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Optimal Bank Reorganization Policies and the Pricing of Federal Deposit Insurance

SANKARSHAN ACHARYA and JEAN-FRANCOIS DREYFUS*

ABSTRACT

Optimal dynamic regulatory policies for closing ailing banks and for deposit insurance premia are derived as functions of the rate of flow of bank deposits, and interest rate on deposits, the economy's risk-free interest rate, and the regulators' bank audit/administration costs. Under competitive conditions, the threshold assets-to-deposits ratio below which a bank should be optimally closed is shown to be greater than or equal to one. Optimal deposit insurance premia and probabilities of bank closure are shown to be nondecreasing in the bank's risk on investment and nonincreasing in the bank's current assets-to-deposits ratio.

SHOULD REGULATORY AGENCIES CLOSE (reorganize) an ailing, federally insured financial institution? If so, when? The recent record rate of failures of Savings and Loans Associations (S&Ls) has renewed the urgency of this question. The regulatory agencies are empowered, but not required, to reorganize financial institutions that are statutorily insolvent (i.e., have zero or negative regulatory net worth).[1] Recently, regulators have in many instances permitted insolvent financial institutions to remain in operation.[2] Also, while debating on the increase of the proposed funding of the federal deposit insurers, the U.S. Congress argued whether it would be desirable, and if so when, to close more S&Ls.

Thus, when regulators should close a (troubled) financial institution has become a major question of bank regulatory policy. While the existing academic literature provides powerful insights into the structure and pricing of the insurers' liability, this literature offers very little guidance regarding the timing of bank reorganizations.[3] Indeed, to the best of our knowledge, the value of the federal

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[1] The regulators may, however, be forced to step in if it is important to maintain a "sound" banking and thrift system.

[2] A national bank is closed by the Comptroller of the Currency on request by the Federal Deposit Insurance Corporation (FDIC). The Federal Home Loan Bank Board (FHLBB) can close an S&L. By closure/reorganization, we mean liquidation, merger with a "willing" partner, or replacement of the current management. We abstract away from the cases in which regulators exercise substantially increased authority and responsibility for the operations of a bank or S&L. For a detailed account of the failure rate of banks and S&Ls, see, e.g., Benston (1985) and the recent *Wall Street Journal* issues.

[3] The pioneering work in pricing of federal deposit insurance is due to Merton (1977, 1978). In these papers, Merton presents a model in which the price of the deposit insurance is equal to the value of a put option written on a bank's assets with a strike price equal to the amount of insured deposits. This approach was extended by Marcus and Shaked (1984), Pennacchi (1987a,b), Pyle (1984), and Ronn and Verma (1986).

deposit insurance has been computed only for exogenously given rules, calling for the bank to be closed if (at the time of an audit) the bank's assets-to-deposits ratio drops below a prespecified minimum.[4] This paper addresses the broader problem of deriving endogenous bank closure rules.

The chosen closure rules, however, impact the actual liability faced by the insurer and in turn the level of risk-adjusted premia. We therefore formulate jointly (i) an optimal bank closure policy for a pure cost-minimizing insurer and (ii) the corresponding pricing of deposit insurance, by developing a dynamic model of a bank.

In our model, the insurer receives periodic audit reports assessing the value of a bank's deposits and the true value of its assets.[5] While the publicly available audit reports do provide some of the information the insurer needs, evaluating these reports and gathering more information about the bank are costly to the insurer. We thus assume that the insurer faces positive audit costs. In addition, the insurer incurs further monetary costs if a bank is closed and the amount of insured deposits exceeds the asset value of the bank on closure.

If the bank is kept in operation, however, the insurer may charge, if feasible, an actuarially fair premium, covering the value of its expected net future liability inclusive of the expected value of future audit costs. (In order to be feasible, the premium must be less than the current value of the bank in operation.) Given this cost structure and the current audit report, the insurer decides whether to keep the bank in operation. Keeping the bank in operation is optimal as long as the value of the insurer's future liability is less than the immediate cost of closing the bank. The insurer's liability must, however, be computed net of the value of premia collected now and in the future (conditional on the bank's continued operation). The timing of the insurer's closure decisions is therefore a function of, among other things, its premium policy, which in turn reflects the insurer's expected future liability.

One of the results of this analysis is that, if the equilibrium spread between the risk-adjusted expected rate of return on the bank's assets and the rate of interest paid on deposits exceeds the insurer's audit and administration costs per dollar of deposits, it is then optimal for the insurer never to close a bank.[6] Given

[4] While Merton (1978) assumes the prespecified minimum level of assets-to-deposits ratio to be one, Pyle (1984) and Ronn and Verma (1986) do allow for any other exogenously specified minimum levels of assets-to-deposits ratio below which to close a bank.

[5] Note that, while we assume that the insurer and the bank share the same information, this information is not perfect. The insurer acts (to close or keep a bank open and to set a fair premium), given the information (contained in audit report) that evolves as a stochastic process over time. As might be seen later, the symmetric information problem is far from being trivial and requires prior study before analyzing the asymmetric information case. Asymmetries of information, while relevant, would at this point only cloud the basic issues regarding the pure insurance pricing problem arising when the insurer acts as a rational cost minimizer. We plan to extend the techniques developed in this paper to study the more complex moral hazard problem which may be faced by the insurer.

[6] Should the bank's assets become insufficient to cover the flow of withdrawals from deposits, the insurer will in such a case elect to meet the shortfall via a direct capital infusion rather than to reorganize the bank. Our model assumes that the deposits are fully insured and that the economic agents are symmetrically informed. There is thus no motive for a "bank run." Consumers are, however, assumed to have a normal demand for liquidity.

free entry into and competition within the banking industry, however, the equilibrium spread between the rates earned and paid by a bank is shown to be necessarily smaller than audit and administration costs per dollar of deposits.

Under such competitive conditions, the insurer is shown to optimally close a bank whenever its assets-to-deposits ratio falls below an endogenously determined threshold level. Thus, this optimal closure policy has the same simple, intuitive structure as the exogenous closure policy specified in the extant literature. The optimal threshold level of the assets-to-deposits ratio obtained in our model is, however, specific to the bank being investigated, unlike the constant (across the banks), exogenous threshold level posited in the previous literature. Under competitive conditions, this optimal threshold assets-to-deposits ratio is shown to be greater than or equal to one. Further, the optimal threshold assets-to-deposits ratio depends on (i) the rate of net inflow of deposits into the bank, (ii) the rate of interest paid on the bank's deposits, (iii) the economy's equilibrium risk-adjusted expected rate of return, and (iv) the insurer's audit and administration costs.

The closure decision (at the optimal threshold level) does not depend on the specific distribution of the future asset value. The optimal deposit insurance premium and the probability of closure of the bank at the time of the next audit do depend, however, on the distribution of the future asset value. In particular, the probability of closure and the premium charged to the bank are monotonically nondecreasing in the riskiness of the bank's investments and nonincreasing in the current assets-to-deposits ratio of the bank.

The monotonicity of the premium with respect to the assets-to-deposits ratio of the bank is particularly interesting in comparison to Merton's (1978) model. Merton assumes that (i) the insurer charges the bank a one-time premium equal to the expected value of the insurer's liability for the infinite future and (ii) the realized audit costs are borne by the insurer. Under these assumptions, Merton finds that the value of the insurer's liability (equal to the insurance premium) is not in general a monotonic function of the insured bank's assets-to-deposits ratio. While a higher assets-to-deposits ratio implies a lower probability of failure in Merton's model, it also implies (given a fixed probabilistic audit frequency) that the insurer will have to bear actual audit costs for a larger expected length of time. Pyle (1983) argues that the resulting nonmonotonicity disappears if it is assumed that audit costs are borne by the bank itself, as long as the bank is solvent, and if the insurer provides insurance (and charges insurance premia) on a period-by-period basis.

In our model, the insurer bears the costs of audits it performs, and the insurer's liability is reevaluated and a new premium charged after each audit, as in Merton (1977). As long as the bank is kept open, the insurer in our model can transfer to the bank (via its premium policy) the realized audit costs it has to bear, whenever doing so is feasible and optimal. This dynamic adjustment is ruled out by Merton's framework, where the one-time premium once paid is effectively sunk.

We specify our model in Section I, derive the insurer's optimal premium and closure policies in Section II, analyze their properties in Section III, illustrate computation of the optimal insurance premium under the assumption that the value of the bank's assets follows a lognormal process in Section IV, and conclude in Section V.

I. The Model

We consider a model of a single bank which, unless declared closed, may remain in operation over a large (countably infinite) number of time periods, $t = 0, 1, 2, \cdots$. The bank holds capital assets and is financed by equity and a variety of other liabilities, collectively referred to as "deposits." Since we assume that all liabilities, except equity, are ultimately guaranteed (insured) by the government or one of its agencies, we need not distinguish among the bank's various debt obligations.

At the beginning of each time period t, the insurer receives an audit report which reveals the value of the bank's assets, X_t, and the value of the bank's aggregate deposits, D_t. Let r_D denote the per-period rate of interest paid on deposits. We assume that depositors withdraw a constant fraction γ of the interest, accrued over the preceding period, the remaining fraction $(1 - \gamma)$ being added to the value of the bank's deposits. Let g denote the rate of growth in deposits, before the accrual of undistributed interest. For simplicity, both r_D and g are assumed to be constant over time. This and similar assumptions below allow for the existence of a well defined, time-stationary optimal policy on the part of the insurer. As we focus on the case of a single bank, the parameters g, r_D, and γ are taken to be bank specific. We make the following two standard assumptions:

Assumption 1: $\delta \equiv g - \gamma r_D < 0$.[7]

Assumption 1 ensures that the bank may not run a "Ponzi game," forever paying out interest financed by newly deposited funds.

Assumption 2: The set of traded securities in every period includes a riskless bond, offering a constant equilibrium yield equal to r. In order to avoid arbitrage by investors, $r \geq r_D$.[8]

The Insurer's Problem

Given the audit report (X_t, D_t) received at time t, the insurer may either (i) close (reorganize) the bank or (ii) keep it in operation for another period. In case

[7] It is similar to an assumption made in Merton (1978, footnote 2). Merton assumes $\gamma = 1$ and $g \leq r_D$. Note that the rate of growth in a bank's deposits may occasionally exceed the flow of withdrawals from deposits. However, having $\delta > 0$ over the entire life of the bank is equivalent to running a Ponzi game.

[8] The fact that $r > r_D$ would require the presence of some market segmentation, where investors are subject to differential transaction costs and some traders are subject to costs in excess of the spread between r and r_D. The latter assumption is, for instance, made in Merton (1978, p. 440) and Pennacchi (1987a, p. 293). It should, however, be noted that we do *not* require that r strictly exceed r_D. Later, we discuss the relationship between r, r_D, and other parameters of our model which is implied by the existence of a competitive equilibrium.

(ii) the insurer must further decide what premium P_t, $P_t \leq X_t$, to charge the bank. Given an audit report (X_t, D_t), and given that the bank is kept in operation at time t after being charged a premium equal to P_t, the values of the bank's deposits and assets at time $t + 1$ are given by

$$D_{t+1} = [1 + (1 - \gamma)r_D + g]D_t \equiv RD_t \tag{1}$$

and

$$X_{t+1} = (X_t - P_t)y_{t+1} + (g - \gamma r_D)D_t = (X_t - P_t)y_{t+1} + \delta D_t, \tag{2}$$

where y_{t+1} denotes the realized return between time t and time $t + 1$ per dollar of the bank's assets at time t. In choosing which course of action to follow, we assume that the insuring agency behaves as a pure cost minimizer.

If the bank is closed at t, the insurer first uses the realized asset value X_t toward payment of the (fully insured) deposits D_t. The insurer then distributes any excess value (if $X_t > D_t$) to the bank's shareholders while covering any shortfall (if $X_t \leq D_t$). Thus, the cost of reorganization to the insurer is $W(X_t, D_t) = \max[0, D_t - X_t]$. The insurer weighs this immediate reorganization cost against the cost incurred and the premia received if the bank is kept in operation for at least one more period. The costs of keeping the bank in operation are twofold: (i) an administrative cost (including the cost of auditing the bank) at time $t + 1$ and (ii) a cost corresponding to the recurrence of this decision process at time $t + 1$. For simplicity, we assume the insurer's audit costs at time $t + 1$ to be equal to a constant fraction $\bar{m}$ of deposits D_{t+1}. Equivalently, these costs may be written as mD_t, where $m \equiv \bar{m}/R$.[9] Let $C_{t+1}(X_{t+1}, D_{t+1})$ be the insurer's cost in state (X_{t+1}, D_{t+1}) at time $t + 1$, given that it adopts optimal closure and premium policies from time $t + 1$ on.

To specify how the insurer evaluates this future cost at time t, we note that the equilibrium price of any traded asset at time t may be expressed as the discounted (conditional) expected value of the asset at time $t + 1$ under an equivalent martingale measure (Harrison and Kreps (1979)). The insurer is assumed to behave competitively in capital markets and to price its liability as the discounted expected value of this liability under a martingale measure consistent with the prices of traded assets, taking this measure and the corresponding implicit contingent claim prices as given. (Henceforth, probability distributions and expectations are defined with respect to this martingale measure.)[10] The net liability incurred by the insurer if the bank is allowed to operate

[9] Note that audit costs may increase with the complexity of audit. Complexity of audit may not, however, be an increasing function of absolute size of deposits. The exact form of this cost function has little impact, however, on the economics of our model as long as costs per dollar of deposits may not grow without bounds. The assumption of linear dependence between audit costs and deposits is thus made for analytical simplicity, as in Merton (1978).

[10] This assumption is natural if the insurer's liability may be hedged using the bank's traded liabilities (e.g., its equity). In this case, the value of the insurer's liability is uniquely priced by arbitrage. The liabilities of smaller banks may be thinly traded, however, as noticed by Marcus and Shaked (1984), for instance. Further, the value of the bank's assets may fail to follow a smooth, fully hedgeable process if deposits and consequently the bank's investments are subject to stochastic jumps. (We thank René Stulz for pointing this out.) In this case, the insurer's liability may fail to be uniquely priced by arbitrage. However, the nonexistence of arbitrage opportunities does require the

one more period after assessment of a premium P is thus valued as

$$Q_t(X_t, D_t, P) = -P + \beta m D_t + \beta E_t[C_{t+1}(X_{t+1}, D_{t+1})],$$

where $\beta \equiv 1/(1 + r)$, (X_{t+1}, D_{t+1}) is given by (1)–(2), and $E_t(\cdot)$ is the expectation operator, given the state of the bank (X_t, D_t) at time t, with respect to the distribution induced by the random variable y_{t+1}. For analytical simplicity, we assume that $\{y_t, t = 0, 1, 2, 3, \cdots\}$ is an i.i.d. sequence and, further, that $\forall\ t, y_t$ is non-negative with probability one.[11]

We define the "optimal" insurance premium at time t (to be charged if the bank is left open) as a solution to the following problem:

$$\text{Minimize}_P\, Q_t(X_t, D_t, P) \quad \text{subject to } P \leq X_t \text{ and } Q_t(\cdot) \geq 0. \tag{3}$$

In (3) a negative P is interpreted as a direct capital infusion by the insurer. Note that X_t measures the value of the bank's assets *after* withdrawals from deposits at time t. Therefore, X_t will be negative when the realized value of the bank's assets (which is equal to $(X_{t-1} - P_{t-1})y_t + gD_{t-1}$ and which is non-negative with probability one) is less than the desired withdrawals at time t, $\gamma r_D D_{t-1}$. When $X_t \leq 0$, the insurer must either close the bank, incurring a cost of $W(X_t, D_t)$, or keep the bank in operation by infusing a minimum capital of $|X_t|$, so as to at least cover the current demand for withdrawals.

The function (originally suggested in Merton (1977, 1978, p. 449)) minimizes cost to the insurer of insuring the representative bank under consideration, subject to the constraint that the insurer not make profits. This objective may, however, imply losses to the insurer with respect to the representative bank. Given many such banks, the insurer may thus face system-wide losses, requiring public funding. An alternative to our objective function is to minimize costs of insuring many (possibly different) banks subject to making no profit from all these banks. With this objective function, the insurer may lose less overall; however, the closure and premium policy may be inequitable as this alternative objective function tends to transfer wealth from better to worse banks.[12]

If there exists a premium $P \leq X_t$ such that $Q_t(X_t, D_t, P) = 0$, then P is conventionally said to be actuarially fair. We may, however, have $Q_t(X_t, D_t, P) > 0$, $\forall P \leq X_t$. In this case, the insurer simply minimizes its net liability $Q_t(X_t, D_t, P)$. Thus, the optimal premium, resulting as a solution to (3), may not always be actuarially fair.

Let now $J_t(X_t, D_t)$ denote the optimized value of Q_t, as the solution of the problem described by (3). The insurer's optimal course of action at time t is then simply to close the bank if $W(X_t, D_t) < J_t(X_t, D_t)$ and to keep it in operation

insurer's liability to be priced under *some* equivalent martingale measure. The reader may therefore note that our characterization of the insurer's optimal closure policy is independent of the specific choice of (nonarbitrage) valuation function by the insurer. The exact valuation of its liability by the insurer, and its consequent premium calculations, will, however, depend on this choice in the absence of perfect hedging opportunities.

[11] This assumption is consistent with the usual lognormal diffusion asset value process assumed in the earlier literature.

[12] Either objective is thus likely to result in some economic distortions. Examining which objective is "better" is beyond the scope of our paper.

if $W(X_t, D_t) \geq J_t(X_t, D_t)$. Thus, given its optimal behavior at time t, the insurer's minimum cost equals

$$C_t(X_t, D_t) = \min[W(X_t, D_t), J_t(X_t, D_t)], \tag{4}$$

which, given (3), may be written as

$$C_t(X_t, D_t) = \min_{P \leq X_t} \min[W(X_t, D_t), -P + \beta m D_t + \beta E_t\{C_{t+1}(X_{t+1}, D_{t+1})\}]$$

$$\text{s.t.} \min[W(X_t, D_t), -P + \beta m D_t + \beta E_t\{C_{t+1}(\cdot)\}] \geq 0. \tag{5}$$

A solution to the insurer's problem is a sequence of functions $\{C_t, t = 0, 1, 2, 3, \cdots\}$ satisfying (3) and (4). These functions and (3)–(4), or equivalently (5), yield the insurer's optimal actions at each point in time. For our results to be applicable to any operating bank, without regard to when it started, we focus on stationary solutions to the insurer's problem. Also, in order to define the closure rule as a function of the assets-to-deposits ratio (for simplicity in application and for analytical tractability), we consider solutions that are homogeneous of degree one in deposits. We shall prove in the next section the existence of a stationary (time-independent) solution $C(X_t, D_t)$ which is homogeneous of degree one in D_t.

Before deriving this solution and its properties, it might be useful to compare the basic structure of our model—characterized by the reduced form (Bellman) equation (5) which defines the insurer's problem at time t—with that of the models developed in the extant literature. Further comparisons will be provided later.

First, as in Merton (1978) and Pennacchi (1987a,b), the insurer in our model bears the audit cost at time $t + 1$ if the bank is in operation at time t. While Merton's (1978) model determines a one-time premium equal to the value of the insurer's potential liability over the infinite future, our model allows for the dynamic reevaluation of the insurer's liability as time goes by, thus allowing the insurer to transfer its audit costs to the insured bank via assessment of sequentially actuarially fair premia (as long as this is feasible). This feature of our model is similar to Pyle's (1983). It should, however, be noted that Pyle (1983) assumes that insured banks directly bear the costs of audits. Banks in our model bear these costs only via the insurer's rational pricing decisions.

Second, it may be noted that Merton (1977) prices the deposit insurance contract as a limited-term, renewable contract (but in the absence of audit costs). Merton's valuation formula, however, assumes that the insurer's net liability is reset at zero at the end of every period, as pointed out by Marcus and Shaked (1984) and Pennacchi (1987b). This need not be feasible, as shown in the context of our model.

Finally, we believe that our model differs from the aforementioned models with respect to the following fundamental and crucial aspect. The insurer in these models is assumed to close the bank and cover its liabilities whenever the bank's assets-to-deposits ratio falls below an *exogenously* specified level. This exogenously specified level of assets-to-deposits ratio, however, may not be optimal. In particular, the insurer may not have any incentive to close the bank if it is feasible to charge an actuarially fair premium, which fully covers the insurer's continuing liability. Hence, in a multiperiod dynamic setting, closure decisions and pricing of the insurance contract at time t must take into account the level of premia which the insurer will assess from time $t + 1$ on.

By formulating the insurer's decision-making process as a standard dynamic programming problem, we endogenize the insurer's optimal closure rule and formulate a sequentially optimal premium policy. The optimal premium in our model, determined by solving (5), is equal to the actuarially fair premium only if feasible. The insurer in our model, pursuing a sequentially optimal premium policy, is able to transmit the audit cost (it initially bears) to the bank as long as it is optimal to do so. As mentioned above, we shall show later that these features of our model remove any nonmonotonicity of the premium with respect to the assets-to-deposits ratio of the bank.

II. Optimal Closure and Premium Policies

We shall now show the existence of a stationary (time-independent) solution $C(X_t, D_t)$ which is linearly homogeneous in D_t. Given such a homogeneous stationary solution, we can rewrite the insurer's decision problem, given by (3) and (4), in terms of the normalized stationary solution as

$$c(x_t) = \min[w(x_t), j(x_t)], \tag{6}$$

where

$$w(x_t) = \max[0, 1 - x_t], \tag{7}$$

and where

$$j(x_t) = \min_{p \le x_t} q(x_t, p) \quad \text{subject to } q(x_t, p) \ge 0, \tag{8}$$

$$q(x_t, p_t) = -p_t + \beta m + R\beta E_t[c(x_{t+1})], \tag{9}$$

$$x_{t+1} = \frac{1}{R}(x_t - p_t)y_{t+1} + \frac{\delta}{R}, \tag{10}$$

$$x_t = \frac{X_t}{D_t}, \quad p_t = \frac{P_t}{D_t}, \quad c(x_t) = C(x_t, 1). \tag{11}$$

Note that $x_t \ge \delta/R$ with probability one, for all t. Also, $R\beta$ may be interpreted as the insurer's effective discount rate in the normalized problem. The bank is then closed whenever $w(x_t) < j(x_t)$, and is left open otherwise after being charged a premium p_t per dollar of deposits as given by (8)–(9). Note that, by Assumptions 1 and 2, $r \ge r_D \ge r_D + (g - \gamma r_D) = (1 - \gamma)r_D + g$, i.e., $R\beta \le 1$. This implies, since $w(x_t)$ is bounded, the existence of a unique, bounded, normalized stationary solution $c(\cdot)$.[13] To this normalized stationary solution $c(\cdot)$ there corresponds a

[13] If $R\beta < 1$, the result simply follows from an application of the contraction mapping theorem to the operator on the space of bounded real functions which transforms a given cost function $\hat{c}(\cdot)$ into the function $c(\cdot)$ defined by $c(\cdot) = \min[w(\cdot)\ \hat{j}(\cdot)]$, where $\hat{j}(\cdot)$ is defined by (8) with $q(\cdot, p_t)$ set equal to $-p_t + \beta m + R\beta E_t[\hat{c}(\cdot)]$. (See, e.g., Blackwell (1965) and Denardo (1967).) In this case $R\beta = 1$, the undiscounted dynamic programming approach given, e.g., in Bertsekas (1976, pp. 294–305) can be used to extend this result.

non-normalized, first-degree homogeneous stationary solution $C(\cdot, \cdot)$, defined by $C(X, D) = Dc(X/D)$.[14] In the following proposition, we derive the explicit form of $c(\cdot)$ after defining for brevity:

$$\tilde{x} \equiv x_{t+1}, \quad \tilde{y} \equiv y_{t+1}, \quad \text{and} \quad x \equiv x_t.$$

PROPOSITION 1: *Define*

$$\bar{x} \equiv \beta m + \beta R \min\left[1 - \frac{\delta}{R}, \frac{\beta m - \delta/R}{1 - \beta R}\right] = \min\left[\beta(m + R - \delta), \frac{\beta(m - \delta)}{1 - \beta R}\right]. \tag{12}$$

Then, (a) *the optimal function* $c(\cdot)$ *is given by*

$$c(x) = \min[\max[0, 1 - x], \max[0, -x + \bar{x}]], \; x \geq \delta/R;$$

(b) *It is optimal to close the bank if* $[\bar{x} \geq 1$ *and* $x < \bar{x}]$ *and to leave it open if* $[\bar{x} < 1]$ *or* $[\bar{x} \geq 1$ *and* $x \geq \bar{x}]$.

Proof: See Appendix A.

Note that the parameter values satisfy

$$\beta(m + R - \delta) > \frac{\beta(m - \delta)}{1 - \beta R} \quad \text{if and only if } \bar{x} < 1 \quad \text{if and only if } m < r - r_D. \tag{13}$$

In Figures 1 and 2, we plot the cost function $c(\cdot)$ for two exhaustive sets of parameter values. In Figure 1, $\bar{x} = \dfrac{\beta(m - \delta)}{1 - R\beta} < 1$, and the cost of keeping the bank in operation, $j(x)$, is less than the cost of closing the bank $w(x)$ for all values of x. The bank is therefore never closed by the insurer. Indeed, when $x \geq \bar{x}$, the insurer is able to assess an actuarially fair premium, thereby fully covering the liability incurred due to the bank's continued operation. When $x < \bar{x}$, however, the insurer's optimal premium policy is interpreted as assessing a premium equal to the value of the bank's assets (per dollar of deposits) when $x > 0$, or injecting funds in an amount $|x|$ (exactly covering current withdrawals) when $x < 0$. The value of the bank's assets is consequently reset to zero at the end of the period. At time $t + 1$, the insurer will thus have to expend administrative costs mD_t and will have to inject additional funds in the amount of $(\gamma r_D - g)D_t = -\delta D_t$ in order to cover the flow of withdrawals in excess of the rate of growth in deposits, for a total expenditure of $(m - \delta)D_t$ dollars. Since deposits grow at a net rate of $(R - 1)$ and the economy's risk-adjusted discount rate is equal to $[(1/\beta) - 1]$, the present value of this perpetual commitment per dollar of deposits is easily seen to be equal to $(m - \delta)/[1/\beta - R] = \beta(m - \delta)/(1 - R\beta) = \bar{x}$. Since this value, $\bar{x}$, plus the immediately required funds (or minus the current premium if $x > 0$) falls short of the cost of liquidating the bank, i.e., $-x + \bar{x} \leq 1 - x$ for all x in $[\delta/R, \bar{x}]$, the insurer optimally elects to support forever the bank's continued operation.

It may, however, be seen from (13) that the policy depicted in Figure 1 in particular relies on the existence of a large enough spread, $r - r_D$, between the economy's risk-adjusted equilibrium rate of return and the rate of interest paid

[14] There may, however, exist other non-normalized solutions which are not linearly homogeneous in D and/or nonstationary.

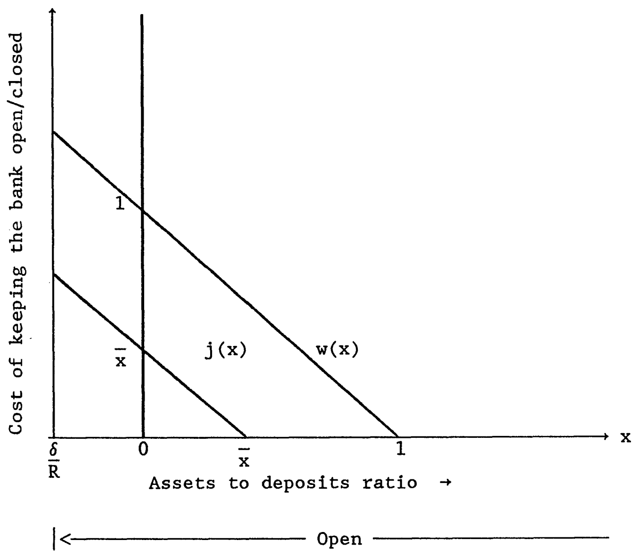

Figure 1. The case in which the bank is optimally never closed. This figure shows the cost of immediate closure, $w(x)$, and the cost of keeping the bank open for at least one more period net of deposit insurance premia, $j(x)$, as functions of the bank's assets-to-deposits ratio, x, when $r - r_D > m$, where r is the risk-adjusted rate of return on the bank's assets, r_D is the interest rate paid on deposits, and m is the audit cost per dollar of deposits. The bank is optimally never closed in this case ($r - r_D > m$).

on deposits. Indeed, as the true residual claimant, albeit one with unlimited liability, the insurer "earns" this spread as long as the bank is kept in operation. (A similar point was made by Merton (1978).) We shall, however, show that the existence of such a large spread is ruled out under competitive conditions in the banking industry.

In Figure 2, $\bar{x} = \beta(m + R - \delta) \geq 1$, the probability of closure is positive, and the bank is closed at time t if the asset–deposit ratio at t, x, is less than the threshold ratio, $\bar{x}$. In this case, the premium charged by the insurer is actuarially fair as long as the bank is optimally kept open. An intuitive understanding of the optimal closure rules in the second case may be gained from the following interpretation of the optimal threshold assets-to-deposits ratio, $\bar{x} = \beta(m + R - \delta) = \beta(1 + r_D + m)$. Indeed, closing the bank "today" would require the insurer to cover current deposits dollar for dollar. If the bank were allowed to operate for at least one more period, then "tomorrow" the insurer would incur a total liability equal to $(1 + r_D + m)$ per dollar of today's deposits, i.e., a sum of deposits, \$1, accumulated interest, r_D, and the auditing cost per dollar, m. Thus, $\bar{x} = \beta(1 + r_D + m)$ is the discounted value of tomorrow's liability per dollar of today's deposits. Proposition 1 then implies that the insurer will optimally close the bank if the bank's current asset value is too low for the insurer to be able to charge an actuarially fair permium, i.e., if $x < \bar{x}$. It should be noted, however, that, if $x = \bar{x}$, the optimal premium charged by the insurer at time t is $p = x$, implying that it will be known at time t that the bank will be closed with probability one at time $t + 1$. (Recall that the not-for-profit insurer effectively

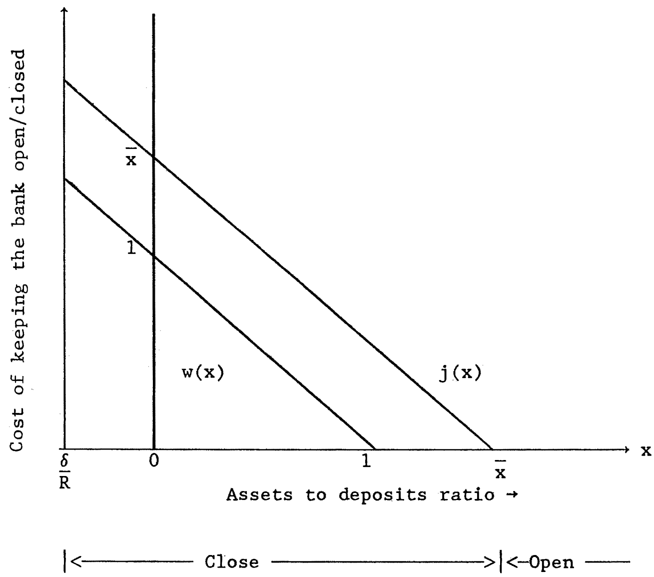

Figure 2. The case in which the bank may be optimally closed. This figure shows the cost of immediate closure, $w(x)$, and the cost of keeping the bank open for at least one more period net of deposit insurance premia, $j(x)$, as functions of the bank's assets-to-deposits ratio, x, when $r - r_D \leq m$, where r is the risk-adjusted rate of return on the bank's assets, r_D is the rate of interest on deposits, and m is the audit cost per dollar of deposits. In this case, the bank is closed when its current assets-to-deposits ratio x is less than the optimal threshold $\bar{x} = \beta(1 + r_D + m)$, where $\beta = 1/(1 + r)$. Note that $\bar{x} \geq 1$ if and only if $r - r_D \leq m$, and, given free entry and competition in the banking industry (Proposition 2), $\bar{x} \geq 1$.

closes the bank at time t if $x \leq \bar{x}$, pays off the depositors, bearing a shortfall of $\max(0, 1 - x)$, and distributes the remaining $\max(0, x - 1)$ to the current shareholders of the bank.)

The optimal threshold assets-to-deposits ratio $\bar{x}$ depends on the bank-specific parameters, δ and r_D, and the economy-wide parameters, r and m. The regulatory optimal closure policy does not depend, however, on the distribution of $\tilde{x}$, given $x - p$. Intuitively, as long as there exists an actuarially fair premium, i.e., as long as there exists $p \leq x$ such that $q(x, p) = 0$, the effective cost borne by the insurer is equal to zero. If $q(x, p) > 0$, for all $p \leq x$, then $p = x$ and the insurer's expected cost is independent of the distribution of $\tilde{x}$. If $p = x$, the bank will be closed with probability one by the time of the next audit.

The current regulatory behavior of allowing a large number of insolvent S&Ls to operate (see footnote 2) corresponds to $\bar{x} \leq 1$ in our model. We show below that, under competitive conditions, preemptive closure on a (marginally) solvent bank ($\bar{x} \geq 1$) would, however, minimize losses to the regulatory agencies.[15] Indeed, $\bar{x} < 1$ if and only if $m < r - r_D$ by (13). This condition means that the spread between the economy's equilibrium rate of return (r) and the rate of interest paid on deposits (r_D) exceeds the audit costs per dollar of current deposits (m). One

[15] Indeed, regulatory agencies are formally empowered to take action against banks with low (3%) but positive capital/asset ratios. See also footnote 1.

might suspect that the existence of such a "large" spread could create profit opportunities to entry into the banking industry. If so, a spread in excess of audit and administration costs, m, cannot survive if the banking industry is characterized by free entry and competitive behavior. This intuition is formalized in the following proposition.

PROPOSITION 2: *Suppose that the banking industry is characterized by free entry and competitive behavior. Then, necessarily, $\bar{x} \geq 1$ in equilibrium.*

Proof: See Appendix A.

It may be noted that Proposition 2 also implies that the case depicted in Figure 1 may not be sustained by a competitive equilibrium. Hence, competitive conditions imply that banks will be subject to a positive probability of regulatory action. In view of Proposition 2, we shall henceforth assume $\bar{x} = \beta(m + R - \delta) \geq 1$ and focus on the closure policies depicted in Figure 2.

Propositions 1 and 2 allow for an interesting comparison of our results with the threshold assets-to-deposits ratio specified by Ronn and Verma (1986). Ronn and Verma consider the possibility that, due to the costs imposed on the system by the liquidation process, the insurer might elect not to close down a bank unless its assets-to-deposits ratio falls below a certain (exogenously specified) level (ρ in their model), specified to be strictly less than one.[16] In the context of our model, however, Proposition 1 shows that the insurer would optimally elect to close the bank whenever $x < \bar{x}$ (with a one-period delay if $x = \bar{x}$), where $\bar{x} > 1$. Indeed, Proposition 2 shows that the resulting equilibrium spread $r - r_D$ will in any case be small enough to induce the insurer to take "early" action (of closing whenever $x \leq \bar{x}$, where $\bar{x} \geq 1$).

Alternatively stated, suppose that reorganization costs are "large," i.e., the value realized upon closure, x, is small (or possibly even negative). One might conjecture that the insurer might in such a case be "reluctant" to close a bank. However, reorganization costs will have to be borne if and when the bank is eventually closed. The insurer will elect to defer these costs only if the presence of a large enough spread $r - r_D$ warrants continued financial support of the bank's operations as well as the attendant administrative costs. Under competitive conditions, such a large spread will not be observed. It should, however, be noted that the existence of sufficiently high barriers to entry might result in the existence of a high monopoly rent (Pennacchi (1987a)), which might lead the regulatory agency to adopt a threshold ratio $\bar{x} \leq 1$.

III. Comparative Statics Results

We now turn to the analysis of the impact of the investment risk assumed by the bank on the probability of closure and on the federal deposit insurance

[16] The insurer in their model is further assumed to remedy immediately any observed asset deficiency (equal to $1 - x$, whenever $\rho \leq x < 1$). It might be noted, however, that such an infusion of funds is not required as long as the bank is kept open, provided that depositors trust that their deposits are fully covered.

premium policy. Given the parameters of the model, we define

$$z(x) \equiv \text{prob}(\tilde{x} \leq \bar{x} \mid x > \bar{x} \geq 1), \tag{14}$$

where $z(x)$ is evaluated given the insurer's optimal premium policy $p(x)$ for the current assets-to-deposits ratio x. We interpret $z(x)$ as the probability that the insurer will, by the time of the next audit, either (i) close the bank (if $\tilde{x} < \bar{x}$) or (ii) take an action such that the bank will be closed, with probability one, one period hence (if $x = \bar{x}$). While $\bar{x}$ is independent of the specific probability distribution governing the return on the bank's assets, the following comparative statics results show that $z(x)$ and $p(x)$ depend on the asset return distribution. (These results assume $x > \bar{x}$ since, if $x \leq \bar{x}$, the bank is effectively being closed.)

PROPOSITION 3: (a) *For $x > \bar{x}$, the current premium, $p(x)$, which is equal to the insurer's expected future liability, and the probability of closure by the next audit time, $z(x)$, are both monotonically nonincreasing functions of x.* (b) *Both $p(x)$ and $z(x)$ are monotonically nondecreasing functions of the risk of the bank's investments.*[17]

Proof: See Appendix A.

The economic interpretation of Proposition 3 should now be clear. By allowing the insurer to dynamically assess premia reflecting its expected liability, inclusive of audit costs, our model removes the source of the nonmonotonicities (of $z(x)$ and $p(x)$ as functions of x) which appear in Merton's (1978) model. Result (b) confirms the intuition that actuarially fair deposit insurance requires assessment of risk-sensitive premia.

IV. Optimal Policies When Asset Values Are Lognormally Distributed

In this section, we derive an explicit algorithm for computing the optimal deposit insurance premium $p(x)$ and the probability of closure $z(x)$ as functions of the asset value of the bank. We do this for parameter values satisfying the competitive equilibrium condition $\bar{x} \geq 1$ (depicted in Figure 2). We assume that the return over $(t, t+1)$ per dollar of the assets of the bank at time t, $y_{t+1} \equiv \tilde{y}$, is lognormally distributed as

$$\ln(\tilde{y}) \sim N\left[r - \frac{\sigma^2}{2}, \sigma^2\right].$$

We numerically solve equation (9), reproduced here for convenience (using

[17] We define "increasing risk" by the Second Order Stochastic Dominance (SSD) criterion (Rothschild and Stiglitz (1970)). Note that the SSD criterion may be characterized purely by distributional properties, without reference to any specific class of utility functions or degrees of risk aversion. Thus, while the distribution of y is evaluated under an equivalent martingale measure, the SSD criterion is well defined relative to such a distribution.

notations $\tilde{x} \equiv x_{t+1}$, $x \equiv x_t$:[18]

$$q(x, p) = -p + \beta m + R\beta E[c(\tilde{x})], \tag{15}$$

to calculate $p \leq x$ for an individual bank, given its current assets-to-deposits ratio x, the rate of interest on deposits r_D, the net rate of flow of deposits δ, the equilibrium rate of interest r, and the insurer's audit cost per dollar of deposits m. (Recall that $R = 1 + r_D + \delta$, and $\beta = 1/(1 + r)$.) By letting $f(\cdot)$ denote the density of $\tilde{x}$, given x (see Appendix B),

$$E_t[c(\tilde{x})] = \int_{\delta/R}^{1} [1 - \tilde{x}]f(\tilde{x})\, d\tilde{x} = \left(1 - \frac{\delta}{R}\right)\Phi[\bar{d}] - \frac{(x - p)}{R} e^r \phi[\bar{d} - \sigma], \tag{16}$$

where $\Phi(\cdot)$ is the cumulative standard normal density function and

$$\bar{d} = \left[\frac{\ln\left(\dfrac{R - \delta}{x - p}\right) - \left(r - \dfrac{\sigma^2}{2}\right)}{\sigma}\right]. \tag{17}$$

(The not-for-profit insurer does not benefit (or lose) if on closure $1 < x < \tilde{x}$, since any excess value $x - 1$ is paid to the bank's shareholders.) Given the solution $p(x)$, $z(x)$ may be calculated as

$$z(x) = \Phi[\bar{d}], \tag{18}$$

where $\bar{d}$ is given in (17).

One may now study the behavior of p and z with respect to the volatility σ. We derive (in Lemma 2, Appendix A) sufficient conditions ($\tilde{x} \geq 1$ and $\ln((\tilde{x}R - \delta)/(x - p)) > r$) for $p(x)$ to be monotonically increasing as a function of σ. Although the intuition of Proposition 3 carries through, it should be noted that increases in σ are not exactly equivalent (for lognormal distributions) to increases in risk defined by the Second Order Stochastic Dominance Criterion. (See Bawa (1975).)

In Figures 3 and 4, we plot the probability of closure and the optimal premium for four values of $\sigma = 0.1, 0.2, 0.3, 0.4$, given the following parameter values: $r_D = 0.099$, $\delta = -0.0001$, $r = 0.10$, $m = 0.002$. These values result in $\tilde{x} = 1.000909$ and thus correspond to Figure 2. In this case, if the bank is open at time t (i.e., $x > \tilde{x}$), then there is a positive probability of closure of this bank by the time $(t + 1)$ of the next audit. If the bank's assets-to-deposits ratio at time t, x, is less than or equal to $\tilde{x} = 1.000909$, the bank is closed at time $t + 1$ with probability one, and premium charged at time t is $p = x$. In this case, the optimal premium is equal to the assets-to-deposits ratio, x, since $q(x, p) \geq 0\ \forall p \in [0, x]$, where the equality holds when $x = \tilde{x}$. The bank is thus effectively kept open at time t if $x > \tilde{x} = 1.000909$, and $p \in [0, x]$ is actuarially fair since $q(x, p) = 0$ for all $x \geq \tilde{x}$. In Figures 3 and 4, we plot $z(x)$ and $p(x)$ for all $x \geq \tilde{x}$.

[18] To solve (16) for $p \leq x$, we use a fairly standard algorithm that (i) starts with an initial value of p, (ii) chooses a step size to search for the interval in which the solution lies, (iii) then halves the step size and repeats stage (ii) to search for the subinterval in which the solution lies, and (iv) repeats stage (iii) until convergence.

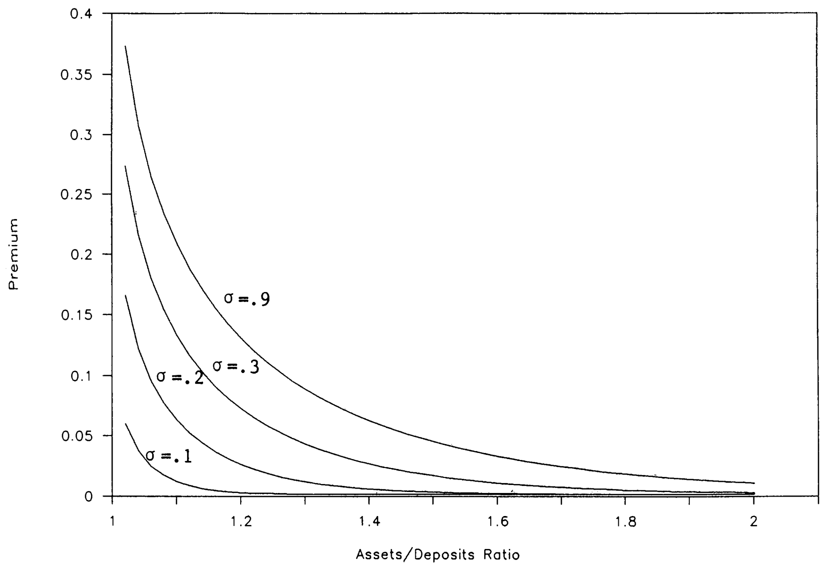

Figure 3. Optimal premia when bank's asset value is lognormal. This figure shows optimal premium over the next time period, $p(x)$, as a function of the current assets-to-deposits ratio, x, when the bank's value of assets in the next period is lognormally distributed. Parameter values used in the simulation are the risk-adjusted rate of return on the bank's assets, $r = 0.10$, the interest rate on deposits, $r_D = 0.099$, the rate of net flow of deposits, $\delta = -0.0001$, and the audit cost per dollar of deposits, $m = 0.002$. These values result in an optimal threshold $\bar{x} = 1.000909$. If x falls below $\bar{x}$, the bank is closed. The plots show $p(x)$ for four different values of volatility σ on the return on the bank's assets.

V. Conclusion

We derived optimal dynamic bank closure and federal deposit insurance premium policies under a fairly general setting. We emphasized that simultaneous determination of the interrelated closure and premium policies was important. The past literature focused on formulating the premium policy, given a policy that a bank be closed if its assets-to-deposits ratio is less than an exogenous minimum level, usually assumed to be less than or equal to one. Within the general setting of our paper, this policy was not found to be necessarily optimal. The optimal threshold level of assets-to-deposits ratio was indeed shown to be greater than or equal to one under competition within and free entry into the banking industry. This threshold level was shown to depend on the bank-specific parameters (the cost of reorganization, the rate of interest paid on deposits, and the rate of net inflow of deposits), the economy-wide parameter (the risk-free rate of interest), and the insurer's audit cost. The insurer's closure policy (i.e., the optimal threshold level) was shown to be independent of the specific probability distribution of the asset value of the bank by the time of the next audit. The optimal deposit insurance premium and the probability of closure of the bank at the time of the next audit, however, did depend on the distribution of the future asset value. In particular, the premium charged to the bank was shown to be a

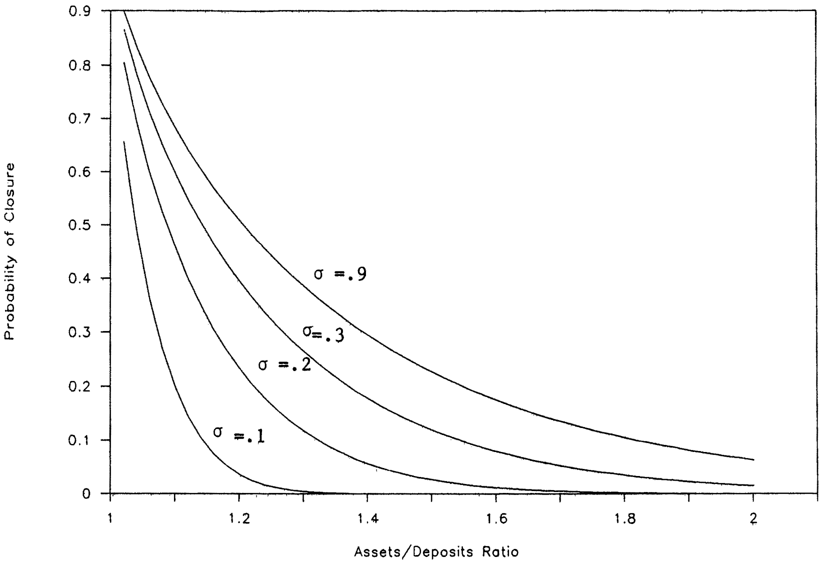

Figure 4. Probability of closure when bank's asset value is lognormal. This figure shows the probability of regulatory closure by the time of next audit, $z(x)$, as a function of the current assets-to-deposits ratio, x, when the next period's asset value of the bank is lognormally distributed for four values of volatility σ of the return on the bank's assets. Other parameter values used in the simulation are the risk-adjusted rate of return on the bank's assets, $r = 0.10$, the interest rate on deposits, $r_D = 0.099$, the rate of net flow of deposits, $\delta = -0.0001$, and the audit cost per dollar of deposits, $m = 0.002$. These values result in an optimal threshold $\bar{x} = 1.000909$. If x falls below $\bar{x}$, the bank is closed.

monotonically increasing function of the riskiness of its investments and a nonincreasing function of the current assets-to-deposits ratio.

We leave two interesting issues for future research. First, although we allowed the insurer to minimize its costs associated with a representative bank, it should be recognized that this might subject the insurer to system-wide losses with a positive probability. The insurer could be alternatively allowed to minimize the total costs of running all federally insured banks. In general, the insurer in our model would face more system-wide losses of running the banking system than the alternative objective function might impose on the insurer. The alternative objective is, however, likely to result in cross-subsidizations from good to bad banks. Which objective yields socially optimal results is not immediately clear. Second, our bank-specific analysis assumes that the insurer's liability on closure of a bank is specific to the bank. The insurer may be, however, exposed to the impact of closure of a given bank on other regulated banks. We leave it for future research to address the impact of such "spill-over effects" on the ongoing values of all regulated banks, on the insurer's effective liabilities of closing a given bank, and then on the optimal deposit insurance premia and closure policies.

Appendix A

To prove Proposition 1, we first characterize the form of $j(\cdot)$, defined for $x \geq \delta/R$, under the assumption that $c(\cdot)$ is piece-wise differentiable, with $c'(\cdot) \geq -1$

wherever the derivative is well defined. The functional form so derived for $j(\cdot)$ is then shown to imply a cost function $c(\cdot)$ which satisfies the above assumptions. This function is thus the unique stationary normalized solution to the insurer's problem.

LEMMA 1: *Suppose that $c(\cdot)$ is piece-wise differentiable, with $c'(\cdot) \geq -1$ wherever the derivative is well defined. Then*

$$j(x_t) = \max[0, q(x_t, x_t)], \tag{A1}$$

where $q(\cdot, \cdot)$ is defined by (9).

Proof: Under the assumptions of Lemma 1, $q(x_t, p_t)$ is continuous and is piece-wise differentiable with respect to p_t wherever $\partial q/\partial p_t$ is well defined (which is almost everywhere under mild regularity conditions on $c(\cdot)$ and on the distribution of y_{t+1}). Also, by construction of the equivalent martingale measure, $E_t(y_{t+1}) = 1 + r = 1/\beta$. (See Harrison and Kreps (1979).) Then, using $c'(\cdot) \geq -1$,

$$\frac{\partial q}{\partial p_t} = -1 - \beta E_t[c'(\cdot)y_{t+1}] \leq -1 + \beta E_t[y_{t+1}] = 0.$$

Thus, $q(x_t, p_t)$ is monotonically nonincreasing in p_t. Note that the premium p_t can be at most equal to x_t, i.e., $p_t \leq x_t$. Therefore, the insurer would optimally increase the premium either up to $\hat{p}_t < x_t$, such that $j(x_t) = q(x_t, \hat{p}_t) = 0$, or up to x_t, such that $j(x_t) = q(x_t, x_t)$. The result follows. Q.E.D.

COROLLARY: *Under the assumptions stated in Lemma 1 (supressing the suffix t for brevity),*

$$j(x) = \max\left[0, -x + \beta m + R\beta c\left(\frac{\delta}{R}\right)\right], \ \forall x \geq \delta/R, \tag{A2}$$

and the form of $c(\cdot)$ is given by

$$\begin{aligned} c(x) &= \min[w(x), j(x)] \\ &= \min\left[\max[0, 1-x], \max\left[0, -x + \beta m + R\beta c\left(\frac{\delta}{R}\right)\right]\right], \ \forall x \geq \frac{\delta}{R}. \end{aligned} \tag{A3}$$

Proof: Substituting x for p in (9) and (10), and using (A1), the result follows. Q.E.D.

Proof of Proposition 1: From (A3), it is clear that the exact form of $c(\cdot)$ will be known, given the value of $c(\delta/R)$. From (A3), however, one may easily check that

$$\left[1 > R\beta \text{ and } \frac{\beta m - \delta/R}{1 - R\beta} \leq 1 - \frac{\delta}{R}\right] \Rightarrow c(\delta/R) = \frac{\beta m - \delta/R}{1 - \beta R}$$

and

$$[1 = \beta R] \text{ or } \left[1 > R\beta \text{ and } \frac{\beta m - \delta/R}{1 - R\beta} > 1 - \frac{\delta}{R}\right] \Rightarrow c(\delta/R) = 1 - \frac{\delta}{R}.$$

It is a simple matter to check that this function satisfies the assumptions of Lemma 1. This proves Part (a) of the proposition. To prove Part (b), recall that the bank will be closed if and only if $j(x) > w(x)$, which holds (since $w(x) \geq 0$) only if $j(x) > 0$. Then, $-x + \bar{x} = j(x) > w(x) = \max(0, 1 - x)$ if and only if $-x + \bar{x} > 0$ *and* $-x + \bar{x} \geq 1 - x > 0$, i.e., if $x < \bar{x}$ *and* $\bar{x} \geq 1$. This proves Part (b). Q.E.D.

Proof of Proposition 2: To prove the desired result, suppose that a potential entrant plans to enter into the banking industry at time t by creating a bank with an initial assets-to-deposits ratio $x = \bar{x}$ and plans to invest this amount in riskless investments, offering a sure return of r over the time interval $(t, t + 1)$. Then, if the optimal premium charged by the insurer is p, the asset value $\tilde{x}$ at time $t + 1$ is given by

$$\tilde{x} = (x - p)\,\frac{1 + r}{R} + \frac{\delta}{R}, \text{ with probability one.} \tag{A4}$$

The proof of Proposition 1 shows that the insurer will charge an actuarially fair premium equal to $\bar{x}$, where it may be recalled that $\bar{x}$ is defined to be such that $q(\bar{x}, \bar{x}) = 0$. Since the insurer thus covers the value of its potential liability, the bank will be kept open at time t. It may, however, be seen that the value of the bank's assets at time $t + 1$ will be equal to δ/R with probability one. The bank will then be either closed or "just maintained afloat," per the discussion of Proposition 1. In either case, one may see that the equity value of the bank at time $t + 1$ is equal to zero with probability one. If $\bar{x} < 1$, the equity value of the bank at time t will equal zero. The net profit to creating such a bank is, therefore, $\pi(\bar{x})$ = (equity value at time t) − (initial equity investment) $= 0 - (\bar{x} - 1) = 1 - \bar{x}$. Nonpositive profits to free entry under competition implies that $\pi(\bar{x}) \leq 0$, i.e., $\bar{x} \geq 1$, which is a contradiction unless $\bar{x} \geq 1$. Q.E.D.

Proof of Proposition 3: (a) We show that $p(x_t)$ is a monotonically nonincreasing function of x_t. Further, so is $z(x_t)$, inclusive of the indirect impact of x_t on $p(x_t)$: Let $x_t^1 > x_t^2 > \bar{x}$. From (9), for $i = 1, 2$, we have $p(x_t^i) = \beta m + \beta R E_t[c(x_{t+1}) \mid x_t^i, p(x_t^i)]$ or, equivalently, $q(x_t^i, p(x_t^i)) \equiv 0$. By (10), the distribution of x_{t+1} given $(x_t^1, p(x_t^2))$ first-order stochastically dominates (FSD) the distribution of x_{t+1} given $(x_t^2, p(x_t^2))$ since $y_{t+1} \geq 0$ with probability one. Thus, $q(x_t^1, p(x_t^2)) \leq q(x_t^2, p(x_t^2)) = 0$ since $c(x_{t+1})$ is a nonincreasing function of x_{t+1}. This implies $p(x_t^1) \leq p(x_t^2)$ since $q(x, p)$ is a decreasing function of p (by Lemma 1). It also trivially follows that $z(x_t^1) \leq z(x_t^2)$ since $x_t^1 > x_t^2$ and $p(x_t^1) \leq p(x_t^2)$, implying that the distribution of x_{t+1}, given $(x_t^1, p(x_t^1))$, FSD the distribution of x_{t+1}, given $(x_t^2, p(x_t^2))$.

(b) Clearly, as the risk of x_{t+1} increases, $E_t[c(x_{t+1}) \mid x_t]$ increases by the Second Order Stochastic Dominance (SSD) as $c(\cdot)$ is convex and nondecreasing. Since $x > \bar{x}$ implies $q(p(x), x) = 0$, $p(\cdot)$ is thus easily seen to be increasing in the risk of x_{t+1}. Q.E.D.

LEMMA 2: *$\partial p/\partial\sigma > 0$ if $\bar{x} \geq 1$, and $\ln[(\bar{x}R - \delta)/(x - p)] > r$.*

Proof of Lemma 2: First we show that

$$k\phi[\bar{d}] = e^r\phi[\bar{d} - \sigma], \tag{A5}$$

where $k = \dfrac{R - \delta}{x = p}$ and $\bar{d} \equiv \dfrac{\ln(k) - (r - \sigma^2/2)}{\sigma}$. To see that (A5) is true, note that

$$\frac{k\phi[\bar{d}]}{e^r\phi[\bar{d}-\sigma]} = k\exp\left[-\frac{\bar{d}^2}{2}\right]\exp(-r)\exp\left[\frac{1}{2}(\bar{d}-\sigma)^2\right]$$

$$= \exp\left[-\frac{1}{2}\left\{-2\ln(k) + \frac{[\ln(k) - r + \sigma^2/2]^2}{\sigma^2} + 2r - \frac{[\ln(k) - r - \sigma^2/2]^2}{\sigma^2}\right\}\right]$$

$$= \exp\left[-\frac{1}{2}\left\{-2\ln(k) + 2[\ln(k) - r] + 2r\right\}\right] = 1.$$

Then note that $q(x, \sigma, p) = -p + \beta m + \beta R\left\{\left(1 - \dfrac{\delta}{R}\right)\Phi[\bar{d}] - \dfrac{(x-p)}{R}\, e^r\Phi[\bar{d} - \sigma]\right\}$.

Since $q(x, \sigma, p(x, \sigma)) \equiv 0$ for $x \geq \bar{x}$ and since $\partial q(x, \sigma, p)/\partial p < 0$, it follows from the Implicit Function Theorem that $\mathrm{sgn}[\partial p(x, \sigma)/\partial\sigma] = \mathrm{sgn}[\partial q(x, \sigma, p)/\partial\sigma]$ evaluated at $p = p(x, \sigma)$. However,

$$\frac{1}{\beta R}\frac{\partial q}{\partial\sigma} = \left(1 - \frac{\delta}{R}\right)\phi(\bar{d})\left[-\frac{\ln(k) - r}{\sigma^2} + \frac{1}{2}\right] - \frac{(x-p)}{R}e^r\phi(\bar{d} - \sigma)\left[-\frac{\ln(k) - r}{\sigma^2} - \frac{1}{2}\right]$$

$$= \phi(\bar{d})\left\{\left(1 - \frac{\delta}{R}\right)\left[-\frac{\ln(k) - r}{\sigma^2} + \frac{1}{2}\right] - \frac{1}{R}(\bar{x}R - \delta)\left[-\frac{\ln(k) - r}{\sigma^2} - \frac{1}{2}\right]\right\}$$

$$= \phi(\bar{d})\left\{\left[\frac{\ln(k) - r}{\sigma^2}\right](\bar{x} - 1) + \frac{1}{2}[1 \pm \bar{x}] - \frac{\delta}{R}\right\} > 0$$

since $\ln(k) > r$ and $\bar{x} \geq 1$. Q.E.D.

Appendix B: (Some Results on the Lognormal Case)

Define $w \equiv \ln(\tilde{y}) \sim N\left[\left(r - \dfrac{\sigma^2}{2}\right), \sigma^2\right]$, $L \equiv \dfrac{1}{\sqrt{2\pi}\,\sigma}$, $g(\cdot) \equiv$ density of $\tilde{y} \equiv y_{t+1}$, $f(\cdot) \equiv$ density of $\tilde{x} \equiv x_{t+1} = (x - p)\dfrac{y}{R} + \dfrac{\delta}{R}$, $x \equiv x_t$, $p \equiv p_t$, $k \equiv \dfrac{R - \delta}{x - p}$. Then

$$\int_0^k \tilde{y}g(\tilde{y})\, d\tilde{y}$$

$$= \int_{-\infty}^{\ln(k)} \exp(w)L\exp\left\{-\frac{1}{2\sigma^2}\left[w - \left(r - \frac{\sigma^2}{2}\right)\right]^2\right\} dw$$

$$= \int_{-\infty}^{\ln(k)} L\exp\left\{-\frac{1}{2\sigma^2}\left[w^2 + \left(r - \frac{\sigma^2}{2}\right)^2 - 2w\left(r - \frac{\sigma^2}{2}\right) - 2w\sigma^2\right]\right\} dw$$

$$= \int_{-\infty}^{\ln(k)} L \exp\left\{-\frac{1}{2\sigma^2}\left[w^2 + \left(r + \frac{\sigma^2}{2}\right)^2 - 2w\left(r + \frac{\sigma^2}{2}\right) - 2r\sigma^2\right]\right\} dw$$

$$= \int_{-\infty}^{\ln(k)} \exp(r) L \exp\left\{-\frac{1}{2\sigma^2}\left[w - \left(r + \frac{\sigma^2}{2}\right)\right]^2\right\} dw$$

$$= \exp(r)\Phi\left[\frac{\ln(k) - \left(r + \frac{\sigma^2}{2}\right)}{\sigma}\right]. \tag{B1}$$

$$\int_0^k g(\tilde{y})\, d\tilde{y} = \Phi\left[\frac{\ln(k) - \left(r - \frac{\sigma^2}{2}\right)}{\sigma}\right]. \tag{B2}$$

$$\int_{\delta/R}^1 [1 - \tilde{x}] f(\tilde{x})\, d\tilde{x}$$

$$= \int_{\delta/R}^1 f(\tilde{x})\, d\tilde{x} - \int_{\delta/R}^1 \tilde{x} f(\tilde{x})\, d\tilde{x}$$

$$= \int_0^k g(\tilde{y})\, d\tilde{y} - \int_0^k \left[(x - p)\frac{\tilde{y}}{R} + \frac{\delta}{R}\right] g(\tilde{y})\tilde{y}$$

$$= \left(1 - \frac{\delta}{R}\right)\int_0^k g(\tilde{y})\, d\tilde{y} - \frac{(x - p)}{R}\int_0^k \tilde{y} g(\tilde{y})\, d\tilde{y}. \tag{B3}$$

By substituting (B1) and (B2) in (B3), (16) follows.

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