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The Purchasing Power of Money and Nominal Interest Rates: A Re-Examination

DILIP K. SHOME, STEPHEN D. SMITH, and JOHN M. PINKERTON*

ABSTRACT

While it has been known for some time that, under uncertainty, the original version of the Fisher hypothesis is not precisely correct, empirical researchers have largely ignored this fact. Such an omission has possibly resulted in erroneous conclusions concerning other hypotheses; most notably the impact of prices on the real economy. This paper clarifies some of the previous interpretations of the existing empirical literature and provides a theoretical version of the relation between prices and interest rates. Empirical tests based on both the Livingston survey data and data from time-series forecasting models provide support for the Fisher effect and the hypothesis that only covariance risk is priced in the Treasury bill market.

ONE OF THE BASIC hypotheses concerning rationality in financial markets is based on statements generally attributed to Fisher [18]. Fisher argued that the nominal or money rate of return on an asset should compensate an investor for his or her required real rate of return and changes in the purchasing power of money. For the uncertainty case, Fisher replaced known purchasing power changes with the expected inflation rate. Whereas the Fisher relationship under certainty is well defined, his basic theorem under uncertainty is not precise. As Fama [15] has argued, it is changes in the purchasing power of money, not the inflation rate, that should concern investors. Under uncertainty, the expected value of one is not, by Jensen's inequality, the inverse of the expected value of the other. Moreover, Fama and Farber [16], Grauer and Litzenberger [20], and Benninga and Protopapadakis [3] have shown that, when investors are risk averse, the Fisher relationship will generally not hold, even after adjustment has been made for the distinction between purchasing power changes and the inflation rate. In particular, these authors show that an adjustment must be made for the fact that the real rate of return on nominal bonds is random.

While these issues have been discussed in the theoretical literature, they have largely been ignored in the empirical work on this topic. Given the fact that nominal rates do not seem to adjust as rapidly to price changes as Fisher hypothesized, numerous empirical researchers have expanded the basic Fisher model in an attempt to explain this empirical anomaly. These extensions include the incorporation of (a) inflation rate variability and the growth in real output

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(Levi and Makin [22]), (b) inflation rate variability and money supply growth (Bomberger and Frazer [5]), (c) tax effects (Peek [26], Peek and Wilcox [27]), and (d) supply considerations, both in the real goods market (Wilcox [30], Peek and Wilcox [27]) and in the market for nominal bonds (Carmichael and Stebbing [10]). It is our view that much of this literature has developed in large part due to the unobservability of movements in the real rate of interest. Such measurement problems cause a basic ambiguity in any attempt to isolate how much of the variation in nominal interest rates can be attributed solely to the fact that the future purchasing power of money is a random variable, which, we believe, is the central issue of this literature.

The purpose of the paper is, therefore, to develop and estimate a model of the Fisher equation under uncertainty and risk aversion. Such a formulation allows us to test (a) the empirical significance of Jensen's inequality and (b) the existence of a real risk premium on default-free government bonds.

The rest of the paper is organized as follows. Section I develops and estimates a log-linear model of the Fisher equation under uncertainty. Section II contains a discussion of our results in the context of some existing models. Concluding remarks are contained in Section III.

I. Asset Returns and Changes in Purchasing Power

In this section the basic model is derived. The framework is one of rational expectations as developed by Beja [2], Lucas [24], and Brock [7]. They have shown, using a representative consumer/investor, that the first-order conditions for maximizing investor utility require that

$$U'(C_t)P_{i,t} = \rho E_t[U'(C_{t+1})(P_{i,t+1} + D_{i,t+1})] \ \forall\ i,\ t, \tag{1}$$

where C_t and C_{t+1} are consumption in period t and $t + 1$, respectively. $P_{i,t}$ and $P_{i,t+1}$ are the prices of asset *in* (in terms of the consumption good) in periods t and $t + 1$, respectively.[1] $D_{i,t+1}$ is the dividend in period $t + 1$, ρ is an impatience factor, $0 < \rho < 1.0$, and $U(\cdot)$ is a twice-differentiable utility function with $U'(\cdot) > 0$ and $U''(\cdot) \leq 0$. The expectation on the right-hand side is taken with respect to the information available at time t.

Denote the price level in periods t and $t + 1$ as I_t and I_{t+1}, respectively, and define $\dfrac{I_t}{I_{t+1}}$ as one plus the change in purchasing power and $\pi_{t+1} \equiv \dfrac{I_{t+1}}{I_t}$ as one plus the inflation rate. In the case of individuals investing some consumption units in nominally risk-free, one-period discount bonds that pay off one unit of money or $1/I_{t+1}$ units of consumption, equilibrium in the market requires that

$$E_t[U'(C_{t+1})/I_{t+1}]\rho R_t - U'(C_t)/I_t = 0, \tag{2}$$

where R_t is one plus the (known at time t) nominal return on the bond.

[1] We consider only one good is this case. Such an assumption does not influence our empirical results since we use a power utility function. However, in other cases the results may differ in the multigood case. See Breeden [6], Section 7, for a discussion.

Much of our analysis can be accomplished by examining equation (2). Many researchers (see, for example, Peek [26], Peek and Wilcox [28], and Wilcox [30]) have used the expected inflation rate as the relevant explanatory variable. More recently, higher order moments (e.g., the standard deviation of inflation rate) have been used as an additional explanatory variable. A theoretical argument in support of the inclusion of the inflation-uncertainty variable is based on statements made by Friedman [19]. According to Friedman, an increase in inflation uncertainty reduces economic efficiency, resulting in a decrease in real output. The reduced aggregate supply causes an increase in the real rate. Levi and Makin [22] provide an alternative argument according to which high uncertainty about inflation rates reduces investment demand and thus puts downward pressure on the real rate. Bomberger and Frazer [5] find an ambiguous theoretical relationship. Finally, it is sometimes argued (see Barnea, Dotan, and Lakonishok [1]) that inflation variability is a measure of risk and should therefore be positively related to the nominal rate, assuming risk aversion. These arguments ignore important theoretical characteristics of the equilibrium presented by equation (2). First, it is the distribution of the purchasing power of money, I_{t+1}^{-1}, which influences the demand for bonds, not the inflation rate per se. In particular, from Jensen's inequality it can be shown that a mean-preserving spread in the inflation rate will cause the expected purchasing power of money to increase. In the absence of risk factors this will result in an increase in the demand for bonds, which causes a decrease in the nominal bond rate, R_t, assuming that nominal bonds are normal goods.

A second potential problem regarding the existing studies involves the theoretical role of risk aversion. As noted earlier, Barnea, Dotan, and Lakonishok [1] argue for a positive relationship between inflation uncertainty and nominal rates because greater uncertainty causes risk-averse investors to add a risk premium to compensate for this additional uncertainty. However, expanding equation (2) suggests that it is the covariance of purchasing power and real output, not the uncertainty in purchasing power per se, that should concern risk-averse investors. In particular, if purchasing-power shocks are positively related to real economic output (i.e., inflation rate and real output are negatively related), then, under the assumption that investors are risk averse, a mean-preserving increase in the purchasing power of money will cause an increase in nominal interest rates.[2]

To develop an empirical test of these results, we assume that investors have power utility functions and that consumption and the price index are joint-lognormally distributed. Equation (2) can be rewritten as

$$E_t\left[\frac{U'(C_{t+1})}{U'(C_t)}\,\pi_{t+1}^{-1}\right] = \rho^{-1}R_t^{-1}. \tag{2a}$$

Under the power utility assumption,

$$U(C_t) = \frac{C_t^{1-\lambda}}{1-\lambda}, \qquad \lambda \neq 1; \quad U(C_t) = \ln(C_t), \quad \lambda = 1.0,$$

[2] Both of these comparative static results can be proved in a general setting. Proofs are available from the authors upon request.

where λ is the degree of relative risk aversion. Therefore, $\frac{U'(C_{t+1})}{U'(C_t)} = g_{t+1}^{-\lambda}$, where $g_{t+1} = \frac{C_{t+1}}{C_t}$ is one plus the rate of growth in consumptin. If g_{t+1} and π_{t+1} are joint-lognormally distributed, then $g_{t+1}^{-\lambda}$ and π_{t+1}^{-1} are lognormally distributed and their product is also lognormal. Furthermore, if X and Y are bivariate lognormal, then

$$E_t[X_{t+1}] = \exp[E_t(\ln X_{t+1}) + 0.5 \operatorname{var}_t(\ln X_{t+1})] \tag{3}$$

and

$$E_t[X_{t+1} Y_{t+1}] = E_t[X_{t+1}]E_t[Y_{t+1}]\exp[\operatorname{cov}_t(\ln X_{t+1}, \ln Y_{t+1})] \quad \text{for all } X \text{ and } Y. \tag{4}$$

Defining $g_{t+1}^{-\lambda} \equiv X_{t+1}$ and $\pi_{t+1}^{-1} \equiv Y_{t+1}$, and using equation (4) above, we have

$$E_t[g_{t+1}^{-\lambda}\pi_{t+1}^{-1}] = E_t[g_{t+1}^{-\lambda}]E_t[\pi_{t+1}^{-1}]\exp[\lambda \operatorname{cov}_t(\ln g_{t+1}, \ln \pi_{t+1})]. \tag{5}$$

Substituting this result into the left-hand side of equation (2a) and rearranging the terms, we have

$$R_t = \rho^{-1}[E_t[g_{t+1}^{-\lambda}]]^{-1}[E_t[\pi_{t+1}^{-1}]]^{-1}\exp[\operatorname{cov}_t[\lambda \ln g_{t+1}, -\ln \pi_{t+1}]]. \tag{6}$$

Equation (6) has a number of interesting properties. Under the lognormality assumption, there is a specific link between the expected change in the purchasing power of money and the expected inflation rate; i.e.,

$$[E_t[\pi_{t+1}^{-1}]]^{-1} = [E_t[\pi_{t+1}]]\exp[-\operatorname{var}_t(\ln \pi_{t+1})],$$

where var(·) is the variance. Notice that holding $E_t[\pi_{t+1}]$ constant while increasing the variance of $\ln(\pi_{t+1})$ results in a ceteris paribus decrease in R_t simply because $E_t[\pi_{t+1}^{-1}]$, the expected purchasing power, has increased. It follows that empirical researchers who use the expected inflation rate and inflation variability as explanatory variables may find that the inflation variability has a spurious negative influence on nominal rates. However, if expected purchasing power changes are used, the variance of the inflation rate is, unambiguously, not a relevant explanatory variable. Therefore, our first testable hypothesis is that the variance of inflation exerts no separate economic influence on nominal rates. The relevance of risk aversion is represented by the last term on the right-hand side of equation (6). Specifically, when the continuously compounded change in the purchasing power of money, $-\ln(\pi_{t+1})$, is positively related to the continuously compounded growth in real output, nominal rates will increase, the more positive or less negative the covariance. Alternatively, nominal rates will decrease, the less positive the covariance of the continuously compounded inflation rate, $\ln(\pi_{t+1})$, and growth in real output. Therefore, our second hypothesis is that it is this covariance risk that is priced by risk-averse investors in the nominal bond market.

The model is reduced to a log-linear form in order to test it. Using equation (4) to replace $E_t[g_{t+1}^{-\lambda}]$ and taking logs of equation (6) yield

$$\ln R_t = \alpha_0 + \alpha_1 \ln E_t[\pi_{t+1}^{-1}] + \alpha_2 E_t[\ln g_{t+1}] + \alpha_3 \operatorname{var}_t[\ln g_{t+1}] + \alpha_4 \operatorname{cov}_t[\ln g_{t+1}, -\ln \pi_{t+1}], \tag{7}$$

where the theoretical restrictions imply that

$$\alpha_0 = -\ln \rho, \quad \alpha_1 = -1.0, \quad \alpha_2 = \lambda = \alpha_4, \quad \text{and} \quad \alpha_3 = -0.5\lambda^2.$$

Under the null hypothesis of risk neutrality, $\alpha_1 = \alpha_3 = \alpha_4 = 0$. The conditional moments of consumption growth, which summarize the information about growth in real activity, will be significant if and only if Treasury bill market participants display risk aversion.

An empirical determination that (a) $\alpha_1 = -1.0$ and that (b) the coefficient of an added inflation uncertainty variable, $[\text{var}_t(\ln\, \pi_{t+1})]$, is insignificant would support our first hypothesis. A determination of a positive risk aversion coefficient $\lambda = \alpha_2 = \alpha_4$ would support our second hypothesis.

A. Data Description

Equation (7) can be estimated using survey data or forecast data obtained from time-series models. The main studies discussed in this paper (Levi and Makin [22], Bomberger and Frazer [5], and Barnea, Dotan, and Lakonishok [1]) have used the Livingston survey data to test the relation between prices and interest rates. Although the Livingston data are widely used, there is considerable disagreement about their efficacy. Cukierman and Wachtel [12] and Bomberger and Frazer [5], among others, argue in favor of ex ante measures based on survey data. Fama and Gibbons [17] and Engle [14], among others, demonstrate the superiority of time-series forecasts. To allow for a direct comparison with related studies, our model is first tested using variables based on the Livingston forecast data. However, given the lack of consensus about the validity of the survey data, the model is also estimated using forecasts from time-series forecasting models. The results are evaluated to determine the sensitivity of the hypotheses tests to alternative specifications.

A.1. Variable Measurements Based on Livingston Data

One problem with the Livingston data concerns the base period. Questionnaires are sent to participants in November and May along with the most recent reported economic data—usually from October and April. There is some ambiguity regarding the information set analysts use in making their forecasts and regarding the time period over which the forecast is made. A technique for correcting the base period problem is suggested by Carlson [9]. Based on a survey of participating analysts, Carlson concludes that analysts predominantly use October and April figures as the base period values and make forecasts for June and December, resulting in eight-month and fourteen-month forecasts. To obtain a six-month rate from an eight-month forecast, Carlson suggests converting the expected eight month percentage change to a monthly rate and then compounding that six times. We follow the widely used Carlson adjustment procedure to obtain a time series of semiannual measures of all the independent (Livingston data-based) variables.

The model calls for ex ante estimates of the moments of the distribution of inflation/purchasing power changes and of the consumption growth. The Livingston data do not contain forecasts of consumption. However, on the basis that GNP appears to be the best estimator of consumption, given the available data, the forecast of *real* GNP is used as a proxy for expected consumption. This approximation will be valid, for example, if the marginal propensity to consume remains constant through time and there is no lag between income and consumption changes. The individual analyst's forecast of the semiannual inflation rate and real GNP growth is estimated, following Carlson, as

$$\pi_{i,t+1} = \left[\frac{(\mathrm{CPI}_i)_{t+1}}{(\mathrm{CPI})_t}\right]^{6/8},$$

$$g_{i,t+1} = \left[\frac{(\mathrm{GNP}_i)_{t+1}}{(\mathrm{GNP})_t}\right]^{6/8},$$

where t is the base period (October or April) and $t + 1$ is the forecast period (June or December). The explanatory variables are then estimated as cross-sectional moments of the forecasts (across the N participants). Thus,

expected purchasing power change $= \ln E_t[\pi_{t+1}^{-1}]$
$= \ln [\sum_{i=1}^{N} (\pi_{i,t+1})^{-1}/N]$,

variance of inflation rate $= \mathrm{var}_t[\ln \pi_{t+1}]$
$= \sum_{i=1}^{N} (\ln \pi_{i,t+1} - E \ln \pi_{t+1})^2/(N-1)$,

expected consumption growth $= E_t[\ln g_{t+1}]$
$= \sum_{i=1}^{N} \ln g_{i,t+1}/N$,

variance of consumption growth $= \mathrm{var}_t[\ln g_{t+1}]$
$= \sum_{i=1}^{N} (\ln g_{i,t+1} - E_t \ln g_{t+1})^2/(N-1)$,

covariance of consumption growth $= \mathrm{cov}_t[\ln g_{t+1}, -\ln \pi_{t+1}]$
and purchasing power $= -\sum_{i=1}^{N} (\ln g_{i,t+1} - E \ln g_{t+1})(\ln \pi_{i,t+1} - E \ln \pi_{t+1})/N$.

As is typical in models of this sort, the use of these ex ante measures of variances and covariances assumes that the cross-sectional measures of dispersion are correlated with the true volatility measures. The dependent variable, $\ln[R]_t$, is measured as the logarithm of one plus the six-month Treasury bill yield, as of October and April, and obtained from the *Federal Reserve Bulletin*. Since real GNP data are available only since June 1971, our estimation horizon is June 1971 to December 1984.

A.2. Variable Measurments Based on Time-Series Forecasting Models

Reduced-form equations for inflation and consumption growth are first specified. Then, to estimate the forecast means of these variables and their variances and covariances at a point in time, the equations are estimated as a system of equations using the Seemingly Unrelated Regression (SUR) technique. To generate a time series of such forecasts, a moving or rolling regression technique is then used.[3]

Following Engle [14], inflation, ln π_t, is specified as a function of a set of lagged independent variables that include lagged inflation, the rate of change of money supply, the rate of change of wage rate, and the rate of change of the import price deflator.[4] Following Hall [21] and Mankiw [25], consumption growth is specified as a function of lagged consumption growth and lagged interest rate.[5] For the sake of comparability with the Livingston survey estimates, semiannual data are used to estimate the system of reduced-form equations. The firt SUR estimate is obtained for a ten-year period ending in June 1971. The next period's predicted values of inflation and consumption growth are obtained from the estimated equations, and the variances and covariances are estimated from the covariance matrix of the residuals. The first observation is then dropped and the December 1971 observation added, and the equations are re-estimated with the twenty observations to obtain the next set of forecasts. In this way a time series of estimates of all the independent variables is generated and used to test the Fisher equation.

The dependent variable, $\ln[R]_t$, is estimated as of December and June.

B. *Empirical Results*

Panel A of Table I presents some descriptive statistics of all the variables entering the model. The most striking characteristic of the data is the almost uniformly high degree of serial correlation, regardless of which estimation method is used. The high autocorrelation coefficients of the regressors raise the possibility of spurious correlation in the regression. This potential problem is avoided by estimating the model with the first differences of the variables.[6] The increased stationarity in he data is evident from the descriptive statistics of the first differences, reported in Panel B of Table I.

Panel A of Table II reports the coefficients obtained from a partially restricted nonlinear regression estimate of equation (7), based on Livingston data. Panel B provides the model estimate using time-series forecast data. The model appears to be quite robust with respect to the measurement of the explanatory variables.

[3] Alternatives to the rolling-regression approach are the Autoregressive Conditional Heteroskedastic (ARCH) technique of Engle [14] and the Generalized Autoregressive Conditional Heteroskedastic (GARCH) technique developed by Bollerslev [4]. These are more sophisticated econometric techniques, but Engle [14] finds that, at least for inflation, the results from the rolling-regression technique are "remarkably similar to these obtained from the ARCH model."

[4] See Engle [14] for details regarding the definition and measurement of the variables in the forecasting model.

[5] The Hall [21] and Mankiw [25] models are based on real consumption per capita. To generate the independent variables needed in our model, we have specified the consumption equation in terms of the logarithm of real consumption growth.

[6] We replicated the estimation using levels of the variables. The Durbin-Watson statistic is 0.78 with the Livingston data and 1.33 with the forecast data, implying high autocorrelation. However, after correcting for autocorrelation using the Durbin [13] procedure, the estimates are not significantly different from those reported in Table II using first differences.

Table I

Descriptive Statistics of Model Variables: June 1971 to December 1984[a]

A: Levels

		Livingston Data			Time-Series Forecast Data		
Variable	n	Mean	Standard Deviation	Autocorrelation Coefficient	Mean	Standard Deviation	Autocorrelation Coefficient
$\ln R_t$	28	4.1140	1.5176	0.6743	4.1503	1.4882	0.6482
$\ln E_t[\pi_{t+1}^{-1}]$	28	−2.9059	0.9913	0.8726	−3.2512	1.3740	0.7941
$\text{var}_t[\ln \pi_{t+1}]$	28	0.7378	0.4284	0.7313	0.7230	0.3906	0.9222
$E_t[\ln g_{t+1}]$	28	2.1904	1.9963	0.6281	0.6091	1.5299	0.3881
$\text{var}_t[\ln g_{t+1}]$	28	1.3807	0.8552	0.4929	1.5673	0.9510	0.9662
$\text{cov}_t[\ln g_{t+1}, -\ln \pi_{t+1}]$	28	0.0196	0.2592	0.1306	0.5716	0.3908	0.9046

B: First Differences

		Livingston Data			Time-Series Forecast Data		
Variable	n	Mean	Standard Deviation	Autocorrelation Coefficient	Mean	Standard Deviation	Autocorrelation Coefficient
$\Delta\ln R_t$	27	0.0861	0.9941	−0.2263	0.0545	1.2083	−0.4160
$\Delta\ln E_t[\pi_{t+1}^{-1}]$	27	−0.0022	0.4981	0.4327	0.0340	0.9318	0.0365
$\Delta\text{var}_t[\ln \pi_{t+1}]$	27	0.0119	0.3116	0.1270	0.0222	0.1051	0.0412
$\Delta E_t[\ln g_{t+1}]$	27	−0.0170	1.7485	−0.1117	−0.0199	1.7037	−0.5737
$\Delta\text{var}_t[\ln g_{t+1}]$	27	−0.0184	0.8726	−0.0805	0.0708	0.3204	0.0656
$\Delta\text{cov}_t[\ln g_{t+1}, -\ln \pi_{t+1}]$	27	0.0070	0.3472	−0.4524	0.0272	0.1676	−0.1402

[a] R_t is one plus the six-month Treasury bill yield, π_{t+1}^{-1} is one plus the expected purchasing power change, g_{t+1} is one plus the consumption growth, and π_{t+1} is one plus the inflation rate. $E(\cdot)$, $\text{var}(\cdot)$, and $\text{cov}(\cdot)$ are the expectations, variance, and covariance operators, respectively, and ln is the logarithm. Δ is the first-difference operator. The number of observations is denoted by n.

Both the time-series and survey data support our two hypotheses. The coefficient, α_1, of the expected purchasing power change is not significantly different from -1.0, and the coefficient, α_5, of inflation variability is not significantly different from zero,[7] at a five percent significance level, using either method. These results support our first hypothesis. That is, if the Fisher relationship is specified in terms of expected purchasing power changes, the inflation uncertainty, as an additional variable, should have no impact on the variation of the nominal interest rates. The risk aversion coefficient $\lambda = \alpha_2 = \alpha_4$ is positive and significant at a five percent significance level, consistent with our second hypothesis.[8] As the covariance between inflation and real output growth decreases, the nominal interest rate increases. Taken together, the results suggest that investors are pricing covariance risk, not price uncertainty per se.

[7] Because of the debate concerning its sign, we use a two-tailed test for the inflation variability coefficient.

[8] While our empirical estimate of λ is statistically significant, the point estimate is smaller than those found in earlier studies (e.g., Litzenberger and Ronn [23]) using common stock data and similar assumptions concerning the form of the utility function.

Table II

Partially Restricted Nonlinear Regression Estimate of the Model: June 1971 to December 1984[a]

Model:

$$\Delta\ln[R_t] = \alpha_1\Delta\ln E_t[\pi_{t+1}^{-1}] + \alpha_2\Delta E_t[\ln g_{t+1}] + \alpha_3\Delta\mathrm{var}_t[\ln g_{t+1}] + \alpha_4\Delta\mathrm{cov}_t[\ln g_{t+1}, -\ln \pi_{t+1}] + \alpha_5\Delta\mathrm{var}_t[\ln \pi_{t+1}]$$

A: Regression Using Livingston Data

α_1	α_2	α_3	α_4	α_5	R^2	DW
−0.8692	0.2735	−0.0374	0.2735	0.8122	0.34	2.4
(−2.29)	(2.62)	(−1.26)	(2.62)	(1.44)		

B: Regression Using Time-Series Forecast Data

α_1	α_2	α_3	α_4	α_5	R^2	DW
−0.7351	0.6760	−0.2285	0.6760	1.2188	0.56	1.8
(−3.43)	(5.83)	(−2.89)	(5.83)	(0.78)		

[a] R_t is one plus the six-month Treasury bill yield, π_{t+1}^{-1} is one plus the expected purchasing power change, g_{t+1} is one plus the consumption growth, and π_{t+1} is one plus the inflation rate. $E(\cdot)$, $\mathrm{var}(\cdot)$, and $\mathrm{cov}(\cdot)$ are the expectations, variance, and covariance operators, respectively, and ln is the logarithm. Δ is the first-difference operator. Restrictions in the estimation are $\alpha_2 = \alpha_4 = \lambda$ and $\alpha_3 = -0.5\,\alpha_4^2$. Likelihood-ratio tests were carried out to validate the use of these restrictions. The chi-square values of the tests for estimations based on the Livingston and forecast data were 2.69 and 0.18, respectively, compared with the critical value, at a five percent significance level, of 5.99. Figures in parentheses are asymptotic t-values. The t-value for α_3 was calculated as $\frac{\alpha_3}{\sqrt{\mathrm{var}[0.5\alpha_4^2]}}$ using the result developed by Stevens [28] for the variance of a product of random variables. The intercept terms are eliminated in the first-difference forms of the model. An estimation of the model with the intercept included produced a statistically insignificant intercept term, with all other parameter estimates essentially unchanged.

II. Discussion of Results

We have shown that, if the Fisher equation under uncertainty is specified, as is appropriate, in terms of changes of purchasing power (rather than inflation), the variance of inflation does not have a significant impact on the nominal interest rate. We have also argued that, if the model is specified in terms of inflation (rather than purchasing power), the variance of inflation can enter the model through the Jensen's inequality. Under lognormality,

$$-\ln E_t[\pi_{t+1}^{-1}] = E_t[\ln \pi_{t+1}] - 0.5\,\mathrm{var}_t[\ln \pi_{t+1}]. \tag{8}$$

Substitution of equation (8) into equation (7) yields a testable inflation-based model; i.e.,

$$\ln R_t = \alpha_0 + \alpha_1' E_t[\ln \pi_{t+1}] + \alpha_2 E_t[\ln g_{t+1}] + \alpha_3 \mathrm{var}_t[\ln g_{t+1}] + \alpha_4 \mathrm{cov}_t[\ln g_{t+1}, -\ln \pi_{t+1}] + \alpha_5' \mathrm{var}_t[\ln \pi_{t+1}], \tag{9}$$

where, under the null hypotheses,

$$\alpha_1' = 1.0 \quad \text{and} \quad \alpha_5' = -0.5$$

and the other coefficients remain the same as in the purchasing power-based specification (equation (7)).

Variations of equation (9) have been estimated by several researchers. Barnea, Dotan, and Lakonishok [1] include just the first two moments of inflation as independent variables and find a significantly positive association between interest rates and the variance of inflation. They interpret inflation variance as a measure of risk and the positive coefficient as the required risk premium, given risk aversion. Bomberger and Frazer [5] include money supply growth and Levi and Makin [22] include GNP growth as additional variables.

To compare our results, equation (9) is estimated using the Livingston data, which are used in all three of these earlier studies. The results support the hypothesis that $\alpha_1' = 1.0$, but the hypothesis that $\alpha_5' = -0.5$ is rejected.[9] The estimated coefficient of inflation variance is significantly greater than −0.5 but not significantly different from zero. Therefore, unlike the purchasing power model, the inflation model yields a coefficient for inflation variability that is significantly greater than its hypothesized value.

A direct comparison of our results with those of earlier studies is not strictly valid since our model specification and estimation period differ from these works. The most comparable results from these studies would be those that are adjusted for serial correlation, include no lagged exogenous variables, and use short-term interest rate as the dependent variable. The incompatibility of studies with lagged exogenous variables is based on the fact that there is no apparent theoretical justification for such a formulation in models such as the ones developed in equations (7) and (9). Given these criteria, we can find no evidence in Bomberger and Frazer [5] to support a negative relation between interest and inflation uncertainty. (See their Table III.) In fact, their coefficient is positive in this case but is insignificantly different from zero, as is our α_5' coefficient. The negative relation found by Levi and Makin [22] may be due to either a lack of adjustment for serial correlation (see Taylor [29], Table II) and/or the time period chosen for the study. Bomberger and Frazer [5] re-estimate the Levi and Makin model, corrected for serial correlation, and find a positive, insignificant association between interest rates and inflation variability. (See their footnote 12.) In short,

[9] The restricted regression results are

$$\begin{aligned}\Delta \ln R_t = {} & \underset{(0.512)}{0.8921}\, \Delta E_t[\ln \pi_{t+1}] + \underset{(0.106)}{0.2405}\, \Delta E_t[\ln g_{t+1}] - \underset{(-0.028)}{0.0289}\, \Delta \operatorname{var}_t[\ln g_{t+1}] \\ & + \underset{(0.106)}{0.2405}\, \Delta \operatorname{cov}_t[\ln g_{t+1}, -\ln \pi_{t+1}] + \underset{(0.581)}{0.9297}\, \Delta \operatorname{var}_t[\ln \pi_{t+1}],\end{aligned}$$

where the numbers in parentheses are standard errors. The t-statistic for H_0: $\alpha_5' = -0.5$ is $\frac{0.9297 + 0.5}{0.581} = 2.46$, and the t-statistic for H_0: $\alpha_1' = +1.0$ is $\frac{0.8921 - 1.0}{0.512} = -0.21$. Thus, α_5' is significantly greater than its hypothesized value of −0.5 and α_1' is not significantly different from its hypothesized value of +1.0—both at a five percent significant level.

given comparable models and estimation techniques, these earlier studies provide little evidence to reject the hypothesis that inflation variability has an insignificant influence on interest rates after adjustment has been made for *covariance* risk. Thus, the significant positive relation found by Barnea, Dotan, and Lakonishok [1] and others may be due to omitted variables that are related either to factors influencing the real rate (such as changes in the level or variability of real growth) or to risk premia for bonds (such as changes in the covariance of real growth and purchasing power). Our model also supports a theoretically justified unitary coefficient on the expected price change variable regardless of whether it is measured in terms of inflation or purchasing power. Combining this result with the irrelevance of inflation variability implies that Jensen's inequality is not a statistically important factor in our data, given the assumption of lognormality. This result is consistent with that found by Campbell [8] in his test of traditional formulations of expectations hypotheses of the term structure against that suggested by Cox, Ingersoll, and Ross [11]. Thus, Jensen's inequality does not appear to be an empirically relevant factor in determining either the level or the structure of nominal interest rates.[10]

III. Summary and Conclusion

It has been known for some time that the original version of the Fisher hypothesis, under uncertainty, is imprecise for at least two reasons. First, while the relevant variable is purchasing power changes, empirical researchers have continued to use expected inflation. Second, when investors are risk averse, they will demand a risk premium, even after adjustment for the distinction between purchasing power and inflation. Moreover, this risk premium is a function of the covariance of real output and future prices, not the total variability of prices that is utilized in many earlier studies and that provides mixed empirical results.

In order to test these propositions, we develop a model of the Fisher relation under the assumptions that investors have power utility functions and that the price level and real output are joint-lognormally distributed. The empirical results, using variable measures based either on the Livingston survey data or on time-series forecast data, generally support our interpretation of the Fisher effect and the risk aversion effect. Given a proper measurement of changes in expected purchasing power or expected inflation, the variance of inflation is insignificant in explaining the variability in interest rates. Such a result also provides additional support for the hypothesis that Jensen's inequality is not an empirically important factor in determining the level or structure of nominal interest rates. Finally, the tests show that participants in the Treasury bill market are risk averse. Therefore, decreases in the covariance of inflation and real output have a positive influence on short-term rates. Together, the results imply that it is not the variability in prices per se that adversely influences nominal rates; rather it is the association between changes in the price level and real output.

[10] We would like to thank an associate editor for suggesting that we pursue the empirical relevance of the Jensen's inequality in more depth.

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