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Information Losses in a Dynamic Model of Credit

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ABSTRACT

This paper examines dynamic information losses associated with loan terminations. We assume that the aggregated returns of current borrowers contain information about the mean returns to future borrowers. In a competitive loan market, the value of this information is not fully internalized by individual borrowers and lenders, and loan decisions fail to be first best. Introducing heterogeneous borrowers, who know their own risk characteristics better than lenders, safer borrowers are less willing to borrow when risk premia rise. As they cease borrowing, the information generated in credit markets becomes noisier and this tends to increase risk premia. The model produces alternating and persistent periods of "tight" and "loose" credit.

IN A DYNAMIC ECONOMY, every business venture explores new terrain. The success or failure of any business holds economic consequences for other possible ventures and reflects on their likelihood of success: At successively aggregated levels, this proposition remains true: information about the current success of export ventures is relevant to future export ventures; the rate of failure of businesses in Montana is useful information to future investors in Montana.

This informational mechanism lies at the heart of the paradigm of competition: in the absence of barriers to entry, firms enter markets which show high returns and dissipate the producer's surplus. The entrant, who initially makes the risky decision to create the market, provides information about this market to other competitors, as a by-product of enterprise. The provision of this information represents an externality. Because it is an externality, firms do not necessarily optimally provide it. Moreover, this information is not provided only by entrants. As long as markets remain uncertain, and current returns are informative about future returns, all present firms provide information to future firms.

This paper presents a model in which aggregate returns from a class of risky projects are important for decisions about future loans. Current credit market decisions depend on the ex ante distribution of project returns, and in turn current decisions affect the production of information about future returns. The dynamic implications of this learning process are traced out in a simple model of credit under uncertainty. Three important results are obtained:

a) When individual borrowers and lenders cannot capture the value (in terms of expected profits) from information generated by their transactions,

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interest rates and credit allocation will differ from the first-best allocation. In general, interest rates on loans are too high and not enough loans are made.

b) The model is consistent with alternating and persistent periods of "loose credit" and "tight credit" markets. In a period of tight credit, economic activity is noisier so that an improvement in underlying economic conditions is more difficult to detect.

c) Market power on the part of lenders may improve allocations if lenders are able to internalize the returns to aggregate information.

The key point and fundamental assumption of the model is that, when a class of borrowers is active, more information about this class becomes available than when all or part of the class is inactive. In other, similar learning models, such as those developed by Jovanovic (1979) and Frank (1988), the information is generated by the only agents interested in the information. In Jovanovic's seminal article on labor markets, employee and firm learn together about a match parameter which represents the value of the employee to firm output. No other firm or employee is directly interested in this parameter, and the issue of the informational externality does not arise. When the relevant agents monopolize the return from information, they value it efficiently in their decisions. In Frank's model of industrial entry and exit, in which entrepreneurs are learning about individual ability parameters, entrepreneurs delay leaving the industry in order to learn more about their ability, even though the expected return from remaining in the industry is negative in the absence of learning effects.

A critical issue in our model is thus whether an individual lender and/or borrower can fully capture returns from the additional information produced by their private transaction. If they can, then a first-best allocation occurs. If they cannot, then the interest rate on loans and the resulting allocation of credit differ from the first best.

If entry can occur into an industry, then the information value of enterprise in that industry will go in part to future consumers and entrants, and investment decisions will fail to be first best. In our paper, we model this by assuming that enterprises last for only one period and that new entrants each period gain the benefits of past information.[1] This information is an externality to the lending decision in a competitive loan market, and thus decisions are second best.

If, on the other hand, the loan market is monopolized, then the lender can capture the gains from information. Such a monopoly lender will subsidize some borrowers during periods when expected returns are low, because the information garnered from doing so will enable the lender to command higher average expected returns via high interest rates when business improves. Noncompetitive lending can in principle overcome the problem of the informational externality. Increased competition in lending could lead to a less efficient allocation of credit.

We then consider the possibility that only some of the borrowers leave the lending market. In general, with fewer borrowers, the information available will become noisier. We model this by considering adverse selection. We assume that

[1] In Lang and Nakamura (1989), this assumption is shown to be the limiting case of a model with many potential entrants without incumbency.

there are two types of borrowers, one riskier than the other. Borrowers know their riskiness, but lenders cannot distinguish between them. When expected returns fall, risk premia rise and safer borrowers drop out of the market. As a consequence, the information generated by lending becomes noisier, reducing the probability of safer borrowers returning to the market. Since periods when safer borrowers are out of the market are period of high risk premia, we identify these as periods of tight credit. This model is consistent with alternating and persistent periods of tight and loose credit.[2]

In our central model, we rule out the possibility of ongoing contractual relationships between the borrower and the lender because borrowers exist for only one period. This assumption is intended to contrast with and complement models of (imperfectly) durable relationships between lenders and borrowers, such as the models depicted in Stiglitz and Weiss (1983), Nakamura (1984), Sharpe (1987), and Hellwig (1977). These latter models reveal a spectrum of effects in which imperfectly durable relationships between borrower and lender can cause low returns in one period to lead to lower expected returns in succeeding periods.

Two versions of the model are presented. In both versions borrowers and lenders are assumed to be risk neutral. When the predicted return on risky projects falls, the risk of default on loans increases, ceteris paribus. As a result, the risk premium on loans increases and expected returns to borrower fall.

In version one of the model, all risky borrowers within a particular risk class are homogeneous. Either all borrowers receive loans or no loans are made to this risk class. Loans to this risk class will be terminated when expected returns for risky projects are sufficiently low. Any information unique to this risk class will be lost when loans are terminated. Once locked out of the market, borrowers may continue to be locked out for a sustained period of time since lenders do not obtain new information from performance on loans to this risk class.[3] If new information about borrower performance was available, predicted future returns might increase sufficiently to make lending to the risk class more attractive. When compared to the results of our model with a first-best allocation of credit, there are states in which loans are terminated in the competitive model but welfare of borrowers and lenders could be improved if loans were made.

Version one is useful in developing the principal insights of our analysis, although it produces some extreme results. These results are modified if new information about a risk class is not completely terminated when loans to that risk class are terminated. This situation occurs in version two of the model. Version two extends the model to the case where there is adverse selection in loan markets. Borrowers have better information about their own risk characteristics than do lenders. Equilibria may exist in which all borrowers receive loans, only some borrowers receive loans, or all loans are terminated. Extending the

[2] A more complete analysis might consider models in which returns from information generated by individual transactions could be partially captured by lenders and/or borrowers.

[3] Of course, there may be other sources of information relevant to forecasting returns from a particular risk class. While we consider the case where only current and past returns are useful information, our results can be generalized (with some modification) if we assume that observed returns are an important part of the information set used in predicting future returns.

model in this way allows us to consider the dynamic properties of the credit market and the efficiency of that market when some loans are terminated while other loans continue. The lending decision not only determines the interest rate and allocation of credit, but it also affects the quality of information generated in the credit markets. This version of the model is not only an extension of version one, but it also is a useful dynamic extension of credit market models with adverse selection. Such extensions are necessary if these models are to be useful in explaining macroeconomic behavior.

Version two follows Stiglitz and Weiss (1981) in that borrowers differ in the riskiness of their projects but have the same expected return. When expected returns on risky projects fall, causing a rise in the risk premium[4] on loans, riskier borrowers may continue to borrow while safer borrowers refuse to borrow.[5] Returns from riskier projects contain more noise, and thus the value of new information obtained from loans is lower when loans to safer borrowers are terminated. This incrased noise lowers the ability of agents to detect a shift in returns. Even though agents are risk neutral, greater uncertainty about returns to risky projects increases the required expected return necessary for safer borrowers to enter the market for loans. This implies that, in the immediate aftermath of a shift to termination of loans to safer borrowers, there is an increase in the predicted return necessary for the reentry of safer borrowers into the market. Lending is therefore path dependent, and the dropout of safer borrowers in one period makes their staying out of the market in the second period more probable.

I. Returns to Borrowers and Lenders in a Static Model

As a basis for understanding the mechanism of the model, it is useful first to develop the principles of a static model of lending. All agents are assumed risk neutral. We will first discuss the case in which there is only one type of borrower, and then we will look at two types of borrowers.

A. Homogeneous Borrowers

Each borrower has a project which requires a loan whose size is equal for all borrowers and normalized to one. In addition, each borrower has an asset which will be worth C dollars when the loan comes due and which the borrower can use as collateral. The return to the project is a random variable, r, distributed normally with mean m and standard deviation s. The loan repayment is R, to be determined endogenously. The expected return to the lender is designated π_L, equal to

$$\pi_L = E \min(R, r + C), \tag{1}$$

[4] The term risk premium is used here and in the rest of the paper to refer to the differential between the interest rate on risky loans and the risk-free rate. Throughout the paper we assume risk neutrality so that this differential is a result of the possibility of default on full payment rather than risk aversion.

[5] This argument should not be confused with credit allocation decisions made concerning different distinguishable risk classes. For simplicity, our model assumes only two distinguishable risk classes—risky and riskless.

where E stands for the mathematical expectation operator. π_L falls with s and rises with m, R, and C. We assume competition among lenders, so that, in equilibrium, R is determined by $\pi_L = R_f$, where R_f is the risk-free rate of return. The variables m, s, and C are common knowledge.

The expected return to the borrower is designated π_B, equal to

$$\pi_B = E \max(r - R, -C). \tag{2}$$

π_B falls with R and C and rises with m and s.

If there is only one type of borrower, the sum of the expected returns to both the borrower and lender from a single loan is simply the expected return on the risky project. That is,

$$\pi_L + \pi_B = m \quad \text{or} \quad \pi_B = m - \pi_L = m - R_f. \tag{3}$$

The loan will be accepted by the borrower whenever

$$\pi_B \geq 0 \quad \text{or} \quad m \geq R_f. \tag{4}$$

As the risk-free rate rises, the risk premium $(R - R_f)$ on loans will also rise. The interest rate on loans, R, will rise with s and fall with m. Figure 1 displays isoprofit curves for lenders in R, s space, with m held constant. Since π_L is decreasing in s and increasing in R, the lender's isoprofit curves are upward sloping. π_L is increasing in the northwest direction. Given m and R_f, the equilibrium interest rate can be found along the curve $\pi_L = R_f$. The equilibrium interest rate will simply be that interest rate (R) along π_L which corresponds with the standard deviation (s) of returns to loans. Expected borrower profits are increasing to the southeast. If there is only one type of borrower, from (3) it is clear that the isoprofit curves for borrowers have the same slope as the isoprofit curves of the lender.

Figure 2 depicts the effects of a decrease in mean return, from m to m'. When mean return is lower, the isoprofit curves for each level of lender profit shift upward; a higher repayment is required to allow lenders to earn R_f. Simultaneously, all the isoprofit curves for borrowers will shift downward. At the new loan

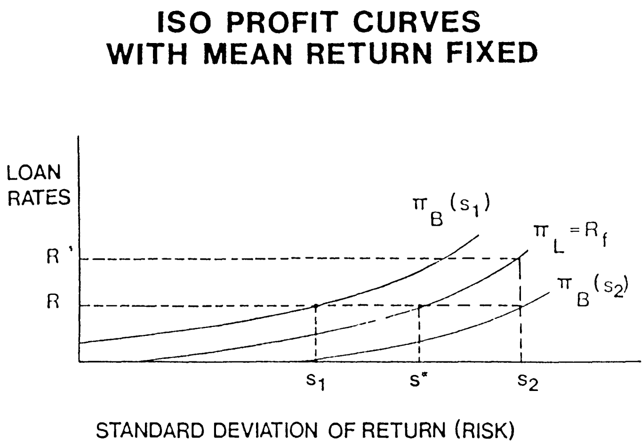

Figure 1. Isoprofit curves for lenders and borrowers with mean return fixed, for borrowers with private information on return variance.

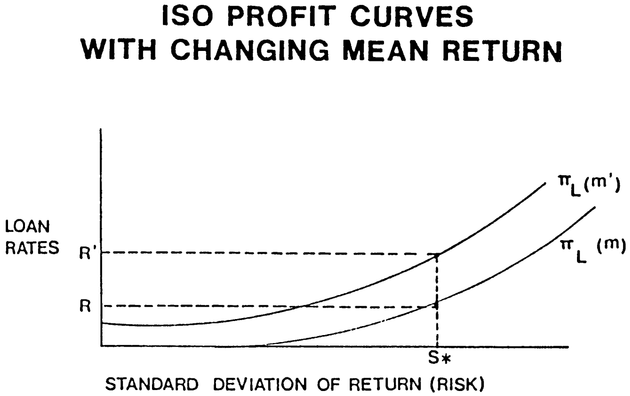

Figure 2. Isoprofit curves for the lender as mean return falls from m to m'.

repayment rate R', borrower profits are lower by exactly $m - m'$. At the point where $m = R_f$, borrower profits reach 0 and borrowers leave the market.

B. Two Types of Borrowers

Now suppose there are two types of borrowers with a common value of m but different values of s, which we will call s_1 and s_2, with $s_1 < s_2$. Lenders know m and the proportions of the two types of borrowers but are unable to distinguish ex ante which borrowers are safer and which are riskier. Referring to Figure 1, type 2 (riskier) borrowers earn higher rates of return for any given loan rate than do type 1 borrowers because there is a lower probability that they will pay back the loan and a greater chance they will earn a high return. R is the equilibrium interest rate if loans are made to both types of borrowers. The effective s value for loans is between s_1 and s_2. The riskier borrowers (s_2) have higher expected returns. Loans will be made to both types of borrowers if the expected returns to the safer borrower are greater than 0 [$\pi_B(s_1) \geq 0$]. R' is the equilibrium interest rate which prevails when only riskier borrowers are willing to borrow. Of course, it is possible that in equilibrium no loans will be made to either type of borrower.

Figure 2 depicts the effect of a change in expected returns to risky projects. When mean return is lower, lenders must charge a higher interest rate at each level of s in order to maintain the competitive expected rate of return. The isoprofit curve ($\pi_L(m')$), at which lenders receive the competitive expected return given $m' < m$, shifts up relative to the old curve (π_L). For the same value of s, the interest rate on loans has risen from R to R'. The combined effect of a higher interest rate and lower mean return implies lower expected profits for both types of borrowers. Given a value for the riskiness of borrower projects, the borrowers' expected profit function will be increasing in m. This relationship between m and π_B is depicted in Figure 3. Both types borrow if $m > m^*$, where m^* is the value of m below which safer borrowers refuse to borrow. If m falls below m^*, the safer borrowers leave the market, and the loan interest rate rises discontinuously. Consequently there is a discontinuous fall in profits for the riskier type borrower. All lending ceases when m falls below R_f.

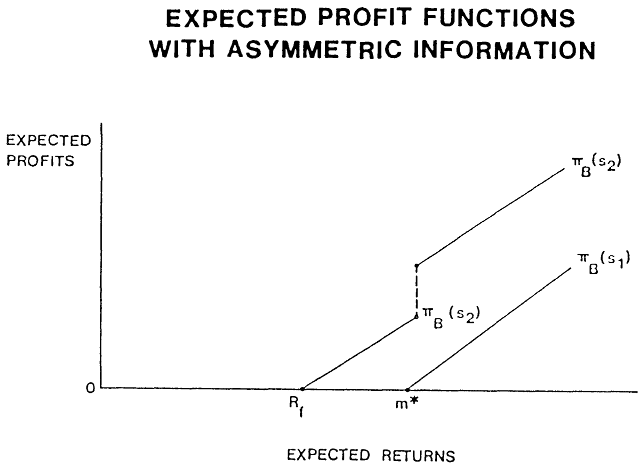

Figure 3. As expected return falls below m^*, safer borrowers drop out of the loan market, risk premia rise discontinuously, and expected profit to riskier borrowers falls discontinuously.

II. A Dynamic Model with Homogeneous Borrowers

A valuable source of information about the expected rates of return of borrowers is the past behavior of similar borrowers. When lending is terminated in a given market, this source of information disappears, handicapping future lending. This point is illustrated in this section.

We begin by assuming that returns from projects for which loans are sought have a common component, r_t. This common component in turn has a permanent component, μ_t, which follows a random walk,[6] and a temporary, white-noise component,[7] η_t, so that past and current values of r are useful in forecasting future returns. Individual borrowers, indexed j, earn $r(j)_t$, equal to the common component, r_t, plus an idiosyncratic shock, $v(j)_t$. We assume that each borrower (and its lender) learns $r(j)_t$ and that the borrower reports this value to a government bureau, which then prints the aggregate average r_t. (We assume that there are a sufficient number of borrowers so that in the aggregate the idiosyncratic noise terms vanish by the law of large numbers and the common component is revealed publicly by the aggregate.)

$$r(j)_t = r_t + v(j)_t, \tag{5}$$

$$r_t = \mu_t + \eta_t, \tag{6}$$

$$\mu_t = \mu_{t-1} + e_t. \tag{7}$$

Here, η_t, e_t, and $v(j)_t$ are zero-mean, white-noise processes with variances s_η^2, s_e^2, and s_v^2.

[6] The main results of the paper with some modifications are robust to more general specifications of the time series properties of the permanent component of returns.

[7] This could be interpreted as an undiversifiable risk component which might result from temporary aggregate shocks to the economy.

At the start of period t, when loans are made, values of r_{t-j} are common knowledge for all $j > 0$, and these are sufficient statistics for forecasts of r_t and $r(j)_t$. The minimum mean square error forecast of r_t, made at the start of period t and designated m_t, is the best forecast of $r(j)_t$ and of μ_t.

The idiosyncratic term plays no important role in the dynamics of the model (other than its effect on the level of expected profits of borrowers and lenders). It is included in (5) to ensure that private information is not a perfect substitute for aggregate information. If there were no idiosyncratic component, then any individual lender could observe $\mu_t + \eta_t$ directly even in the absence of other lending. In the absence of observable aggregate returns (i.e., if lending ceases), the outcome of an individual loan will provide useful information about future returns. The weight this private information is given in the forecasting equation of future returns will be a declining function of the variance of idiosyncratic risk relative to the variance of aggregate risk. The critical point is that individual lenders cannot observe $\mu_t + \eta_t$ through their own private transactions. When private information is not a perfect substitute for aggregated information, there is an informational externality. Individual lenders and borrowers do not take into account their role in producing aggregate information. For simplicity we have assumed that only aggregate returns are used in forecasting the future.

The minimum mean squared error estimator for r_t, given r_{t-1}, if loans have never been terminated is calculated recursively by

$$m_t = m_{t-1} + b[r_{t-1} - m_{t-1}], \tag{8}$$

where

$$b = (q/2)[(1 + 4/q)^{0.5} - 1] \quad \text{and} \quad q = s_e^2/s_\eta^2. \tag{9}$$

The variance of the forecast error for r_t is

$$E[(r_t - m_t)^2] = s_\eta^2/(1 - b). \tag{10}$$

The variance of the change in forecasts is

$$E[(m_t - m_{t-1})^2] = s_e^2. \tag{11}$$

The variance of returns to individual projects is then

$$s_\eta^2/(1 - b) + s_v^2.$$

Now assume that borrowers live only for one period and that in each period a new cohort of borrowers appears. In this case, no current borrower can capture the future value of information from current projects.

A. Competitive Lenders

With competitive lenders, the interest rate R will be set so that the lender's expected return, π_L, will equal the risk-free rate. In this case the profit-maximizing decision rule by borrowers and lenders is the same as in the static or myopic case. As seen in equation (4), the loan will be made as long as $m_t \geq R_f$. If $m_t < R_f$, then loans are terminated. If loans are terminated in period t, then no further information becomes available about the quality of loans, so, as time passes, mean return remains fixed:

$$m_{t+k} = m_t < R_f \quad \text{for } k > 0. \tag{12}$$

When loans are terminated in period t, then no loans will be made in the future to this risk class since there will be no new information to base a change in forecasted returns. In the absence of new information, the variance associated with future forecasts of r increases as both lenders and borrowers find their information deteriorating over time.[8]

The permanent termination of lending to this group of borrowers is inefficient. From the point of view of future borrowers, information about current borrowing has a positive value. Borrowers at time $t + 1$ would be willing to subsidize borrowers at time t, if they could, because knowledge of the state at time t increases borrowers' expected returns at time $t + 1$. The same is true for borrowers in all future periods.

B. Monopoly Lenders

Assume instead that there is a single lender, infinitely lived, who monopolizes the loan market to this group of borrowers. The borrower lives one period so that the information produced in lending is relevant only to the lender. When the lender evaluates whether to accept a loan and engage in the risky project, the lender must consider the value of possible future loans. By accepting the loan, the lender learns more about the value of future projects. It is possible that, although the current return to a project has a negative expected return, the actual realization may be higher than predicted. There is some probability that, if projects are undertaken today, future projects will have expected returns greater than zero. Since the lender can always refuse to act on new projects, the information to be gained has non-negative value, and its expected value is strictly positive. The lender will offer loans in some states where the expected return from those loans is strictly negative. The capture and internalization of the value of information which comes bundled with the act of lending lead to a first-best decision rule.

As long as $m_t \geq R_f$, the lender sets R so that $\pi_B = 0$ and the lender receives positive expected profits. However, if $m_t < R_f$, the lender may be willing to accept negative expected profits and to subsidize some borrowers in order to learn more about future values of m. Under the first-best decision rule, there still exists a cutoff rate of return, designated as $m^*(0)$, below which the lender will refuse to subsidize lending. Once lending has been terminated, as the variance of forecast error increases, the cutoff rate of return will fall. We designate the cutoff rate of return for a lender who has not lent for j periods as $m^*(j)$. It can be shown that $m^*(j)$ increases without limit as j increases, so that the monopoly lender will always return to the market, no matter how low m_t has fallen.[9]

[8] The extreme result of *permanent* loan termination is due to both the random walk assumption and the lack of any relevant information about returns to this class of borrowers other than average returns from borrower projects. If these assumptions are modified (i.e., if returns follow a stationary stochastic process), then loan termination will not be permanent.

[9] Similar assumptions can be made about borrowers, which would lead to a first-best allocation.

THEOREM: *In a first-best decision rule, there exists some constant $m^*(0)$ such that*

a) $m^*(0) < R_f$ *and*

b) if $m_t < m^*(0)$ *is observed, then lending is terminated.*

Proof: See Appendix.

Since $m^*(0)$ is strictly below R_f, policies which create at least small added incentives for lending will improve welfare. Thus, subsidies to lending could improve welfare, and so could regulations (or jawboning by the Fed) which encourage loans to small and risky borrowers during recessions. In addition, a reduction in the risk-free rate generated by Fed policy may also improve welfare. Increased competition among lenders may reduce welfare if it lessens the ability of lenders to capture future returns from lending. Finally, it may be that one of the reasons for the existence of financial intermediaries is that a financial intermediary, by reducing competition among lenders, may be better able to capture the future returns from lending.

This latter justification for financial intermediation is separate from the delegated intermediation justification provided by Diamond (1984). In Diamond's case, small lenders can conserve on monitoring costs by hiring an agent to act for a group of them. In our case, the lender acts as a bridge between borrowers at different points in time; monopoly power is crucial to the incentive to the lender to accurately value information.

III. A Dynamic Model with Nonhomogeneous Borrowers

Now consider the possibility that there are two or more types of borrowers within a risk class. Different types of borrowers have the same mean return, but some are riskier than others. As the loan rate of interest increases, safer borrowers refuse to borrow.

We will examine transitions between two regimes, regime 1 (loose credit), which occurs when both types of borrowers borrow and interest rates and risk premia are relatively low, and regime 2 (tight credit), when only riskier borrowers borrow and interest rates and risk premia are relatively high. In the tight credit regime, the market provides a relatively noisy signal about future expected returns, and this makes a transition to loose credit regime less likely, for two separate reasons. The first is that, with information being less reliable, the risk premia in loan rates become larger. The second is that the information-updating formula places greater weight on information from the more reliably informative loose credit regime than it does on information from the noisier tight credit regime. As a consequence, good news that occurs in the tight credit regime, which otherwise might cause a return of safer borrowers to the market, is given less credence.

For simplicity, we assume that there are two equally probable types of borrowers, who last for only one period. The jth type 1 borrower has return

$$r(1, j)_t = \mu_t + z_t + v(j)_t, \tag{13a}$$

while the j th type 2 borrower has return

$$r(2, j)_t = \mu_t + z_t + h_t + v(j)_t, \tag{13b}$$

$$\mu_t = \mu_{t-1} + e_t, \tag{14}$$

Here, e_t, z_t, h_t, and $v(j)_t$ are independent and serially uncorrelated stationary random variables, normally distributed with zero mean and variances s_e^2, s_z^2, s_h^2, and s_v^2. Note that the borrowers differ in terms of the variance of the temporary random component of returns $(z_t, z_t + h_t)$.

If interest rates are sufficiently high, then type 1 borrowers refuse to borrow, while type 2 borrowers may continue to borrow. Depending on the interest rate, we may have both types borrowing, only type 2 borrowing, and no borrowing. Having addressed the problem in which no borrowing occurs above, we will discuss here the transitions between both types and type 2 borrowing.

There are N borrowers of each type and N is large. As before, we define r_t as the average return from projects which are undertaken. The temporary random component of r_t is denoted as η_t, and its distribution is dependent on which types borrow. Therefore,

$$r_t = \mu_t + \eta_t, \tag{15}$$

where

$$s_{\eta t}^2 = \begin{matrix} s_z^2 + s_h^2/4 & \text{when both types borrow,} \\ s_z^2 + s_h^2 & \text{when only type 2 borrows.} \end{matrix} \tag{16}$$

Define

$$\mu_t^e = \text{estimator for } \mu_t \text{ given } r_t,\ r_{t-1},\ \cdots,$$

$$\mu_{t\,|\,t-1}^e = \text{estimator for } \mu_t \text{ given } r_{t-1},\ r_{t-2},\ \cdots,$$

$$V_t = E[(\mu_t - \mu_t^e)^2],$$

$$V_{t\,|\,t-1} = E[(\mu_t - \mu_{t\,|\,t-1}^e)^2] = E[(\mu_t - \mu_{t-1}^e + e_t)^2] = V_{t-1} + s_e^2.$$

Again denoting by m_t the estimator for r_t given the lagged values of r, it is straightforward that

$$m_t = \mu_{t\,|\,t-1}^e = \mu_{t-1}^e, \tag{17}$$

$$r_t - m_t = \mu_t - \mu_{t\,|\,t-1}^e + \eta_t, \tag{18}$$

$$E[(r_t - m_t)^2] = s_{\eta t}^2 + V_{t\,|\,t-1}. \tag{19}$$

Each borrower knows its type, but lenders do not. After each period, r_t is public information. Each lender lends to one borrower.

The estimator for the permanent component of returns can be found by the updating equation:

$$\mu_t^e = \mu_{t-1}^e + b_t[r_t - \mu_{t-1}^e], \tag{20}$$

$$b_t = V_{t\,|\,t-1}/(s_{\eta t}^2 + V_{t\,|\,t-1}). \tag{21}$$

Finally, we have the updating formula for V_t:

$$V_t = V_{t|t-1} - V_{t|t-1}^2/(s_{\eta t}^2 + V_{t|t-1})$$
$$= s_{\eta t}^2(V_{t-1} + s_e^2)/(s_{\eta t}^2 + V_{t-1} + s_e^2). \tag{22}$$

From the discussion of Figure 2, it is clear that there exists a benchmark μ_t^*—which may evolve over time—such that, if $\mu_t^e \geq \mu_t^*$, safer borrowers borrow and, if $\mu_t^e < \mu_t^*$, safer borrowers do not borrow.

Note from (22) that, for a given value for V_{t-1}, the value of V_t is an increasing function of $s_{\eta t}^2$. In other words, for a given previous history, the variance of the estimator for μ_t is greater if loans are terminated for safer borrowers than if both types of borrowers remain in the market. It follows that, if in period t loans to safer borrowers are terminated, then the variance of the estimator for expected returns in period $t + 1$ given r_t will be higher than if both types borrow.

Equation (22) describes a nonlinear first-order difference equation with a variable coefficient, $s_{\eta t}^2$. Since $s_{\eta t}^2$ takes on only two values, the time path for V alternates between the path described by two stationary nonlinear difference equations with constant coefficients. Let $V^*(2)$ define the stationary value for V given that only type 2 borrowers take out loans and this situation continues indefinitely. $V^*(1)$ is the stationary value for V given that loans continue to be made to both types of borrowers and $V^*(1) < V^*(2)$. Starting from some initial point (V_0) between $V^*(1)$ and $V^*(2)$, V will always be increasing during periods when type 1 borrowers drop out of the market and decreasing when both types are borrowing.[10]

The variance relevant for lending and borrowing decisions is the forecast error variance of returns (r_t) given information at time $t - 1$ which was given in equation (19). If V_t is increasing (decreasing) over time, then the forecast error variance of r_t will be increasing (decreasing).

What ultimately concerns us is the evolution of μ_t^e and μ_t^*. These variables will influence the time path of interest rates, and perhaps more importantly they will determine whether loans are made to both types of lenders or only to riskier lenders, or whether no risky loans are made.

Now consider the dynamic evolution of μ_t^*. In the Appendix we show that the effect of an increasing forecast error variance (when only type 2 borrows) will be to increase the benchmark value μ^*, provided that the probability of default is less than one half. In period t both types of borrowers will borrow if m_t is greater than or equal to μ_t^*. In that case, the forecast error variance will fall and μ^* will be lower next period. If m_t is below μ_t^*, then only type 2 borrows (assuming $m_t > R_f$ and some loans are still made). The value of μ^* will be higher next period.

The result demonstrates that, when expected returns fall below μ_t^* by a certain distance, m will have to rise by more than this initial absolute value in order for a return to a regime in which both types borrow. Further, the benchmark mean return continues to increase as long as mean return remains below the benchmark so that the probability of rising above the benchmark decreases, and the contrary holds when mean return rises above the benchmark. These changes in the

[10] $V^*(1)$ and $V^*(2)$ are lower and upper bounds for V given that V_0 is between these values initially.

benchmark mean return (increasing when only type 2 borrows, decreasing when both types borrow) lead to longer waiting times for transitions from one regime to another.

Now consider the evolution of μ_t^e. From the updating equation for μ_t^e (20), we have

$$E[(\mu_t^e - \mu_{t-1}^e)^2] = b_t^2 E[(r_t - \mu_{t-1}^e)^2] \tag{23}$$

$$= V_{t|t-1}^2/(s_{\eta t}^2 + V_{t|t-1}). \tag{24}$$

Using equation (22) to substitute for the right-hand side of (24),

$$E[(\mu_t^e - \mu_{t-1}^e)^2] = V_{t-1} - V_t + s_e^2. \tag{25}$$

Equation (25) shows that the variability of the estimator for μ will be higher if V is decreasing (both types borrow) and lower if V is increasing (only type 2 borrows). Note that, in the absence of transitions (if both types borrow or type 2 borrows forever), the model reduces to the model with homogeneous risky borrowers and

$$E[(\mu_t^e - \mu_{t-1}^e)^2] = s_e^2.$$

This result occurs because, in the updating formula,

$$m_t = m_{t-1} + b_t(r_{t-1} - m_{t-1}).$$

The value of b decreases while the noisiness of r_t increases, and the two offset each other as b_t approaches its stationary value. However, the same result does not hold for the transition process. During the transition process, the value of $E[(\mu_t^e - \mu_{t-1}^e)^2]$ is smaller when we have switched from both types of borrowing to type 2 borrowing only, and it is larger when we have switched from type 2 borrowing to both types borrowing. That is, the estimates of μ are generally less variable from one period to the next when only type 2 borrows. This result shows that the predicted returns to borrower projects will be less subject to change based on new information when we are in a period of tight credit.

IV. Conclusion

This paper has examined the effects of dynamic information losses associated with loan terminations. We have assumed that the risk-free rate of return and the set of alternative safe projects for borrowers are fixed, neither stochastic nor subject to policy variation. Moreover, we have not examined the general equilibrium possibility that the risk-free rate of return may be influenced by developments in the market for risky loans. Finally, we have not set up a model in which the demand for risky projects influences aggregate output. Clearly, much work needs to be done before the aggregate possibilities of this model are fleshed out and ready for testing.

Nevertheless, some speculations on the aggregate implications of the model may be useful since the intention of this research is in part to understand credit market sources of asymmetry and their aggregate effects. In particular, recent work on asymmetries in aggregate output over the course of the business cycle has struck us as being possibly supportive of theories of credit which once went under the rubric "pushing on a string."

For example, banks which survived the Great Depression in America were notoriously conservative in their lending practices for an extended period after 1933. Banks shifted their portfolios away from commercial loans toward government securities. For evidence of this shift, see, e.g., Goldsmith (1958). This sustained period of conservative banking has been attributed to fear of runs, to the lack of availability of investments, and to the crowding out of private investments by government deficits.[11] However, the FDIC greatly reduced the probability of runs, and under any plausible model this fear of runs appears irrational. The low rate of bank failure is documented in FDIC annual reports.

In this paper, we propose an alternative set of reasons for alternating periods of conservative and expansive lending practices, one which may tend to occur in the aftermath of any recession. This reason is that a reduction in lending leads to a loss of information about the behavior of borrowers, which makes a return to lending less probable.

As a practical matter, termination of loans to a risk class or division may involve closing of the relevant departments or divisions of lenders. In this case, not only is current information lost, but access to a whole history may disappear. As an example, during the decline in exports associated with the strong dollar from 1981 to 1985, many banks terminated or curtailed their export loan departments. Once the dollar fell in value, potential exporters found that obtaining loans was more difficult than would have been the case if those departments had remained in existence.

Future research in this area, which first requires placing these results within a more complete model of the economy, may provide a theoretical foundation for empirical findings of business cycle asymmetries of the type investigated by Sichel (1987). It is noteworthy that this theory appears to suggest that an activist monetary policy could improve welfare. If we think of information as an input to production, the intertemporal losses of information from volatility may lower average expected output, and it thus may be useful to reduce volatility even though the source of the volatility is "real," since this reduces information loss.

Our paper has examined two cases of informational loss in a dynamic model of credit. In both cases, credit allocation differs from the first best with too few loans being made. In addition, the model suggests a rationale for alternating periods of conservative and expansive lending behavior. It remains to be seen whether these results prove to be robust to a more complete specification of the aggregate economy.

Appendix

Proof of Theorem: The value of the loan at time t has two parts: the expected value of the loan, m_t, and the expected value of future loans, given m_t, designated $Q_{t+1}[m_t]$.

$Q_{t+1}[m_t] \geq 0$, since future loans need not be made unless they are expected to

[11] For overviews of this period, see Friedman (1982) and Friedman and Schwartz (1963).

earn non-negative returns. As long as $m_t + Q_{t+1}[m_t] \geq R_f$, then the loan will be made. What we need to do is to analyze the properties of $Q(\cdot)$, showing that it is finite for finite m_t and that it is continuous and monotonically increasing. Then there will exist a unique m^* such that the theorem holds, by the intermediate value theorem.

The only nonstandard element to the analysis of Q is that, when no loans are made, variance increases so that the value of the information derived from making a loan increases over time. It is critical that this value be bounded. To show that Q is finite for finite m_t, we consider the counterfactual case in which the loan information r_t is known even when the loan is not made and the project not undertaken. In this case the loan is optimally accepted according to the myopic decision rule $m_t > R_f$, since the information from the loan is not affected by loan termination. Under this counterfactual case, the present value of future lending is the present value of the infinite sum of $r^e_{t+\iota}$, conditional on $r^e_{t+\iota} \geq R_f$. It is straightforward to show that this is finite via the ratio test, since the ratio of successive elements approaches the discount factor. In fact, an explicit bound is easily obtained. The present value under the counterfactual case is greater than Q, so Q must be finite.

To show that Q is continuous and monotonically increasing, we adopt the familiar constructive method of approaching Q using a series of finite horizon functions, Q_i, where i is the number of future periods considered. The transform T defined by $Q_{i+1} = T(Q_i)$ discounts and is monotone, so that T is a contraction (in the metric space of continuous functions defined by the norm of uniform convergence). Since the metric space is complete, Q exists and is continuous. Monotonicity of Q follows, and, by the intermediate value theorem, m^* exists and is unique. Q.E.D.

To show that μ^* increases when we move from both types borrowing to only type 2 borrowing, note that the only effect of such a move is to increase the variance of the next period's returns to loans because of the greater forecast error variance—that is, $V_{t\mid t-1}$ rises. This means that the s values for each type in Figure 2 will increase equally, as will the average s value which is relevant to the lender. If the slope of the lenders' isoprofit curve is steeper than the slope of the safer borrowers' isoprofit curve, then a shift in s^* will lead to a sufficiently higher loan rate and type 1 borrowers will have lower profits (move to a higher isoprofit curve).

When returns are normal, the slope of the isoprofit curves for both borrowers and lenders is equal to

$$\frac{\partial R}{\partial s} = \int_k^\infty \frac{z\phi(z)\ dz}{1 - \Phi(k)},$$

$$k = \frac{(R - m - C)}{s}.$$

The point at which both types of borrowers have exactly a 50% probability of default is $k = 0$. z is a standard normal random variable, and ϕ and Φ are the density and cumulative distribution functions, respectively. When k is negative, the slope of the isoprofit curves will be increasing in s (for any given value of R).

The increase in risk of all types of borrowers in the given risk class causes a large enough increase in R to lower profits of the safer borrowers and possibly terminate loans to safer borrowers.

When k is positive and the probability of default exceeds 50%, this result does not hold. In this case, the borrower with the higher value of s actually is more likely to repay the loan in full; consequently, an increase in the loan repayment, R, is more costly to the riskier borrower. It remains true that the safer borrower pays a higher total cost for borrowing and will leave the market at a lower value of R than the riskier borrower. However, in this case μ^* decreases when we move from both types borrowing to type 2 borrowing.

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