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The Equilibrium Valuation of Risky Discrete Cash Flows in Continuous Time

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ABSTRACT

This paper values a contingent claim to discrete stochastic cash flows generated by a Poisson arrival process with a randomly varying intensity parameter. In the most general case, both the size and the arrival intensity of cash flows may correlate wih state variables in a continuous time economy. Assuming the conditions of an intertemporal capital aset pricing model, solutions for the value of the contingent claim can be found using various techniques. The paper suggests immediate applications to the valuation of insurance contracts, the decision to build a firm with unknown future investment opportunities, and the pricing of mortgage-backed securities.

THE THEORY OF CAPITAL budgeting can be traced to Fisher (1930). Several researchers have since adapted the net present value model to financial decisions; indeed, one can view equilibrium asset pricing models as attempts to determine an appropriate discount rate for risky cash flows. These methods apply equilibrium models to the valuation of projects whose benefits can be described by conditional probability distributions of future cash flows.

The modern tradition in capital budgeting stems from the work of Samuelson (1965). Brennan (1973) modeled cash flows explicitly within a continuous time setting. Constantinides (1978) developed a valuation model for stochastic continuous cash flows using Merton's (1973) intertemporal asset pricing framework. Dothan and Williams (1980) adapted the valuation procedures to include stochastic interest rates. Merton (1971) introduced jump processes to the finance literature; some jumps in asset value occur as a result of cash flows. Merton assumed that the jump components of cash flows were diversifiable and was able to value certain assets with these characteristics. Cox and Ross (1976) studied option pricing applications in a similar framework.

This paper allows cash flows generated by Poisson processes to bear systematic risk, without giving up the computational advantage of independently determined jump and continuous processes. The analysis extends the study of this type of cash flow while it retains the appeal and tractability of previous models that use Poisson processes. Like Merton's analysis, the jump itself can be diversified; unlike Merton's analysis, the expected frequency of jumps cannot.

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If the firm can diversify away the "pure" idiosyncratic timing risk, or investors have portfolios composed of sufficiently small percentage shares of assets, this diversification can take place. In either case, jumps in asset value disappear; asset dynamics become continuous. Under the assumptions of a general equilibrium capital asset pricing model, a differential equation for the value of the contingent claim can then be derived. The conclusions of this paper have direct analogies to Constantinides' (1978) study of "Market Risk Adjustment in Project Valuation," which considers stochastic cash flows of the continuous type but does not consider jump processes.

Section I presents the model of cash flows heuristically, while Section II formalizes the model and derives the major pricing result of this paper. Section III couches the methodology in the general equilibrium framework of Cox, Ingersoll, and Ross (1985a). Section IV provides some important applications and examples.

I. A Model of Cash Flows

Consider a project which gives a firm the right to receive arbitrary discrete or continuous, certain or stochastic cash flows over an arbitrary period of time. Cash flows may take several forms, many of which we can value easily given modern finance techniques. Without loss of generality, this paper concentrates on cash flows generated by jump processes, whose arrival frequencies correlate with state variables in the economy. We can value these specialized cash flows, with the understanding that these contingent claims may represent only a part of those generated by a particular project.

The valuation of these cash flows can be made simplest by assuming that the timing of the cash flows is independent of the state variables in the economy. Previous models assumed independence of the jump process and corresponding Wiener processes (e.g., see Merton (1971 or 1976)). This paper maintains this independence assumption but allows the probability distribution of jumps to depend on continuous state variables in a specific manner. Suppose that the *intensity* of the cash arrival follows its own continuous time stochastic process and allows this process to be correlated with state variables. Then, at any point in time, *conditional* on the level of the intensity, the jump is independent of the movement of the state variables. Then the familiar version of Itô's Lemma can be applied.

One can imagine many practical applications of this model of cash flows. Consider a health insurance contract, for example, where an insurance company must pay its policyholder cash conditional on the incidence of a medical expense. The expenditure accrues discretely at random claim times by an amount equal to some function of the loss.[1] The frequency of health claims accelerates when

[1] An example is the difference between the expense and the deductible, provided that this difference is positive.

the economy sags and decelerates when the economy flourishes. The *level* of health claims relates to economy-wide state variables as well. By issuing health insurance contracts, then, the insurance company bears systematic risk and must be compensated for doing so. Fairley (1979) and Hill (1979) examined systematic risk in underwriting portfolios.[2] Smith (1979) and Kraus and Ross (1982) have both addressed systematic risk in insurance contract pricing. Doherty and Garven (1986) and Cummins (1988) employed similar assumptions in insurance valuation problems.

As a second example, consider the decision to incur the costs of starting a firm with uncertain future growth opportunities. Management adopts future projects with positive net present values but does not know ex ante when these projects will be adopted or what their net present values might be. Furthermore, the occurrence frequency of profitable projects depends on state variables—profit opportunities present themselves more frequently when the economy does well and less often when the economy does poorly. Changes in the interest rate also affect the arrival frequency of profitable projects. Ceteris paribus, positive net present value projects should be generated with greater frequency when the capitalization rate is low than when it is high.[3]

Finally, consider the pricing of a mortgage-backed security. A mortgagor can prepay his or her mortgage according to some optimal rule which suggests prepayment when contract interest rates fall below a critical interest rate. Given the stochastic character of interest rates, one can theoretically value the mortgage with the mortgagor's right to prepay. However, mortgagors prepay their mortgages for other reasons. For example, higher mobility, higher unemployment, and lower wealth cause prepayments to occur.

Autonomous (non-interest rate) prepayments flow with the economic tide. Defaults, for example, vary systematically with the business cycle. Default means prepayment for the owner of an insured mortgage pool. Mortgagees, therefore, must be compensated for bearing systematic prepayment risk. This issue was recognized by Dunn and McConnell (1981), who valued mortgage-backed securities when the intensity of prepayments can be represented as a deterministic function of time and the interest rate. Brennan and Schwartz (1985) complement and extend the Dunn and McConnell framework.[4] Schwartz and Torous (1989) empirically examine autonomous prepayment behavior in this framework.

[2] On the other hand, the presence of systematic risk in *investment* portfolios has been documented substantially.

[3] If the timing of the investment can be chosen as well, we should expect that, in periods of high interest rates, firms may inventory investment plans until financing can be arranged that guarantees that the project(s) will have positive net present value(s). This would suggest that the arrivals of (adopted) projects take place according to a mixed Poisson-stopping time process; an idea is generated according to a Poisson process, and the project is adopted when rates fall sufficiently low. If the market observes the arrival of ideas instead of the arrival of projects, then share values should jump when ideas occur and not when projects are adopted.

[4] The intensity of autonomous prepayments may vary stochastically and systematically. For the pricing of mortgages in this context, see Shimko (1988).

II. Poisson-Generated Cash Flows: The Mathematical Statement

We begin by assuming the conditions of the Cox, Ingersoll, and Ross (CIR (1985a)) intertemporal general equilibrium model of asset prices (IGEM).[5] The IGEM disallows assets with values that jump at discrete points in time. Ahn and Thompson (1988) have generalized the CIR term structure treatment (not the general equilibrium model) to include jumps in the state variables. Brennan and Schwartz (1982) generalized the CIR framework to allow for jumps in the state variables but required that these jumps be independent of aggregate wealth. Our first task is to demonstrate conditions under which the IGEM can be applied to the valuation of discrete cash flows, where some state variables follow jump processes.

To CIR's original eight assumptions we add:

(A9) *Intermediary existence.* There exists a financial intermediary which invests in an arbitrarily large number of projects (M) that generate cash flows according to a Poisson arrival process. The firm has no other assets. Alternatively, shareholders may create this intermediary.

(A10) *Atomistic equity financing.* There exist an arbitrarily large number of shareholders (one shareholder per share, N shares) who own the intermediary of (A9) and receive proportional cash flows from the set of projects.

(A11) *Common continuous cash arrival intensities.* The intensity of the arrival process for cash flows from every project follows the common process:

$$d\lambda = \alpha_\lambda(X, \lambda, t)dt + \sigma_\lambda(X, \lambda, t)dZ_\lambda, \tag{1}$$

where dZ_λ is a standard Wiener process; α_λ and σ_λ are deterministic functions of other state variables X, λ, and time. Sufficient conditions for the non-negativity of λ require that $\alpha_\lambda \geq 0$ and $\sigma_\lambda = 0$ when evaluated at $\lambda = 0$.

(A12) *Continuous conditional cash flow levels.* At an arrival time for project i, the cash flow C_i is realized, where C_i equals the contemporaneous level of the conditional cash flow generating process:

$$dC_i = \alpha_i(X, \lambda, C_i, t)dt + \sigma_i(X, \lambda, C_i, t)dZ_i, \qquad i \in (1, \cdots, M), \tag{2}$$

where dZ_i is a standard Wiener process; $dZ_i^2 = dt$, $dZ_i dZ_j = \rho_{ij}dt$, $dZ_i dZ_\lambda = \rho_{i\lambda}dt$. The cash flow may be positive or negative.

In spite of the fact that individual project values follow mixed diffusion-jump

[5] The IGEM assumptions include (i) existence of a single consumption/investment good, (ii) existence of a set of linear production activities where changes in productivity follow exogenously specified Wiener processes, (iii) existence of a finite-dimensional state variable vector whose components follow Wiener processes, (iv) free entry and exit with competitive price-taking individuals and firms, (v) common riskless borrowing and lending rates, (vi) markets for contingent claims where values of claims follow Wiener processes with endogenously determined parameters, (vii) a fixed number of identical individuals with homogeneous expectations and von Neumann-Morgenstern utility functions, and (viii) continuous and frictionless trading.

processes, we can now show that, under assumption (A1)–(A12), the following theorem holds:

THEOREM: *The value of a single share of the intermediary follows a continuous diffusion process.*

Proof: The value of an individual project follows a mixed diffusion-jump process; let $V_i = V_i(X, \lambda, C_i, t)$, $i \in \{1, \cdots, M\}$, represent the value of an individual project, and calculate dV_i according to Itô's Lemma and its extension to jump processes. Define dq_i as the jump component of V_i. At any moment in time, the jump size conditional on the occurrence of a jump is known and equal to $g(C_i, V_i)$. The value of the firm, $V = \sum_1^M V_i$, also follows a mixed jump-diffusion process. Without loss of generality, let $M = \kappa N$, with κ constant. The impact of the jump components of project values can then be transferred to the individual shares:

$$\begin{aligned} \lim_{M,N\to\infty} \frac{1}{N} \textstyle\sum_{i=1}^M dq_i &= \lambda_t \kappa \frac{1}{M} \textstyle\sum_{i=1}^M g(C_i, V_i)dt \\ &\equiv \lambda_t \kappa \hat{g}(C, V)dt, \end{aligned} \tag{3}$$

where $\hat{g}$ is the average jump size, given the distribution of jump sizes at time t. This is a version of the law of large numbers. The variance of the mean goes to zero. See Merton (1976) for a similar argument in a different context. The share price has no jump component, as claimed. Q.E.D.

Modigliani and Miller (1958) proved that, in perfect markets, combining and dividing cash flows cannot affect total value. In that case, we can use continuous methods to value the share of stock, S. The firm value is $N \times S$, so, if the projects are identically parameterized ex ante, $V_i = N \times S/M$; project value is proportional to share price. Therefore, once the systematic risk of the jump frequency has been determined, the jump component can be ignored for pricing purposes.

The proof can be generalized in at least two directions. Assumption (A12) requires continuity of the conditional cash flow level. A weaker assumption yielding the same result (A12′) requires continuity of the mean of the conditional cash flow process. Similarly, (A11) can be generalized to allow differing cash arrival intensities, provided they are determined by processes like equation (1).

III. The Fundamental Differential Equation for Contingent Claims: An Extension to Jump Processes

This section continues with the CIR methodology, as exposited by Brennan and Schwartz (1985). Let V represent the value of an individual project; we need not concern ourselves with the distinction between projects, so the subscript i has been dropped. Allow m continuous state variables to exist, indexed by i. The variables X follow the joint process:

$$dX_i = \alpha_i dt + \sigma_i dz_i, \qquad i \in \{1, \cdots, m\}, \tag{4}$$

where dz_i represents a standard Wiener process. The instantaneous correlation between any two state variables i and j is given by ρ_{ij}.

Let subscripts denote partial derivatives. Then the value of the contingent claim must satisfy the following partial differential equation:

$$\tfrac{1}{2}\textstyle\sum_{i=1}^{m}\sum_{k=1}^{m} V_{ik}\rho_{ik}\sigma_i\sigma_k + \sum_{i=1}^{m} V_i(\alpha_i - \varphi_i\sigma_i) - V_\tau + C' - rV = 0, \tag{5}$$

where C' represents the instantaneous cash flow from the security. The parameter φ_i represents the market price of risk for state variable i, which can be determined through specific characterization of individuals' utility functions or limitations on the distributions of outcomes for the state variables. The relevant boundary condition for a finitely lived asset composed only of future stochastic discrete cash flows requires that $V(\tau = 0) = 0$ since the security cannot generate cash past time T. In the infinite case, the transversality condition $V_\tau = 0$ applies. Define $\alpha_i^* = \alpha_i - \varphi_i\sigma_i$, which represents the risk-adjusted growth rate for state variable i. This paper will not discuss the nature of φ_i; one can consult Merton (1973), Breeden (1979), or CIR (1985a) for discussion of the determination of the market price of risk.

The extension from CIR to the present study requires that we make two small adjustments:

(i) Substitute the expected cash flow for the actual cash flow in the fundamental partial differential equation; that is, $C' = \lambda_t g_i$.

(ii) Include the cash flow arrival rate in the set of relevant state variables.

It is important to distinguish between two types of jumps in asset value. One type of cash flow causes the asset to become valueless; this is a "terminating" cash flow. In this case, $g(C, V) = C - V$ since the bearer receives C and surrenders V. A death benefit in a life insurance policy exemplifies this type of cash flow. A second type of cash flow has no effect on the value of the asset per se. For example, in the event of an automobile collision, the insurance policy reimburses the insured party but generally cannot cancel the policy immediately. The jump in the value of the asset equals the amount of the cash flow in this case; $g(C, V) = C$.

IV. Examples and Applications

A. Constant State Variables

When all state variables X, C, and λ are constant, the fundamental PDE reduces to the following:

$$rV = \lambda g(C, V) - V_\tau. \tag{6}$$

The asset value is completely determined by its claims to stochastic cash flows; each asset can be likened to a stochastic annuity or perpetuity. The form of g is determined by the nature of the cash flow. The boundary condition in the finite case is $V|_{\tau=0} = 0$. In the infinite case, the transversality condition $V_\tau = 0$ applies.

The solution in the finite renewing case can be given by

$$V = \lambda C(1 - \exp(-r\tau))/r. \tag{7}$$

For the terminating case, substitute $r + \lambda$ for r. For the infinite case, let $\tau \to \infty$. In all cases, discounting takes place with respect to the clock of the arrival rate process; we discount at the rate r/λ for stochastic times and at rate r for deterministic times.

Now consider a claim to the kth cash flow in an infinite or finite sequence of stochastic cash flows. On the occurrence of the first jump, the asset value becomes the next highly ranked cash flow. Therefore, if V_k is the value of the kth cash flow in a sequence, its value must satisfy the recursive system

$$\begin{aligned} rV_k &= \lambda(V_{k-1} - V_k) - V_\tau, \qquad k = 1, 2, \cdots, \\ V_0 &= C. \end{aligned} \tag{8}$$

The solution can be verified by substitution:

$$V_k = \frac{C}{(1+\gamma)^k}\varphi[k, r+\lambda, \tau], \tag{9}$$

where $\gamma = r/\lambda$ and φ is the cumulative gamma density with parameters n and $r + \lambda$. When $k = 1$, the gamma distribution simplifies to an exponential distribution, where $\varphi = 1 - e^{-(r+\lambda)\tau}$, and the pricing result is as in equation (7).

B. Stochastic Cash Flows and Intensity: Joint Lognormality

We consider the extension of the simple example to one in which the conditional cash flows follow a joint lognormal diffusion process:

$$dC_i = \alpha_i C_i dt + \sigma_i C_i dZ_i, \qquad i \in \{1, \cdots, M\}, \tag{10}$$

with $dZ_i dZ_j = \rho_{ij} dt$. Furthermore, assume that the arrival intensity of cash flows also follows a lognormal diffusion process:

$$d\lambda = \alpha_\lambda \lambda dt + \sigma_\lambda \lambda dZ_\lambda, \tag{11}$$

common across cash flows. Assume the conditions of Merton's (1973) intertemporal capital asset pricing model with a constant opportunity set to determine the nature of the market price of risk, φ_i for each of these state variables. Under this version of the ICAPM, $\varphi_i = (\alpha_m - r)\rho_{im}/\sigma_m$, where m represents the market portfolio, and α_m and σ_m are the instantaneous growth rate and standard deviation of the proportional changes in the level of the market portfolio.

To value the asset, we must solve the specialized version of the fundamental partial differential equation:

$$\tfrac{1}{2}V_{CC}\sigma_C^2C^2 + \tfrac{1}{2}V_{\lambda\lambda}\sigma_\lambda^2\lambda^2 + V_{C\lambda}\rho_{C\lambda}\sigma_C\sigma_\lambda C\lambda + V_C\alpha_C^*C + V_\lambda\alpha_\lambda^*\lambda - V_\tau + \lambda g(C, V) - rV = 0. \quad (12)$$

The i subscript has been dropped for clarity. Asterisks identify certainty-equivalent growth rates of the corresponding state variables.

We need only solve for the case of a renewing finite cash flow; the values in the other cases can be found quickly from this one. Therefore, recall the boundary condition $V(\tau = 0) = 0$, and set $g(C, V) = C$. Discount certainty-equivalent cash flows at the riskless rate. (See Constantinides (1978).) Since expected cash, $L = C\lambda$, is found by taking the product of two lognormal diffusion processes, expected cash itself must follow a lognormal diffusion process. In fact,

$$dL = \alpha_L L dt + \sigma_L L dZ_L, \quad (13)$$

where $\alpha_L = \alpha_i^* + \alpha_\lambda^* + \rho_{i\lambda}\sigma_i\sigma_\lambda$; $\sigma_L dZ_L = \sigma_i dZ_i + \sigma_\lambda dZ_\lambda$. The expected value of the cash flow at time $u \in [t, T]$ then can be found easily; $E[L_u] = L_t\exp(\alpha_L(u - t))$. Discounting this expectation at the riskless rate of interest, we find the asset value (finite renewing case):

$$V = \frac{C\lambda}{r - \alpha_L}[1 - e^{-(r-\alpha_L)\tau}]. \quad (14)$$

The value would equal the value in the deterministic C and λ case if α_L were equal to zero. Therefore, when C and λ follow lognormal diffusion processes, we can think of substituting a risk-adjusted interest rate $r - \alpha_L$ for the riskless rate, r. To value the terminating cash flow, we must further substitute an intensity-adjusted interest rate of $r + \lambda$ for r. Finally, for the perpetual opportunity cases, we allow $\tau \to \infty$.

C. Risky Bond Valuation with Stochastic Interest Rates

In this section, we examine the valuation of a discount bond which may suddenly become worthless (as in Merton (1976)) with stochastic default intensity and stochastic interest rates. In particular, we will look at a risky discount bond whose default likelihood and instantaneous returns are affected by economy-wide state variables. We will make the simplifying assumption that the correlation between the interest rate and default rate is zero. This assumption rules out cases where the discount rate and the default rate depend nontrivially on the same state variables but allows us to examine stochastic default when default likelihood depends on non-interest rate factors, e.g., industry cycles, mismanagement, or other firm-specific factors.

Merton (1976) discovered that, for the constant interest rate case, one could substitute $r + \lambda$ for the riskless rate and thereby determine the price of a risky discount bond. When interest rates vary, this simple rule will not apply. Consider, for example, the single-factor term structure model in CIR (1985b). The discount bond was required to satisfy the following differential equation:

$$\tfrac{1}{2}\sigma_r^2 rP_{rr} + \kappa(\theta - r)P_r - P_\tau - \varphi_r rP_r - rP = 0, \tag{15}$$

with $P(r, \tau = 0) = 1$. The notation has been adjusted to maintain consistency in this paper. The bond price was found to be given by

$$P(r, \tau) = A(\cdot)\exp[-rD(\cdot)], \tag{16}$$

where

$$A[\tau;\, \kappa,\, \theta,\, \sigma_r,\, \varphi_r,\, \gamma(\kappa, \sigma_r, \varphi_r)] \equiv \left[\frac{2\gamma e^{[(\kappa+\varphi_r+\gamma)\tau/2]}}{(\gamma + \kappa + \varphi_r)[e^{\gamma\tau} - 1] + 2\gamma}\right]^{2\kappa\theta/\sigma_r^2},$$

$$D[\tau;\, \kappa,\, \sigma_r,\, \varphi_r,\, \gamma(\kappa, \sigma_r, \varphi_r)] \equiv \left[\frac{2[e^{\gamma\tau} - 1]}{(\gamma + \kappa + \varphi_r)[e^{\gamma\tau} - 1] + 2\gamma}\right],$$

$$\gamma(\kappa, \varphi_r, \sigma_r) \equiv [(\kappa + \varphi_r)^2 + 2\sigma^2]^{1/2}.$$

When we admit the possibility that the discount bond could jump instantaneously to zero, the coefficient of P in (15) becomes $r + \lambda$.

The price of a discount bond (P^*) which may drop to zero with constant probability λdt can be given by $P^* = P \times \exp(-\lambda\tau)$. We calculate the price as if we were certain to receive the final payment but then discount again at the rate λ to determine its value.[6]

When λ itself follows mean reversion (17),

$$d\lambda = \mu(\nu - \lambda)dt + \sigma_\lambda \sqrt{\lambda} dZ_\lambda, \tag{17}$$

the pricing function must be revised substantially. The resulting bond price can be given by

$$P = A_1(\tau) \times A_2(\tau) \times \exp[-rD_1(\tau) - \lambda D_2(\tau)], \tag{18}$$

where

$$A_1(\tau) = A[\tau;\, \kappa,\, \theta,\, \sigma_r,\, \varphi_r,\, \gamma(\kappa, \sigma_r, \varphi_r)],$$

$$A_2(\tau) = A[\tau;\, \lambda,\, \mu,\, \sigma_\lambda,\, \varphi_\lambda,\, \gamma(\nu, \sigma_\lambda, \varphi_\lambda)],$$

$$D_1(\tau) = D[\tau;\, \kappa,\, \sigma_r,\, \varphi_r,\, \gamma(\kappa, \sigma_r, \varphi_r)],$$

$$D_2(\tau) = D[\tau;\, \nu,\, \sigma_\lambda,\, \varphi_\lambda,\, \gamma(\nu, \sigma_\lambda, \varphi_\lambda)],$$

where $\gamma(\cdot)$ is as defined in (16).

To value a risky coupon bond, one can tabulate the values of its component discount bonds. The realization that payments may terminate before τ will automatically be included in the price of the bond.

[6] For a similar result in option pricing, see McDonald and Siegel (1984).

V. Conclusion

This study represents a simple but important step in the study of stochastic discrete cash flows and the valuation of assets backed by cash flows of this type. The intensity of arrivals in a discrete process (λ) can be likened to the growth rate of a continuous process (r). A fundamental symmetry, therefore, must exist between the concepts of an interest rate and an arrival rate. This study has revealed some of those symmetries.

The potential applications of this analysis are many. In this paper, I have suggested applying these procedures to the valuation of insurance contracts, a firm with uncertain future growth opportunities, mortgage-backed securities, and risky bonds.

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