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# Changes in Expected Security Returns, Risk, and the Level of Interest Rates

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**ABSTRACT**

Regressions of security returns on treasury bill rates provide insight about the behavior of risk in rational asset pricing models. The information in one-month bill rates implies time variation in the conditional covariances of portfolios of stocks and fixed-income securities with benchmark pricing variables, over extended samples and within five-year subperiods. There is evidence of changes in conditional "betas" associated with interest rates. Consumption and stock market data are examined as proxies for marginal utility, in a general framework for asset pricing with time-varying conditional covariances.

RECENT EVIDENCE OF PREDICTABILITY in security returns has stimulated the study of asset pricing models with time-varying expected returns. The use of conditional expectations is required to exploit the rich empirical predictions of these models. Unfortunately, studies are only beginning to document and characterize the behavior of conditional distributions of returns.

This paper shows how regressions of security returns on treasury bill rates provide insight about the behavior of risk in asset pricing models. The regressions restrict the conditional covariances of returns with "benchmark" pricing variables or "marginal utility," in a rational expectations pricing model. Such restrictions are useful in formulating models with time-varying expected returns. Since the results do not rely on a specific benchmark, they are relevant for many models.

An example follows from the classic paper of Fama and Schwert (1977), who found negative slope coefficients in regressions of common stock returns on treasury bill rates. The regressions imply that a model should allow the conditional covariances of monthly returns with marginal utility to change as the level of the interest rate changes. There is also evidence that a single "beta" model should allow for interest rate-related changes in the relative risks of common stocks and fixed-income securities.

This paper is organized as follows. Section I explains how asset return-interest rate regressions restrict the behavior of security risk measures. Section II reports

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regressions illustrating the idea. The third section presents extensions. The extensions consider "real" and nominal risk measures and models with conditional betas. Empirical proxies for marginal utility are examined, using a general framework for asset pricing with time-varying expected returns and covariances. The benchmark variables include stock index returns and a measure of aggregate consumption. The models with stochastic variation in the conditional covariances attain an improved cross-sectional fit for average excess returns, but the models are rejected. Thus, it appears that variation in risk, modelled by conditional covariances with these variables, is not sufficient to "explain" the asset return-interest rate relation. Section IV discusses beta pricing models. Section V is the conclusion.

## I. The Model

This paper makes inferences about security risk, maintaining "weak" assumptions about market equilibrium. The assumptions are (i) equilibrium is consistent with no arbitrage in a perfect market and (ii) rational expectations in the sense that observed returns differ from equilibrium expected returns by a pure forecast error.

Define a nominal "benchmark" variable for asset pricing as a random variable $m_{t+1}$ for which equation (1) holds:

$$E\{m_{t+1}(1 + R_{t+1}) \mid \Omega_t\} = \underline{1}, \tag{1}$$

where $R_{t+1}$ is the rate of return on a vector of $n$ assets observed at time $t + 1$. $\Omega_t$ represents public information at time $t$, and $\underline{1}$ is a vector of ones. $m_{t+1}$ is a nominal benchmark because $R_{t+1}$ is a nominal return. ("Real" benchmarks are discussed later.) Equation (1) is equivalent to saying that the time-$t$ price of each asset is public information and is equal to the conditional expected value of the product of its time-$t + 1$ payoff (market value plus any cash flow) with $m_{t+1}$. There exists a positive benchmark $m_{t+1}$ if and only if the no-arbitrage assumption (i) holds. Lack of arbitrage does not uniquely identify $m_{t+1}$ unless markets are complete. (In that case, $m_{t+1}$ is equal to primitive state prices divided by state probabilities.) Asset pricing models specialize (1) by specifying benchmark variables.[1]

If prices are consistent with utility maximization, a function of marginal utility is a benchmark. For example, given a time-additive utility function with time discount factor $\rho$, equation (1) follows from the first-order condition for utility maximization, where

$$m_{t+1} = \rho \frac{U'(t + 1)}{U'(t)} I(t, t + 1). \tag{2}$$

$U'(t)$ is the marginal utility of real payoffs at time $t$ and $I(t, t + 1)$ is a ratio of price level deflators. The consumption-based asset pricing model is one example, where measures of consumption proxy for marginal utility (e.g., Breeden (1979)).

[1] See Harrison and Kreps (1979) for one interpretation. See also Beja (1971), Rubinstein (1976), Ross (1977), and Hansen and Richard (1987).

Empirical tests of asset pricing models specify benchmark variables and make statistical assumptions about the covariances of returns with the benchmark. For example, some tests of consumption-based models assume that the conditional variances and covariances of returns with consumption are constant parameters (e.g., Ferson (1983) and Hansen and Singleton (1983)). Tests of the Capital Asset Pricing Model (CAPM) often assume that portfolio betas relative to a market proxy are fixed parameters. Empirical tests of arbitrage pricing (APT) models typically make a similar assumption about factor "loadings." Obviously, inappropriate statistical assumptions can cause a model to be rejected even if a valid benchmark has been identified. The objective of this paper is to provide some insight about the time-series behavior of measures of risk that holds for any benchmark variable.

Expected excess returns or "risk premiums" are typically the focus of empirical work. Write equation (1) for an asset return $R_{i,t+1}$ and for a treasury bill with rate of return $RF_{t+1}$. Subtracting the two implies

$$E\{m_{t+1}(R_{i,t+1} - RF_{t+1}) \mid \Omega_t\} = 0. \tag{3}$$

The one-period treasury bill return $RF_{t+1}$ is known at time $t$. Use the definition of covariance to expand (3), and substitute the price of the treasury bill, $(1 + RF_{t+1})^{-1}$, for $E\{m_{t+1} \mid \Omega_t\}$. Equation (1) implies

$$E\{R_{i,t+1} - RF_{t+1} \mid \Omega_t\} = \mathrm{cov}(-m_{t+1}; R_{i,t+1} \mid \Omega_t)(1 + RF_{t+1}). \tag{4}$$

Given a benchmark variable, the conditional covariance of return with $m_{t+1}$ is a general measure of "systematic risk." If the conditional covariance is zero for a particular asset, the expected excess return should be zero. Securities' expected returns should be ranked at each date according to their conditional covariances with a benchmark. Any benchmark variable should "explain" expected returns, so the behavior of expected returns can be used to restrict the class of benchmarks. Equation (4) is the basis of a reinterpretation of asset return-interest rate regressions.

Take the expected value of equation (1) given $Z_t$, a subset of $\Omega_t$ that includes the treasury bill rate $RF_{t+1}$. Equation (4) holds when $Z_t$ replaces $\Omega_t$. Rational expectations imply the following regression:

$$R_{i,t+1} - RF_{t+1} = C_{it}\,(1 + RF_{t+1}) + u_{i,t+1}, \tag{5}$$

where $u_{i,t+1}$ is a forecast error with conditional mean zero given $Z_t$ and $C_{it} = \mathrm{cov}(-m_{t+1}; R_{i,t+1} \mid Z_t)$.

Equation (5) shows how "statistical" assumptions about conditional measures of risk can be investigated implicitly. Assume there is a benchmark variable $m_{t+1}$ for which the conditional covariances are constant parameters over time ($C_{it} = C_i$). Then, a time-series regression of excess returns on the riskless interest rate should have slopes and intercepts equal to $C_i$.

A constant $C_i$ model has some theoretical justification. Kraus and Litzenberger (1975) derive such a model for the market portfolio, assuming that the ratio of the payoff on the market to the expected payoff is identically distributed over time and that utility is logarithmic. Rubinstein (1973) argues, using comparative statics in a single-period model, that $C_i$ should be "stationary" in some cases where expected excess returns are time varying.[2] Empirically, the evidence of Keim and Stambaugh (1986) suggests that expected returns are inversely related to security price levels. A constant $C_i$ model implies that expected risk premiums are inversely proportional to the price of a treasury bill. However, evidence also suggests that the riskiness of asset returns is related to interest rate levels. Fama and Schwert (1977), for example, regressed asset returns on the treasury bill rate. They found large, negative slope coefficients and positive intercepts for common stocks, using 1953–1971 data. This suggests that the conditional covariances of stock returns with a benchmark variable are changing over time.[3]

Accepting the regression model (5) does not provide evidence that a first-order condition is satisfied, because equation (1) will hold for some random variable even when there are arbitrage opportunities. However, rejecting equal slopes and intercepts can provide information. This interpretation of the regressions assumes that they are consistent with a rational expectations pricing model. If they are, then rejecting equal slopes and intercepts implies there is no benchmark variable for which the conditional covariances are constant parameters.[4]

Equation (4) implies that, if conditional covariances given the interest rate are not constant, then conditional covariances given the "market's" information set cannot be constant.[5] Choosing the riskless rate as the information set probably results in tests with low statistical power. However, finding implicit movements in conditional covariances given the interest rate is a powerful result because it implies variation in the conditional covariances with any benchmark variable, using any information set that includes the interest rate.[6]

## II. Initial Regression Results

Monthly returns of six "long-term" security portfolios (government and corporate bonds and four stock indices, 1926–1985), treasury bills with two to six months to maturity (1959–1985), and seven- to twelve-month bills (1964–1985) are examined. The data are described in Fama (1984) and Ibbotson and Sinquefield (1982) and are provided by the Center for Research in Security Prices (CRSP)

[2] These authors had in mind an ex ante "real" riskless rate not observable in U.S. data. Conditional covariances with real benchmarks are discussed in the next section.

[3] See also Fama (1976), Breen, Glosten, and Jagannathan (1989), and Shanken (1987).

[4] An alternative interpretation is that market imperfections (e.g., taxes) result in an equilibrium that does not satisfy equality of the slopes and intercepts, even with constant covariances. Ferson (1985) examines a simple, after-tax form of the model, finding that the implicit marginal tax rates for the long-term assets in Table I all exceeded 100% in absolute magnitude. This does not rule out that more complex tax models (e.g., Constantinides (1984)) can explain the data.

[5] A standard decomposition of covariance implies

$$E(C_{it} \mid RF_{t+1}) = \text{cov}(-m_{t+1}; R_{i,t+1} \mid RF_{t+1}) + \text{cov}\{E(R_{i,t+1} \mid Z_t); E(m_{t+1} \mid Z_t) \mid RF_{t+1}\}.$$

Because $E(m_{t+1} \mid Z_t) = 1/(1 + RF_{t+1})$, the second term above is zero.

[6] Hansen and Hodrick (1983) reject constant $C_i$ for currency futures contracts. They use the information in lagged forward rate "forecast errors" and the U.S. treasury bill rate. Their tests imply that conditional covariances of futures given the market information set are time varying. If they had used only the interest rate as an instrument, they would have a less powerful test, but a rejection would rule out more models.

at the University of Chicago. The one-month treasury bill return is subtracted from the other assets' total monthly returns to form excess returns.

Table I summarizes monthly regressions of excess returns on one-month treasury bill rates. The coefficients of determination are typically small, indicating that the variation of expected excess returns captured by the treasury bill rate is small relative to the total variation of excess returns. The predicted portion of monthly return variation is smaller for the longer term assets. The regressions suggest, as observed by Fama and Schwert (1977), that the expected excess returns of the long-term bonds and stocks are negatively related to the one-month interest rate. The excess returns of the treasury bills are positively related to the interest rate, but the slope coefficient estimates become less precise as the maturity of the bill increases.

A model in which the conditional covariances with a benchmark variable are fixed parameters implies equality of the regression slopes and intercepts. Imposing this equality, the estimates of the $C_i$ coefficients are positive and typically several standard errors from zero. Thus, such a model predicts positive expected risk premiums for all of the assets. However, there is evidence that the slopes and intercepts are not equal. Standard tests reject equality for the shortest maturities of treasury bills and the stock indices. Joint tests across the treasury bill equations, using the generalized method of moments (GMM) approach of Hansen (1982), produce $\chi^2(5) = 27.35$ (right-tail $p$-value $= 0.000$) in the second panel and $\chi^2(6) = 15.55$ ($p$-value $= 0.016$) in the third panel. The cross-sectional patterns confirm that a constant $C_i$ model is misspecified. There is a negative relation between the assets' slope coefficients and the average excess returns. The constant $C_i$ model predicts that securities with larger coefficients should earn higher expected returns.

There is residual autocorrelation in many of the equations. If expected risk premiums vary with information that is not captured in treasury bill rates, then autocorrelated residuals are anticipated. (See, e.g., Huizinga and Mishkin (1986).) Autocorrelation implies that ordinary least squares (OLS) standard errors are biased. The standard errors of the slopes and intercepts and the test statistics are adjusted for autocorrelation and heteroskedasticity.[7]

Of course, there are further caveats to be considered in Table I. The regressions are subject to potential biases.[8] The one-month treasury bill rates do not correspond to pure ex ante returns because bills maturing on the last trading day of each month are not consistently available. For example, the Ibbotson and

[7] Table I reports standard errors using the consistent estimator proposed by Newey and West (1987), with three moving average terms. (There were few large autocorrelations at longer lags.) Standard errors were also calculated using Hansen's (1982) variance estimator with zero, one, and three moving average terms. (The first are equivalent to White's (1980) standard errors.) Any of these leads to identical inferences in Table I.

[8] For example, Stambaugh (1987) studies a bias in time-series regressions with lagged stochastic regressors, when future values of the independent variable are correlated with the error term. In a simple example, this bias depends on the autocorrelation of the right-hand-side variable and on its correlation with the regression error. Stambaugh provides an approximate correction for finite sample bias. Estimates of the bias using the sample moments in Table I are on the order of $-0.145$ to $-0.133$ standard errors for the bills' slope coefficients. Estimates for the long-term assets are all between zero and $-0.048$ standard errors.

**Table I**

**Regressions of Excess Returns of Various Assets on Nominal One-Month Treasury Bill Rates: Monthly Data; 1926–1985**

The data in the first panel are from Ibbotson and Sinquefield (1982) as updated by the Center for Research in Security Prices at the University of Chicago, covering the 1926–1985 period. Rates of return are nominal arithmetic returns (income plus capital gains). The number of observations is 720. The returns in the second and third panels are described in Fama (1984), updated through 1985, and converted to discrete compounding. $R^2$ is the coefficient of determination, and $p$-value is the right-tail probability value for the significance of the regression based on the $F$-statistic. Standard errors (SE) of the OLS regression coefficients are autocorrelation, heteroskedasticity-consistent estimates suggested by Newey and West (1987), using three moving average terms.

| | | Unrestricted Regression Equation | | | | | | | | Restricted Regression[a] | Tests of Hypothesis That Slope and Intercept Are Equal in a Particular Equation: | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Partial Residual Autocorrelations | | | | | | |
| Asset $\tilde{R}_{i,t+1} - RF_{t+1}$ | Average Excess Return (% per month) | Intercept [Decimal Units] (SE) | Slope (SE) | $R^2$ | $p$-Value | $\rho_1$ | $\rho_2$ | $\rho_3$ | $\rho_4$ | Intercept = Slope (SE) | Corrected $F$-Statistic[b] | $p$-Value |
| Government Bond Portfolio | 0.075 | 0.0016 (0.0010) | −0.33 (0.49) | 0.002 | 0.130 | 0.03 | 0.01 | −0.13 | 0.01 | 0.10 (0.06) | 0.44 | 0.506 |
| Corporate Bond Portfolio | 0.130 | 0.0026 (0.0010) | −0.48 (0.51) | 0.004 | 0.040 | 0.15 | −0.02 | −0.07 | 0.01 | 0.16 (0.05) | 0.90 | 0.342 |
| Standard & Poor's Stock Index | 0.677 | 0.0114 (0.0036) | −1.70 (0.75) | 0.006 | 0.019 | 0.10 | −0.03 | −0.13 | 0.06 | 0.45 (0.17) | 5.16 | 0.024 |
| CRSP Value Weighted Stock Index | 0.641 | 0.0109 (0.0035) | −1.63 (0.75) | 0.006 | 0.020 | 0.11 | −0.02 | −0.13 | 0.06 | 0.44 (0.18) | 4.77 | 0.028 |
| CRSP Equally Weighted Stock Index | 1.035 | 0.0165 (0.0054) | −2.24 (1.09) | 0.006 | 0.020 | 0.18 | −0.03 | −0.12 | −0.02 | 0.70 (0.23) | 4.25 | 0.040 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Small Capitalization Stock Index | 1.118 | 0.0171<br>(0.0063) | −2.17<br>(1.28) | 0.004 | 0.045 | 0.15 | 0.01 | −0.11 | −0.04 | 0.76<br>(0.23) | 2.89 | 0.090 |
| March 1959 to December 1985 (322 Observations) | | | | | | | | | | | | |
| 2-month bill | 0.033 | −0.0001<br>(0.0001) | 0.1972<br>(0.0251) | 0.115 | 0.000 | 0.33 | −0.02 | 0.08 | −0.06 | 0.0339<br>(0.0028) | 14.94 | 0.000 |
| 3-month bill | 0.051 | −0.0001<br>(0.0002) | 0.1239<br>(0.0482) | 0.067 | 0.000 | 0.22 | −0.00 | 0.02 | 0.01 | 0.496<br>(0.0045) | 6.58 | 0.010 |
| 4-month bill | 0.049 | −0.0001<br>(0.0003) | 0.1296<br>(0.0740) | 0.034 | 0.000 | 0.17 | 0.04 | −0.04 | −0.01 | 0.0463<br>(0.0063) | 3.05 | 0.080 |
| 5-month bill | 0.076 | 0.0001<br>(0.0004) | 0.1400<br>(0.1019) | 0.023 | 0.003 | 0.22 | 0.02 | −0.04 | 0.01 | 0.0728<br>(0.0083) | 1.87 | 0.172 |
| 6-month bill | 0.081 | 0.0002<br>(0.0005) | 0.1309<br>(0.1183) | 0.013 | 0.019 | 0.18 | 0.00 | −0.03 | −0.04 | 0.0790<br>(0.0104) | 1.21 | 0.270 |
| August 1964 to December 1985 (257 Observations) | | | | | | | | | | | | |
| 7-month bill | 0.072 | 0.0005<br>(0.0007) | 0.1193<br>(0.1546) | 0.006 | 0.100 | 0.19 | −0.03 | −0.08 | −0.05 | 0.0614<br>(0.0154) | 0.59 | 0.442 |
| 8-month bill | 0.092 | −0.0001<br>(0.0008) | 0.1862<br>(0.1774) | 0.011 | 0.043 | 0.19 | −0.02 | −0.11 | −0.03 | 0.0754<br>(0.0178) | 1.09 | 0.296 |
| 9-month bill | 0.097 | −0.0003<br>(0.0011) | 0.2266<br>(0.2257) | 0.012 | 0.038 | 0.21 | −0.05 | −0.12 | −0.04 | 0.0761<br>(0.0210) | 1.00 | 0.316 |
| 10-month bill | 0.069 | −0.0008<br>(0.0011) | 0.2738<br>(0.2735) | 0.014 | 0.029 | 0.20 | −0.07 | −0.09 | −0.01 | 0.0459<br>(0.0239) | 1.32 | 0.250 |
| 11-month bill | 0.074 | −0.0004<br>(0.0012) | 0.2063<br>(0.2638) | 0.007 | 0.095 | 0.19 | −0.06 | −0.11 | −0.04 | 0.0550<br>(0.0253) | 0.61 | 0.436 |
| 12-month bill | 0.070 | −0.0001<br>(0.0014) | 0.1432<br>(0.2915) | 0.003 | 0.206 | 0.18 | −0.11 | −0.09 | −0.09 | 0.0597<br>(0.0283) | 0.24 | 0.624 |

[a] The regressions imposing the restriction that the slope and intercepts are equal are estimated by generalized method of moments [Hansen (1982)]. Unity plus the one-month bill rate is the explanatory variable, and the intercept is suppressed. The instruments are a constant and the one-month bill rate. These regressions allow conditional heteroskedasticity of the residual but are not adjusted for autocorrelation. A chi-square statistic tests the restriction that the model errors are orthogonal to the instruments. A joint test across the equations produces $\chi^2(6) = 6.94$ ($p$-value $= 0.673$) in the first panel, $\chi^2(5) = 27.35$ ($p$-value $= 0.000$) in the second panel, and $\chi^2(6) = 15.55$ ($p$-value $= 0.016$) in the third panel.

[b] The corrected $F$-statistic allows conditional heteroskedasticity of an unknown form and autocorrelation of the residual, using the Newey-West standard errors.

Sinquefield data measure the return of a bill with at least one month to maturity. This creates a measurement error that could bias the regressions. Measurement error bias could be present even if exact, one-month maturities were available, due to error in the recorded price of the bill. To minimize correlated errors, a different riskless rate series is used to compute the excess return on the left-hand side of a regression from that used on the right-hand side, when possible.[9] To obtain information about the sensitivity of the results to inexact maturities, the regressions are re-estimated using alternative data for the one-month bill.[10]

"Structural shifts" in the relation of expected rturns to interest rates could imply that the parameters are not fixed over the sample. Table I spans a period including many "events" that could be associated with such parameter shifts.[11] Empirical studies that examine conditional risk measures might assume constant parameters for shorter time periods than Table I.

Table II summarizes results for five-year subperiods. Evidence against equality of the slopes and intercepts is depicted symbolically to provide a feel for the patterns across the assets and time periods. The tests detect fluctuations in the conditional covariances of common stocks within five-year subperiods. The intervals including World War II and the Korean War produce such evidence. Prior to 1940 and in the first half of the 1960s, there is little evidence of changing covariances for stocks.[12] "Small" and large stocks seem to have differing risk behaviors between the mid-1960s and 1980. In the first half of the 1970s, including the Arab oil embargo period, only the S&P 500 and the CRSP value weighted stock index (skewed toward the shares of larger firms) produce significant test statistics. In the second halves of the decades, the small stock and equally weighted indices seem to have changing covariances. There is evidence against the model for each of the stock indices in the 1980s and for short-term bills and corporate bonds in the early 1960s. The slopes and intercepts of short-term bills also differ after 1975.

Aggregate test statistics are shown in the right-hand column of Table II. These examine the hypothesis that conditional covariances with a benchmark are constant parameters within each of the five-year subperiods. The tests reject the

[9] The regressions reported in the tables use the one-month return from Fama's short-term bill file as the right-hand-side variable where possible (e.g., Panels 2 and 3 of Table I and in the latter part of the sample in Panel 1). The excess return is computed relative to the Ibbotson and Sinquefield one-month rate in Panels 1 and 2 of Table I and relative to the one-month rate from original issue twelve-month bills in the third panel.

[10] In about 42% of 322 observations, the one-month bill in the second panel of Table I matures in less than one month. Thirty percent mature on the last day of the month, and 28% have more than one month to maturity. The third panel uses the excess returns of bills that were originally issued as twelve-month bills. These regressions were re-estimated using the one-month rate from the original issue twelve-month bills on the right-hand side. The actual maturities are more variable and the measurement error problem should be more severe. The explanatory power of regressions was slightly lower, but none of the inferences was affected.

[11] For example, the Great Depression, World War II, the Korean War, the Treasury-Federal Reserve accord of the early 1950s, the oil embargo of the mid-1970s, shifts in Federal Reserve operating procedures in the late 1970s and early 1980s, and several business cycles.

[12] Of course, nominal bill rates were not very volatile in early parts of the sample period, so it may not be surprising to find little evidence of changes in risk associated with interest rate movements in those periods.

hypothesis, for the common stock portfolios, the corporate bonds, and two- to three-month bills. There is no power to reject the model for the long-term government bonds or the longer maturity bills.

## III. Extensions

### A. Real Covariances with Marginal Utility

Since nominal, not "real," riskless interest rates are observed, the evidence characterizes conditional covariances of nominal excess returns with nominal benchmark variables. The behavior of nominal risks can be affected by fluctuations in inflation. Possibly, conditional covariances of real returns with real marginal utility are better approximated by constant parameters. To investigate this further, rewrite equation (1) as

$$E\{m^*_{t+1}(1 + R^*_{i,t+1}) \mid \Omega_t\} = 1, \tag{6}$$

where

$$m^*_{t+1} = m_{t+1}/I(t, t + 1)$$

and

$$(1 + R^*_{i,t+1}) = (1 + R_{i,t+1})I(t, t + 1).$$

The variables with asterisks can be thought of as "real" rates of return and a real marginal utility ratio. For example, if $m_{t+1}$ is given by equation (2), then $m^*_{t+1} = \rho U'(t + 1)/U'(t)$. Define a real benchmark variable $m^*_{t+1}$ as a variable which satisfies (6). The symmetry of equations (6) and (1) implies that, if a real, riskless rate were observable, it could be used to identify the conditional mean of $m^*_{t+1}$ and simple regressions of returns on this rate would provide information about the conditional covariances of deflated returns. A real riskless rate is not observable, but it is possible to investigate hypotheses about covariances with real benchmarks, conditional on observable information.

Consider the expectation of equation (6) given the nominal interest rate. Take the difference of this expression for two returns, and expand using the definition of covariance to obtain

$$E\{R^*_{i,t+1} - RF^*_{t+1} \mid RF_{t+1}\} = \text{cov}(-m^*_{t+1}; R^*_{i,t+1} - RF^*_{t+1} \mid RF_{t+1})/E(m^*_{t+1} \mid RF_{t+1}). \tag{7}$$

If "inflation" is fully revealed by the level of the nominal rate (so that $I(t, t + 1)$ can be taken outside the conditional expectation operators), then the conditional covariances of the deflated variables are the same as the nominal variables. However, fluctuations in nominal bill rates are not perfectly correlated with inflation rates.[13]

Define $\text{cov}(-m^*_{t+1}; RF^*_{t+1} \mid RF_{t+1}) \equiv \lambda$. Expand the conditional mean of equation (6) for the treasury bill into the product of conditional expectations given the nominal rate, minus $\lambda$. Substitute from this expression for the conditional mean

[13] See, for example, Fama and Gibbons (1984).

**Table II**

## Summary of Tests for Equal Slopes and Intercepts in Regressions of Monthly Excess Returns of Various Assets on Nominal One-Month Treasury Bill Rates

The data are from Ibbotson and Sinquefield (1982) as updated by the Center for Research in Security Prices at the University of Chicago, covering the 1926–1985 period. $F$-statistics testing the hypothesis of constant covariances within five-year subperiods are shown. Two $F$-ratios are indicated for each case. The first is based on standard OLS assumptions; the second uses heteroskedasticity-consistent standard errors for the regression coefficients. Degrees of freedom are (1, 58). Right-tail probability values are indicated symbolically in place of the $F$-ratio where significant at conventional levels (see below).

| Period[a] / Asset | 1926:1–1930:12 | 1931:1–1935:12 | 1936:1–1940:12 | 1941:1–1945:12 | 1946:1–1950:12 | 1951:1–1955:12 | 1956:1–1960:12 | 1961:1–1965:12 | 1966:1–1970:12 | 1971:1–1975:12 | 1976:1–1980:12 | 1981:1–1985:12 | Overall $p$-Value[b] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Government Bond | 0.10 | 0.13 | 0.02 | 1.12 | 0.00 | 0.00 | 2.28 | 2.60 | 1.32 | 0.16 | 0.05 | 0.00 | 0.783 |
| Portfolio | 0.16 | 0.15 | 0.08 | 0.51 | 0.00 | 0.01 | 1.61 | 1.93 | 1.03 | 0.17 | 0.02 | 0.00 | 0.921 |
| Corporate Bond | *** | ** | 0.66 | 0.12 | 0.04 | 0.05 | 2.07 | *** | 0.99 | 0.61 | 0.02 | 0.05 | 0.004 |
| Portfolio | *** | 2.67 | 1.19 | 0.14 | 0.04 | 0.06 | 2.24 | *** | 1.09 | 0.75 | 0.00 | 0.04 | 0.014 |
| Standard & Poors | 0.09 | 0.27 | 0.45 | **** | * | *** | ** | 0.37 | 1.16 | *** | 1.53 | * | <0.0001 |
| Stock Index | 0.12 | 0.39 | 0.49 | **** | * | ** | *** | 0.35 | 1.35 | *** | 0.02 | * | <0.0001 |
| CRSP Value | 0.10 | 0.25 | 0.27 | **** | * | *** | ** | 0.37 | 1.44 | ** | 1.86 | * | <0.0001 |
| Weighted Stock Index | 0.10 | 0.36 | 0.30 | **** | * | *** | ** | 0.32 | 1.70 | *** | 0.05 | * | <0.0001 |
| CRSP Equally | 0.00 | 0.02 | 0.11 | **** | ** | *** | *** | 0.05 | * | 1.87 | * | ** | <0.0001 |
| Weighted Stock Index | 0.00 | 0.01 | 0.15 | **** | * | *** | *** | 0.02 | ** | 2.46 | 0.85 | ** | <0.0001 |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Small Capitalization | 0.01 | 0.00 | 0.08 | **** | * | *** | *** | 0.01 | ** | 1.00 | ** | ** | <0.0001 |
| Stock Index | 0.03 | 0.00 | 0.10 | **** | * | *** | *** | 0.04 | ** | 1.13 | 1.49 | ** | <0.0001 |
| 2-month bill | — | — | — | — | — | — | *** | *** | 0.02 | 1.11 | *** | ** | <0.0001 |
| | | | | | | | *** | *** | 0.66 | 0.57 | * | * | 0.0005 |
| 3-month bill | — | — | — | — | — | — | *** | ** | * | 2.05 | * | 2.71 | 0.001 |
| | | | | | | | ** | * | 1.99 | 1.03 | 0.74 | 1.90 | 0.024 |
| 4-month bill | — | — | — | — | — | — | 0.38 | 0.74 | 0.63 | 1.45 | 0.30 | 1.33 | 0.566 |
| | | | | | | | 0.11 | 0.67 | 0.50 | 1.00 | 0.37 | 1.04 | 0.733 |
| 6-month bill | — | — | — | — | — | — | — | *** | 0.26 | 0.26 | 0.81 | 0.34 | 0.039 |
| | | | | | | | | *** | 0.11 | 1.16 | 0.22 | 0.24 | 0.066 |
| 7-month bill | — | — | — | — | — | — | — | — | 1.20 | 0.31 | 0.05 | 0.01 | 0.814 |
| | | | | | | | | | 0.89 | 0.24 | 0.02 | 0.01 | 0.885 |
| 10-month bill | — | — | — | — | — | — | — | — | 0.42 | 0.36 | 0.25 | 0.01 | 0.904 |
| | | | | | | | | | 0.31 | 0.30 | 0.08 | 0.00 | 0.953 |
| 12-month bill | — | — | — | — | — | — | — | — | 0.76 | 0.05 | 0.12 | 0.03 | 0.916 |
| | | | | | | | | | 0.51 | 0.04 | 0.04 | 0.02 | 0.962 |

[a] —means not available.
[b] The overall $p$-value approximates the $F$-statistics as chi-square variates and sums these across subperiods, assuming independence.
* $p$-Value less than 0.10, greater than 0.05.
** $p$-Value less than 0.05, greater than 0.01.
*** $p$-Value less than 0.01.
**** $p$-Value less than 0.001.

of $m^*_{t+1}$ into equation (7) to obtain

$$E\{R^*_{i,t+1} - RF^*_{t+1} \mid RF_{t+1}\} = C^*_{it}E\{(1 + RF^*_{t+1}) \mid RF_{t+1}\}, \tag{8}$$

where

$$C^*_{it} \equiv \text{cov}(-m^*_{t+1}; R^*_{i,t+1} - RF^*_{t+1} \mid RF_{t+1})[1/(1 + \lambda)].$$

If the conditional covariances of real returns with a real benchmark are constant parameters over time, then $C^*_{it} = C^*_i$ is a constant parameter, and expected real risk premiums move in proportion to one plus the expected real return of a nominal riskless asset.

An empirical test for constant $C^*_i$ in equation (8) examines a bivariate regression for each asset. The dependent variables are the real return of a one-month bill and the excess real return of an asset. These are regressed on the nominal bill rate and a constant. The model implies that the coefficients in the two regressions are proportional. $C^*_i$ is the proportionality coefficient. The results of this experiment are similar to the results for the constant nominal covariance model.[14] The experiment was repeated for five-year subperiods, and the patterns in the rejections of the model were similar to those reported in Table II. (These results are available by request.)

*B. Modeling Changes in Conditional Covariances*

The implicit relation of conditional covariances to the interest rate can be further examined by hypothesizing a functional form. The $C_{it}$'s are conditional expectations and therefore are functions of the information $RF_{t+1}$. Consider a first-order Taylor series, approximating the $C_{it}$'s as linear functions.[15] A linear approximation implies a quadratic relation between expected excess returns and the interest rate. Such a model allows several interesting experiments. A linear specification of conditional covariances can be tested for individual assets. Given a linear model, the sign of the coefficient on the squared interest rate determines the correlation between a conditional covariance and the interest rate. The cross-sectional implications for the unconditional mean returns differ from a constant-covariance model.

Regressions of the excess returns on the one-month treasury bill rate and its square are examined. The incremental explanatory power of the squared term is significant (based on the increment to the $R$-squares) for the shorter term bills. A test using GMM for excluding the squared bill rate from the system of long-term equations produces a right-tail $p$-value of 0.047. If the conditional covariance of asset $i$ with a benchmark can be expressed as a linear function of the interest rate, then $\alpha_i - \beta_i + \gamma_i = 0$, where $\alpha_i$ is the intercept for asset $i$, $\beta_i$ is the regression coefficient on the bill rate, and $\gamma_i$ is the coefficient on the square of the bill rate.[16]

[14] The systems are estimated by GMM, using the right-hand-side variables as instruments. The minimized value of the GMM criterion function provides a test statistic for the restricted model. Under the null hypothesis, the statistic is asymptotically distributed as a chi-square variable with one degree of freedom.

[15] Linear models for conditional variances and covariances are used in Bodurtha and Mark (1988), Campbell (1987), Cumby (1988), and Shanken (1987).

[16] This restriction follows from substituting a linear function for $C_{it}$ in equation (5) and equating

There is little power to reject this hypothesis. (These results are available by request.)

The coefficients on the squared interest rate are estimated precisely only for the shortest term bills, but the positive point estimates suggest a positive relation between the level of the interest rate and the covariances of bills, bonds, and small-firm stock portfolios. Negative $\gamma_i$ coefficients are observed for the large stocks (the S&P and value weighted stock indices). If conditional covariances move with the level of the interest rate, then the regressions in Table I have a left-out-variables bias. The bias in the slope coefficient is negative (positive) for a security whose $C_{it}$ is negatively (positively) correlated with the interest rate.[17] If $C_{it}$ is correlated with $RF_{t+1}$, equation (4) implies that the unconditional mean excess return is

$$E\{R_i - RF\} = E\{C_{it}\}E(1 + RF_{t+1}) + \mathrm{cov}(C_{it}; RF_{t+1}). \tag{9}$$

Fitted mean excess returns are computed using equation (9). The fitted values assume the conditional covariances are linear. The slope coefficients $\beta_i$ and $\gamma_i$ of the quadratic regressions are used, without reference to the intercepts.[18] The sample means are not used in forming the fitted mean returns. Although the fitted values are not very close to the sample means, there is no longer a perverse negative relation between the two as there was for the constant covariance model.[19] Accounting for variation in conditional covariances with a benchmark, as the level of the interest rate changes, results in an improved fit for the sample mean returns.

### C. A General Model of Time-Varying, Conditional Covariances

This section uses consumption and stock return data as empirical proxies for benchmark variables. A functional form for the conditional covariances of returns with these variables is not specified. Instead, a conditional covariance is approximated using the conditional expected value of the product of deviations from conditional means. Equation (10) summarizes the asset pricing model:

$$\begin{aligned} -m_{t+1} &= -E(m_{t+1} \mid Z_t) + v_{t+1}, \\ r_{i,t+1} &= (r_{i,t+1}\, v_{t+1})(1 + RF_{t+1}) + w_{i,t+1}, \end{aligned} \tag{10}$$

where $r_{i,t+1} \equiv R_{i,t+1} - RF_{t+1}$ is an excess return. The first equation of (10) identifies the unanticipated part of the benchmark variable $m_{t+1}$ as equal to

coefficients. A similar approach can be used to examine other functional forms. (See Ferson (1985).) One example includes a dummy variable for January on the slopes and intercepts of the regressions in Table I. A simple model allows the conditional covariances to take on a constant value in January different from the value in other months. The pattern of rejections observed for this model is similar to that reported in Table I.

[17] Fama (1976) makes related observations. See Stulz (1986) for a theoretical model in which risks can respond in different directions to interest rate shifts. See Chang and Pinegar (1987) for recent evidence of left-out-variables bias related to risk.

[18] Given the quadratic regression coefficients, linearity of $C_{it}$ implies $\alpha_i = \beta_i - \gamma_i$, $\mathrm{cov}(C_{it}; RF) = \gamma_i \mathrm{var}(RF)$, and $E(C_i) = [\beta_i - \gamma_i + \gamma_i E(RF)]$.

[19] A regression of the sample mean excess returns on the fitted means has a slope of 0.2, and the $R$-squared is 0.31.

$-v_{t+1}$. $E(m_{t+1} \mid Z_t)$ stands for a model of the conditional mean (to be specified). The second equation imposes the restriction that expected excess returns are related to conditional covariances with a benchmark, according to equation (4). The conditional expected value of $(r_{i,t+1}\, v_{t+1})$ given $Z_t$ is the conditional covariance, $\mathrm{cov}(r_{i,t+1}; -m_{t+1} \mid Z_t)$.

The system (10) is estimated by GMM. Define the combined error term (the subscript $i$ is omitted and the equations for several assets are stacked together) as

$$\eta_{t+1} = \begin{bmatrix} v_{t+1} \\ w_{t+1} \end{bmatrix}. \tag{11}$$

The asset pricing model implies that $E(\eta_{t+1} \mid Z_t) = 0$. The specification is completed by choosing a benchmark variable $m_{t+1}$ and a model for the conditional mean of the benchmark.

A linear regression model for the conditional mean of a benchmark is

$$E(m_{t+1} \mid Z_t) = B_m{}'Z_t, \tag{12}$$

where $B_m$ is a regression coefficient vector, and the instruments $Z_t$ include a constant term. Using this regression model for a given benchmark, the first equation of (10) is exactly identified. The overidentification that allows the model to be tested comes from the asset return equations. A second approach identifies the conditional mean of a benchmark using equation (1), which implies

$$E(m_{t+1} \mid Z_t) = 1/(1 + RF_{t+1}). \tag{13}$$

Using this model, the first equation of (10) provides overidentifying restrictions for the system.

Two empirical proxies for benchmark variables are examined. The first proxy follows the literature on consumption-based asset pricing. Assume a representative agent with a time-additive power utility function $u(c) \propto c^{1-\alpha}$ and consumption equal to $c_t$ at time $t$. Equation (2) specializes, implying

$$m_{t+1} = \rho[(c_{t+1}/c_t)^{-\alpha}]I(t, t+1). \tag{14}$$

The first proxy for $m_{t+1}$ uses equation (14) and monthly data on real, per capita expenditures for consumer nondurable goods.

A second example uses stock market data to proxy for marginal utility, following the empirical literature on the CAPM. Brown and Gibbons (1985) study a model in which the growth of consumption is proportional to the real return on the market. Using unconditional moments, they find evidence to support the model. In such a model, the following is a benchmark variable:

$$m_{t+1} = \kappa(1 + R_{m,t+1})^{-\alpha}I(t, t+1), \tag{15}$$

where $R_{m,t+1}$ is the real rate of return on a market index and $\kappa$ is a proportionality parameter.[20] Rubinstein (1976) discusses similar models. The CRSP equally

[20] $\kappa$ depends on the fraction of wealth consumed, the risk aversion parameter, and the time-discount parameter. The present example follows Brown and Gibbons by assuming that $\kappa$ is a constant parameter, although this is not generally required in a constant relative risk aversion model.

weighted and value weighted stock indices are used to measure the market return in equation (15).

The preceding sections suggest that conditional covariances of returns with a benchmark variable should be time varying. A modification of system (10) is used to test directly for constant conditional covariances with the specific proxies:

$$-m_{t+1} = -E(m_{t+1} \mid Z_t) + v_{t+1},$$
$$\epsilon_{i,t+1} = (r_{i,t+1}\, v_{t+1}) - C_i. \tag{16}$$

Given constant conditional covariances and a model for the conditional mean of a proxy, then $\epsilon_{i,t+1}$ has conditional mean zero and $C_i$ is the conditional covariance for asset $i$. If the conditional covariance is time varying as a function of the instruments, then $\epsilon_{i,t+1}$ does not have conditional mean zero, violating the GMM orthogonality condition.

Table III presents results for four models. Models 1 and 2 use equation (14) with monthly consumption data for 1964:8 to 1985:12. (The data are from Citibase and DRI.) The conditional mean of $m_{t+1}$ is estimated using equation (12) in model 1 and using equation (13) in model 2. Models 3 and 4 use equation (15) with the value weighted and equally weighted stock indices, deflated by the CPI. In these models, equation (13) identifies the conditional means.[21] The instrument vector $Z_t$ is a constant and the one-month treasury bill rate.

The top portion of Table III summarizes the tests for constant conditional covariances. When nondurables consumption is the benchmark proxy, the tests do not reject the hypothesis that the covariances are constant parameters. This is consistent with earlier studies which conclude that measures of consumption are too "smooth" to explain asset returns. The results suggest that conditional covariances of returns with consumption do not fluctuate enough to capture the movements in expected returns that are related to interest rates.[22] The tests can reject the hypothesis that covariances with the stock indices are constant parameters.

The bottom portion of Table III reports results for the asset pricing models with time-varying conditional covariances. Estimates of the parameter $\alpha$, which measures the concavity of the utility function, are reported at the bottom of the table. When consumption is the benchmark proxy, the estimates are at least two standard errors below zero (indicating risk-loving behavior or "negative" intertemporal substitution). The stock market indices produce more reasonable-looking estimates of $\alpha$.

Two parameters are not reported in the table. When equation (13) is used to model the mean of $m_{t+1}$ in a consumption model, the time discount parameter $\rho$ is identified. The estimates of $\rho$ are slightly below unity, as economic theory suggests, and seem to be precisely estimated. The estimates of $\kappa$ in the stock market models are close to one.

The $\chi^2$ statistics imply strong rejections of the restrictions, for each of the

[21] The regression coefficients $B_m$ and the scale factors $\kappa$ and $\rho$ are not both identified when a regression model is used for the mean of $m_{t+1}$. Models 3 and 4 were estimated using regression models for the mean of $m_{t+1}$, setting $\kappa = 1.0$, with similar results.

[22] See Singleton (1986) for a review of other explanations.

**Table III**

**Asset Pricing Models with Time-Varying Conditional Covariances and Benchmark Variables: Monthly Data, 1964:8–1985:12**

The asset pricing equations are:

$$-m_{t+1} = -E(m_{t+1} \mid Z_t) + v_{t+1},$$

$$r_{i,t+1} = (r_{i,t+1} v_{t+1})(1 + RF_{t+1}) + w_{i,t+1};$$

where $r_{i,t+1}$ is a return in excess of the one-month treasury bill, $m_{t+1}$ is a benchmark pricing variable, and $RF_{t+1}$ is a one-month treasury bill return. The conditional covariance with the benchmark is:

$$C_{it} \equiv \operatorname{cov}(r_{i,t+1}; -m_{t+1} \mid Z_t) = E(r_{i,t+1} v_{t+1} \mid Z_t),$$

where the $Z_t$ are predetermined instrumental variables. The model implies that the error terms $v_{t+1}$, $w_{i,t+1}$ are orthogonal to $Z_t$. The $\chi^2$ statistic examines this restriction. The asset pricing equations are estimated by Generalized Method of Moments, using a constant and the one-month treasury bill return $RF_{t+1}$ as the instruments $Z_t$. SE is the standard error of the estimate. Nondurables is the growth rate of real, per capita personal consumption expenditures for nondurable goods. VW-Stocks is the return of the CRSP value weighted common stock index; EW-Stocks is the return of the equally weighted index. The regression model for $E(m|Z)$ uses a linear regression of $m$ on the instruments as a proxy for the conditional mean of $m$. The mean pricing errors are the differences between the sample mean excess returns of the assets and the sample average of the values implied by the right-hand side of the asset pricing equation. The assets are denoted as follows: GB = Government Bond Portfolio, CB = Corporate Bond Portfolio, SP = Standard and Poor's Stock Index, SS = Small-Capitalization Stock Index, TB$n$ = Treasury bill with $n$ months to maturity at the beginning of the period.

| Model Number<br>Benchmark Variable<br>Model for $E(m\|Z)$ | 1<br>Nondurables<br>Regression | 2<br>Nondurables<br>$1/(1+RF)$ | 3<br>VW-Stocks<br>$1/(1+RF)$ | 4<br>EW-Stocks<br>$1/(1+RF)$ |
|---|---|---|---|---|
| Estimates and Tests for constant $C_{it} = C_i$: | $\hat{C}_i \times (10000)$<br>(SE) | $\hat{C}_i \times (10000)$<br>(SE) | $\hat{C}_i \times (10000)$<br>(SE) | $\hat{C}_i \times (10000)$<br>(SE) |
| Asset: TB2 | −0.000 | 0.005 | −0.001 | −0.010 |
| | (0.002) | (0.004) | (0.001) | (0.011) |
| TB6 | 0.009 | 0.012 | −0.033 | −0.040 |
| | (0.014) | (0.016) | (0.025) | (0.031) |
| TB12 | 0.037 | 0.004 | −0.087 | −0.109 |
| | (0.063) | (0.039) | (0.056) | (0.077) |
| GB | 0.310 | 0.077 | −0.312 | −0.401 |
| | (0.936) | (0.223) | (0.242) | (0.030) |
| CB | 0.010 | 0.343 | −0.714 | −0.805 |
| | (0.265) | (0.238) | (0.408) | (0.501) |
| SP | 0.129 | −0.002 | −6.952 | −8.211 |
| | (0.194) | (0.432) | (2.838) | (2.851) |
| SS | 0.182 | 0.466 | −14.167 | −12.849 |
| | (0.248) | (0.604) | (8.764) | (6.721) |
| Joint Test Across Asset Equations of $C_{it} = C_i$ ($p$-value) | 0.478 | 0.311 | 0.009 | 0.018 |

**Table III**—*Continued*

| Model Number | | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Benchmark Variable | | Nondurables | Nondurables | VW-Stocks | EW-Stocks |
| Model for $E(m\|Z)$ | | Regression | $1/(1 + RF)$ | $1/(1 + RF)$ | $1/(1 + RF)$ |
| Asset Pricing with Time-Varying $C_{it}$: Mean Pricing Errors (standard errors in parentheses) | | | | | |
| Asset: | TB2 | 0.000352 | 0.000348 | 0.000329 | 0.000348 |
| | | (0.000059) | (0.000052) | (0.000058) | (0.000062) |
| | TB6 | 0.000907 | 0.000796 | 0.000585 | 0.000654 |
| | | (0.000225) | (0.000194) | (0.000218) | (0.000232) |
| | TB12 | 0.000891 | 0.000632 | −0.000006 | 0.000107 |
| | | (0.000484) | (0.000392) | (0.000455) | (0.000444) |
| | GB | −0.00013 | −0.000609 | −0.004480 | −0.004618 |
| | | (0.00225) | (0.001590) | (0.002212) | (0.002151) |
| | CB | −0.00042 | −0.000301 | −0.004546 | −0.004577 |
| | | (0.00225) | (0.001845) | (0.002073) | (0.002029) |
| | SP | 0.00371 | 0.002374 | −0.013767 | −0.014144 |
| | | (0.00390) | (0.002654) | (0.004004) | (0.003840) |
| | SS | 0.01123 | 0.009058 | −0.018184 | −0.012643 |
| | | (0.00752) | (0.004324) | (0.007026) | (0.006013) |
| $\alpha$ - estimate[a] | | −26.33 | −4.67 | 7.02 | 8.44 |
| (standard error) | | (7.60) | (2.23) | (0.93) | (1.08) |
| $\chi^2$ Test Statistic | | 70.610 | 80.336 | 76.183 | 73.419 |
| ($p$-value) | | (0.000) | (0.000) | (0.000) | (0.000) |

[a] $\alpha$ is the concavity parameter of the power utility function $u(x) \propto x^{1-\alpha}$.

models. Finding rejections of the consumption-based models is consistent with previous empirical studies. Such rejections should be expected if the conditional covariances with consumption do not vary enough with the interest rate. Previous studies also reject conditional mean-variance efficiency of stock market indices, and equation (4) is more restrictive than mean-variance efficiency.

Although the goodness-of-fit statistics are large, the models produce an improved fit for average returns. Table III reports mean "pricing errors" for each model. These are the time-series means of the residuals $w_{i,t+1}$ in system (10).[23] The pricing errors are large, but the within-sample fit is better than both the constant covariance model and the model assuming linear covariances.

Figures 1 through 4 are time series plots of the fitted conditional covariances. These are generated by projecting the fitted values of $(r_{i,t+1}\ v_{t+1})$ on a constant, the bill rate, the inverse of the bill rate, and the square of the bill rate using linear regression. The projections are an approximation to the ex ante part of the fitted values. Of course, these may be poor approximations to the conditional covariances, given that the models are rejected. However, the plots seem to confirm that the fitted covariances with consumption fluctuate much less than the covariances with the stock market benchmarks. The plots also suggest patterns similar to those inferred in the previous sections. They suggest that the

[23] Approximate standard errors for the pricing errors are obtained from the inverse of the GMM weighing matrix, as described in Hansen (1982).

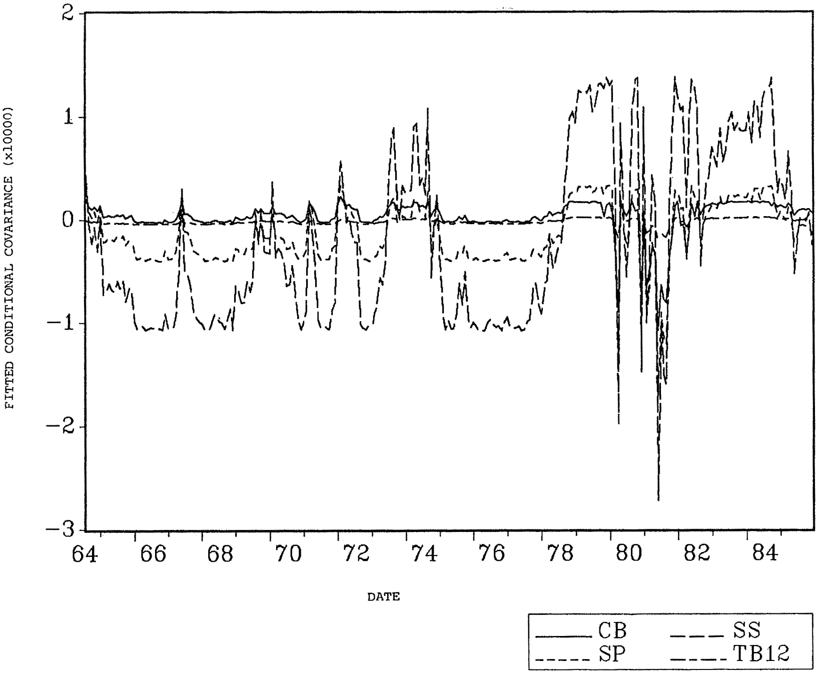


**Figure 1. Monthly fitted values of conditional covariances with benchmark pricing variables.** The proxy benchmark is nondurables consumption. The conditional mean of the proxy is modelled using linear regression. The asset returns are denoted as follows: CB = corporate bond portfolio. SS = small-stock portfolio. SP = Standard and Poor's stock index. TB12 is a treasury bill with twelve months to maturity at the beginning of the month.

conditional covariances of bonds and stocks diverge and fluctuate widely with interest rates in the 1979–1982 period. The movements in the covariances of the small stocks appear different than the large stocks after 1979. There is also a hint of patterns related to the business cycle. The covariance of common stocks tends to drop sharply just prior to a business cycle trough and then fluctuates during the expansion, ending up at a high level near the business cycle peak.

## IV. Beta Pricing Models

Systematic risk is often measured using "betas" relative to reference portfolios. This section relates the results to such beta pricing models. Models of conditional betas are implied by equation (4). Consider a reference portfolio with return $R_{p,t+1}$. Taking ratios implies

$$E\{R_{i,t+1} - RF_{t+1} \mid \Omega_t\} = \beta_{ipt} E\{R_{p,t+1} - RF_{t+1} \mid \Omega_t\},$$

$$\beta_{ipt} = \frac{\operatorname{cov}(R_{i,t+1};\ -m_{t+1} \mid \Omega_t)}{\operatorname{cov}(R_{p,t+1};\ -m_{t+1} \mid \Omega_t)}. \tag{17}$$

If $R_{p,t+1}$ is conditionally perfectly correlated with $m_{t+1}$, then the "beta" reduces

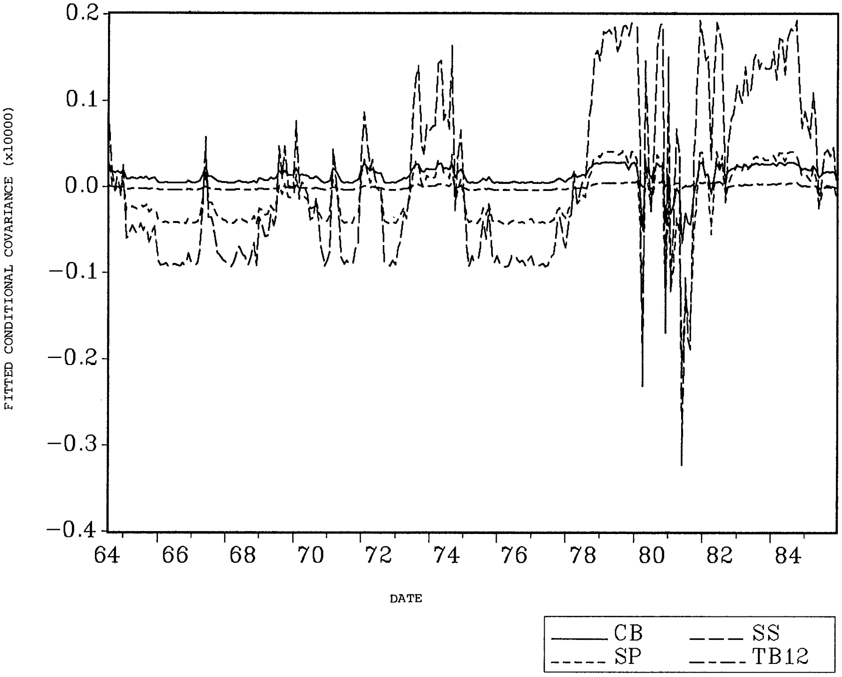


**Figure 2. Monthly fitted values of conditional covariances with benchmark pricing variables.** The proxy benchmark is nondurables consumption. The conditional mean of the proxy is modelled as the price of a one-month treasury bill. The asset returns are denoted as follows: CB = corporate bond portfolio. SS = small-stock portfolio. SP = Standard and Poor's stock index. TB12 is a treasury bill with twelve months to maturity at the beginning of the month.

to the familiar form:[24]

$$\beta_{ipt} = \frac{\mathrm{cov}(r_{i,t+1};\, R_{p,t+1} \mid \Omega_t)}{\mathrm{var}(R_{p,t+1} \mid \Omega_t).}. \tag{18}$$

Equation (17), using (18), is equivalent to conditional mean-variance efficiency of $R_{p,t+1}$ when there is a riskless asset.[25]

The simplest conditional beta pricing model uses a single, constant-beta for each asset. Donaldson and Mehra (1984) and Connor and Korajczyk (1987) present theoretical models consistent with changing risk premiums and constant conditional betas. Chan and Chen (1988) propose a model in which ratios of conditional betas can be constant, while the betas shift proportionately over time. Hansen and Hodrick (1983) and Gibbons and Ferson (1985) estimate constant-beta models with time-varying risk premiums. Some recent empirical papers seem to support such models, while other studies reject them.[26]

[24] Such a return can be written in the form $R_{p,t+1} = \alpha(\Omega_t) + b(\Omega_t)\, m_{t+1}$. Substitution into (4) confirms the claim. Note that the beta pricing model is less restrictive than equation(4).

[25] Grinblatt and Titman (1987) and Huberman, Kandel, and Stambaugh (1987) study relations between mean-variance efficiency and models of conditional expected returns.

[26] Empirical support can be found in Fama and French (1986), Ferson, Kandel, and Stambaugh

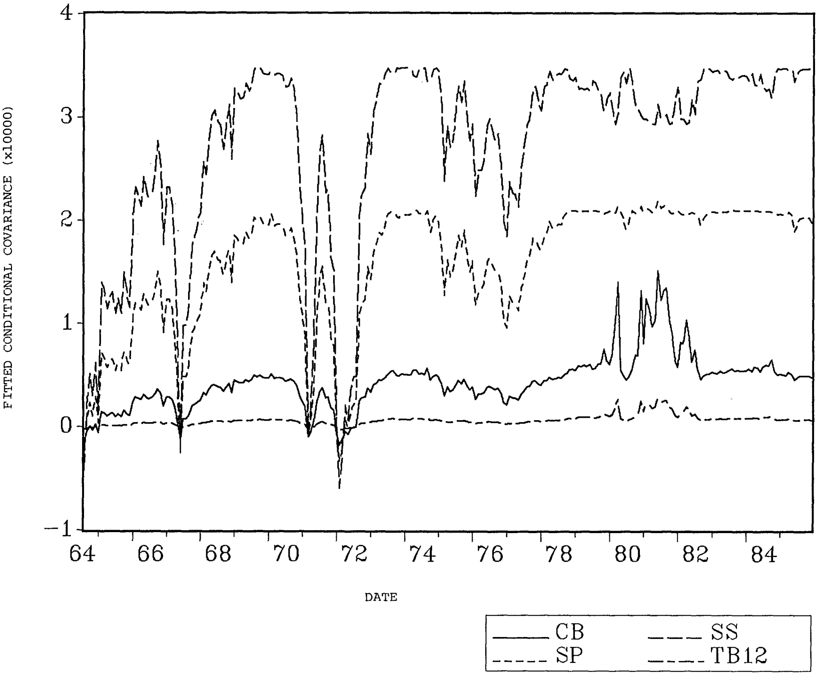


**Figure 3. Monthly fitted values of conditional covariances with benchmark pricing variables.** The proxy benchmark is the value weighted stock index. The conditional mean of the proxy is modelled as the price of a one-month treasury bill. The asset returns are denoted as follows: CB = corporate bond portfolio. SS = small-stock portfolio. SP = Standard and Poor's stock index. TB12 is a treasury bill with twelve months to maturity at the beginning of the month.

It may seem that a constant-beta model of stock returns is ruled out by the results of studies that measure betas directly. Many studies observe that "market model" betas are not constant parameters. However, these studies typically investigate simple (unconditional) regression coefficients, and they rely on a specific market proxy. Such evidence does not rule out a benchmark variable with constant conditional betas.

If ratios of conditional betas relative to a benchmark are constant over time, then expected risk premiums will expand and contract proportionately with the expected premium on a reference portfolio. Figures 5 and 6 plot fitted excess returns from the regressions in Table I. Figure 5 illustrates the fitted values for the two-, five-, nine-, and twelve-month treasury bills. The bills have positive regression slopes in Table I, so the fitted values are high when short-term, nominal interest rates are high. A careful examination of the figure reveals shifts in the ordering of the fitted values during the relatively low nominal interest rate periods corresponding to the business cycle expansions of 1967–1968 and 1971–1973.

(1987), Gibbons and Ferson (1985), and Hansen and Hodrick (1983). Rejections of such models are in Campbell (1987), Ferson, Foerster, and Keim (1988), Hodrick and Srivastava (1984), and Stambaugh (1988).

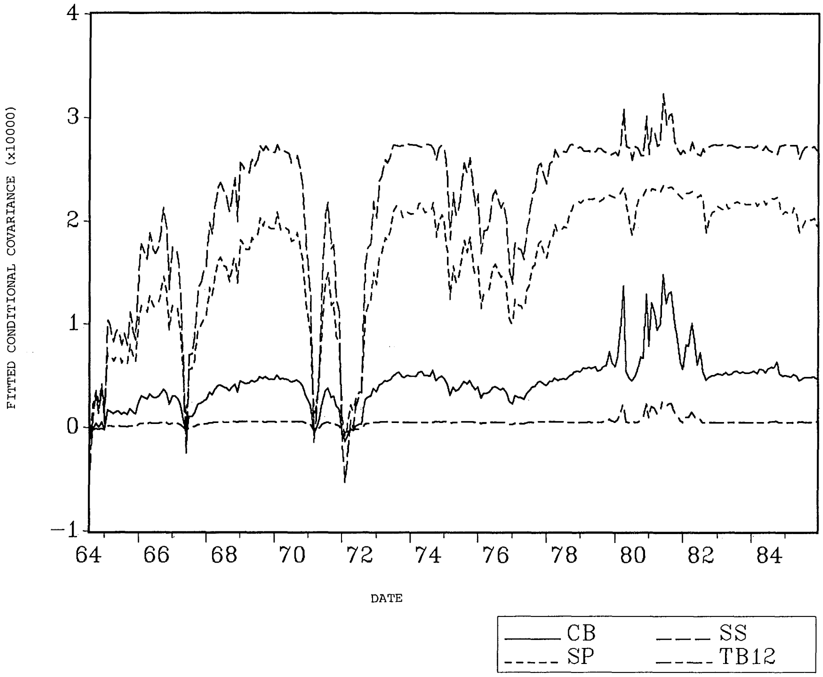


**Figure 4. Monthly fitted values of conditional covariances with benchmark pricing variables.** The proxy benchmark is the equally weighted stock index. The conditional mean of the proxy is modelled as the price of a one-month treasury bill. The asset returns are denoted as follows: CB = corporate bond portfolio. SS = small-stock portfolio. SP = Standard and Poor's stock index. TB12 is a treasury bill with twelve months to maturity at the beginning of the month.

Figure 6 shows the government and corporate bonds and the equally and value weighted stock indices. The figure suggests proportional movements in the expected premiums, except around the 1979–1982 period. The ordering of the fitted returns across the assets shifts erratically in this period.

Figures 5 and 6 suggest that a single, constant-beta model might be a reasonable approximation for the bills or for the long-term assets over much of the sample. However, such a model might not capture the relative risk of stocks and bonds. Of course, the fitted values are subject to estimation error. Statistical tests for constant-beta models are reported in Table IV. The tests examine the restriction that the coefficient vectors in the excess return regressions are proportional across the assets. This restriction is derived in Gibbons and Ferson (1985). The test procedure is similar to that described in Section III.*B*, except that one of the excess returns is used as a reference asset instead of the real return of the one-month bill.

Table IV summarizes results for subsets of the assets, using the full sample and subperiods chosen to isolate October 1979 to September 1982.[27] The tests

[27] Clarida and Friedman (1984), Huizinga and Mishkin (1986), Sanders and Unal (1988), and other studies document shifts in the stochastic process of interest rates and expected returns in the 1979–1982 period. Table IV reports results for the GMM criterion function with no moving average

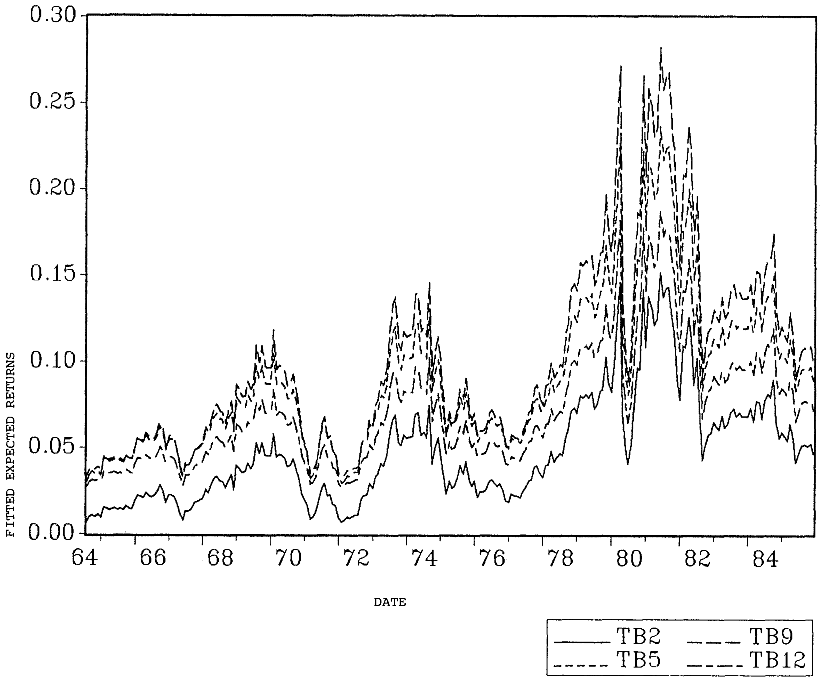


**Figure 5. Treasury bills. Monthly fitted values from regressions of excess returns on one-month treasury bill rates.** TB$n$ is a treasury bill with $n$ months to maturity at the beginning of the month. The return is in excess of a one-month bill rate.

are unable to detect time variation in relative betas among the long-term assets. Any nonproportional changes in expected risk premiums suggested by Figure 6 are not estimated precisely enough to reject the model. There is evidence against a single, constant-beta model in the treasury bill market. The post-1979 period is influential. Repeating the tests for bills using the 1968–1979 subperiod produces no $p$-value smaller than 12.5%. However, the influence of the post-1979 period is not confined to the subperiod of the Federal Reserve "policy experiment." Results similar to those for 1968–1985 are obtained when 1979:10–1982:9 is omitted from the sample.

The bottom panels of Table IV investigate systems with bonds, stocks, and bills. In the overall sample a single, constant-beta model is rejected at conventional levels of significance. Again, the post-1979 data are influential, but this influence does not seem to depend exclusively on the 1979–1982 subperiod. There is evidence of changes in relative betas when the systems include only three assets: a single stock, bond, and bill return. Thus, the information in one-month treasury bill yields is enough to reject a single, constant-beta model for the relative risks of common stocks and fixed-income securities.[28]

adjustment. A subset of the tests was replicated using the Newey-West weighing matrix with three moving average terms, and the chi-square statistics were typically identical to two decimal places. Using alternative risk-free rate measures had little impact on the results. Results similar to those reported for the 1926:1–1985:12 period are also obtained for 1964:8–1985:12.

[28] Additional instruments are required to test a two-beta model. Tests for constant conditional

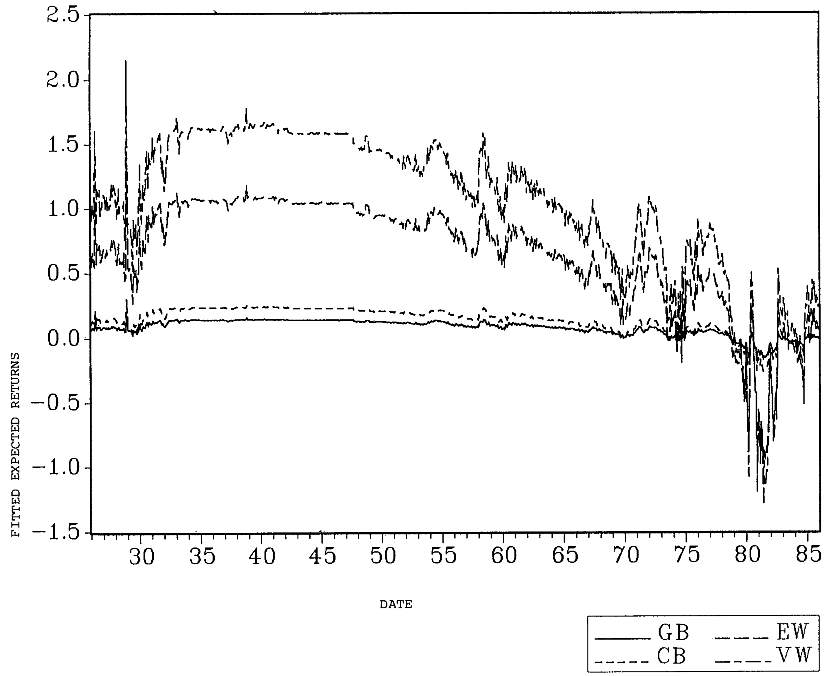


**Figure 6. Long-term assets. Monthly fitted values from regressions of excess return on one-month treasury bill rates.** The returns are in excess of a one-month treasury bill rate. Gl = treasury bond portfolio; CB = corporate bond portfolio; VW = value weighted stock index. EW = equally weighted stock index.

## V. Conclusions

Regressions of asset returns on treasury bill rates can provide insights about the behavior of risk in asset pricing models. Assuming a rational pricing model, the information in one-month rates implies shifts in the conditional covariances of returns with "marginal utility." Changes in risk can be inferred for stocks and fixed-income securities, in real and in nominal terms, over extended samples and within five-year subperiods. Shifts in the relative betas of bonds, stocks, and bills are associated with interest rate movements after 1979. These inferences are made without an explicit measure of marginal utility, and they hold for any benchmark pricing variable. Consumption and stock market data are used to proxy for benchmark variables. A general framework allows time-varying conditional covariances without specifying their functional form. The models with stochastic variation in the conditional covariances attain an improved cross-sectional fit for average excess returns, but the models are rejected. Thus, it appears that changes in risk, as measured by conditional covariances with these variables, are not sufficient to "explain" the asset return-interest rate relation.

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betas, like those in Table IV, were conducted using the quadratic interest rate regressions described in Section III.*B*. The results for single-beta models were similar to Table IV. A two-beta model could not be rejected.

**Table IV**

**Tests of the Nonlinear Restriction Across Regressions of Excess Returns on the One-Month Treasury Bill Rate Implied by the Hypothesis that Conditional Betas Are Constant**

Securities are denoted as follows: GB = Government Bond Portfolio, CB = Corporate Bond Portfolio, SP = Standard and Poor's Stock Index, VW = CRSP value weighted stock index, EW = CRSP equally weighted stock index, SS = Small-Capitalization Stock Index, TB$n$ = Treasury bill with $n$ months to maturity at the beginning of the month. Estimation is by Generalized Method of Moments. The instruments are a constant and the one-month bill rate. $\chi^2$ is the asymptotic chi-square statistic, equal to the minimized value of the objective function. *df* is the degrees of freedom, and $p$-value is the probability that a chi-square variate exceeds the sample value.

| Monthly Sample Period | Securities Included in System | Test Results $\chi^2$ | Test Results *df* | Test Results $p$-Value |
|---|---|---|---|---|
| | Long-Term Bond and Stock Systems: | | | |
| 26:1–85:12 | GB,CB,SP,VW,EW,SS | 1.05 | 5 | 0.958 |
| | GB,CB,SP,VW,EW | 0.69 | 4 | 0.953 |
| | GB,CB,SP,EW | 0.58 | 3 | 0.902 |
| | GB,CB,SP | 0.01 | 2 | 0.993 |
| | GB,CB,EW | 0.06 | 2 | 0.972 |
| | Bills Only Systems: | | | |
| 64:8–85:12 | TB2,TB3,TB4,TB5,TB6,TB7,TB8,TB9,TB10,TB11,TB12 | 28.23 | 10 | 0.002 |
| | TB2,TB4,TB6,TB8,TB10,TB12, | 23.10 | 5 | 0.000 |
| | TB2,TB5,TB9,TB12 | 5.92 | 3 | 0.116 |
| | TB2,TB7,TB12 | 4.41 | 2 | 0.110 |
| 64:8–79:9 | TB2,TB3,TB4,TB5,TB6,TB7,TB8,TB9,TB10,TB11,TB12 | 11.70 | 10 | 0.305 |
| | TB2,TB4,TB6,TB8,TB10,TB12 | 7.81 | 5 | 0.167 |

| | | | | |
|---|---|---|---|---|
| | TB2,TB5,TB9,TB12 | 5.71 | 3 | 0.125 |
| | TB2,TB7,TB12 | 3.34 | 2 | 0.188 |
| 64:8–79:9 and 82:10–85:12 | TB2,TB3,TB4,TB5,TB6,TB7,TB8,TB9,TB10,TB11,TB12 | 24.07 | 10 | 0.007 |
| | TB2,TB4,TB6,TB8,TB10,TB12 | 18.81 | 5 | 0.002 |
| | TB2,TB5,TB9,TB12 | 8.14 | 3 | 0.043 |
| | TB2,TB7,TB12 | 1.27 | 2 | 0.531 |
| | Bills, Bonds, and Stock Systems: | | | |
| 64:8–85:12 | —All 17 Securities— | 23.98 | 16 | 0.090 |
| | GB,CB,SP,SS,TB2,TB4,TB6,TB8,TB10,TB12 | 21.91 | 9 | 0.009 |
| | GB,CB,VW,EW,TB2,TB4,TB6,TB8,TB10,TB12 | 19.94 | 9 | 0.018 |
| | CB,SS,TB2,TB12 | 11.01 | 3 | 0.012 |
| | GB,SP,TB2 | 7.42 | 2 | 0.025 |
| | CB,SS,TB3 | 9.76 | 2 | 0.008 |
| 64:8–79:9 | —All 17 Securities— | 17.35 | 16 | 0.363 |
| | GB,CB,SP,SS,TB2,TB4,TB6,TB8,TB10,TB12 | 10.28 | 9 | 0.328 |
| | GB,CB,VW,EW,TB2,TB4,TB6,TB8,TB10,TB12 | 9.31 | 9 | 0.409 |
| | CB,SS,TB2,TB12 | 5.44 | 3 | 0.142 |
| | GB,SP,TB2 | 3.65 | 2 | 0.161 |
| | CB,SS,TB3 | 4.76 | 2 | 0.092 |
| 64:8–79:9 and 82:10–85:12 | —All 17 Securities— | 25.35 | 16 | 0.064 |
| | GB,CB,SP,SS,TB2,TB4,TB6,TB8,TB10,TB12 | 18.21 | 9 | 0.033 |
| | GB,CB,VW,EW,TB2,TB4,TB6,TB8,TB10,TB12 | 17.50 | 9 | 0.041 |
| | CB,SS,TB2,TB12 | 10.05 | 3 | 0.018 |
| | GB,SP,TB2 | 2.79 | 2 | 0.248 |
| | CB,SS,TB3 | 5.58 | 2 | 0.061 |

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