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ABSTRACT

We develop a multiperiod rational expectations model of securities market equilibrium in which equilibrium prices may move between periods even though it is common knowledge that no new information has arrived about ultimate security payoffs. This happens because investors know they have imperfect information about the endowments of other investors and this knowledge affects their probability beliefs about the prices that will prevail at the intermediate trading date. These beliefs are reflected in the equilibrium at the initial trading date when investors focus on the probabilities of intermediate capital gains and losses, rather than ultimate payoffs.

VIRTUALLY ALL MODELS OF the pricing of securities employ some notion of market equilibrium, so for prices to fluctuate something in the equilibrium must change. Since equilibrium in an informationally efficient market implies that market clearing prices reflect all available information about future securities payoffs, it seems to be frequently assumed that observed jumps in prices ought, under the "efficient market hypothesis," to reflect the arrival of new information about future cash payoffs to securities.

The major theme of this paper is to examine a multiperiod model of securities market equilibrium in which equilibrium prices may change sharply between periods even if it is common knowledge that no new information about ultimate payoffs has arrived and all investors are rational and make full use of all available information. In our model, the new information that leads to a price change is information about how much of the known total supply of risky securities is held by investors of different preferences.

For this model to produce intertemporal price fluctuation in equilibrium, two phenomena must occur. Firstly, the initially unknown composition of the market must not be revealed by equilibrium prices at the initial trading date. Secondly, despite this, the true composition of the market must (at least partially) determine prices at a later trading date. They key to both these phenomena lies in the difference in investor behavior when they anticipate revising their current portfolio holdings in response to future price changes as compared to when they choose portfolios on the basis of receiving the ultimate cash payoffs to the securities they buy.

In practice, few investors anticipating holding financial securities to their eventual liquidation by the issuer; short-term debt or debt close to maturity would be the major exception. Few equity securities relative to the total supply

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are liquidated in any year, and then usually only by being sold to another buyer as in a takeover. Investors' expected returns on financial assets over the investors' expected holding periods for those assets are primarily based on capital gains, where uncertainty arises from not knowing the future price.

Uncertainty about future prices can involve uncertainty about future information that may change beliefs concerning the ultimate cash payoffs to the security. This aspect is the basis of widely used multiperiod securities equilibrium models such as Merton's intertemporal capital asset pricing model. However, uncertainty about future prices can also reflect uncertainty about what we call the "state of the market": the beliefs, preferences and endowments of the other participants in the economy. Even if all investors' probability beliefs about ultimate payoffs were common knowledge, as well as the knowledge that these beliefs would not change in the future, uncertainty about future prices would still be present as long as investors had imperfect information about the state of the market. We refer to this source of uncertainty as "market created risk" to emphasize that its source is investors themselves, rather than the stochastic process describing the ultimate cash payoffs to securities. Payoff-related risk respecting future equilibrium prices could be present in a single-investor world, whereas market created risk requires multiple investors who are imperfectly informed about each other.

To focus most clearly on some of the important effects that market created risk can have on the movement of prices over time, and to do this in the context of a reasonably tractable model, we abstract from payoff-related uncertainty and examine the effects of uncertainty concerning the amounts of wealth held by investors of differing risk preferences. As noted earlier, for market created risk to lead to sharp movements in price it is necessary not only that uncertainty about the state of the market not be resolved by observing the market clearing prices in trading at the initial date, but also that prices at a later date be different for different states of the market. As was indicated, both phenomena can occur when investors at the initial date know there will be a subsequent round of trading, and so are directly concerned only with anticipating the prices that will prevail at this later round, while at some later date investors expect no further trading to take place, and so choose portfolios on the basis of beliefs only about final payoffs. Obviously, backward recursion implies that prices at the initial date are indirectly dependent on beliefs about final payoffs, but initial prices are also affected by uncertainty about the state of the market since the true composition of investors will affect prices at the final round of trading.

In the following sections we examine a model in which it is assumed to be common knowledge that there will be exactly two rounds of trading. At the final trading date uncertainty about the state of the market plays no role in investors' demands and the market clearing prices will, in general, reveal the state of the market.

At the initial trading date investors know that future prices will vary depending on the actual state of the market and they form their current demands based on their prior beliefs about the possible states of the market and the future prices these states imply. In general, investors' demands are also affected by investors' wealth and risk preferences and these differ across the states of the market.

There are two general possibilities with respect to the nature of the market clearing prices at the initial date. One possibility is a separating equilibrium in which these prices differ at the initial trading date for different states of the market (and prices remain the same at the final trading date). We show that there is also the possibility, however, of a pooling equilibrium in which the same prices may clear the market at the initial date for different states of the market. In this case, prices will change at the second trading date (and portfolio changes will generate positive trading volume) even though no exogenous information signals are received.

Relative to the separating equilibrium, the pooling equilibrium appears to exhibit "excess" volatility. In this equilibrium, the pooling prices change to separating prices without any new information about final payoffs. Thus, the fundamentals of the assets, namely the probability and magnitude of final payoffs, are unchanged, but the prices change stochastically.

Endowment uncertainty is important in both equilibria, but it is only in the pooling equilibrium that investors can insure themselves against this risk. Before the initial trading begins, investors do not know the future values of their endowments. Risk-averse investors prefer to have the expected value of their endowments for sure rather than face the market created risk. The pooling equilibrium, in which initial trading takes place at pooling prices, allows investors to insure partially against the risk. However, the pooling equilibrium is not necessarily welfare improving compared to the separating equilibrium. In the separating equilibrium, initial prices reveal the value of the risk averse investors' endowments, giving them no chance to share risk through trading. In the first round of trading in the pooling equilibrium, fundamental value, i.e., the future trading price, is unknown to investors. Some investors may be better off not knowing it immediately but, because expected wealth is not the same in the two equilibria for the two types of investors, the equilibria are not Pareto comparable.

I. Assumptions

There are two types of investors: risk neutral and risk averse; for tractability the latter are assumed to possess logarithmic utility. There are three dates, $t = 1, 2, 3$, and two possible states of nature affecting the cash flows of assets. The state of nature is revealed at $t = 3$, which is when asset payoffs and consumption occur. There are two financial assets: "cash" and "stock." A unit of cash pays one unit of the consumption good in both states of nature. A share of stock pays one unit of the consumption good if one of the states of nature occurs and pays nothing if the other state occurs. The states of nature will be assumed to have equal probabilities of occurring.

Investors are endowed with certain amounts of cash and stock and trade to an equilibrium at $t = 1$. At $t = 2$ the market reopens and trading may again take place but a well defined market clearing price is established whether or not investors change their portfolios.

In addition to the two states of nature there are two alternate states of the market (described in more detail below), labeled $e = A, B$, with probabilities π^A, π^B $(=1 - \pi^A)$ of occurring. Let price refer to the price of a share of stock in units of cash. Then at $t = 2$ in the state A market the price is P^A, and similarly P^B is the price at $t = 2$ if the state of the market is B.

The difference between the two states of the market concerns the distribution across the two investor types of the aggregate endowments of the securities. All investors have the same prior probability beliefs about which state of the market prevails and no investor learns anything about the state of the market from observing his or her own risk preferences and endowment. All this is assumed to be common knowledge. There are various ways to arrange such a situation; for simplicity, we assume that there are different numbers of investors with different endowment patterns but for each possible risk preference/endowment combination there is at least one investor in each market state having that combination. An investor observing his or her own preferences and endowment learns only that there is at least one investor with that combination. Since that fact was known ex ante by everyone, all investors have the same posterior beliefs.[1]

For simplicity of exposition and notation, we will treat the aggregate of each investor type as a single investor. However, it should be noted that this simplification relies on two assumptions. One is the assumption described above that these aggregates, which are different in the two states of the market, are actually composed of different numbers of particular individual endowment patterns such that at least one investor has each pattern in each state of the market, so that observation of one's own endowment is not informative regarding the state of the market. The other assumption is similar: that $t = 1$ trading among investors within an aggregate combines with $t = 1$ trading between different types to render the total volume of $t = 1$ trading also uninformative as a signal of the state of the market.

We denote the log utility investors as type L and the risk-neutral investors as type N. We let Z_{jk} denote the aggregate initial endowment of security j, where $j = C$ denotes cash and $j = S$ denotes stock, for aggregate investors k, where $k = L, N$. At $t = 1$, a type k investor chooses holdings Y_{Ck}, Y_{Sk}, and these are held until the final round of trading at $t = 2$, when the type k investor chooses final holdings X_{Ck}, X_{Sk}.

II. Types of Equilibria

Since $t = 2$ is the last date at which trading can take place, individual demand decisions are independent of beliefs about the state of the market because forecasts of future prices are irrelevant. The decisions at the intial trading date, $t = 1$, are more complicated. Basically, two types of equilibria are possible. One

[1] For example, in market state A the group of log utility investors might contain 20 individuals and consist of 10 investors endowed with 0.5 shares of stock each (plus some cash) and another 10 investors endowed with 2.5 units shares of stock each. In market state B, on the other hand, the group of log utility investors might contain only eight individuals, with five investors endowed with 0.5 shares of stock each and three investors endowed with 2.5 shares of stock each. Thus, the aggregate holdings of stock by log utility investors are 30 shares ($10 \times 0.5 + 10 \times 2.5$) in market state A and 10 shares ($5 \times 0.5 + 3 \times 2.5$) in market state B. A given log utility individual's observation of his or her endowment of, say, 0.5 shares of stock provides no information to that individual about the true state of the market.

possibility is a separating equilibrium price: $\tilde{P}^A$ or $\tilde{P}^B$. In this case, investors trade immediately to their final portfolios and there is no price change or further trading at $t = 2$.

The other possibility is that the $t = 1$ equilibrium takes the form of a pooling price, P_1, which is the same no matter which state of the market actually prevails and so does not reveal the market state at $t = 1$. For this equilibrium to occur, the price P_1 must clear the market at $t = 1$ in both market states, given forecasts of the two possible values of the price that will prevail at $t = 2$, depending on the state of the market, and given prior probability beliefs at $t = 1$ about the state of the market.[2]

It should be noted that, in general, the two possible $t = 2$ prices in the $t = 1$ pooling equilibrium are different from the corresponding $t = 2$ prices in the equilibrium where market state revelation occurs at $t = 1$. This is because $t = 1$ trades take place at different prices in the two equilibria, so the distribution of wealth across investor types when the market reopens at $t = 2$ is also different.

III. Portfolio Optimization

Suppose the log utility investor faces a stock price of P^e at $t = 2$. Since this is the last trading date, the investor chooses security holdings based on final payoffs to solve

$$\max_{X_{CL}, X_{SL}} \tfrac{1}{2} \log(X_{CL} + X_{SL}) + \tfrac{1}{2} \log(X_{CL}) \tag{1}$$

$$\text{s.t.} \quad X_{CL} + P^e X_{SL} = Y_{CL} + P^e Y_{SL}.$$

The solutions to the first-order conditions for optimality are

$$X_{CL} = \frac{Y_{CL} + P^e Y_{SL}}{2(1 - P^e)},$$

$$X_{SL} = \frac{(1 - 2P^e)(Y_{CL} + P^e Y_{SL})}{2P^e(1 - P^e)} \tag{2}$$

At the price $P^e = \tfrac{1}{2}$, the log utility investor desires to hold only cash, while the risk neutral investor is indifferent between cash and stock at this price, given the beliefs about final payoffs. We assume that initial endowments of securities are such that a cash-only portfolio for the log utility investor at $t = 2$ is infeasible (because it would imply negative cash holdings for the risk neutral investor) in at least one state of the market.

Provided the demands of the risk neutral investor, to be examined below, are such that the same price does not clear the market in both market states, P^e will equal P^A or P^B depending on the market state.

At $t = 1$ two types of equilibria are potentially possible, as noted earlier. One

[2] As noted below, we assume that parameter values rule out a special case of the pooling equilibrium in which the price equals ½ at $t = 1$ and in both market states at $t = 2$, log utility investors hold only cash at both dates, and risk neutral investors bear all risk of final payoffs. Since price never changes in this case, there is no market created risk.

possibility is a separating equilibrium, in which the $t = 1$ price, $\tilde{P}^A$ or $\tilde{P}^B$, reveals the state of the market at the first round of trading. Our main interest is in the other possible equilibrium, of the pooling sort, in which the state of the market is not revealed in trading at $t = 1$ but investors know that the $t = 2$ price will be P^A or P^B with probabilities π^A and π^B, respectively, and these prices determine the relevant $t = 2$ payoffs (i.e., capital gains or losses) on the securities they choose at $t = 1$. The portfolio choice at $t = 1$ is made to maximize the expected derived utility of $t = 2$ wealth. For the log utility investor the derived utility of wealth function, as is well known, is also logarithmic. This can be seen by substituting from (2) in the objective function in (1).

In the separating equilibrium at $t = 1$, investors trade directly to their final holdings. Letting $\tilde{P}^e$ denote the $t = 1$ price in market state e in the separating equilibrium, the problem analogous to (1) for the $t = 1$ holdings of the log utility investor in the separating equilibrium yields

$$Y_{CL} = \frac{Z_{CL} + \tilde{P}^e Z_{SL}}{2(1 - \tilde{P}^e)},$$

$$Y_{SL} = \frac{(1 - 2\tilde{P}^e)(Z_{CL} + \tilde{P}^e Z_{SL})}{2\tilde{P}^e(1 - 2\tilde{P}^e)}. \tag{3}$$

In the separating equilibrium the price remains the same at $t = 2$. Since (3) implies that $Y_{CL} + \tilde{P}^e Y_{SL} = Z_{CL} + \tilde{P}^e Z_{SL}$, comparison of (2) and (3) shows that, when $P^e = \tilde{P}^e$, the $t = 2$ holdings of the log utility investor are the same as the $t = 1$ holdings: $X_{CL} = Y_{CL}$, $X_{SL} = Y_{SL}$.

In the pooling equilibrium, on the other hand, the $t = 1$ price faced by the log utility investor is P_1 and the $t = 1$ portfolio holdings are chosen to solve

$$\max_{Y_{CL}, Y_{SL}} \textstyle\sum_{e=A,B} \Pi^e \log(Y_{CL} + P^e Y_{SL}) \tag{4}$$

$$\text{s.t.} \quad Y_{CL} + P_1 Y_{SL} = Z_{CL} + P_1 Z_{SL}.$$

Substituting from the budget constraint, the first-order condition for optimality can be written

$$0 = \textstyle\sum_{e=A,B} \dfrac{\Pi^e(P_1 - P^e)}{(P_1 - P^e)Y_{CL} + P^e(Z_{CL} + P_1 Z_{SL})}. \tag{5}$$

The solution to (5) yields

$$Y_{CL} = \left[\frac{\Pi^A P^B}{P_1 - P^B} + \frac{\Pi^B P^A}{P_1 - P^A}\right](Z_{CL} + P_1 Z_{SL}),$$

$$Y_{SL} = \left[\frac{-\Pi^A}{P_1 - P^B} + \frac{-\Pi^B}{P_1 - P^A}\right](Z_{CL} + P_1 Z_{SL}). \tag{6}$$

It will turn out that an important role in the equilibrium is played by the case when the solution to (6) is $Y_{CL} = 0$, $Y_{SL} = (Z_{CL}/P_1) + Z_{SL}$, i.e., when the log utility investors chooses to invest all wealth in stock. From (6), it can be seen that this case occurs when the $t = 1$ price equals the harmonic mean of the $t = 2$

prices:

$$\frac{1}{P_1} = \frac{\Pi^A}{P^A} + \frac{\Pi^B}{P^B}. \tag{7}$$

We note in passing that it can be seen from (5) that this harmonic mean result would imply $Y_{CL} = 0$ as well in a model where the number of states of the market exceeded two. We also note that if we had defined price as number of shares of stock per unit of cash then (7) would have expressed P_1 as the ordinary arithmetic mean of P^A and P^B. It seems more natural, however, to have price be in terms of the cash numeraire, as we have done.

Now consider the $t = 2$ problem of the risk neutral investor:

$$\max_{X_{CN}, X_{SN}} \ \tfrac{1}{2}(X_{CN} + X_{SN}) + \tfrac{1}{2}X_{CN} \tag{8}$$

$$\text{s.t.} \quad X_{CN} + P^e X_{SN} = Y_{CN} + P^e Y_{SN}.$$

We assume that negative holdings—i.e., short sales—of securities are prohibited. Since our model has two representative investors, short sales would require an investor to be able to issue a security, because there is no third party to borrow it from. If we confine our attention to parameter values such that $P^e < \tfrac{1}{2}$, this implies that the optimal solution to (8) is for the risk-neutral investor to invest solely in stock at $t = 2$:

$$X_{CN} = 0,$$

$$X_{SN} = \frac{Y_{CN}}{P^e} + Y_{SN}. \tag{9}$$

From (8) and (9), we see that the derived utility of $t = 2$ wealth for the risk-neutral investor, in states of the market such that $P^e < \tfrac{1}{2}$, is linear in the quantity $(Y_{CN}/P^e) + Y_{SN}$, which is the number of shares of stock that the risk-neutral investor can buy at $t = 2$ in market state e by investing all wealth in stock.

In the separating equilibrium, consideration of (8) and (9) shows that the risk-neutral investor's $t = 1$ demands are

$$Y_{CN} = 0,$$

$$Y_{SN} = \frac{Z_{CN}}{\tilde{P}^e} + Z_{SN}. \tag{10}$$

These demands remain the same at $t = 2$ in the separating equilibrium.

In the pooling equilibrium, still assuming that parameter values are such that $P^e < \tfrac{1}{2}$ at $t = 2$ for all states of the market, the risk-neutral investor's portfolio problem at $t = 1$ is

$$\max_{Y_{CN}, Y_{SN}} \ \textstyle\sum_{e=A,B} \Pi^e\left(\frac{Y_{CN}}{P^e} + Y_{SN}\right) \tag{11}$$

$$\text{s.t.} \quad Y_{CN} + P_1 Y_{SN} = Z_{CN} + P_1 Z_{SN}.$$

Substituting for Y_{CN} from the budget constraint, the problem in (11) can be rewritten as

$$\max_{Y_{SN}} Y_{SN} \sum_{e=A,B} \Pi^e \left(1 - \frac{P_1}{P^e}\right) + (Z_{CN} + P_1 Z_{SN}) \sum_{e=A,B} \frac{\Pi^e}{P^e}. \tag{12}$$

Suppose P_1 equals the harmonic mean of the $t = 2$ prices as in (7). Then it is immediately seen in (12) that the risk-neutral investor is indifferent at $t = 1$ to the composition of his or her portfolio between cash and stock. At this price the risk-neutral investor is willing to supply the log utility investor's demand to invest completely in stock, as long as doing so does not violate the restriction against going short in cash. Assuming this restriction can be obeyed in both states of the market, P_1 will be an equilibrium price in both market states. In this case, investors will learn nothing about the state of the market by observing the price at $t = 1$ and so will use their prior beliefs, π^A and π^B, as was assumed in (4) and (11).

A $t = 1$ equilibrium at the price P_1 may at first seem counter intuitive, since it has the risk averse log utility investors holding only the more risky security in terms of final payoffs—viz., the stock. The desire of the log utility investors to hold stock over cash is easy to explain. With respect to $t = 2$ net payoffs on $t = 1$ holdings—i.e., capital gains and losses—cash gives a zero payoff in both market states. The capital gain/loss on stock is $P^A - P_1$ in market state A and $P^B - P_1$ in market state B. When P_1 satisfies (7), the expected net payoff on stock is positive since an arithmetic mean exceeds a harmonic mean. So stock offers a risk premium relative to (riskless) cash just large enough for an investor with log utility to prefer stock to cash.

The situation with respect to the risk neutral investor is a bit more subtle. The stock offers a risk premium relative to cash, so one might suppose a risk-neutral investor would prefer stock to cash even more strongly than would a risk-averse investor. The key is that the risk-neutral investor is risk neutral with respect to final payoffs, but not necessarily with respect to $t = 2$ wealth. As we observed earlier, the risk-neutral investor's derived utility is linear in the maximum number of shares of stock that can be bought with $t = 2$ wealth, where wealth is measured in units of cash. Because the number of shares that can be bought depends on P^e and this price varies across market states, the risk-neutral investor's derived utility of wealth, viewed at $t = 1$, is state dependent across $t = 2$ market states. At $t = 1$, this state dependence has an effect on portfolio choice similar to that of risk aversion over final payoffs. In the present case, the state dependence effect is stronger than the risk aversion of logarithmic utility as regards the risk premium required to prefer stock to cash.

IV. Separating Equilibrium

The separating equilibrium is straight forward. Since the risk-neutral investor holds no cash at $t = 1$ (see (10)), the log utility investor's $t = 1$ cash holding must equal the total supply of cash. Setting Y_{CL} in (3) equal to $Z_{CL} + Z_{CN}$, yields

$$\tilde{P}^e = \frac{2Z_C - Z_{CL}^e}{2Z_C + Z_{SL}^e} \quad (e = A, B), \tag{13}$$

where $Z_C = Z^e_{CL} + Z^e_{CN}$, the superscript on endowments indicates the state of the market, and the total supply of cash, Z_C, has no superscript since it is the same in both market states. Since price reveals the true state of the market in the separating equilibrium, the price in (13) does not depend on the prior beliefs about the identity of the market state.

V. Pooling Equilibrium

Putting together earlier results, we can also describe the pooling equilibrium in which the state of the market is not revealed at the first round of trading. We know that such an equilibrium has the risk-neutral investor holding a stock-only portfolio at $t = 2$, so the $t = 2$ cash holding of the log utility investor, X_{CL}, must equal the total supply of cash. Furthermore, we know that the log utility investor will choose a stock-only portfolio at $t = 1$, so that $Y_{CL} = 0$ and $Y_{SL} = (Z_{CL}/P_1) + Z_{SL}$. Substituting the latter values in (2) and equating X_{CL} to the total supply of cash, $Z_{CL} + Z_{CN}$, we obtain

$$P^e = \frac{2Z_C}{2Z_C + \dfrac{Z^e_{CL}}{P_1} + Z^e_{SL}} \quad (e = A, B). \tag{14}$$

Substituting from (14) in the harmonic mean condition (7) for P_1 and simplifying gives

$$P_1 = \frac{2Z_C - \Pi^A Z^A_{CL} - \Pi^B Z^B_{CL}}{2Z_C + \Pi^A Z^A_{SL} + \Pi^B Z^B_{SL}} \tag{15}$$

The combination of (14) and (15) relates P^A, P^B, and P_1 to the exogenous endowments and prior probability beliefs. As noted earlier, the pooling equilibrium described by (14) and (15) is valid only for sets of parameter values such that $P^A < 1/2$ and $P^B < 1/2$, since these conditions were assumed in deriving the equilibrium.

VI. Numerical Example

To illustrate the separating and pooling equilibria, consider the following numerical example. The two states of the market have the same total amounts of cash and stock outstanding but differ in how much of the total supplies is in the hands of each investor type. For simplicity, we will assume that the two states of the market are equally likely (i.e., $\pi^A = 1/2$). The endowments of the two aggregate investor types are as follows:

Endowments	Cash	Stock
Market state A		
Log utility	$Z^A_{CL} = 3$	$Z^A_{SL} = 30$
Risk neutral	$Z^A_{CN} = 2$	$Z^A_{SN} = 70$
Market state B		
Log utility	$Z^B_{CL} = 1$	$Z^B_{SL} = 10$
Risk neutral	$Z^B_{CN} = 4$	$Z^B_{SN} = 90$

Substituting the assumed values in (13), the separating equilibrium prices in the two states of the market are

$$\tilde{P}^A = 0.175, \qquad \tilde{P}^B = 0.450.$$

In the separating equilibrium, the investors trade at $t = 1$ to their final portfolios. Substituting in (3) and (10), the final holdings are:

Separating equilibrium holdings	Cash	Stock
Market state A		
Log utility	$X_{CL} = Y_{CL} = 5$	$X_{SL} = Y_{SL} = 18.57$
Risk neutral	$X_{CN} = Y_{CN} = 0$	$X_{SN} = Y_{SN} = 81.43$
Market state B		
Log utility	$X_{CL} = Y_{CL} = 5$	$Y_{SL} = Y_{SL} = 1.11$
Risk neutral	$X_{CN} = Y_{CN} = 0$	$Y_{SN} = Y_{SN} = 98.89$

The more interesting case is when market created risk exists because the $t = 1$ equilibrium is of the pooling type. Substituting in (14) and (15), the $t = 1$ price and the $t = 2$ prices in the two market states are

$$P_1 = 0.267, \qquad P^A = 0.195, \qquad P^B = 0.421.$$

Let us consider the pooling equilibrium in more detail. The price at $t = 1$ is 0.267, regardless of the true state of the market, while investors forecast that the $t = 2$ price will be 0.195 or 0.421, depending on whether the state of the market is A or B, respectively. Since P_1 is the harmonic mean of P^A and P^B, we know that the log utility investor desires to hold only stock at $t = 1$, and we know that the risk-neutral investor is indifferent between cash and stock at $t = 1$. The $t = 1$ equilibrium holdings from (6) and (12) are:

Pooling equilibrium $t = 1$ holdings	Cash	Stock
Market state A		
Log utility	$Y_{CL} = 0$	$Y_{SL} = 41.25$
Risk neutral	$Y_{CN} = 5$	$Y_{SN} = 58.75$
Market state B		
Log utility	$Y_{CL} = 0$	$Y_{SL} = 13.75$
Risk neutral	$Y_{CN} = 5$	$Y_{SN} = 86.25$

Comparing these holdings with the original endowments, we see that when the true market state is A the log utility investor sells 3 of cash for $3/0.267 = 11.25$ shares of stock and thus has a $t = 1$ stock holding of $30 + 11.25 = 41.25$ shares. When the true state is B the log utility investor sells 1 of cash for $1/0.267 = 3.75$ shares of stock, giving a $t = 1$ stock holding of $10 + 3.75 = 13.75$ shares.

At $t = 2$, investors form portfolios based on the distribution of final payoffs. This causes the $t = 2$ price to depend on the true market state: different prices clear the $t = 2$ market in different market states. If the market state is A, then the price drops from 0.267 at $t = 1$ to 0.195 at $t = 2$; if the state of the market is B, then the price rises from 0.267 at $t = 1$ to 0.421 at $t = 2$. Substituting in (2) and (9), the investors' holdings at $t = 2$ are:

Pooling equilibrium $t = 2$ holdings	Cash	Stock
Market state A		
Log utility	$X_{CL} = 5$	$X_{SL} = 15.62$
Risk neutral	$X_{CN} = 0$	$X_{SN} = 84.38$
Market state B		
Log utility	$X_{CL} = 5$	$X_{SL} = 1.87$
Risk neutral	$X_{CN} = 0$	$X_{SN} = 98.13$

In either market state, risk-neutral investors hold only the more risky security at $t = 2$, as we would expect, while log utility investors hold the total supply of the less risky security.

Comparing the $t = 2$ holdings in the two equilibria shows that the variability across market states of final payoffs to the risk-averse log utility investor is smaller in the pooling equilibrium than in the separating equilibrium. The payoffs are the same (i.e., 5) in the state of the world when only the bond pays off. In the state of the world when both securities pay off, the total payoffs to the log utility investor depending upon whether the market state is A or B are 23.57 and 6.11 in the separating equilibrium, as compared with 20.63 and 6.87 in the pooling equilibrium. This effect is a general phenomenon for any value of π^A and represents the fact that the pooling equilibrium allows the log utility investor to hedge partially the endowment risk across states of the market.

While the pooling equilibrium gives the log utility investor less variability across market states of final consumption, this does not imply that the log utility investor always achieves a higher level of expected utility in the pooling equilibrium as compared to the separating equilibrium. The reason is that the log utility investor may have a lower expected return in the pooling equilibrium.

In the example, the expected payoff to the log utility investor across states of the market and states of the world is $(5 + 20.63 + 5 + 6.87)/4 = 9.38$ in the pooling equilibrium versus $(5 + 23.57 + 5 + 6.11)/4 = 9.92$ in the separating equilibrium. This drop in expected return has a stronger effect on the expected utility of the log utility investor than does the lowering of variability of return. The log utility investor's expected utility is 2.043 in the pooling equilibrium versus 2.047 in the separating equilibrium. The risk-neutral investor, on the other hand, has a higher expected utility in the pooling equilibrium than in the separating equilibrium. However, other numerical examples produce the opposite preference ordering.

VII. Market Completeness

Clearly, the market in our model is complete with respect to the two states of the world. Less obviously, it is also complete at $t = 1$ with respect to the two states of the market which will affect prices at $t = 2$. Provided the risk-neutral investor has a large enough endowment of stock, as is the case in our numerical example, there is no need to impose a short sale prohibition at $t = 1$. Therefore, consider a portfolio at $t = 1$ containing $P^B/(P^B - P^A)$ of cash and $-1(P^B - P^A)$ shares of stock. This portfolio will pay off one unit of cash at $t = 2$ if the market state is A and nothing if the market state is B. Similarly, a portfolio containing $-P^A/(P^B - P^A)$ in cash and $1/(P^B - P^A)$ shares of stock will pay off one unit of cash at $t = 2$ if the market state is B and nothing if the market state is A. Thus, incompleteness of the market with respect to the states of the market is not in general a factor in the existence of market created risk.

VIII. Concluding Comments

The particular model that we have used to show how price can change through the effects of market created risk, even in the absence of new information about asset payoffs, is clearly dependent on the assumed existence of a group of risk neutral investors. It is the indifference of these investors about portfolio composition, when current price is the harmonic mean of future prices, that allows the pooling price to clear the market. However, the possibility of a pooling equilibrium that possesses the qualitative features of the equilibrium in our model can also exist for models in which all investors are risk averse, although the details are complicated.

What we have in mind is a model, more closely approximating the real world, in which the number of possible market states, representing possible distribution of holding across different investor types, is large. To have prices affected by market created risk in such a model means only that there are two or more different market states such that, given the possible prices that will prevail when the actual market state is revealed, the different distributions of holdings across investor types in these market states all are consistent with the same price clearing the market at the current date. Although it may be difficult to find simple combinations that achieve this result exactly, we see nothing in principle that precludes such situations. When one considers the noise of other factors in the real world, such that investors can draw only rough influences from the current market price, then the possibilities for uncertainty about the true state of the market are quite real. At the limit, with a continuum of market states, pooling equilibria would seem to be generic.

This calls attention to the major role of aggregation in distinguishing between models of financial markets where market uncertainty is an important factor and those models where it is not. Aggregation is important in standard models of financial markets because when the conditions permitting aggregation hold, demand equations for all investors in the market have the same structure. Asset pricing equations can then be developed of a very tractable form, based on the pricing that would be implied by just a single representative investor in the market. When the conditions for aggregation hold, however, the importance of market created risk is diminished if not eliminated. The reason is that market uncertainty is concerned with an investor's uncertainty about the true nature of financial markets that will be setting future prices. When it is common knowledge that aggregation holds, investors have a substantial amount of information about the nature of the financial markets and in models having fully revealing rational expectations equilibria prices can reveal the rest.

As a final note, we call attention to one feature of our model for which the real world counterpart seems much richer in implications for dynamic price patterns, but more difficult to incorporate analytically, than our simplified case. (There are, of course, other features in this category.) This is our assumption that uncertainty about the state of the market will be completely resolved at some date. In the real world, uncertainty about the state of the market is both reduced and augmented by the unfolding of events. Just as uncertainty about asset payoffs is never finally resolved, market created risk is never finally eliminated.

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