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Common Stochastic Trends in a System of Exchange Rates

RICHARD T. BAILLIE and TIM BOLLERSLEV*

ABSTRACT

Univariate tests reveal strong evidence for the presence of a unit root in the univariate time-series representation for seven daily spot and forward exchange rate series. Furthermore, all seven spot and forward rates appear to be cointegrated; that is, the forward premiums are stationary, and one common unit root, or stochastic trend, is detectable in the multivariate time-series models for the seven spot and forward rates, respectively. This is consistent with the hypothesis that the seven exchange rates possess one long-run relationship and that the disequilibrium error around that relationship partly accounts for subsequent movements in the exchange rates.

A GENERAL CONSENSUS HAS emerged in recent years that many macroeconomic time series, such as GNP, consumption expenditures, disposable income, etc., can be characterized by a stochastic trend model. In particular, Nelson and Plosser [22] described this property as one of being “difference stationary” so that the first difference of a time series is stationary. An alternative “trend stationary” model, where a stationary component is added to a deterministic trend term, has generally been found to be less appropriate.¹ Similarly, it has long been recognized that many financial time series, such as foreign exchange rates, are nonstationary, e.g., in Meese and Singleton [21].

The issue of nonstationary is not merely a statistical curiosity but has several important implications for the modeling of exchange rates. For instance, many models of exchange rate determination under rational expectations require stationarity assumptions when solving out the expected future values of the fundamentals. Further, there has been some controversy over the appropriate transformations to use when conducting tests of whether the forward rate is an unbiased and efficient predictor of the future spot exchange rate.² Some authors, e.g., Frenkel [11, 12], have conducted tests in levels, while Geweke and Feige [14] and Hansen and Hodrick [17] have used variables such as $(s_t - f_t)$ or $(s_{t+k} - f_t)$, where s_t and f_t are the logarithms of the spot and forward rates, respectively, and k is the time to maturity of the forward contract. Depending on the degree of nonstationarity of the different variables involved, standard inference procedures may or may not apply.

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¹ See Nelson and Plosser [22], Harvey [18], Schwert [27], and Watson [31], among others.

² See, for instance, Baillie [1] and Hodrick [19].

All previous studies addressing these issues have essentially conducted the tests of nonstationarity in a univariate time-series framework. However, in the context of a system of exchange rates or other asset prices, it seems more realistic to consider the joint determinants or fundamentals of the system. Variants of the monetary model of exchange rate determination generally assume separate sets of fundamentals for each currency. At the same time it frequently appears that many freely floating exchange rates share common long-run temporal movements, while others are formally linked to the U.S. dollar or are in the European Monetary System (EMS). Thus, a potentially important question concerns how many stochastic trends are responsible for driving a system of exchange rates and also how these trends contribute to the apparent nonstationarity of the univariate time-series representations for exchange rates.

Centrally related to the issue of nonstationarity of economic and financial time series is the concept of cointegration.³ In this context, if a linear combination of nonstationary variables is stationary, the variables are said to be cointegrated and the stationary linear combination can be interpreted as being an equilibrium error. As explained in Appendix B, nonstationarities in a vector process can be equivalently modeled in terms of a vector of common stochastic trends.⁴ The number of such stochastic trends, or common long-run components, is equal to the dimension of the system minus the number of linearly independent cointegrating vectors. Following some recent theoretical results by Johansen [20], a relatively simple test for the dimensionality of the common stochastic trend process is now available. In Section III we implement Johansen's [20] procedure and find that six unit roots appear to be present in the vector autoregressions for a set of seven daily spot and seven one-month forward rates, respectively. This has interesting implications for the determinants of a system of floating exchange rates and is consistent with the notion that the system of exchange rates is tied to one long-run equilibrium path. Furthermore, the error around this equilibrium is an important ingredient of the random innovations in successive changes in the exchange rates.

As already mentioned, the finding of cointegration among a set of variables also has important consequences for the proper specification and statistical testing procedures to employ when simultaneously modeling the variables. Campbell and Shiller [4] discuss some of these ideas as they relate specifically to models of the term structure of interest rates and present value models of stock prices. For example, first differencing of all the nonstationary variables imposes too many unit roots, and any potentially important long-run relationship between the variables will be obscured. Estimating the models in levels involving nonstationary variables, however, invalidates standard asymptotically based statistical theory and severely complicates the inference procedures.⁵ One solution to this problem is based on the so-called error correction formulation, as described in more detail in Appendix B.

³ See Engle and Granger [9] and Granger [15].

⁴ See also Stock and Watson [30].

⁵ See Sims, Stock, and Watson [28].

The plan for the rest of the paper is as follows. In order to motivate the multivariate analysis, a series of tests for unit roots in the univariate time-series representations of the seven daily spot and one-month forward foreign exchange rates is described in Section I. The testing procedures are based on work by Perron [24], Phillips [25], and Phillips and Perron [26] and are applicable to a wide variety of weakly dependent and heterogeneously distributed time series. Details of the test statistics are deferred to Appendix A. Given the documented nonstationary nature of each of the univariate series, Section II deals with testing for cointegration between the seven pairs of spot and forward rates. In the present context with daily data the one-month forward rate forecast errors are serially correlated through time. However, the Phillips-Perron tests are ideally suited to testing for cointegration in relationships with overlapping data and variables related under the rational expectations hypothesis, so that the exact order of the serial correlation is known a priori. The results discussed above relating to the joint determination and to the common long-run components for the system of exchange rates are presented in Section III, along with a more detailed discussion of the implications of the empirical findings. The paper ends with a brief conclusion.

I. Univariate Tests for Unit Roots

The original work on testing for unit roots in exchange rates was done by Meese and Singleton [21], who considered weekly data for the Swiss, Canadian, and West German exchange rates against the U.S. dollar for the period from January 1976 to July 1981. The Meese and Singleton [21] study applied the original tests for unit roots due to Dickey and Fuller [6, 7].

There is now a substantial body of documented evidence that many exchange rate and financial market series exhibit time-dependent heteroskedasticity.⁶ The work of Phillips and Perron [26] provides tests for unit roots which are robust to a wide variety of serial correlation and time-dependent heteroskedasticity. A recent study by Corbae and Ouliaris [5]⁷ has applied these tests to weekly spot and forward rate series where the forward rates have thirteen-week maturity time. However, as noted by Baillie and Bollerslev [3], the degree and persistence of the time-dependent heteroskedasticity and the level of kurtosis are far more pronounced in daily than in weekly exchange rate data. Consequently, as a preliminary diagnostic check, it seems worthwhile applying the Phillips-Perron tests to daily exchange rate data, also.

The tests involve computing one of three OLS regressions defined from

$$y_t = \hat{\alpha}y_{t-1} + \hat{u}_t, \quad (1)$$

$$y_t = \mu^* + \alpha^*y_{t-1} + u_t^*, \quad (2)$$

$$y_t = \tilde{\mu} + \tilde{\beta}(t - n/2) + \tilde{\alpha}y_{t-1} + \tilde{u}_t, \quad (3)$$

⁶ See Baillie and Bollerslev [3] and the references therein.

⁷ We are grateful to an anonymous referee for bringing this paper to our attention.

where n denotes the sample size and the innovation sequences $\hat{u}_t$, u_t^* , and $\tilde{u}_t$ could be stationary ARMA processes with time-dependent variances.

In model (1) the null hypothesis of a unit root, i.e., $H_0^1: \hat{\alpha} = 1$, is tested against the stationary alternative, $H_A^1: \hat{\alpha} < 1$, by the adjusted t -statistic $Z(t_{\hat{\alpha}})$, given by equation (A1) in Appendix A. Fuller [13] shows that at the one percent and five percent levels the critical values of $Z(t_{\hat{\alpha}})$ are -2.58 and -1.95 , respectively.

In model (2) the null hypotheses of a unit root, with or without a drift, i.e., $H_0^2: \alpha^* = 1$ and $H_0^3: \mu^* = 0, \alpha^* = 1$, are tested against the stationary alternatives by means of the adjusted t - and F -statistics $Z(t_{\alpha^*})$ and $Z(\Phi_1)$ given by (A2) and (A3) in Appendix A. At the one percent and five percent levels the critical values of $Z(t_{\alpha^*})$ are -3.43 and -2.86 , respectively, and those of $Z(\Phi_1)$ are 6.43 and 4.59 , respectively.⁸

Given equation (3), which allows for a deterministic trend, we can test the hypotheses $H_0^4: \tilde{\alpha} = 1$, $H_0^5: \tilde{\beta} = 0, \tilde{\alpha} = 1$, and $H_0^6: \mu = 0, \tilde{\beta} = 0, \tilde{\alpha} = 1$ by means of the test statistics $Z(t_{\tilde{\alpha}})$, $Z(\Phi_3)$, and $Z(\Phi_2)$. Again, the details of these statistics are given by the formulas (A4), (A5), and (A6) in Appendix A. Under the null hypotheses the one percent and five percent critical values of $Z(t_{\tilde{\alpha}})$, $Z(\Phi_3)$, and $Z(\Phi_2)$ will be -3.96 and -3.41 , 8.27 and 6.25 , and 6.09 and 4.68 , respectively.⁹

In the following analysis we took daily spot and thirty-day forward exchange rate data from the New York Foreign Exchange Market between March 1, 1980 and January 28, 1985, which constitute a total of 1,245 observations. The data were provided by Data Resources Incorporated (DRI) and are opening bid prices for the UK, West Germany, France, Italy, Switzerland, Japan, and Canada vis-à-vis the U.S. dollar. The six different test statistics were calculated for all seven currencies. In evaluating the test statistics a truncation lag, l , corresponding to the maximum order of nonzero autocorrelations in the disturbances had to be chosen; see Appendix A for details. The statistics were computed for $l = 0, 2, 4, 6$, and 10 but were found to be remarkably similar for different values of l . For reasons of space, only the results for $l = 10$ are reported in Table I. Along these lines, it is interesting to note that, in this situation with approximately uncorrelated innovations, the Monte Carlo evidence reported in Schwert [27] indicates that the finite sample distributions of the Phillips-Perron tests are reasonably close to the asymptotic Dickey-Fuller results.

Turning to the results, both the simple unit root tests of the t -statistic type, $Z(t_{\alpha^*})$ and $Z(t_{\tilde{\alpha}})$, are insignificant for all fourteen series. At the same time the $Z(\Phi_1)$ statistic rejects the random walk without a drift at the usual five percent level or lower for the UK, France, and Italy. This might explain the significant value of $Z(t_{\hat{\alpha}})$ for the UK. For none of the other currencies does the adjusted t -test based on the regression in (1) reject the null of a unit root against a stationary alternative at any level. In fact, for all the currencies but the UK, the estimated value of $\hat{\alpha}$ is in excess of one. However, given the large appreciation of the dollar over the sample period, the results based on $Z(t_{\hat{\alpha}})$ and the regression in (1) excluding a constant term should be interpreted very carefully. The inclusion of a time trend as in (3) and the use of the $Z(\Phi_2)$ statistic do not change the

⁸ See Dickey and Fuller [7] and Fuller [13].

⁹ See Dickey and Fuller [7] and Fuller [13].

Table I
Tests for Unit Roots in the Logarithms of Spot and Forward Exchange Rates^a

Country	y_t	$Z(t_c)$	$Z(t_{cr})$	$Z(\phi_1)$	$Z(t_c)$	$Z(\phi_2)$
UK	s_t	-2.544*	0.727	4.663*	-2.495	5.780*
	f_t	-2.553*	0.807	4.829*	-2.427	5.803*
West Germany	s_t	2.009	-0.908	2.853	-2.398	3.621
	f_t	2.050	-0.910	2.953	-2.399	3.691
France	s_t	2.935	-0.674	4.918*	-2.573	5.540*
	f_t	3.012	-0.711	5.203*	-2.426	5.463*
Italy	s_t	3.576	-0.969	6.995**	-2.127	6.199**
	f_t	3.561	-1.024	6.999**	-2.131	6.107**
Switzerland	s_t	1.469	-0.490	1.370	-2.271	2.731
	f_t	1.549	-0.569	1.573	-2.272	2.799
Japan	s_t	0.074	-1.796	1.620	-2.582	2.584
	f_t	0.089	-1.741	1.524	-2.535	2.520
Canada	s_t	1.380	-1.145	1.971	-2.793	3.530
	f_t	1.421	-1.196	2.115	-2.773	3.548

^a The data are daily spot and one-month forward exchange rates against the U.S. dollar for 1980-1985. The six different statistics all test for a unit root in the univariate time-series representation for each of the seven spot rates, s_t , and forward rates, f_t , against a stationary or trend-stationary alternative. The exact forms of the tests are given in Appendix A.

* Rejection of a unit root occurs at the five percent level.

** Rejection of a unit root occurs at the one percent level.

qualitative results. The overall conclusion is that there is strong evidence for the presence of a unit root for all seven currencies, spot and forward rate series alike.

Although none of the innovation series from the three sets of regressions exhibit any significant serial correlation, not surprisingly, Lagrange Multiplier (LM) tests for time-varying conditional heteroskedasticity, not reported here, were found to be highly significant for all the currencies.¹⁰ Hence, all the series appear to be stationary in their first differences, i.e., integrated of order one, or $I(1)$ in the terminology of Engle and Granger [9], and well approximated by a martingale difference sequence, but with time-dependent heteroskedasticity.

II. Tests of Cointegration

Since both spot and forward rate series exhibit $I(1)$ behavior for all seven currencies examined, a natural corollary concerns whether there exists some long-run equilibrium relationship between the seven pairs of spot and forward rates. Following Engle and Granger [9], a linear combination of $I(1)$ variables,

$$y_{t+k} = s_{t+k} - a - bf_t, \quad (4)$$

will also generally be $I(1)$. However, if y_{t+k} is covariance stationary, i.e., $y_{t+k} \sim I(0)$, then s_{t+k} and f_t are defined to be cointegrated of order 1, 1, i.e., $CI(1, 1)$, and y_{t+k} is readily interpreted as being a transitory equilibrium error; see Appendix B for further discussion of these ideas. The property of being cointegrated will be valid for all integer values of k , although in the context of daily exchange rates and thirty-day forward rates it is natural to think in terms of $k = 22$ (i.e., twenty-two working days) since this is the time period over which the expectation typically takes place.¹¹

Thus, we estimated equation (4) by OLS to obtain:

$$s_{t+22} = \hat{a} + \hat{b}f_t + \hat{y}_{t+22}, \quad (5)$$

where $\hat{y}_{t+22}$ denotes the OLS residuals. Under the null hypothesis of no cointegration, $\hat{y}_t$ will be $I(1)$. In order to test for cointegration, we therefore want to test for a unit root in $\hat{y}_t$. Since, by construction, the OLS residuals $\hat{y}_t$ have mean zero, the tests for a unit root should be based on $Z(t_{\hat{\alpha}})$ and the regression in (1) excluding an intercept.

However, even though the OLS estimates of $\hat{a}$ will converge to the true value of a at rate n , as opposed to $\sqrt{n}$ with stationary regressors (see Stock [29]), the asymptotic Dickey-Fuller critical values are no longer appropriate when testing for a unit root in the regression residuals, $\hat{y}_t$. In particular, in finite samples, the estimated residuals $\hat{y}_t$ will appear "more stationary" than the true value of y_t , and the Dickey-Fuller critical values will be numerically too small, leading to a rejection of a unit root in $\hat{y}_t$, i.e., finding cointegration, too often. Some Monte Carlo evidence along these lines is reported in Engle and Granger [9] and Engle

¹⁰ See also Baillie and Bollerslev [3].

¹¹ While this generally matches the forward rate with the spot rate in the future that would be used to cover an open forward position, the alignment could be one or two days off around the beginning of a new month; see Hodrick [19].

Table II

Tests for Cointegration between the Logarithms of Spot and Forward Exchange Rates^a

Country	$\hat{a}$	$\hat{b}$	$\hat{a}, \hat{b}$	$a = 0, b = 1$	
			$Z(t_{\hat{a}})$	$Z(t_{\hat{a}})$	$Z(t_{\hat{a}^*})$
UK	-0.0187	1.0135	-5.111**	-4.655**	-5.009**
West Germany	-0.0301	0.9802	-5.705**	-5.463**	-5.778**
France	-0.0379	0.9852	-5.597**	-5.450**	-5.637**
Italy	-0.0892	0.9886	-5.692**	-5.641**	-5.740**
Switzerland	-0.0298	0.9756	-5.582**	-5.408**	-5.668**
Japan	-0.8347	0.8476	-4.955**	-5.073**	-5.106**
Canada	-0.0095	0.9599	-5.953**	-6.096**	-6.087**

^a Daily spot, s_t , and one-month forward, f_t , exchange rates for 1980–1985. $\hat{a}$ and $\hat{b}$ denote the OLS estimates from a regression of s_t on a constant and f_{t+22} . $Z(t_{\hat{a}})$ and $Z(t_{\hat{a}^*})$ are tests for a unit root in $s_t - a - bf_{t+22}$ evaluated at $(a, b) = (\hat{a}, \hat{b})$ and $(a, b) = (0, 1)$.

** Rejection of a unit root (i.e., of no cointegration) occurs at the one percent level.

and Yoo [10]. For instance, in a simple bivariate system as here but with only 200 observations, Engle and Yoo [10] report the one percent and five percent critical values for the Dickey-Fuller t -test for $\hat{\alpha} = 1$ in (1) to be -4.00 and -3.37 , respectively, as opposed to -2.59 and -1.95 with y_t observable.

The OLS estimates of $(\hat{a}, \hat{b})$ along with the $Z(t_{\hat{a}})$ statistic calculated for $\hat{y}_t$ are presented in Table II. Given the overlapping nature of the data in (5), it follows that y_t will exhibit serial correlation through lag 21 (see Hansen and Hodrick [17]), a fact which the Phillips-Perron tests are ideally suited to handle by the choice of the truncation lag, l . Thus, when computing the test statistic $Z(t_{\hat{a}})$, the estimated residual variance S_{nl}^2 in (A7) was calculated with $l = 21$.¹² Using the simulated critical values from Engle and Yoo [10], the results in Table II imply rejection of the hypothesis $\hat{\alpha} = 1$ in (1) for $\hat{y}_t$ at levels much lower than one percent for all seven sets of residuals. Hence, we conclude that there is strong evidence of cointegration between daily spot and thirty-day forward exchange rates. Interestingly, the value of the test statistic is very similar across currencies. Also, the OLS estimates of $(\hat{a}, \hat{b})$ are generally very close to $(0, 1)$, except for the case of Japan.¹³

Of course, if the forward rate is an unbiased predictor of the corresponding future spot rate, it follows that $(a, b) = (0, 1)$. Thus, we also calculated the two test statistics $Z(t_{\hat{a}})$ and $Z(t_{\hat{a}^*})$ for y_t defined from (4) with $(a, b) = (0, 1)$ imposed. Again, the test statistics reported in Table II are calculated with $l = 21$ to accommodate the moving-average error structure in y_t . Note, in this situation with y_t observable, the usual Dickey-Fuller critical values are applicable when carrying out the tests. Not surprisingly, the results in Table II do not change the qualitative conclusions obtained when actually estimating the cointegrating relationship. For all seven pairs of spot and forward rates, $Z(t_{\hat{a}})$ and $Z(t_{\hat{a}^*})$ firmly

¹² Very similar results were obtained for higher values of l .

¹³ Note, standard inference procedures regarding $(\hat{a}, \hat{b})$ do not apply in this situation with $I(1)$ regressors.

reject the null hypothesis of a unit root in the forward rate forecast errors, i.e., find cointegration. This is in accordance with the results reported in Corbae and Ouliaris [5], who use weekly data on the three-month forward rate forecast errors. Of course, these findings do not imply the absence of a time-varying risk premium. However, the deviations from the unbiasedness hypothesis are stationary, or transitory, in nature. Thus, when actually modeling time-varying risk premia in the forward foreign exchange market, these observations should be taken into account.¹⁴

III. Multivariate Tests for Unit Roots

The preceding sections have provided reassuring new evidence that daily spot and forward exchange rates have a unit root or a stochastic trend in their univariate time-series representations but that the forward rate forecast errors are stationary; i.e., the spot and forward rates are cointegrated. However, foreign exchange markets involve the simultaneous determination of several exchange rates, and in many situations it is more natural to consider the joint modeling of two or more currencies. This therefore raises the additional question of whether the long-run movements of a set of exchange rates are determined by some common driving fundamentals or whether each currency reacts to its own particular set of fundamentals or forcing variables. In particular, when simultaneously modeling a set of exchange rates, a logical first step should involve a test for the number of independent unit roots, or stochastic trends, to impose. Previous univariate unit root tests, as discussed above, are not well suited to answering this question. Accordingly, in this section a new multivariate test due to Johansen [20] is implemented in testing for the number of common unit roots in the set of seven daily spot and forward rates, respectively. The details of the test statistic and the relationship to the cointegrating framework developed by Engle and Granger [9], and the common trends representation in Stock and Watson [30], are summarized in Appendix B.

Table III reports the results of calculating the $-2\ln Q_r$ test statistic in (A14) for a VAR(1) model for each of the two sets of exchange rates. In this situation the statistic is simply based on the $7 - r$ smallest squared canonical correlations of y_{t-1} with respect to Δy_t , where y_t denotes the mean adjusted 7×1 vector of spot or forward rates, respectively; see Appendix B for details. Almost identical results were obtained for higher order vector autoregressions (VARs). This is hardly surprising given the approximate martingale difference behavior of both spot and forward rates discussed in the previous sections. Full details of the estimated models are available from the authors on request but are omitted for reasons of space.

The results are striking. Using a one percent significance level, we cannot reject the hypothesis that six stochastic trends are present in each of the full seven-dimensional systems determining the daily spot and forward rates or,

¹⁴ Hodrick [19] contains a recent survey on the evidence of risk premia in the foreign exchange market.

Table III
Multivariate Tests for Unit Roots in the Logarithms
of Spot and Forward Exchange Rates^a

<i>r</i>	$-2 \ln Q_r$		Quantiles	
	<i>s_i</i>	<i>f_i</i>	95%	99%
6	0.686	0.803	4.2	5.2
5	4.352	4.633	12.0	15.6
4	11.012	10.662	23.8	28.5
3	21.195	21.444	38.6	44.5
2	45.593	43.953	57.2	63.9
1	76.286	74.358	78.1	86.6
0	119.703**	119.789**	103.1	112.7

^a Tests for the number of linearly independent cointegrating vectors *r*, or equivalently the number of common unit roots $7 - r$, in a VAR(1) for the set of seven daily spot and seven one-month forward rates over the 1980–1985 period. The details of the likelihood ratio test, $2\ln Q_r$, for there being at most $7 - r$ independent unit roots are given in Appendix B. The ninety five percent quantiles for $r = 2, \dots, 6$ are the simulated values from Johansen [20]. The ninety nine percent quantiles and the ninety five percent quantiles for $r = 0, 1$ were calculated using the $c\chi^2(f)$ approximation in Appendix B.

** Significant at the one percent level.

alternatively, that only one cointegrating factor, or one long-run equilibrium relationship, exists between each of these two sets of nominal rates.

This result implies that many previous studies, such as Hakkio [16] and Baillie, Lippens, and McMahon [2], were incorrect in assuming a finite-dimensional autoregressive representation for the vector of first differences of spot and forward rates when testing the unbiasedness hypothesis in the forward foreign exchange market. From Appendix B, if the two sets of rates are cointegrated, no invertible-vector moving-average representation exists for either set of rates and, therefore, no standard finite-dimensional autoregressive approximation is feasible. In particular, an error correction term should be included in order to guarantee that the rates do not drift too far apart. Thus, for the *i*th spot or forward rate, y_{it} , a reasonable representation might be

$$\Delta y_{it} = \mu_i + \gamma_i \sum_j \alpha_j y_{jt-1} + \epsilon_{it}, \quad (6)$$

so that the daily change in each of the rates is partly determined by an $I(0)$ disequilibrium error which is common among all the currencies. This error represents the degree to which the rates are away from a long-run alignment.

Note that, in the absence of a time-varying risk premium, this result indicates a perceptible deviation from weak form efficiency for each of the exchange rates. In efficient speculative markets with no risk premia, the logarithmic change in each of the asset prices should be unpredictable. However, as previously noted by Granger [15] and illustrated by the first-order error correction model above, if two or more prices on different assets are cointegrated, part of the price changes will in general be predictable.

IV. Conclusion

The use of recent tests in Perron [24], Phillips [25], and Phillips and Perron [26], which are robust to a wide variety of weakly dependent and heterogeneously distributed disturbances, reveals convincing evidence for the presence of a unit root for seven currencies' spot and forward exchange rates. Equally strong evidence is provided that the seven pairs of series are cointegrated, so that $(s_t - f_t)$ or $(s_{t+k} - f_t)$ will be stationary quantities. In many cases it is natural to model exchange rates in terms of this forward premium, or forward rate forecast error, in which case the results presented here indicate that the variables will be stationary and traditional large sample inferential procedures will be valid.

At the same time, the application of a multivariate test for unit roots due to Johansen [20] suggests the presence of at least one common unit root, or stochastic trend, among the set of seven daily spot and forward foreign exchange rates, respectively. This result implies that the seven exchange rates are tied together in one long-run relationship. The disequilibrium error from this relationship is an important component in next period's change in the exchange rate and can be interpreted as a violation of weak form efficiency or as indirect evidence for a time-varying risk premium. Thus, while many models of exchange rate determination have assumed separate sets of fundamentals for each currency, our results suggest that there are strong trend components binding the exchange rates together.

Also, when simultaneously modeling a set of foreign exchange rates, it is generally not appropriate simply to first difference the data in order to achieve stationarity. This fairly standard procedure imposes too many unit roots. Estimating a model in levels involving only nonstationary variables, however, imposes too few unit roots, and standard asymptotically based inference procedures will not apply. The so-called error correction model provides one simple solution to the problem. Although the empirical results presented here are specific to the foreign exchange market, similar results are likely to carry over to other financial asset markets.

Appendix A

Following Perron [24], Phillips [25], and Phillips and Perron [26], the six univariate test statistics used in the paper are defined as follows.

For equation (1),

$$Z(t_{\hat{\alpha}}) = (S_o/S_{nl})t_{\hat{\alpha}} - 1/2(S_{nl}^2 - S_o^2)[n^{-1}S_{nl}(\sum y_{t-1}^2)^{1/2}]^{-1}, \quad (A1)$$

where $t_{\hat{\alpha}}$ is equal to the standard t -test statistic for $\hat{\alpha} = 1$ obtained from the OLS regression in (1) and S_o^2 denotes the sample variance under the null hypothesis. Here, and throughout Appendix A unless otherwise noted, all the summations are defined for t from 1 to n , and S_{nl}^2 refers to a consistent estimator for the variance of $\sum u_t$ under the null hypothesis as defined more formally below. Note, in the absence of any serial correlation in u_t , that $Z(t_{\hat{\alpha}})$ is asymptotically

equivalent to the standard t -test statistic, $t_{\hat{\alpha}}$. A similar result is true for the other adjusted test statistics presented below.

For equation (2),

$$Z(t_{\alpha^*}) = (S_o/S_{nl})t_{\alpha^*} - (1/2S_{nl})(S_{nl}^2 - S_o^2)[n^{-2} \sum (y_{t-1} - \bar{Y}_{-1})^2]^{-1/2}, \quad (A2)$$

$$Z(\Phi_1) = (S_o^2/S_{nl}^2)\Phi_1 - (1/2S_{nl}^2)(S_{nl}^2 - S_o^2) \cdot \{n(\alpha^* - 1) - 1/4(S_{nl}^2 - S_o^2)[n^{-2} \sum (y_{t-1} - \bar{Y}_{-1})^2]^{-1}\}, \quad (A3)$$

where

$$\Phi_1 = (2S^{*2})^{-1}n(S_o^2 - S^{*2}),$$

and t_{α^*} denotes the standard t -test for $\alpha^* = 1$ from (2), S^{*2} the residual variance evaluated at the OLS estimates, S_o^2 the residual variance under the appropriate null hypothesis, and $\bar{Y}_{-1}$ the mean of $y_o, y_1, \dots, y_{n-1}$.

Finally, for equation (3),

$$Z(t_{\tilde{\alpha}}) = (S_o/S_{nl})t_{\tilde{\alpha}} - (n^3/4\sqrt{3}D_x^{1/2}S_{nl})(S_{nl}^2 - S_o^2), \quad (A4)$$

$$Z(\Phi_3) = (S_o^2/S_{nl}^2)\Phi_3 - (1/2S_{nl}^2)(S_{nl}^2 - S_o^2) \cdot [n(\tilde{\alpha} - 1) - (n^6/48D_x)(S_{nl}^2 - S_o^2)], \quad (A5)$$

$$Z(\Phi_2) = (S_o^2/S_{nl}^2)\Phi_2 - (1/3S_{nl}^2)(S_{nl}^2 - S_o^2) \cdot [n(\tilde{\alpha} - 1) - (n^6/48D_x)(S_{nl}^2 - S_o^2)], \quad (A6)$$

where

$$\Phi_2 = (3\tilde{S}^2)^{-1}n(S_o^2 - \tilde{S}^2),$$

$$\Phi_3 = (2\tilde{S}^2)^{-1}n[S_o^2 - (\bar{Y} - \bar{Y}_{-1})^2 - \tilde{S}^2],$$

and again, $t_{\tilde{\alpha}}$, $\tilde{S}^2$, and S_o^2 denote the t -test, the OLS residual variance, and the variance under the appropriate null hypothesis, respectively. Furthermore, $\bar{Y}$ refers to the sample mean of y_t , and D_x is equal to the determinant of $(X'X)$, where X denotes the $n \times 3$ matrix of explanatory variables in the OLS regression defined by (3).

In all the above expressions, a consistent estimator of $\sigma^2 = \lim n^{-1} E(\sum u_t)^2$, S_{nl}^2 , is called for. In this paper the version of S_{nl}^2 used is

$$S_{nl}^2 = n^{-1} \sum_{t=1}^n u_t^2 + 2n^{-1} \sum_{\tau=1}^l \sum_{t=\tau+1}^n \omega_{\tau l} u_t u_{t-\tau}. \quad (A7)$$

When $\omega_{\tau l} = 1$ for all τ , Phillips [25] has shown that (A7) is consistent for σ^2 under a wide variety of behavior of u_t . In particular, u_t might follow a moving-average process of order l , with possible time-varying conditional heteroskedasticity but finite unconditional second moment. Following a suggestion of Newey and West [23], in implementing (A7) we choose $\omega_{\tau l} = 1 - \tau/(l+1)$ in order to guarantee a positive estimate of the variance.

Appendix B

This appendix briefly summarizes the relationship between cointegration, common trends, and error correction models, along with the ideas behind a new multivariate test for cointegration due to Johansen [20].

Following Engle [8] and Engle and Granger [9], consider the g -dimensional time-series vector y_t , where every component is integrated of order one; i.e., all g components in y_t contain a unit root in the autoregressive polynomial in their univariate time-series representations. Then, $y_t \sim I(1)$ and $\Delta y_t \sim I(0)$, and the process will have a vector moving-average representation

$$\Delta y_t = C(L)\epsilon_t, \quad (\text{A8})$$

where $C(L)$ denotes a possibly infinite order matrix polynomial in the lag operator L , $E(\epsilon_t) = 0$, $E(\epsilon_t \epsilon_s') = 0$ for $s \neq t$, and a normalization rule imposes $C(0)$ lower triangular and $E(\epsilon_t \epsilon_t') = I$. On using the identity $C(L) = C(1) + \Delta C^*(L)$, it follows that

$$\Delta y_t = C(1)\epsilon_t + \Delta C^*(L)\epsilon_t. \quad (\text{A9})$$

Thus, if there exists a $g \times r$ matrix α with rank r such that $\alpha' C(1) = 0$, by direct multiplication and integration in (A9),

$$z_t = \alpha' y_t = \alpha' C^*(L)\epsilon_t, \quad (\text{A10})$$

which will generally be $I(0)$. The r columns of α are called the cointegrating vector, and z_t represents the vector of equilibrium errors. Therefore, a necessary condition for cointegration is that $C(1)$ has reduced rank equal to $g - r$ or, equivalently, that the spectral density matrix for Δy_t is singular at zero frequency. This is what "holds" together the elements of y_t in the long run.

In the alternative common trends representation due to Stock and Watson [30], y_t is given by the sum of a stationary component and a $g - r$ dimensional random walk. Defining

$$\tau_t = \tau_{t-1} + J' \epsilon_t,$$

where $\tau_0 = 0$, $C(1) = GJ'$, and G and J denote $g \times (g - r)$ matrices, it follows by integrating (A9) that

$$y_t = y_0 + G\tau_t + C^*(L)\epsilon_t, \quad (\text{A11})$$

so y_t is driven by only $g - r$ stochastic trends, or unit roots. Hence, $g - r$ unit roots are consistent with r linearly independent cointegrating vectors.

Finally, consider the autoregressive representation for y_t :

$$A(L)y_t = \epsilon_t, \quad (\text{A12})$$

where by implication $A(1)$ has reduced rank r . Then, rewriting $A(L) = A(1)L + \Delta A^*(L)$, it is shown in Engle and Granger [9] that the autoregressive representation for Δy_t takes the so-called error correction form,

$$A^*(L)\Delta y_t = -\gamma \alpha' y_{t-1} + \epsilon_t. \quad (\text{A13})$$

Note that (A13) does not permit a standard vector autoregression for Δy_t . The inclusion of the error correction term, $\alpha' y_{t-1}$, is crucial in enforcing the long-run

relationship between the different elements in y_t . Also, opposed to (A12), (A13) involves only stationary variables.

The Engle and Granger [9] test for the presence of at least one cointegrating factor, i.e., $r \geq 1$, was briefly discussed in Section II of the paper. However, it is often of interest to test for the number of cointegrating factors r , or the number of unit roots $g - r$, where $1 \leq r < g$. Following Johansen [20], suppose the vector autoregression in (A12) is of order p ; i.e., $A(L) = A_0 + A_1L + \dots + A_pL^p$. Rearranging terms, an equivalent representation is then given by

$$\Delta y_t = \sum_{i=1}^{p-1} \Phi_i \Delta y_{t-i} + \Phi_p y_{t-p} + \epsilon_t,$$

where

$$\Phi_i = -I + \sum_{j=1}^{p-1} A_j, \quad i = 1, \dots, p.$$

Thus, $A(1) = \Phi_p$, and under the null hypothesis of r cointegrating vectors the rank of Φ_p equals r . By concentrating the corresponding likelihood with respect to the lagged values of Δy_t , Johansen [20] shows that a simple test of this hypothesis can be based on the OLS residuals from the two auxiliary regressions:

$$\Delta y_t = \sum_{i=1}^{p-1} \Gamma_{oi} \Delta y_{t-i} + \nu_{ot},$$

and

$$y_{t-p} = \sum_{i=1}^{p-1} \Gamma_{li} \Delta y_{t-i} + \nu_{lt}.$$

Calculating the residual sample second-moment matrices,

$$\hat{S}_{ij} = n^{-1} \sum_{t=1}^n \hat{\nu}_{it} \hat{\nu}_{jt}', \quad i, j = 0, 1,$$

the likelihood-ratio test of there being at most r cointegrating vectors takes the form

$$-2\ln Q_r = -n \sum_{i=r+1}^g \ln(1 - \hat{\lambda}_i), \quad (\text{A14})$$

where $\hat{\lambda}_{r+1}, \dots, \hat{\lambda}_g$ denotes the $g - r$ smallest eigenvalues of $\hat{S}_{10} \hat{S}_{00}^{-1} \hat{S}_{01}$ with respect to $\hat{S}_{11}$. These are also called the squared canonical correlations of ν_{1t} with respect to ν_{0t} . Under the null hypothesis of at most r cointegrating vectors, or $g - r$ unit roots, the asymptotic distribution of the test statistic in (A14) is given by

$$-2\ln Q_r \sim \text{tr} \left[\int_0^1 B dB' \left(\int_0^1 B(u) B(u)' du \right)^{-1} \int_0^1 dB B' \right], \quad (\text{A15})$$

where B denotes a $g - r$ dimensional standard Brownian motion.

Note that, when testing for $r = g - 1$ cointegrating vectors, or one unit root, (A15) reduces to the square of the Dickey-Fuller unit root t -distribution; see Phillips [25]. Although the derivation of the $-2\ln Q_r$ test statistic in Johansen [20] relies on ϵ_t being i.i.d normal, following Phillips [25] the same test statistic and corresponding asymptotic distribution are likely to apply in a much wider context with heterogeneously distributed errors. So far this is conjecture. Various quantiles of the distribution in (A15) have been simulated in Johansen [20] for $r = 1, \dots, 5$, and a suitable approximation is found to be $c\chi^2(f)$, where $\chi^2(f)$ denotes a central chi-square distribution with $f = 2(g - r)^2$ degrees of freedom, c

$= 0.85 - 0.58 f^{-1}$, and $f = 2(g - r)^2$. It is also worth stressing that, opposed to the Engle-Granger procedure employed in testing $r \geq 1$, the asymptotic distribution of $-2\ln Q_r$ is free of nuisance parameters.

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