Dynamic Capital Structure Choice: Theory and Tests
Author(s): Edwin O. Fischer, Robert Heinkel, Josef Zechner
Source: *The Journal of Finance*, Vol. 44, No. 1 (Mar., 1989), pp. 19-40
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2328273
Accessed: 26/11/2009 07:24

THE JOURNAL OF FINANCE • VOL. XLIV, NO. 1 • MARCH 1989

Dynamic Capital Structure Choice: Theory and Tests

EDWIN O. FISCHER, ROBERT HEINKEL, and JOSEF ZECHNER*

ABSTRACT

This paper develops a model of dynamic capital structure choice in the presence of recapitalization costs. The theory provides the optimal dynamic recapitalization policy as a function of firm-specific characteristics. We find that even small recapitalization costs lead to wide swings in a firm's debt ratio over time. Rather than static leverage measures, we use the observed debt ratio *range* of a firm as an empirical measure of capital structure relevance. The results of empirical tests relating firms' debt ratio ranges to firm-specific features strongly support the theoretical model of relevant capital structure choice in a dynamic setting.

A LIMITATION OF CURRENT single-period capital structure models is that they ignore the firm's optimal restructuring choices in response to fluctuations in asset values over time.[1] In particular, in the absence of transactions costs, firms could carry large amounts of debt and, by the appropriate repurchase strategy, capture large tax shields while keeping the debt essentially riskless.[2] The existence of transactions costs complicates the problem but makes its study even more interesting. As Myers [26] points out:

> Large adjustment costs could possibly explain the observed wide variation in actual debt ratios, since firms would be forced into long excursions away from their initial debt ratios. . . . If adjustment costs are large, so that some firms take extended excursions away from their targets, then we ought to give less attention to refining our static tradeoff stories and relatively more to understanding what the adjustment costs are, why they are so important and how rational managers would respond to them (page 587).

* Fischer is from Karl-Franzens University of Graz; Heinkel and Zechner are from the Faculty of Commerce and Business Administration, University of British Columbia. An earlier version of this paper was presented at the WFA and EFA meetings in 1987. We are grateful to Michael Brennan, Espen Eckbo, Alan Kraus, Robert McDonald, Neal Stoughton, Sheridan Titman, Joe Williams, two anonymous referees, and the Editor, René Stulz, for their helpful comments.

[1] Examples of one-period models include Kraus and Litzenberger [20], Castanias [7], Modigliani [24], Brennan and Schwartz [5], Ross [28], Zechner and Swoboda [33], Jensen and Meckling [17], Myers [25], Titman [31], Ross [27], Leland and Pyle [21], Heinkel [15], and Darrough and Stoughton [13].

[2] See Brennan [4] for an exposition of this behavior. Thus, some predictions from static capital structure theories may not hold. For example, high-risk firms may not necessarily have lower optimal capital structures, as predicted by Bradley, Jarrell, and Kim [2]; instead, these firms might adopt a more active debt management policy, thereby counterbalancing the higher firm risk.

In this paper we model capital structure choice in a continuous-time framework. We derive closed-form solutions for the value of the firm's debt and equity as a function of its dynamic recapitalization decisions. The resulting optimal dynamic capital structure policy depends upon the benefit of debt financing (e.g., a tax advantage), potential costs of debt financing (e.g., bankruptcy costs), underlying asset variability, the riskless interest rate, and the size of the costs of recapitalizing.

An important insight from examining optimal dynamic capital structure choice is to refine the meaning of an optimal debt ratio. As in Miller's [23] model of capital structure irrelevance, we might observe similar firms with very different leverage ratios. In a Miller world, this would be interpreted as a "neutral mutation." In our model, firms will allow their financial structure to change over time due to costs of recapitalizing. Any ratio lying within a set of boundaries is optimal, so similar firms could have different leverage ratios at any point in time. However, similar firms have similar recapitalization criteria, and, thus, their capital structures exhibit similar intertemporal behavior. Many of the existing empirical tests of optimal capital structure have taken year-end debt ratios or averages of year-end debt ratios as optimal and uniquely determined by firm-specific features. Their results are weakened by the variability in debt ratios introduced by firms following dynamic optimal behavior. The noise introduced by the dynamic behavior creates bias since the observed leverage ratios, in general, depend upon past growth of the firms' assets.

Our model generates predictions about firms' capital structure decisions that are not based on static leverage ratios, and, hence, the empirical tests are not subject to the problems mentioned above. According to our results, different firms allow the actual leverage ratio to deviate from the target ratio by different amounts. We determine the leverage ratio range for a sample of COMPUSTAT firms and find a statistically significant relationship between these debt ratio ranges and the firm characteristics that enter our model.

Related to the theoretical model of this paper is work by Brennan and Schwartz [6], who analyze a firm's intertemporal investment and capital structure policy. In their model, capital structure decisions are driven by a tax advantage and agency costs created by debt. Since there are no transactions costs in their model, firms recapitalize continuously and the results focus on the optimal rate of change in the face value of debt. In our model, investment decisions are exogenous and costs of recapitalization prevent a firm from adjusting its capital structure continuously.

Kane, Marcus, and McDonald [18 and 19] develop a model which is the basis for the theoretical framework in this paper. In their model, a firm cannot adjust its capital structure before the outstanding debt becomes due. In our paper, firms can recapitalize at any point in time and we determine the critical upper and lower financial leverage ratios at which transactions costs are incurred to rebalance the firm's financial structure.

This paper is structured in the following way. Section I introduces the model. In Section II we analyze a firm's optimal recapitalization criteria and derive comparative statics. Section III consists of an empirical test of our theoretical results. In Section IV we summarize our main findings.

I. The Model

In the spirit of the traditional finance literature we isolate the impact of capital structure decisions on firm value by taking a firm's investment decisions as exogenous and independent of its capital structure. We fix the firm's capital budgeting decisions such that an unlevered firm can finance its investments by retaining its earnings.[3] We then model the value of a levered firm as a function of the market value of the unlevered assets and of its leverage ratio.

Since coupon payments are tax deductible, the total rate of return on a levered firm has two sources: one is related to the change in the market value of its unlevered assets, and the other is a cash payment to investors due to debt-related tax shields.

In equilibrium, a firm following an optimal financing policy offers a "fair" risk-adjusted rate of return to its investors. Then, assuming that leverage is advantageous, the unlevered assets without debt-related tax shields must offer a "below fair" risk-adjusted rate of return. This risk-adjusted rate of return on the unlevered assets is determined endogenously by a no-arbitrage relationship that must hold between the value of the optimally levered firm and the value of the unlevered assets. As pointed out by Kane, Marcus, and McDonald [18 and 19], the value of the unlevered assets reflects the potential to optimally lever them.[4] Therefore, the value of an optimally levered firm can only exceed the value of its unlevered assets by the amount of transactions costs incurred in order to lever them. This no-arbitrage relationship only holds for a particular risk-adjusted drift on the unlevered asset value. The difference between the risk-adjusted rate of return on the unlevered assets and the total rate of return on the levered firm is due to the benefits from leverage. If the firm chooses the optimal dynamic capital structure policy, this rate of return to leverage is maximized. In our model, we simultaneously solve for the firm's optimal recapitalization policy and the equilibrium rate of return on the unlevered assets.

A. *The Dynamic Valuation Equation*

In this section we derive the dynamic valuation equations for the firm's debt and equity securities for any given recapitalization policy. Then, in the subsequent section, we choose the initial debt ratio and the recapitalization criteria which maximize firm value subject to a constraint that the value of equity is non-negative due to limited liability.

Define

A = market value of a firm's unlevered assets (i.e., the market value of a firm's land, machines, buildings, etc.);

[3] This implies that an all-equity firm issues shares only once to finance its initial asset base. Hence, when an existing levered firm issues or redeems securities in the financial market, it will result in a pure change in capital structure.

[4] Kane, Marcus, and MacDonald [19] use the analogy of a competitive market for houses or apartment buildings. The value of a house will reflect the present value of the tax shields associated with tax-deductible interest payments on the mortgage. If the house is fully equity financed, the owner earns a below-equilibrium rate of return on the investment since he does not realize the tax reductions.

E = market value of a firm's equity;
D = market value of a firm's debt;
B = face value of a firm's debt;
V = total market value of a levered firm;
y = A/B;
σ^2 = instantaneous variance of an unlevered firm's rate of return;
k = proportional transactions costs associated with issuing new debt;
g = proportional bankruptcy costs;
τ_c = corporate tax rate;
τ_p = personal tax rate on ordinary income;
r = instantaneous riskless interest rate;
i = instantaneous coupon rate.

We assume that the value of a firm's unlevered assets, A, follows a geometric Wiener process of the form

$$\frac{dA}{A} = \mu dt + \sigma dz. \tag{1}$$

Then, for a fixed face value of debt, B, the value-to-debt ratio, $y = A/B$, follows the stochastic process

$$\frac{dy}{y} = \mu dt + \sigma dz. \tag{2}$$

We assume that the firm's debt is a callable consol bond and that the values of the firm's debt and equity securities, D and E, between recapitalization dates depend only on firm-specific state variables.[5] Then, for a given recapitalization policy,

$$E = E(y, B), \tag{3}$$

$$D = D(y, B). \tag{4}$$

To simplify the notation we generally suppress the recapitalization bounds as arguments for the value of equity and debt until Section II, when we explicitly analyze the recapitalization policy.

We assume a very simple tax regime. Corporations are taxed instantaneously at a constant rate, τ_c, with full loss offset provisions. Investors pay no tax on capital gains or dividends and are taxed instantaneously at a constant rate, τ_p, on interest income.[6] Using this simple tax structure allows us to adapt the fundamental valuation equation from Cox, Ingersoll, and Ross [11] to after-personal-tax rates of return. It follows that the market values of debt and equity satisfy the ordinary differential equations:

$$\tfrac{1}{2}\sigma^2 y^2 D_{yy} + \hat{\mu} y D_y - r(1 - \tau_p)D + (1 - \tau_p)iB = 0, \tag{5}$$

$$\tfrac{1}{2}\sigma^2 y^2 E_{yy} + \hat{\mu} y E_y - r(1 - \tau_p)E - (1 - \tau_c)iB = 0, \tag{6}$$

[5] Brennan and Schwartz [6] provide the necessary and sufficient conditions for a firm's securities to depend only on firm-specific state variables.

[6] We could also assume that equity income is taxed. τ_p would then be interpreted as the tax rate representing the tax disadvantage of interest income relative to equity income.

where $\hat{\mu}$, the risk-adjusted expected rate of return on the firm's unlevered assets, will be discussed in Section II. In contrast to Miller [23], an equilibrium where firms issue both debt and equity requires that τ_p be less than τ_c. This is because the existence of transactions costs requires a positive tax subsidy to justify the use of debt.[7]

B. Boundary Conditions

When the value-to-debt ratio, y, increases, i.e., the leverage ratio drops, the firm foregoes an increasing amount of debt-related tax shields, and at some point it is optimal to recapitalize. When y decreases over time, i.e., the firm's leverage ratio increases, a recapitalization could occur for two reasons. First, it may be optimal to recapitalize in order to avoid bankruptcy costs. Second, a recapitalization might be necessary because of equityholders' limited liability. Since investment decisions are fixed, equityholders cannot sell assets to make coupon payments. Coupon payments can therefore be thought of as decreasing equityholders' dividends. For simplicity we have assumed that an unlevered firm pays no dividend, so that coupon payments constitute a negative dividend to equityholders equal to the after-tax coupon payment, $-(1 - \tau_c)iB$. Even if an unlevered firm would pay a positive dividend, say a constant proportion of its value, the dividend of a levered firm with the same investment policy would become negative if the value of the assets decreased sufficiently. For sufficiently small asset values, equityholders will find it in their own best interest to default. Debtholders will then take over the firm and recapitalize.

To characterize an empirically reasonable recapitalization policy, we employ a simple strategy in the spirit of the "(s, S)" inventory control problem: capital structure is adjusted when y, the firm's value-to-debt ratio, reaches a critical upper bound $\bar{y}$ or lower bound $\underline{y}$. The optimality of this type of policy has been proven under fairly restrictive conditions.[8] Conforming with the approach used by Constantinides [9] in the context of a similar problem, we restrict our attention to such strategies although their optimality has not yet been proven for the more general conditions in both of our models.

An important implication of the assumed diffusion process in equation (1) is that y will never lie outside the bounds of $[\underline{y}, \bar{y}]$. By contrast, employing a two-barrier policy under the assumption of a jump process would result in realizations of y strictly outside the "no recapitalization" region. This has implications for the existence of riskless debt, discussed in Section I.*D*.

We assume that the recapitalization policy is defined in the bond indenture and that y is continuously observable by all parties. This is consistent with our use of no-arbitrage conditions in solving the continuous-time valuation equations below. Hence, the bond indenture can be contingent upon y (or A) and allows a "first-best" recapitalization policy in that equityholders rationally choose a firm-value-maximizing policy.

[7] Suppose there are no recapitalization costs and $\tau_p < \tau_c$. In this case bankruptcy costs are not sufficient for an interior optimal capital structure since firms could be fully levered and still avoid bankruptcy by recapitalizing continuously.

[8] For discrete-time control problems with transactions costs, optimality of policies with two barriers has been proven by, for example, Iglehart [16] and Constantinides [8]. A proof in continuous time is given by Taksar, Klass, and Assaf [30].

Alternatively, we could prohibit such indenture provisions. In this case, equityholders cannot precommit to the first-best policy and so will recapitalize when it is in their best interest, resulting in a "second-best" policy. Even under the second-best policy, equityholders have an incentive to issue additional debt at sufficiently high y. However, the incentives for equityholders to recapitalize in order to reduce debt might differ substantially from the first-best solution. Under the first-best policy it may be optimal to specify a leverage reduction before bankruptcy occurs; under the second-best solution equityholders will not have incentives to reduce debt early. Thus, riskless debt is infeasible in the second-best solution since such a policy is suboptimal to equityholders ex post. Equityholders will not, however, postpone bankruptcy indefinitely since they are paying a negative dividend to support the continuous bond interest payment. At a sufficiently low asset value, equityholders find it in their best interest to default.

So, as discussed above, our model employs a "region of no recapitalization" bounded by two barriers. The optimal decision when y reaches a boundary depends upon the form of the costs of recapitalization. For example, if recapitalization costs were proportional to the change in the face value of debt, then the optimal control policy would be to make very small, continuous adjustments, holding y at the boundary until the process moved y back into the region of no recapitalization.[9] However, neither the above form of recapitalization costs nor the resulting continuous recapitalizations are empirically reasonable.

Bond indentures may, for example, restrict additional debt issues. Many indentures contain a call provision that makes it possible to retire existing debt at very low cost. In light of such institutional features, we assume that, when y reaches a bound, the existing debt is costlessly retired and that costs of issuing new debt are proportional to the amount of new debt sold. Because the costs of recapitalization are proportional to the amount of *new* debt rather than the *change* in the debt level, it is clear that the optimal recapitalization policy returns y to the same level in the interior of the region of no recapitalizations, regardless of which boundary induced the change. Under this assumed cost structure, the debt level changes will occur irregularly and will be "lumpy" rather than very small and continuous.

Consider the firm value at a recapitalization date. As discussed above, the costs of recapitalization are a proportion k of the face value of the new debt issue. Define y_0 as the optimal *initial* value-to-debt ratio after a recapitalization.[10] For

[9] If recapitalization costs are proportional to the change in debt, we can only solve for the case of risky debt. The optimal policy in this case does not exhibit a discrete change in debt ratio to a point between these two boundaries, but rather uses infinitesimal changes to prohibit the ratio from crossing the boundary. One interpretation of this policy is that there are two initial leverage ratios: the lower and upper boundaries. Even in this case the resulting optimal bounds are functions of firm-specific features, similar to what we report below for our model.

[10] This optimal initial value-to-debt ratio differs from the optimal ratio that would obtain in a model without transactions costs. In the latter case it is optimal to adjust capital structure continuously in order to maintain full debt financing. In the former case, transactions costs induce an interior optimal initial value-to-debt ratio.

$y = \bar{y}$,

$$V(\bar{y}, B) = V\left(y_0, \frac{B\bar{y}}{y_0}\right) - k\frac{B\bar{y}}{y_0}. \tag{7}$$

Equation (7) states that, when a firm reaches a leverage ratio such that it will recapitalize and issue additional debt, the firm value must be equal to the value of an optimally levered firm, $V\left(y_0, \frac{B\bar{y}}{y_0}\right)$, minus the costs of recapitalizing. From the definition of y it follows that the optimal face value of debt immediately after recapitalization is $B\bar{y}/y_0$, and so recapitalization costs are $kB\bar{y}/y_0$.

If the firm's leverage ratio reaches its maximum allowable value, i.e., if $y = \underline{y}$, the boundary condition depends on whether the recapitalization is associated with bankruptcy. Bankruptcy occurs if equityholders choose to stop making negative dividend payments to cover the coupon payment and the value of the firm is insufficient to repurchase the existing bonds at par. Assuming that bankruptcy costs are a proportion, g, of the face value of the debt outstanding, B, the value of the firm at $y = \underline{y}$ is given by (8):

$$V(\underline{y}, B) = \begin{cases} \max\left[V\left(y_0, \frac{B\underline{y}}{y_0}\right) - k\frac{B\underline{y}}{y_0} - gB, 0\right] & \text{if } V\left(y_0, \frac{B\underline{y}}{y_0}\right) - k\frac{B\underline{y}}{y_0} < B, \\ V\left(y_0, \frac{B\underline{y}}{y_0}\right) - k\frac{B\underline{y}}{y_0} & \text{otherwise.} \end{cases} \tag{8}$$

As discussed above, the absence of arbitrage requires that firm value immediately after recapitalization be equal to the market value of the firm's unlevered assets plus the costs associated with the optimal debt issue:

$$V(y_0, B) = y_0 B + kB.$$

Recognizing that the optimal face values of debt after recapitalization at $y = \bar{y}$ and $y = \underline{y}$ are given by $B\bar{y}/y_0$ and $B\underline{y}/y_0$, respectively, we get the no-arbitrage conditions (9) and (10):

$$V\left(y_0, \frac{B\bar{y}}{y_0}\right) = \bar{y}B + k\frac{B\bar{y}}{y_0}, \tag{9}$$

$$V\left(y_0, \frac{B\underline{y}}{y_0}\right) = \underline{y}B + k\frac{B\underline{y}}{y_0}. \tag{10}$$

Inserting (9) in (7) and (10) in (8) yields

$$V(\bar{y}, B) = \bar{y}B, \tag{11}$$

$$V(\underline{y}\ B) = \begin{cases} \max[(\underline{y} - g)B, 0] & \text{if } \underline{y} < 1, \\ \underline{y}B & \text{otherwise} \end{cases}. \tag{12}$$

Equation (12) implies that, if $\underline{y} < 1$, then $V(\underline{y}, B)$ is less than B and bankruptcy occurs.

We can now divide the firm value at recapitalization in equations (11) and (12) between the equity and the debtholders. We assume that the debt is callable at par.[11] Therefore, the debtholders receive the face value of the bond whenever the firm does not default. In the case of bankruptcy, debtholders pay the bankruptcy costs and become the new owners of the firm if the firm value exceeds bankruptcy costs. The value of the equity is always the value of the firm minus the value of the debt.

$$E(\bar{y}, B) = B(\bar{y} - 1), \tag{13}$$

$$E(\underline{y}, B) = \begin{cases} 0 & \text{if } \underline{y} < 1, \\ B(\underline{y} - 1) & \text{otherwise;} \end{cases} \tag{14}$$

$$D(\bar{y}, B) = B, \tag{15}$$

$$D(\underline{y}, B) = \begin{cases} \max[(\underline{y} - g)B, 0] & \text{if } \underline{y} < 1, \\ B & \text{otherwise.} \end{cases} \tag{16}$$

C. The Case of Risky Debt

If the lower bound on the value-to-debt ratio is less than one, $\underline{y} < 1$, then the solution to the ordinary differential equations (5) and (6), subject to the boundary conditions (13) through (16), is:

$$D(y, B) = D_1 y^{m_1} + D_2 y^{m_2} + \frac{i}{r} B, \tag{17}$$

$$E(y, B) = E_1 y^{m_1} + E_2 y^{m_2} - \frac{(1 - \tau_c)i}{(1 - \tau_p)r} B, \tag{18}$$

where

$$D_1 = \frac{B}{\Delta} \left\{ \left(1 - \frac{i}{r}\right) \underline{y}^{m_2} - \left[(\underline{y} - g)^+ - \frac{i}{r} \right] \bar{y}^{m_2} \right\},$$

$$D_2 = \frac{B}{\Delta} \left\{ \left[(\underline{y} - g)^+ - \frac{i}{r} \right] \bar{y}^{m_1} - \left(1 - \frac{i}{r}\right) \underline{y}^{m_1} \right\},$$

$$E_1 = \frac{B}{\Delta} \left[(e + \bar{y} - 1) \underline{y}^{m_2} - e \bar{y}^{m_2} \right],$$

$$E_2 = \frac{B}{\Delta} \left[e \bar{y}^{m_1} - (e + \bar{y} - 1) \underline{y}^{m_1} \right],$$

$$\Delta = \bar{y}^{m_1} \underline{y}^{m_2} - \bar{y}^{m_2} \underline{y}^{m_1},$$

[11] Depending upon the tax treatment of call premia, this is not necessarily optimal. The effects of call premia when the firm cannot precommit to a recapitalization policy are examined in Fischer, Heinkel, and Zechner [14].

$$e = \frac{(1 - \tau_c)i}{(1 - \tau_p)r},$$

$$m_1 = a + b,$$

$$m_2 = a - b,$$

$$a = \frac{1}{2} - \frac{\hat{\mu}}{\sigma^2},$$

$$b = \sqrt{a^2 + \frac{2r(1 - \tau_p)}{\sigma^2}}, \text{ and}$$

$$(\underline{y} - g)^+ = \max\{(\underline{y} - g), 0\}.$$

From equations (17) and (18) it can be seen that, for given firm characteristics, the values of debt and equity depend not only on the current capital structure but also on the dynamic recapitalization criterion. The three key variables are the firm's current capital structure and the upper and lower bounds at which the firm either reduces or increases the debt level.

D. The Case of Riskless Debt

In the case of riskless debt, the lower bound on the value-to-debt ratio, $\underline{y}$, is greater than or equal to one. Here the assumptions of a diffusion process in equation (1) and of constant monitoring of firm value by bondholders imply that firm value is always sufficient to repurchase the old debt at par when the firm recapitalizes and hence debt becomes riskless.[12] Property 1 summarizes this point, and the proof contains the solution to the differential equation (5) subject to boundary conditions (15) and (16) for $\underline{y} \geq 1$.

PROPERTY 1: *If $\underline{y} \geq 1$, debt is riskless, regardless of the current value-to-debt ratio or firm characteristics.*

Proof: Solving (5) subject to (15) and (16) for $i = r$ we get

$$D(y, B) = B. \tag{19}$$

Q.E.D.

The value of a firm's equity securities for $\underline{y} \geq 1$ is given by

$$E(y, B) = C_1 y^{m_1} + C_2 y^{m_2} - \frac{(1 - \tau_c)}{(1 - \tau_p)} B, \tag{20}$$

where

$$C_1 = \frac{B}{\Delta}[(f + \bar{y} - 1)\underline{y}^{m_2} - (f + \underline{y} - 1)\bar{y}^{m_2}],$$

[12] This important implication was recently pointed out by Brennan [4].

$$C_2 = \frac{B}{\Delta}\left[(f + \underline{y} - 1)\bar{y}^{m_1} - (f + \bar{y} - 1)\underline{y}^{m_1}\right],$$

$$f = \frac{1 - \tau_c}{1 - \tau_p},$$

and Δ, m_1, and m_2 are defined above.

II. Optimal Recapitalization Policy

In this section we derive a firm's optimal recapitalization policy and examine its relationship with specific firm characteristics. In doing so, we assume that the coupon rate is set such that the debt being issued is sold at par:

$$D(y_0, B) = B. \tag{21}$$

At a recapitalization date, we choose B, $\underline{y}$, and $\bar{y}$ to maximize the total firm value net of transactions costs, subject to a non-negativity constraint on equity value due to limited liability. Since we are now concerned with the determination of the optimal recapitalization decisions, we explicitly include $\bar{y}$ and $\underline{y}$ as arguments of V, D, and E.

It follows from equation (12) that firm value is discontinuous at $\underline{y} = 1$. Therefore, we perform two optimizations, one for risky debt and one for riskless debt; the global optimum is the solution that maximizes the present value of the debt tax subsidy, net of transactions and bankruptcy costs.

A. The Case of Risky Debt

If a firm issues risky debt, the values of its securities are given by equations (17) and (18). In this case the limited liability constraint on equity would be binding for a sufficiently small asset base since equityholders are not allowed to change the investment policy and therefore cannot sell off assets to finance coupon payments. Given equation (14), the necessary and sufficient condition for the value of equity, E, to be non-negative is

$$E_y(y = \underline{y}\, B, \underline{y}, \bar{y}) \geq 0. \tag{22}$$

Because of the no-arbitrage condition outlined above, the firm value after recapitalization must be equal to the value of the unlevered assets plus transactions costs, $y_0B + kB$. This implies that there is only one endogenous, risk-adjusted expected growth rate of the market value of a firm's unlevered assets, $\hat{\mu}$, that is consistent with this condition.

If there is an advantage to leverage, then $\hat{\mu}$ will be less than the after-personal-tax rate of return on the riskless asset, $r(1 - \tau_p)$, which is the equilibrium risk-adjusted growth rate of a firm following the *optimal* capital structure policy. We define this advantage of leverage as δ:[13]

$$\delta = r(1 - \tau_p) - \hat{\mu}. \tag{23}$$

[13] Kane, Marcus, and McDonald [19] define δ as the deficiency in the expected rate of return to unlevered assets.

A capital structure equilibrium is defined by the variables, $\underline{y}$, $\bar{y}$, B, δ, and i chosen to satisfy the following conditions:

$$\text{maximize } V(y_0, B, \underline{y}, \bar{y}) - kB$$

subject to

$V(y_0, B, \underline{y}, \bar{y}) = B(y_0 + k)$ (no-arbitrage),

$E_y(y = \underline{y}, B, \underline{y}, \bar{y}) \geq 0$ (limited liability), and

$D(y_0, B, \underline{y}, \bar{y}) = B$ (issue at par).

B. The Case of Riskless Debt

Property 1 states that a firm's debt is riskless if $\underline{y} \geq 1$. In this case the values of debt and equity are given by equations (19) and (20).

Of all the $\underline{y}$ consistent with riskless debt, the optimal $\underline{y}$ must be equal to one. At any $\underline{y}$ greater than this, the firm incurs transactions costs earlier than necessary and foregoes tax shields relative to allowing y to move to one. This result is stated as Property 2.

PROPERTY 2: *It is not optimal to decrease a firm's leverage ratio before its value-to-debt ratio drops to one.*

A capital structure equilibrium with riskless debt is defined by the variables $\underline{y}$, $\bar{y}$, B, δ, and i chosen to satisfy the following conditions:

$$\text{maximize } V(y_0, B, \underline{y}, \bar{y}) - kB$$

subject to

$V(y_0, B, \underline{y}, \bar{y}) = B(y_0 + k)$ (no-arbitrage),

$\underline{y} = 1$ (riskless debt), and

$D(y_0, B, \underline{y}, \bar{y}) = B$ (issue at par).

Whether the optimal recapitalization policy involves risky debt (i.e., $\underline{y} < 1$) or riskless debt (i.e., $\underline{y} = 1$) depends upon which one of the two maximization problems defined above yields the higher return to leverage, i.e., the higher δ.

C. Numerical Results

Numerical solutions have been calculated for different parameter values. Table I contains the parameter values for the base case. This case yields the following optimal dynamic capital structure policy:

optimal initial debt ratio $(B/A_0) = 0.62$;
maximum debt ratio $(1/\underline{y}) = 1.75$;
minimum debt ratio $(1/\bar{y}) = 0.29$;
optimal debt ratio range $(1/\underline{y} - 1/\bar{y}) = 1.46$;
return to leverage $(\delta) = 0.28\%$;
coupon rate $(i) = 4.45\%$.

The initial debt ratio (B/A_0) for this firm is 0.62, but even a small recapitalization cost ($k = 1\%$) causes the firm to delay restructuring, leading to wide swings in the debt ratio.

This case demonstrates a problem with much of the existing literature that empirically tests for capital structure relevance by relating observed debt ratios to firm-specific features: the firm's "optimal" debt ratio varies over a wide range and may take on any value between 0.29 and 1.75 on the measurement date. Such wide variation injects significant noise into capital structure observations.

Tables II through VII provide numerical results for comparative statics on six model parameters: τ_c, τ_p, σ^2, k, r, and g. Tables II, III, and VI demonstrate that raising τ_c or r or decreasing τ_p increases the tax advantage of debt, causing a decrease in the optimal debt ratio range and a higher initial debt ratio. The increased value of tax shields relative to the recapitalization cost makes more active debt management optimal. A higher return to leverage (δ) is observed.

Table IV indicates that increasing σ^2 causes the optimal leverage *range* to increase and the initial debt ratio to fall to avoid very frequent costly recapitalizations. An interesting and counterintuitive result is that, in a model where bankruptcy costs and recapitalization costs drive optimal capital structure decisions, the advantage of leverage, δ, increases with σ^2. This occurs because the increased debt risk leads to a higher coupon rate. This larger tax shield more than offsets the higher expected bankruptcy costs and recapitalization costs.

Table VII exhibits the comparative statics for the bankruptcy parameter, g. For relatively low bankruptcy costs, g, increasing g reduces the initial debt ratio.

Table I

Parameter Values for the Base Case

Parameter	Value
Corporate Tax Rate τ_c	50%
Personal Tax Rate τ_p	35%
Variance σ^2	0.05%
Transactions Costs k	1%
Riskless Interest Rate r	2%
Bankruptcy Costs g	5%

Table II

Optimal Recapitalization Policy for Different Corporate Tax Rates τ_c

	Leverage Ratio[a]					
Corporate Tax Rate	Lower Bound	Upper Bound	Leverage Ratio Range[b]	Initial Leverage Ratio[a]	Delta[a,c] (%)	Coupon Rate[a] (%)
0.38	0.03	2.44	2.41	0.17	0.005	2.51
0.40	0.08	2.20	2.12	0.31	0.020	2.88
0.42	0.13	2.05	1.92	0.41	0.051	3.22
0.44	0.18	1.94	1.76	0.49	0.092	3.54
0.46	0.22	1.86	1.64	0.54	0.144	3.85
0.48	0.26	1.80	1.54	0.59	0.207	4.15
0.50	0.29	1.75	1.46	0.62	0.281	4.45

[a] Values are the solutions to the optimization problem in Section II.
[b] Leverage ratio range is the difference between the upper and lower leverage ratio bounds.
[c] Delta is the advantage to leverage defined by $\delta = r(1 - \tau_p) - \hat{\mu}$ [equation (23)].

Table III

Optimal Recapitalization Policy for Different Personal Tax Rates τ_p

Personal Tax Rate	Leverage Ratio[a] Lower Bound	Leverage Ratio[a] Upper Bound	Leverage Ratio Range[b]	Initial Leverage Ratio[a]	Delta[a,c] (%)	Coupon Rate[a] (%)
0.00	0.50	1.48	0.98	0.81	1.76	5.52
0.05	0.48	1.49	1.01	0.79	1.53	5.47
0.10	0.46	1.51	1.05	0.78	1.30	5.40
0.15	0.44	1.53	1.09	0.76	1.08	5.31
0.20	0.42	1.56	1.14	0.74	0.864	5.18
0.25	0.38	1.60	1.22	0.72	0.656	5.01
0.30	0.34	1.66	1.32	0.68	0.459	4.78
0.35	0.29	1.75	1.46	0.62	0.281	4.45
0.40	0.21	1.90	1.69	0.53	0.131	3.97
0.45	0.10	2.25	2.15	0.34	0.028	3.23

[a] Values are the solutions to the optimization problem in Section II.
[b] Leverage ratio range is the difference between the upper and lower leverage ratio bounds.
[c] Delta is the advantage to leverage defined by $\delta = r(1 - \tau_p) - \hat{\mu}$ [equation (23)].

Table IV

Optimal Recapitalization Policy for Different Variances σ^2

Variance	Leverage Ratio[a] Lower Bound	Leverage Ratio[a] Upper Bound	Leverage Ratio Range[b]	Initial Leverage Ratio[a]	Delta[a,c] (%)	Coupon Rate[a] (%)
0.02	0.36	1.49	1.13	0.69	0.213	2.96
0.04	0.30	1.68	1.38	0.64	0.258	3.96
0.06	0.28	1.80	1.52	0.61	0.305	4.94
0.08	0.26	1.87	1.61	0.60	0.352	5.90
0.10	0.25	1.93	1.68	0.58	0.400	6.86
0.12	0.25	1.97	1.72	0.57	0.447	7.82
0.14	0.24	2.01	1.77	0.57	0.495	8.77
0.16	0.24	2.03	1.79	0.56	0.542	9.72
0.18	0.23	2.06	1.83	0.56	0.590	10.67
0.20	0.23	2.07	1.84	0.55	0.637	11.62

[a] Values are the solutions to the optimization problem in Section II.
[b] Leverage ratio range is the difference between the upper and lower leverage ratio bounds.
[c] Delta is the advantage to leverage defined by $\delta = r(1 - \tau_p) - \hat{\mu}$ [equation (23)].

Table V

Optimal Recapitalization Policy for Different Transactions Costs k

Transactions Costs	Leverage Ratio[a] Lower Bound	Leverage Ratio[a] Upper Bound	Leverage Ratio Range[b]	Initial Leverage Ratio[a]	Delta[a,c] (%)	Coupon Rate[a] (%)
0.02	0.22	1.84	1.62	0.61	0.222	4.10
0.04	0.14	2.02	1.88	0.56	0.147	3.64
0.06	0.10	2.20	2.10	0.50	0.100	3.33
0.08	0.07	2.38	2.31	0.44	0.067	3.08
0.10	0.04	2.55	2.51	0.37	0.044	2.89

[a] Values are the solutions to the optimization problem in Section II.
[b] Leverage ratio range is the difference between the upper and lower leverage ratio bounds.
[c] Delta is the advantage to leverage defined by $\delta = r(1 - \tau_p) - \hat{\mu}$ [equation (23)].

Table VI

Optimal Recapitalization Policy for Different Riskless Interest Rates r

Riskless Interest Rate	Leverage Ratio[a] Lower Bound	Leverage Ratio[a] Upper Bound	Leverage Ratio Range[b]	Initial Leverage Ratio[a]	Delta[a,c] (%)	Coupon Rate[a] (%)
0.02	0.29	1.75	1.46	0.62	0.281	4.45
0.04	0.34	1.55	1.21	0.68	0.447	6.43
0.06	0.37	1.44	1.07	0.71	0.619	8.38
0.08	0.40	1.38	0.98	0.73	0.796	10.32
0.10	0.42	1.33	0.91	0.75	0.976	12.27
0.12	0.44	1.29	0.85	0.77	1.160	14.21
0.14	0.46	1.26	0.80	0.78	1.347	16.16

[a] Values are the solutions to the optimization problem in Section II.
[b] Leverage ratio range is the difference between the upper and lower leverage ratio bounds.
[c] Delta is the advantage to leverage defined by $\delta = r(1 - \tau_p) - \hat{\mu}$ [equation (23)].

Table VII

Optimal Recapitalization Policy for Different Bankruptcy Costs g

Bankruptcy Costs	Leverage Ratio[a] Lower Bound	Leverage Ratio[a] Upper Bound	Leverage Ratio Range[b]	Initial Leverage Ratio[a]	Delta[a,c] (%)	Coupon Rate[a] (%)
0.02	0.37	1.53	1.16	0.77	0.442	5.52
0.04	0.31	1.68	1.37	0.67	0.320	4.70
0.06	0.27	1.80	1.53	0.59	0.251	4.25
0.08	0.23	1.91	1.68	0.52	0.205	3.95
0.10	0.21	1.99	1.78	0.47	0.173	3.74
0.12	0.18	2.07	1.89	0.43	0.149	3.58
0.14	0.17	2.14	1.97	0.39	0.130	3.45
0.16	0.15	2.20	2.05	0.35	0.114	3.35
0.18	0.14	2.25	2.11	0.32	0.102	3.27
0.20	0.19	1.00	0.81	0.54	0.095	2.00
0.22	0.19	1.00	0.81	0.54	0.095	2.00
0.24	0.19	1.00	0.81	0.54	0.095	2.00

[a] Values are the solutions to the optimization problem in Section II.
[b] Leverage ratio range is the difference between the upper and lower leverage ratio bounds.
[c] Delta is the advantage to leverage defined by $\delta = r(1 - \tau_p) - \hat{\mu}$ [equation (23)].

This result is consistent with traditional static models of capital structure. Increasing g also makes it optimal to allow the debt ratio to vary over a wider range. In this range it is optimal to issue risky debt. The return to leverage decreases with g. For higher levels of g, debt is optimally riskless so that the initial debt ratio and the optimal debt ratio range are independent of g. For these levels of g, the optimal debt ratio range is lower than the optimal range for low levels of g. This result implies that firms with low bankruptcy costs have, on average, higher leverage ratio ranges than firms with very high bankruptcy costs.

The above relationship between bankruptcy costs and debt ratio range critically relies upon the possibility of issuing riskless debt. In a second-best world in which the bond contract cannot specify the recapitalization policy, only risky debt is possible. In contrast to the relationship discussed above, the debt ratio range is then monotonically increasing in the bankruptcy cost parameter, g. Thus, while the other comparative statics are the same under either a first- or second-best world, the predicted relationship between bankruptcy costs and debt ratio range is reversed.

Finally, referring to Table V, increasing recapitalization costs results in a monotonically increasing optimal debt ratio range. One interpretation of varying recapitalization costs is their relative importance for big and small firms. Small debt issues are observed to incur larger proportional transactions costs than large issues;[14] for one sample, underwriters' compensation was 13.6% for small issues and 0.8% for large issues. Since issue size is positively related to firm size, small firms should have a wider range of debt ratios.

It is interesting to note that issuing *risky* debt yields a larger advantage to leverage than does issuing riskless debt for a wide range of parameter values. These results are obtained although we assumed a thirty-five percent personal tax disadvantage of debt, thereby considerably decreasing the value of the debt-related tax shields.

III. Predictions and Tests

In a dynamic setting, debt ratio observations are not adequate measures of the firm's capital structure policy. The above analysis suggests that the debt ratio *range* is a more relevant measure of a firm's dynamic debt policy. Thus, our model leads to the following hypothesis about capital structure relevance in terms of the optimal debt ratio range for a firm following an optimal dynamic policy:

Hypothesis: Firms that allow wide swings in their debt ratios, i.e., firms with large debt ratio ranges, have a low effective corporate tax rate, τ_c, a high variance of underlying asset value, σ^2, a small asset base (i.e., small firms), and low bankruptcy costs.

This hypothesis results from our dynamic generalization of the traditional tax/bankruptcy cost static model.[15] The alternative hypothesis is that the tax/bankruptcy cost tradeoff cannot explain dynamic capital structure behavior.[16]

Care must be taken in interpreting the comparative static result on the corporate tax rate. The valuation model assumes one homogeneous tax rate, so varying the tax rate is interpreted as comparing distinct production economies, each with its own homogeneous tax rate. Thus, when we admit differences in tax rates across firms in our empirical investigation, we implicitly rule out tax arbitrage, treating each set of firms with the same effective tax rate as a distinct production economy.[17]

[14] For example, see Brealey and Myers [3].

[15] As discussed above, the prediction on bankruptcy costs assumes the ability to issue riskless debt. Assuming the alternative second-best case reverses the prediction.

[16] Additional theoretical modeling would lead to competing dynamic analogs of other static models, such as Jensen and Meckling [17] or Myers [25]. These models might generate other null hypotheses.

[17] For a discussion of joint restrictions on asset pricing and tax rates that preclude tax arbitrage, see Ross [29] and Dammon and Green [12].

To test the predictions regarding firms' dynamic capital structure policies we obtained quarterly accounting and market value data for a sample of 999 firms on the Industrial Quarterly COMPUSTAT tape. These firms were selected randomly from among all COMPUSTAT-listed firms with complete quarterly debt ratio data over the period 1977-III to 1985-IV.[18]

The proxies for the relevant firm-specific parameters of our model are as follows:

(1) debt ratio range:[19]

(a) $LEVR1$ = the difference between the maximum and the minimum debt ratio over the thirty-four-quarter sample period, with debt ratio defined as total liabilities divided by the sum of total liabilities and equity market value.[20]

(b) $LEVR2$ = the difference between the maximum and the minimum debt ratio over the thirty-four-quarter sample period, with debt ratio defined as long-term debt divided by the sum of long-term debt and equity market value.

(2) corporate tax rate:

Under U.S. tax law, most corporations face the same statutory tax rate. However, the effective tax rate can differ significantly across firms.[21] These differences arise for several reasons, such as progressivity of tax rates, limitations on the use of investment tax credits, and limitations on the use of tax-loss carry-backs. We use two proxies for a firm's effective tax rate.

(a) $TXRT1$ = the ratio of the total (thirty-four quarters) reported income tax over the total pre-tax income.

(b) $TXRT2$ = the average of the ratio of the quarterly total reported income tax over the quarterly pre-tax income.[22]

(3) size:

$SIZE$ = the average of the quarterly total liabilities plus common equity market value, in millions of dollars.

(4) standard deviation rate:

SD = the sample standard deviation of the logarithm of the ratio of $SIZE(t)$ over $SIZE(t-1)$, where t is measured in quarters.

[18] All the income statement data are from the forty-item, forty-quarter Industrial file. The quarterly balance sheet leverage measures were acquired from COMPUSTAT.

[19] Our model defines the debt ratio as the *book* value of debt over the *market* value of the firm's assets. Because the market value of the firm's total liabilities is not available, we approximate the total value of the firm's assets by the sum of the market value of equity and the book value of total liabilities.

[20] In using this measure we assume that the observed minimum and maximum debt ratios represent the recapitalization bounds, or at least that the distance between the unobserved bounds is positively correlated with the difference between the observed minimum and maximum ratios.

[21] Cordes and Sheffrin [10] estimate effective tax rates and find considerable variation across firms.

[22] These two measures are weighted averages of thirty-four ratios of taxes paid over taxable income. The weight assigned to a particular quarter in $TXRT1$ is given by that quarter's earnings divided by total earnings over the thirty-four quarters. Alternatively, in $TXRT2$ all quarters are weighted equally.

As discussed in Section II, our model implies that the debt ratio range is discontinuous and nonmonotonic in the bankruptcy cost parameter. As suggested there, however, high-bankruptcy-cost firms should, on average, have narrower debt ratio ranges than firms with relatively low bankruptcy costs. Titman [31] and Titman and Wessels [32] argue that machine and equipment manufacturing firms will impose especially high bankruptcy costs on customers, suppliers, and workers. Rational agents anticipate imposition of these costs in the pricing of the firm's products, wage levels, etc. The imposition of these costs in firm value is one interpretation of the bankruptcy costs in our model. To be consistent with the discontinuity of debt ratio range in bankruptcy costs, we use a two-level dummy variable, rather than a continuous bankruptcy cost proxy. Following Titman and Wessels [32], we define an industry dummy variable to capture the effect of bankruptcy costs.[23]

(5) bankruptcy costs:

$IND = 1$ if $3400 \leq$ SIC code ≤ 4000;
$IND = 0$ otherwise.

Table VIII contains summary statistics for the proxy variables. The mean leverage ratio range $LEVR1$ (based upon total liabilities) of 0.341 is the difference between an average maximum debt ratio of 0.655 and an average minimum of 0.314. Similarly, the mean $LEVR2$ (defined in terms of long-term debt) of 0.337 follows from an average maximum debt ratio of 0.465 and an average minimum debt ratio of 0.128. Table VIII demonstrates significant variability in firms' debt ratios over time, so that observations of a firm's capital structure at a fixed point in time may contain little information about optimal capital structure choice.

The minimum and maximum values for the tax rates and $LEVR1$ in Table VIII indicate the presence of outliers, although the means for these variables are

Table VIII

Summary Statistics for Proxies[a]

	Minimum	Maximum	Mean	Standard Deviation
$LEVR1$	0.010	2.84	0.341	0.190
$LEVR2$	0	0.946	0.337	0.195
$TXRT1$	−17.68	16.72	0.367	1.07
$TXRT2$	−6.87	35.56	0.406	1.19
$SIZE$ (in $ million)	4.28	99,341	1,673.5	5,769.8
SD	0.025	1.07	0.132	0.079

[a] $LEVR1$: maximum minus minimum ratio of total liabilities divided by total liabilities plus equity market value.

$LEVR2$: maximum minus minimum ratio of long-term debt divided by long-term debt plus equity market value.

$TXRT1$: ratio of the total reported income tax over the total pre-tax income.

$TXRT2$: average of the quarterly ratios of reported income tax over pre-tax income.

$SIZE$: average quarterly total liabilities plus equity market value ($ million).

SD: standard deviation of the logarithm of the ratio of $SIZE(t)$ over $SIZE(t-1)$.

[23] Bradley, Jarrell, and Kim [2], among others, report debt ratio commonalities by industry.

Table IX

OLS Regressions

Variables	Regressions[a] 1	2	3	4
Dependent:	*LEVR*1	*LEVR*1	*LEVR*2	*LEVR*2
Explanatory:				
Constant	0.159	0.160	0.259	0.251
	(15.90)[b]	(15.81)	(21.12)	(20.29)
SIZE	-0.334×10^{-5}	-0.334×10^{-5}	-0.289×10^{-5}	-0.289×10^{-5}
	(−4.05)	(−4.05)	(−2.87)	(−2.88)
*TXRT*1	-0.337×10^{-3}	—	−0.005	—
	(−0.076)		(−1.01)	
*TXRT*2	—	−0.002	—	0.013
		(−0.42)		(2.63)
SD	1.459	1.460	0.761	0.770
	(24.13)	(24.09)	(10.28)	(10.40)
IND	−0.022	−0.022	−0.056	−0.056
	(−2.12)	(−2.13)	(−4.42)	(−4.41)
R^2	0.385	0.385	0.120	0.126

[a] *LEVR*1: maximum minus minimum ratio of total liabilities divided by total liabilities plus equity market value.
*LEVR*2: maximum minus minimum ratio of long-term debt divided by long-term debt plus equity market value.
*TXRT*1: ratio of the total reported income tax over the total pre-tax income.
*TXRT*2: average of the quarterly ratios of reported income tax over pre-tax income.
SIZE: average quarterly total liabilities plus equity market value ($ million).
SD: standard deviations of the logarithm of the ratio of $SIZE(t)$ over $SIZE(t-1)$.
IND: dummy variable with value one when SIC code is within 3400 to 4000, and zero otherwise.

[b] *t*-statistics in parentheses.

not unreasonable. We will subsequently identify outlier tax rates and demonstrate their impact on our tests. The large maximum value for *LEVR*1 is due to two firms in the COMPUSTAT sample that had, at one point in time, negative reported total liabilities. Removing these firms from the sample left the results shown below unchanged.

Table IX reports the results of OLS regressions of the leverage range on the set of explanatory variables. The results are consistent with our hypothesis, except for the corporate tax rate. In three of the four regressions, the coefficient on the tax rate is of the correct sign, but insignificant. When *LEVR*2 is regressed on *TXRT*2, the coefficient is significant and has the wrong sign.

Since the theoretical model does not necessarily predict a linear relationship between leverage ratio range and the explanatory variables, we examine the residuals plotted against both predicted values and against the explanatory variables. The only significant nonlinearity seems attributable to the standard deviation rate, *SD*. Therefore, *SD* squared is included in subsequent tests:

$$SDSQ = SD^2.$$

Also, a Goldfeld-Quandt test indicates the presence of heteroscedasticity. A generalized least-squares regression method will be applied below to correct for the heteroscedasticity. Finally, two observations yielded exceedingly large residuals, attributable to extreme tax rate values. A regression method designed to isolate the impact of extreme explanatory variable values will be employed below.

To deal with the problems of heteroscedasticity and economically unreasonable sample tax rates, we employ a generalized least-squares regression of a debt ratio range measure ($LEVR1$ or $LEVR2$) on $SIZE$, SD, $SDSQ$, IND, and two tax measures:[24]

$$GTXRT1 = \begin{cases} TXRT1 & \text{if } 0 \leq TXRT1 \leq 1, \\ 0 & \text{otherwise,} \end{cases}$$

$$BTXRT1 = \begin{cases} 0 & \text{if } 0 \leq TXRT1 \leq 1, \\ TXRT1 & \text{otherwise.} \end{cases}$$

A similar procedure provides the variables $GTXRT2$ and $BTXRT2$.

The generalized least-squares procedure involves using the residuals from an OLS regression to estimate the variance-covariance matrix of the error term, e. Analysis of the OLS residuals indicates that the error variance is related to SD and $SDSQ$. We assume multiplicative heteroscedasticity, leading us to the OLS regression

$$\ln(e^2) = a + b_1(SD) + b_2(SDSQ) + v. \tag{24}$$

The GLS procedure then involves dividing each observation by

$$(\exp(\hat{a} + \hat{b}_1(SD) + \hat{b}_2(SDSQ)))^{1/2}. \tag{25}$$

Table X contains the results of the four OLS regressions corresponding to the four OLS regressions shown in Table IX. All of the predicted relationships from our hypothesis are supported in all four regressions; larger, less risky firms with higher effective tax rates have narrower observed debt ratio ranges. In addition, firms in the manufacturing industry, where bankruptcy costs are more likely to be high, also have narrower debt ratio ranges.[25] Thus, the time-series behavior of firms' financial leverage ratios is convincingly consistent with capital structure relevance.

For the most part, both leverage ratio range measures, $LEVR1$ and $LEVR2$, yield similar results. However, the explanatory power of our model is greater when using the leverage range measure based upon total liabilities (i.e., $LEVR1$) instead of long-term debt (i.e., $LEVR2$). The theoretical model is based upon total liabilities and is unable to discriminate between liabilities with differing maturities. Even using only long-term debt in the leverage range measure yields strongly supportive results. The bankruptcy proxy is only significant when the leverage ratio range is measured by long-term debt only. Apparently, liquidation costs impact the long-term debt decision more than the short-term debt decision.

In all four regressions in Table X, the economically reasonable tax rate proxies (i.e., $GTXRT1$ and $GTXRT2$) are significant as predicted. The impact of the

[24] For an application of this regression technique, see Bilson [1].

[25] This result is consistent with our model under the assumption that first-best recapitalization is feasible. We also fitted the regression equations for industry subsamples and four subperiods. The signs of the slope coefficients were unchanged, but significance levels were reduced.

Table X

WLS Regressions[a]

Variables	Regressions 1	2	3	4
Dependent:	*LEVR*1	*LEVR*1	*LEVR*2	*LEVR*2
Explanatory:				
$1/WT^{b}$	0.131	0.136	0.306	0.307
	(9.56)[c]	(10.18)	(15.39)	(15.77)
SIZE	-0.257×10^{-5}	-0.248×10^{-5}	-0.269×10^{-5}	-0.239×10^{-5}
	(−4.23)	(−4.15)	(−2.98)	(−2.73)
*GTXRT*1	−0.064	—	−0.159	—
	(−2.63)		(−4.54)	
*BTXRT*1	−0.002	—	−0.002	—
	(−0.44)		(−0.39)	
*GTXRT*2	—	−0.080	—	−0.183
		(−3.40)		(−5.43)
*BTXRT*2	—	−0.001	—	0.016
		(−0.23)		(4.38)
SD	2.028	2.022	0.840	0.914
	(19.37)	(19.31)	(5.68)	(6.01)
SDSQ	−1.156	−1.177	−0.198	−0.317
	(−12.76)	(−12.81)	(−1.44)	(−2.15)
IND	−0.012	−0.012	−0.049	−0.048
	(−1.41)	(−1.38)	(−3.91)	(−3.91)
R^2	0.848	0.867	0.300	0.240

[a] Explanatory variables are as defined below and then divided by the WLS weight, *WT*.

*LEVR*1: maximum minus minimum ratio of total liabilities divided by total liabilities plus equity market value.

*LEVR*2: maximum minus minimum ratio of long-term debt divided by long-term debt plus equity market value.

*GTXRT*1: *TXRT*1 as defined in Table IX if $0 \leq TXRT1 \leq 1$, zero otherwise.

*BTXRT*1: *TXRT*1 as defined in Table IX if $TXRT1 \notin [0, 1]$, zero otherwise.

*GTXRT*2: *TXRT*2 as defined in Table IX if $0 \leq TXRT2 \leq 1$, zero otherwise.

*BTXRT*2: *TXRT*2 as defined in Table IX if $TXRT2 \notin [0, 1]$, zero otherwise.

SIZE: Average quarterly total liabilities plus equity market value ($ million).

SD: Standard deviation of the logarithm of the ratio of $SIZE(t)$ over $SIZE(t-1)$.

IND: Dummy variable with value one when SIC code is within 3400 to 4000, and zero otherwise.

SDSQ: Square of *SD*.

[b] WLS weights given by $WT = (\exp(\hat{a} + \hat{b}_1\ (SD) + \hat{b}_2\ (SDSQ)))^{1/2}$ [equation (25)].

[c] *t*-statistics in parentheses.

sample tax rate outliers is insignificant in the first three regressions. The significant coefficient on *BTXRT*2 in the fourth regression is due to one firm with an unreasonably large positive tax rate combined with a leverage ratio range of over ninety percent.

IV. Summary and Conclusions

This paper develops a model of dynamic optimal capital structure choice in the presence of recapitalization costs. The analysis builds upon the traditional tax/

bankruptcy cost theory of capital structure relevance. The results demonstrate the dangers of viewing observed debt ratios as "optimal"; to avoid these problems we employ a different measure of capital structure relevance, namely the range over which the firm allows its debt ratio to vary.

The model provides distinct predictions relating firm-specific properties to the range of optimal leverage ratios: smaller, riskier, lower-tax, lower-bankruptcy-cost firms will exhibit wider swings in their debt ratios over time. As noted above, some of the results depend upon the assumed form of transactions costs and upon the ability to precommit to a first-best recapitalization policy. Clearly, additional work is required to explore the impact of alternative functional forms for recapitalization costs and to extend other static theories to a dynamic setting. However, while our predictions come from one particular way of characterizing the benefits and costs of debt financing, they may prove more general than the underlying model.

We view our simple tests as only a first attempt to study empirically firms' dynamic capital structure behavior. The supportive results, given the simplicity of our data and tests, are encouraging.

REFERENCES

1. John F. O. Bilson. "The Speculative Efficiency Hypothesis." *Journal of Business* 54 (July 1981), 435–52.
2. Michael Bradley, Gregg A. Jarrell, and E. Han Kim. "On the Existence of an Optimal Capital Structure: Theory and Evidence." *Journal of Finance* 39 (July 1984), 857–78.
3. Richard Brealey and Stewart Myers. *Principles of Corporate Finance.* New York: McGraw-Hill, 1984.
4. Michael J. Brennan. "Costless Financing Policies under Asymmetric Information." University of British Columbia working paper, June 1986.
5. ——— and Eduardo S. Schwartz. "Corporate Income Taxes, Valuation and the Problem of Optimal Capital Structure." *Journal of Business* 51 (January 1978), 103–14.
6. ———. "Optimal Financial Policy and Firm Valuation." *Journal of Finance* 39 (July 1984), 593–607.
7. Richard Castanias. "Bankruptcy Risk and Optimal Capital Structure." *Journal of Finance* 38 (December 1983), 1617–35.
8. George Constantinides. "Multiperiod Consumption and Investment Behavior with Convex Transactions Costs." *Management Science* 25 (November 1979), 1127–37.
9. ———. "Capital Market Equilibrium with Transaction Costs." *Journal of Political Economy* 94 (December 1986), 842–62.
10. Joseph J. Cordes and Steven M. Sheffrin. "Estimating the Tax Advantage of Corporate Debt." *Journal of Finance* 38 (March 1983), 95–105.
11. John C. Cox, Jonathan E. Ingersoll, Jr., and Stephen A. Ross. "An Intertemporal General Equilibrium Model of Asset Prices." *Econometrica* 53 (March 1985), 363–407.
12. Robert M. Dammon and Richard C. Green. "Tax Arbitrage and the Existence of Equilibrium Prices for Financial Assets." *Journal of Finance* 42 (December 1987), 1143–66.
13. Masako N. Darrough and Neal M. Stoughton. "Moral Hazard and Adverse Selection: The Question of Financial Structure." *Journal of Finance* 41 (June 1986), 501–13.
14. Edwin O. Fischer, Robert Heinkel, and Josef Zechner. "Dynamic Recapitalization Policies and the Role of Call Premia and Issue Discounts." University of British Columbia and University of Graz working paper, June 1988.
15. Robert Heinkel. "A Theory of Capital Structure Relevance under Imperfect Information." *Journal of Finance* 37 (December 1982), 1141–50.

16. Donald L. Iglehart. "Optimality of (s, S) Policies in the Infinite Horizon Dynamic Inventory Problem." *Management Science* 9 (March 1963), 259–67.
17. Michael C. Jensen and William H. Meckling. "Theory of the Firm: Managerial Behavior, Agency Costs and Capital Structure." *Journal of Financial Economics* 4 (October 1976), 305–60.
18. Alex Kane, Alan J. Marcus, and Robert L. McDonald. "How Big is the Tax Advantage to Debt?" *Journal of Finance* 39 (July 1984), 841–53.
19. ———. "Debt Policy and the Rate of Return to Leverage." *Journal of Financial and Quantitative Analysis* 20 (December 1985), 479–99.
20. Alan Kraus and Robert Litzenberger. "A State Preference Model of Optimal Financial Leverage." *Journal of Finance* 28 (September 1973), 911–21.
21. Hayne Leland and David Pyle. "Information Asymmetries, Financial Structure and Financial Intermediation." *Journal of Finance* 32 (May 1977), 371–87.
22. Robert C. Merton. "Theory of Rational Option Pricing." *The Bell Journal of Economics and Management Science* 4 (Spring 1973), 141–83.
23. Merton Miller. "Debt and Taxes." *Journal of Finance* 32 (May 1977), 261–75.
24. Franco Modigliani. "Debt, Dividend Policy, Taxes, Inflation and Market Valuation." *Journal of Finance* 37 (May 1982), 255–73.
25. Stewart Myers. "The Determinants of Corporate Borrowing." *Journal of Financial Economics* 4 (November 1977), 147–75.
26. ———. "The Capital Structure Puzzle." *Journal of Finance* 39 (July 1984), 575–92.
27. Stephen Ross. "The Determination of Financial Structure: The Incentive-Signalling Approach." *The Bell Journal of Economics* 8 (Spring 1977), 23–40.
28. ———. "Debt and Taxes and Uncertainty." *Journal of Finance* 40 (July 1985), 637–58.
29. ———. "Arbitrage and Martingales with Taxation." *Journal of Political Economy* 95 (April 1987), 371–93.
30. M. Taksar, M. Klass, and D. Assaf. "Diffusion Model for Optimal Portfolio Selection in Presence of Brokerage Fees." Technical Report No. 4, Stanford University Department of Operations Research, 1983.
31. Sheridan Titman. "The Effect of Capital Structure on a Firm's Liquidation Decision." *Journal of Financial Economics* 13 (March 1984), 137–51.
32. ——— and Roberto Wessels. "The Determinants of Capital Structure Choice." *Journal of Finance* 43 (March 1988), 1–19.
33. Josef Zechner and Peter Swoboda. "The Critical Implicit Tax Rate and Capital Structure." *Journal of Banking and Finance* 10 (December 1986), 832–48.