Thus, as TRA increases, the equity premium increases.

III. Conclusions

It is known that the usual risk aversion makes consumption smoothing over time valuable. (See Michener (1982).) Consumption smoothing is, however, less attractive for a temporally risk-averse consumer. Spreading a "windfall income" over periods is not optimal for him or her. This result occurs because the marginal utility of future consumption is decreased by a current "windfall income" and it gives less incentive to postpone the income into the future. In our context, smoothing consumption means equalizing expected consumption over time—that is, $E[dC^*] \cong 0$. Our result shows that, as TRA increases, the absolute value of the mean of the growth (or growth rate) of optimal consumption, $|E[dC^*]|$ or $|E[dC^*/C^*]|$, increases.

This result holds even for a general nonparametric utility function with

[7] If preference is time-additive preference (i.e., $a = 0$ in (9) and $\gamma = 0$ in (15)), time patience has no effect on the equilibrium interest rate.

negative cross-partials. Consider a utility function with consumption in two periods as arguments. By the second order Taylor's expansion, we have[8]

$$U(C + \Delta, C - \Delta) \cong U(C, C) + U_1\Delta - U_2\Delta - U_{12}\Delta^2 + (1/2)U_{11}\Delta^2 + (1/2)_{22}\Delta^2, \quad (16)$$

where Δ denotes a small change in consumption and U_1 denotes the derivative of the utility function with respect to consumption in the first period, etc. We assume that U_1 and U_2 are equal. Let $U(C + \Delta, C - \Delta)$ and $U(C, C)$ denote utility from unsmoothed and smoothed consumption streams, respectively. From (16), we see that concavity of the utility function (the usual risk aversion) makes consumption smoothing over time more attractive. TRA, however, makes consumption smoothing less attractive. Our result of the effect of TRA on consumption smoothing is exactly opposite to Ronn (1988), who claims that TRA makes consumption smoothing more attractive.

TRA has a consistent negative effect on the equilibrium interest rate. For both utility indices, as TRA increases, the equilibrium interest rate decreases. This result is intuitively plausible. In a portfolio problem, as TRA increases, the optimal proportion of wealth invested in the risk-free asset increases.[9] A temporally risk-averse consumer is extremely averse to the situation where he or she has to consume little in all periods. In order to avoid this worst case, the consumer can buy bonds which will promise certain payoffs at the maturity dates. Consequently, as TRA increases, the demand for bonds increases and the price of the asset increases.[10] The equilibrium interest rate decreases. This effect also increases the equity premium.

In what follows, we examine the effect of time preference on the equilibrium interest rate. First, for Utility Index B, as time impatience increases, the equilibrium interest rate increases. This is a common result. As time impatience increases, the consumer will not postpone consumption to later periods. Hence, a higher interest rate must be obtained to induce the consumer to defer consumption and buy bonds. The opposite result, however, is obtained under Utility Index A. This is contrary to what we would expect. Nonseparability of variable time preference yields this unconventional result. Obviously, as time impatience

[8] We thank Richard Green for providing this example.

[9] In a portfolio problem where the interest rate is given exogenously, it can be shown that, under Utility Index A, the optimal proportion of wealth invested in the production technology is

$$w^* = (\alpha - r)/\phi\sigma^2,$$

where $\phi = (1/2)[(\eta r + a) + \sqrt{(\eta r + a)^2 + 2\eta(\alpha - r)^2/\sigma^2}]$. Thus, $\partial w^*/\partial a < 0$. Under Utility Index B, the optimal proportion is

$$w^* = [(\alpha - r)\beta]/[(\ln\beta + \gamma\beta^t)\sigma^2],$$

as shown by Pye (1973). Thus, we have that $\partial w^*/\partial\beta < 0$ and $\partial w^*/\partial\gamma > 0$. It follows that, as TRA increases, the optimal proportion of wealth invested in the risky production technology decreases but the optimal proportion of wealth invested in the riskless asset increases.

[10] The equilibrium condition, $w = 1$, is imposed. Therefore, nothing is invested in the riskless asset. The fact that the consumer would want more bonds drives up the price of the bond to the point that the demand is zero. We thank Howard Thompson for providing this argument.

increases, current consumption increases. Under variable time preference, an increase in current consumption lowers the discount factor for future consumption. Thus, the consumer becomes more impatient and prefers to have further increases in current consumption. This type of intertemporal dependence does not exist under separable preferences. It causes a relatively large decrease in marginal utility of current consumption compared to marginal utility of future consumption. This is why the higher equilibrium bond price and the lower equilibrium interest rate are obtained with a higher time impatience.

The underlying intuition of these results can be easily understood in the famous Fisherian diagram. Figure 1 shows the case of Utility Index B (separable preference). The indifference curve II exhibits a higher time impatience than the indifference curve I. It is known that the slope of the line tangent to the indifference curve and the production opportunity frontier, P, is $-(1 + r)$, where r is the interest rate. As time impatience increases from I to II, the slope of the tangential line becomes steeper. Thus, Figure 1 gives the implication that the higher time impatience, the higher the equilibrium interest rate. Figure 2 shows the case of Utility Index A (nonseparable preference). As time impatience increases from I to II, there will be a double effect on the increase in current consumption, as mentioned earlier. This effect makes a shift of the production opportunity frontier from P to P'. Thus, Figure 2 provides the implication that the higher the time impatience, the lower the equilibrium interest rate.

An interesting future study about TRA would be to investigate what happens for general concave nonparametric utility functions with negative cross-partials.

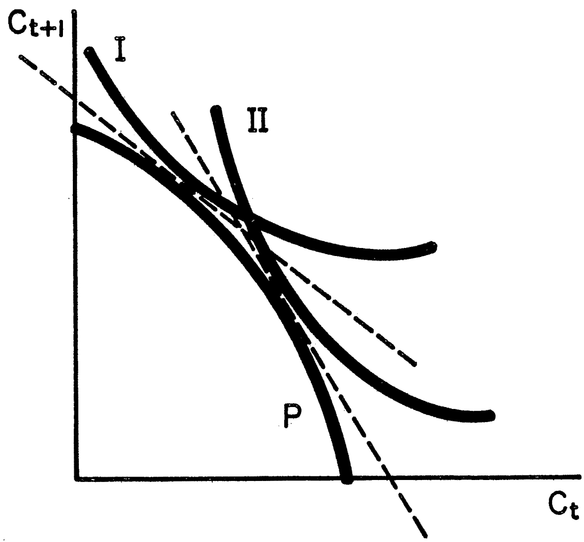

Figure 1. The effect of time impatience on the interest rate under separable preference. $C_t \equiv$ consumption at time t; $C_{t+1} \equiv$ consumption at time $t + 1$. The production opportunity frontier P will not be changed even though time impatience increases from indifference curve I to indifference curve II. The slope of the tangential line is $-(1 + r)$, where r is the interest rate. An increase in time impatience raises the equilibrium interest rate.

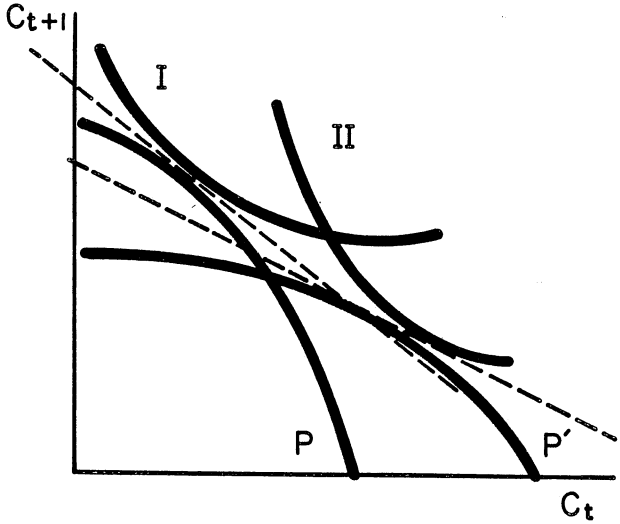

Figure 2. The effect of time impatience on the interest rate under nonseparable preference. $C_t \equiv$ consumption at time t; $C_{t+1} \equiv$ consumption at time $t + 1$. As time impatience increases from indifference curve I to indifference curve II, the production opportunity frontier will be shifted from P to P' because of the interaction between consumption and time impatience. The slope of the tangential line becomes less steep. An increase in time impatience lowers the equilibrium interest rate.

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