American Finance Association

Is the Real Interest Rate Stable?
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Source: *The Journal of Finance*, Vol. 43, No. 5 (Dec., 1988), pp. 1095-1112
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2328208
Accessed: 26/11/2009 07:41

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THE JOURNAL OF FINANCE • VOL. XLIII, NO. 5 • DECEMBER 1988

Is the Real Interest Rate Stable?

ANDREW K. ROSE*

ABSTRACT

Univariate time-series models for consumption, nominal interest rates, and prices each appear to have a single unit root before 1979. If nominal interest rates have a unit root but inflation and inflation forecast errors do not, then ex ante real interest rates have a unit root and are therefore nonstationary. This deduction does not depend on the properties of the unobservable ex post observed real return, which combines the ex ante real interest rate and inflation-forecasting errors. The unit-root characteristic of real interest rates is puzzling from at least two perspectives: many models imply that the growth rate of consumption and the real interest rate should have similar time-series characteristics; also, nominal returns for other assets (e.g., stocks and bonds) appear to have different times-series properties from those of treasury bills.

THIS ESSAY IS CONCERNED with the stability of the ex ante short American real interest rate. The paper can be motivated in a number of ways. A number of authors have investigated the question of whether financial markets fluctuate excessively. Insofar as the stationarity of returns is crucial in addressing this issue (Shiller [23]), an assessment of the stationarity of both nominal and real interest rates is potentially valuable. Another group of economists has investigated the question of whether trends in economic time series are deterministic or stochastic in nature (e.g., Nelson and Plosser [19]). To date, none of the analysis has been concerned with real interest rates: it seems interesting to extend the work. Another topic investigated recently is the viability of consumption-based asset-pricing models (e.g., Hansen and Singleton [11, 12]). In this paper, the time-series characteristics of a number of relevant variables are examined in a nonstructural setting; these provide an implicit test of the model. Finally, the fact that the real interest rate is a crucial determinant of investment, savings, and indeed virtually all intertemporal decisions renders its characteristics of intrinsic interest.

This paper attempts to provide robust evidence relevant to these issues. In particular, no structural model for either interest rates or prices is assumed; the behavior of the ex ante real rate is not inferred from the ex post observed real rate. Further, the sensitivity of the results is checked extensively. The results, though based on simple techniques, appear to be inconsistent with a number of popular theoretical models.

In Section I, the relevant theory is presented; the literature is briefly surveyed in the section that follows. Section III contains the empirical work; some

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implications of this analysis follow in the fourth section. The paper ends with some caveats and conclusions.

I. Introduction

The nominal interest rate is traditionally decomposed into two parts: the expected inflation rate and the ex ante real interest rate. Assuming that both of the latter are low over the period in question, one can write

$$i_t \cong r_t^e + \dot{p}_t^e, \tag{1}$$

where i_t is the nominal return on a given security issued at period t, maturing in one period, r_t^e is the expected ex ante real return, and $\dot{p}_t^e$ is the inflation rate expected at t for the next period; all figures are in percentages.

Suppose that the expected ex ante inflation rate differs from the actual inflation rate observed ex post by a stationary forecasting error term ε_t:

$$\dot{p}_t^e = \dot{p}_t + \varepsilon_t, \qquad r_t \equiv r_t^e + \varepsilon_t = i_t - \dot{p}_t. \tag{2}$$

For $\dot{p}_t^e$ to be a "good" predictor of $\dot{p}_t$, one usually requires much more stringent conditions than the stationarity of ε_t; for instance, it is often assumed that expectations are "rational"; i.e., $\dot{p}_t^e = \mathrm{E}(\dot{p}_t \mid \Omega_t)$, where $\mathrm{E}(\cdot \mid \cdot)$ denotes the conditional expectations operator and Ω_t is an information set available when expectations are formed at t. However, the stationarity and, hence, finite unconditional variance of (ε) seems to be a minimal requirement.

Combining (1) and (2), one obtains

$$i_t = r_t^e + \dot{p}_t + \varepsilon_t. \tag{3}$$

If one is willing to assume a process for r_t^e, then (3) is estimable with standard techniques. For instance, if r_t^e is a first-order autoregressive process $[r_t^e = \rho r_{t-1}^e + n_t$, $|\rho| < 1$, n_t stationary], then taking quasi-differences of (3) with the prefilter $(1 - \rho L)$, where L is the lag operator $[L^n x_t \equiv x_{t-n}]$, yields

$$i_t(1 - \rho L) = \dot{p}_t(1 - \rho L) + \varepsilon_t(1 - \rho L) + n_t. \tag{4}$$

If $V(\eta) = 0$, then the real rate is constant, but r_t^e can also be a function of other relevant variables. This framework has traditionally been used to test the "Fisher hypothesis," the simplest form of which is

$$\mathrm{corr}(i_{t+j}, \dot{p}_t^e) = 1 \quad \text{for all } t, \quad \text{some } j \geq 0, \tag{5a}$$

where $\mathrm{corr}(\cdot, \cdot)$ is the correlation operator and j may be a large integer. Much empirical work has been concerned with testing this hypothesis. (The analysis in this paper assumes the validity of the Fisher hypothesis, although allowing for a stationary risk premium in (5a) leads to identical results.) The "Gibson Paradox" will also be of interest below; it corresponds to

$$\mathrm{corr}(i_t, p_t) > 0, \tag{5b}$$

where p_t is the price level at time t.

Another matter of some interest has been the decomposition of the variance of the nominal interest rate. If r_t^e and $\dot{p}_t^e$ are stationary stochastic processes, one can write

$$V(i) = V(r^e) + V(\dot{p}^e) + 2\,\mathrm{cov}(r^e, \dot{p}^e), \tag{6}$$

where $V(\cdot)$ and $\mathrm{cov}(\cdot, \cdot)$ denote the unconditional variance and covariance operators. Researchers have often constructed ex post real rates and found that the variance of ex ante real rates is typically much smaller than the variance of actual inflation (e.g., Mishkin [17]). This inequality is also found for variances conditional on the preceding year's information.

"Standard" statistical treatment of, e.g., (4) and (6) requires the stochastic processes in (3) (i, r^e, $\dot{p}$, ε) to be covariance stationary. The validity of the assumption of stationarity has been questioned recently. In particular, Nelson and Plosser [19] estimated univariate *ARIMA* (autogressive integrated moving-average) models of the form:

$$\phi(L)(1 - L)^d x_t = \theta(L)\nu_t,$$

where $\phi(L) = (1 - \phi_1 L - \cdots - \phi_P L^P)$ and $\theta(L) = (1 + \phi_1 L + \cdots + \theta_q L^q)$ have all zeros outside the unit circle and ν_t is white noise. The "order of integration" in this model or number of unit roots is d. Nelson and Plosser argue that the hypothesis that both the nominal interest rate and the log of the price level contain a unit root (i.e., $d = 1$) cannot be rejected with American data. This unit root is apparent even after detrending the variables with a deterministic linear time trend, and it renders the variables nonstationary in levels with infinite unconditional variances, but stationary after first differencing.

However, if the log of prices is stationary after differencing, then the inflation rate is stationary and has a different order of integration from the nominal interest rate (which has a unit root and is hence nonstationary). It is reasonable to assume that ex ante and ex post inflation differ only by a stationary forecast-error term. As the nominal rate is defined as the sum of the inflation rate, inflation-forecast errors, and the ex ante real interest rate, the ex ante real interest rate can be deduced to be a nonstationary process with a unit root.

It is easy to verify this argument empirically. One only needs to demonstrate empirically that the nominal interest rate has a unit root and that the inflation rate does not have a unit root.

II. Survey of the Literature

In this section, I briefly survey the literature on real interest rate movements. The focus of the survey is on the orders of integration of prices, nominal interest rates, and real interest rates. In particular, I ask the question: has the literature adequately addressed the issue of nonstationarity?

In an important and controversial paper, Fama [3] found evidence that monthly evidence for short Treasury Bills and the CPI from 1953 to 1971 supported a number of hypotheses. These included the hypotheses that (a) the bill market was efficient, (b) the Fisher hypothesis held in the short run, and (c) real rates were constant, so that variations in nominal interest rates were due to variability in expected inflation. Fama does not explicitly test for (non)stationarity of nominal interest and inflation rates, though his evidence seems consistent with the hypotheses that both variables are nonstationary. However, the characteristics of the inflation rate do not seem robust with respect to the sample chosen. Indeed, as Shiller [24] and Mishkin [17] point out, Fama's sample is extremely unrepresentative of the twentieth century in that it contains little variation in the variables of interest. This problem is serious and deserves careful attention; in the context of this paper, explicit testing for the stationarity of the inflation rate over different samples and variables is desirable. The literature clearly indicates that the nominal interest rate is nonstationary (Fama [3, 4, 5], Fama and Gibbons [6], and Mankiw and Miron [16]), but there is conspicuous ambiguity on the issue of whether the inflation rate contains a unit root.

A number of critics of Fama's original paper have argued that the real interest rate is, in fact, nonconstant; some have argued that the ex post real interest rate cannot be distinguished from a random-walk ex ante real interest rate entangled in stationary inflation-forecast errors (Garbade and Wachtel [9], Nelson and Schwert [18], and Fama and Gibbons [6]). However, if has proven difficult to provide definitive evidence concerning the ex ante real interest rate, as it is inherently unobservable. Any analysis of the real interest rate must account for this feature of the data. Further, the implications of random-walk real interest rates have not been discussed to date.

III. Do Prices and Nominal Rates Have Unit Roots?

To test for unit-root nonstationarity, one can use a variety of methods. One simple pair of tests are the "Dickey-Fuller" tests for a unit root in the autoregressive representation of a variable. Suppose that a variable can be modeled as autoregression of order $(P + 1)$:

$$(1 - \phi_1 L)(1 - \phi_2 L) \cdots (1 - \phi_{p+1} L)x_t = w_t + \mu, \tag{7}$$

where w_t is an i.i.d. process. Without loss of generality, order the roots $\phi_1, \cdots, \phi_{p+1}$ in order of descending size. To test the hypothesis H_0: $\phi_1 = 1$ (i.e., x_t contains a unit root), one estimates the following regression with OLS:

$$(1 - L)x_t = \alpha + \beta x_{t-1} + \textstyle\sum_{i=1}^{p} \gamma_i (1 - L)x_{t-i} + \nu_t. \tag{8}$$

The ratio of $\hat{\beta}$ to its standard error is not distributed as Student's t-distribution, but the empirical distribution of this "t-like" statistic is tabulated in Fuller [8]. Another statistic that has proven useful is $T \times \hat{\beta}$; its distribution is also tabulated in Fuller [8]. A large negative value of either the "t-like" statistic or $T \times \hat{\beta}$ is evidence that is inconsistent with the null hypothesis of a unit root in x_t.

A deterministic linear trend can be added to (8), or the constant term can be deleted, so long as one then compares the statistics with the appropriate tables. The former modification does not seem relevant for the data in question, and the latter is retained for generality. Phillips [22] also suggests modifying the statistics to account for more general w_t processes; his procedures have been used and do not lead to different conclusions. (Also see Perron [21].)

To check the hypotheses that the nominal interest rate and the price level each have a single unit root, I use a variety of data frequencies, samples, data transformations, and variables.

A. Annual Results

To test for unit roots with annual American data, I use four measures of prices and two measures of nominal interest rates. $P1$ is the GNP deflator; $P2$ is the consumer price index; $P3$ is an implicit price deflator; $P4$ is a wholesale price index. $P1$ and $P2$ have been used by Nelson and Plosser [19]; $P3$ has been used by Friedman and Schwartz [7]. The data appendix provides further data documentation.

I use two measures of nominal interest rates. $I1$ is the yield on high-grade long-term corporate bonds. $I2$ is the short-term commercial-paper rate. Both series are taken from the Friedman and Schwartz volume and are in percentiles (e.g., $I1 = 3.30$ in 1900). An expectational argument can be used to show that the long rate should in general be a martingale (at least as a first approximation). The long rate is included, therefore, solely for comparison with the short rate.

I test for unit roots in the six series, augmenting (8) by choosing $P = 1$. All the tests have been calculated using $P = 2$; the results do not change, and the second lag is almost always insignificant. I calculate both the "t-like" statistic for $\hat{\beta}$ in (8) and $T \times \hat{\beta}$; the two give similar results, though $T \times \hat{\beta}$ rejects the null hypothesis slightly more often. I also calculate the tests for both logs and levels of the data; the tests universally yield the same results. I therefore report only the results for logs (so that Δp is an inflation rate). The results are reported for two samples: 1892 to 1970 (so that $T = 79$) and 1901 to 1950 ($T = 50$). Both samples yield the same result; others have been checked and yield identical conclusions. (I also checked for unit roots in short nominal rates after tax and found a unit root.) I test for both one and two unit roots in the logs of both prices and interest rates. The tests for one unit root are the "t-like" test for $\hat{\beta}$ and $T \times \hat{\beta}$ in (8); the tests for two unit roots are the same statistics when each variable in (8) has been differenced.

Table I contains the results as well as the critical values. The results seem relatively clear-cut. While the series for both prices and nominal interest rates do not reject the hypothesis of a single unit root, a second unit root can be firmly rejected for both variables. These results are consistent with those of Shiller and Siegel [25]. Neither of these conclusions seems particularly sensitive to choice of variable, choice of sample, choice of statistic, degree of regression augmentation, or the log transformation. Thus, inflation seems not to have a unit root, but nominal interest rates apparently do.[1]

[1] As another check on the nonstationarity of real interest rates, I checked for co-integration between nominal interest rates and inflation. Two variables are said to be co-integrated if each variable individually is integrated of order one (say) but a linear combination of the variables is integrated of lower order (say zero). I tested for co-integration using the techniques suggested by

Table I
Annual Unit-Root Tests[a]

	1892 to 1970				1901 to 1950			
	One Root		Two Roots		One Root		Two Roots	
	$t(\hat{\beta})$	$T\hat{\beta}$	$t(\hat{\beta})$	$T\hat{\beta}$	$t(\hat{\beta})$	$T\hat{\beta}$	$t(\hat{\beta})$	$T\hat{\beta}$
$P1$	0	0	−4.62	−44.23	−0.81	−1.00	−3.72	−29.15
$P2$	−0.19	−0.16	−5.00	−40.72	−1.32	−1.55	−4.08	−26.7
$P3$	−0.05	−0.08	−4.32	−41.11	−0.99	−1.35	−3.48	−26.9
$P4$	−0.77	−1.34	−5.65	−62.87	−1.35	−3.65	−4.54	−40.75
$I1$	−0.35	−1.03	−4.16	−50.54	−0.38	−0.80	−4.38	−44.75
$I2$	−1.28	−4.27	−7.93	−104.29	−1.13	−2.20	−5.56	−50.7
0.05 Crit. value	−2.91	−13.4	−2.91	−13.5	−2.93	−13.3	−2.93	−13.3
0.01 Crit. value	−3.55	−19.4	−3.55	−19.4	−3.58	−18.9	−3.58	−18.9

[a] Definitions for all tables are in the Data Appendix.

B. Quarterly Results

An interesting way to confirm the results of the previous section is to test the hypothesis of a single unit root in both interest rates and prices on international data. Verification of the hypothesis on the basis of evidence from countries with similar economies lends credence to the hypothesis of nonstationary American real rates. In this section, quarterly evidence from eighteen OECD countries is used to corroborate the annual American results presented above.

The data are from the IMF's "International Financial Statistics." The data, which run from 1957, represent the three-month Treasury bill returns and the seasonally adjusted GDP price deflators for the eighteen countries. Both series are transformed by natural logarithms. For each test, equation (8) is estimated in both levels and first differences; $P = 4$ in all cases.

Table II reports the Dickey-Fuller $T \times \hat{\beta}$ tests for both one and two unit roots in (the logs of) interest rates and prices. Rejection of the null hypothesis at the 0.05 (0.01) level is marked by one (two) asterisk(s). In no case is the null hypothesis of a single unit root rejected for either nominal interest rates or prices. However, the hypothesis of two unit roots in interest rates is rejected at the 0.01 confidence level in all cases. The hypothesis of a unit root in inflation is rejected at the 0.05 level for all eighteen countries; the rejection is significant at the 0.01 level for sixteen of these countries. That is, inflation does not appear to have a unit root, but nominal interest rates do. These results are again quite robust to the choice of statistic, sample, degree of augmentation, log transform, and country considered.

Granger and Engle. The nominal rate was regressed on a constant and the inflation rate with least squares. There are two relevant test statistics: the Durbin-Watson statistic of this regression and the augmented Dickey-Fuller test for stationarity of the residuals of this regression. Tests were run for all interest rate and price combinations, in both logs and levels, for all the samples used in Table I. Evidence for co-integration is meager. At the annual frequency, none of the sixty-four tests (two interest rates × four prices × two logs or levels transformations × two samples × two test statistics) indicated co-integration at even the ten percent significance level.

Table II
Quarterly Unit-Root Tests

	Interest Rates			Prices		
	T	One Root	Two Roots	T	One Root	Two Roots
Australia	112	−0.73	−87.33**	56	−2.83	−56.71**
Austria	112	−2.06	−83.19**	110	0.21	−49.91**
Belgium	105	−1.39	−33.78**	111	−0.10	−54.22**
Canada	112	−2.89	−99.04**	111	−0.13	−15.80*
Denmark	112	−2.24	−100.99**	67	−0.94	−96.48**
Finland	111	−0.19	−62.68**	105	0.50	−98.80**
France	112	−1.89	−52.02**	111	0.05	−23.91**
Germany	111	−9.95	−72.90**	112	−0.34	−31.17**
Ireland	112	−2.06	−83.19**	110	0.21	−49.91**
Italy	111	−1.36	−64.38**	59	−0.21	−25.30**
Japan	112	−0.19	−85.65**	111	−0.25	−20.42**
Netherlands	110	−2.43	−74.83**	112	−0.47	−62.90**
New Zealand	111	2.32	−126.11**	110	0.39	−58.14**
Spain	96	−1.19	−32.31**	83	−0.06	−44.80**
Sweden	112	−1.54	−73.35**	109	0.26	−81.80**
Switzerland	112	−5.05	−65.23**	102	−0.21	−27.05**
U.K.	111	−2.25	−94.12**	87	0.04	−26.22**
U.S.	112	−2.38	−76.83**	112	−0.10	−18.17*

* Null hypothesis rejected at the 0.05 level.
** Null hypothesis rejected at the 0.01 level.

C. Monthly Results

To check results further, I use monthly American data. Four price series are used. $P1$ is the seasonally adjusted producer price index from CITIBASE (mnemonic PWFSA); $P2$ is its non-seasonally adjusted counterpart. $P3$ is the seasonally adjusted consumer price index for all urban consumers (PUNEW), while $P4$ is the non-seasonally adjusted index. These data are available from 1947M1 to 1986M6. All data are described in an appendix. Four measures of annualized short interest rates are used. $I1$ is the one-month interest rate on finance paper placed directly (FYFN1M); $I2$ is the nominal rate on one-month certificates of deposit (FYCD1M); $I3$ is the Euro dollar deposit rate (FYUR30). These data are available from 1971M1 to 1986M11. I is the one-month Treasury bill return from Ibbotson and Sinquefeld [14]; it is available from 1926M1 through 1983M12.

I calculated both the t-tests and $T \times \hat{\beta}$ tests for all variables but only report the latter for brevity. (The two yield almost identical conclusions, though the latter have slightly higher power.) I also report only the result for the logs of the variables for convenience, though again results are quite similar for both logs and levels. All regressions are augmented with twelve lags of the regressand (i.e., $P = 12$ in (8)).

I report tests for both one and two unit roots in all four price variables for five different samples. The results are reported in Table III; the appropriate critical values are at the bottom of the table. Only one sample yields results that are inconsistent with the hypothesis of a single unit root in the log of prices. This is the period following the change in monetary policy in October 1979. Other investigators have also found structural breaks at this point (e.g., Huizinga and Mishkin [13]). For all other samples, the evidence seems to be strongly consistent with the hypothesis that inflation is stationary, in contrast with Patel [20].

In Table IV, the unit-root test results are tabulated for the first three measures of interest rates $I1$ to $I3$ and their logs, $LI1$ to $LI3$. Two samples are examined: 1971M2 through 1986M1 and 1971M2 through 1979M9. Again, t-tests and $T \times \beta$ tests yield identical results, so only the latter are recorded, and all regressions are augmented with twelve lags of the regressand. Both of the samples indicate that short interest rates appear to have a single unit root.

I investigate the robustness of this result further in Table V. Five different samples are used to see whether the hypothesis of a single unit root in the one month interest rate is consistent with American data. The data used are the one month Treasury bill returns found in Ibbotson and Sinquefeld [14]. The samples examined are 1927M2/3 to 1983M12, 1951M4 to 1973M12, 1934M1 to 1951M2 1951M4 to 1958M12, and 1959M1 to 1979M9. Table V contains $T \times \hat{\beta}$ tests for one unit root (augmented with twelve lags of the regressand), along with the

Table III

Monthly Unit-Root Tests for Prices

	1948M3 1986M6		1948M3 1972M12		1973M1 1986M6		1953M1 1971M12		1979M10 1986M6	
T	460		297		162		228		81	
	One Root	Two Roots	One Root	Two Roots	One Root	Two Roots	One Root	Two Roots	One Root	Two Roots
$P1$	0.23	−103	0.48	−162	−0.75	−21.6	0.70	−103	−3.42	−5.38
$P2$	0.14	−106	0.28	−150	−0.91	−22.4	0.42	−110	−4.63	−6.54
$P3$	0.28	−65	0.48	−110	−0.28	−21.5	0.57	−32	−1.73	−8.26
$P4$	0.28	−64	0.36	−105	−0.31	−22.1	0.37	−33	−1.69	−8.77
.05	−14	−14	−14.1	−14.1	−13.8	−13.8	−13.9	−13.9	−13.5	−13.5
.01	−20.5	−20.5	−20.4	−20.4	−20	−20	−20.2	−20.2	−19.2	−19.2

Table IV

Monthly Unit-Root Tests for Interest Rates

	1971M2/3 1986M11		1971M2/3 1979M9	
T	190	189	104	103
	One Root	Two Roots	One Root	Two Roots
$I1$	−5.62	−178	−6.82	−48.0
$I2$	−6.02	−178	−7.20	−45.0
$I3$	−7.22	−239	−9.81	−87.0
$LI1$	−5.86	−143	−7.37	−58.0
$LI2$	−6.12	−141	−7.94	−54.0
$LI3$	−7.96	−209	−10.10	−93.0
.05	−13.9	−13.9	−13.7	−13.7
.01	−20.2	−20.2	−19.8	−19.8

Table V

Unit-Root Tests for the Ibbotson-Sinquefeld Asset-Return Data[a]

Sample	1927M2/3 1983M12	1951M4 1973M12	1934M1 1951M2	1951M4 1958M12	1959M1 1979M9
T	683/682	273	206	93	249
M	−609	−246	−224	−69	−248
S	−609	−215	−220	−61	−203
I	−6.8	−7.1	−6.5	−12.6	−8.6
J	−466	−230	−133	−65	−228
B	−505	−247	−76	−105	−197
G	−614	−328	−237	−142	−252
P	−110	−71	−71	−60	−6.6
.05	−14.0	−14.0	−14.0	−13.7	−14.0
.01	−20.6	−20.3	−20.2	−19.8	−20.3

[a] Key: M—S&P 500 return.
S—small stocks return.
I—Treasury bill return.
J—below Baa bond return.
B—long-term corporate bond return.
G—long-term Government bond return.
P—CPI inflation.

relevant critical values. For the purpose of comparison, unit-root tests for the other data recorded in Ibbotson and Sinquefeld are also included: the return on the S&P 500 (M), the return on stocks of small firms (S), the return on bonds rated below Baa (J), the return on corporate bonds (B), the return on long-term government bonds (G), and CPI inflation (P). Tests for two unit roots are not reported, as they universally reject the hypothesis of two unit roots at any reasonable significance level.

The results are basically consistent with the hypothesis that only the Treasury-bill-returns variable has a unit root. This conclusion seems quite robust to sample size, an interesting result in light of the different monetary regimes. Inflation appears to have a unit root for the last sample considered, but this test statistic is very sensitive to choices of both p and the sample.

In an appendix, I decompose the conditional variance of nominal interest rates into inflation and real interest rate innovations, analogously to equation (6). The results support the hypothesis that the real interest rate has a unit root, which is of substantive importance in determining the variance of nominal interest rates.

To sum up, if one excludes the last few years of American data the results seem striking. A single unit root seems to exist in both prices and the short nominal interest rate. Inflation and nominal interest rates therefore are not integrated of the same order; inflation does not have the unit root that nominal rates do. One can reasonably deduce that real ex ante rates must have a unit root. This deduction can be made without analyzing the ex post real rate, which combines the ex ante rate with the inflation-forecast error. Finally, if inflation is stationary but nominal rates are not, then the Gibson Paradox (5b) can be easily explained as a "spurious regression" between two nonstationary series (Granger and Newbold [10]; also see Shiller and Siegel [25]).

IV. Implications

Two stylized facts have emerged from the empirical analysis. First, time-series models of short nominal interest rates appear to have a unit root. Second, the existence of a unit root in comparable models for the inflation rate can be rejected for many sample periods. This is *prima facie* evidence that the ex ante real interest rate has a unit root. In this section of the paper, I explore some possible implications of these results in the context of a standard consumption-based capital asset pricing model (CCAPM). First, I briefly review the CCAPM, as exposited by Lucas [15].

In the CCAPM that I consider, consumers hold dividend-yielding assets to maximize expected additively separable utility over an infinite horizon. The utility function is restricted to be constant relative risk aversion; no production or goods storage is considered. Consumers face an intertemporal budget constraint in that their asset income must match their expenditures on consumption and assets for future savings. Thus, consumers solve the folowing problem:

$$\max_{x_{i\tau}} \mathrm{E}_t[\textstyle\sum_{\tau=0}^{\infty} \beta^{\tau} c_{t+\tau}^{\alpha+1}/\alpha + 1], \quad \alpha < 0, \tag{9a}$$

subject to

$$p_{it} \times x_{it+1} + c_t = (p_{it} + d_{it}) \times x_{it}, \tag{9b}$$

where E_0 denotes the mathematical expectation conditional on information at time 0, β is the rate of time preference, c_t denotes consumption at t, p_{it} is the ex-dividend price of asset i at t, x_{it} is the share of asset i held at t, and d_{it} is the dividend of asset i held at t. The first-order Euler equation, obtained by substituting (9b) into (9a) and differentiating, is

$$\mathrm{E}_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{\alpha} r_{it+1}\right] = 1, \tag{10}$$

where $r_{it+1} \equiv (p_{it+1} + d_{it+1})/p_{it}$. Many economists have emphasized the generality of equation (10).

Testable implications for (10) are easily derived using the logarithmic analogue (Hansen and Singleton [11]):

$$\begin{aligned} &\mathrm{E}_t(\log(\beta(c_{t+1}/c_t)^{\alpha} r_{it+1})) = 0 \\ &\qquad = \log \beta + \alpha \mathrm{E}_t(\log(c_{t+1}/c_t)) + \mathrm{E}_t(\log(r_{it+1})). \end{aligned} \tag{11}$$

This equation allows one to determine the implications that (10) has for the consumption and return processes. Suppose that the growth rate of consumption and the real short interest rate can be represented as infinite-order (moving-average) processes:

$$\log(c_t/c_{t-1}) = \mu_c + \psi_0 \varepsilon_t + \psi_1 \varepsilon_{t-1} + \cdots = \mu_c + \psi_0 \varepsilon_t + \psi(L)\varepsilon_{t-1}, \tag{12a}$$

$$\log(r_{it}) = \mu_{ir} + \lambda_{i0}\eta_{it} + \lambda_{i1}\eta_{it-1} + \cdots = \mu_{ir} + \lambda_{i0}\eta_t + \lambda_i(L)\eta_{it-1}, \tag{12b}$$

where both $\{\varepsilon_t\}$ and $\{\eta_{it}\}$ are white-noise processes. Combining (12) with (11) yields

$$-\log \beta - \alpha\mu_c - \mu_{ir} = \alpha\psi(L)\varepsilon_t + \lambda_i(L)\eta_{it}, \tag{13}$$

so that

$$\alpha\psi(L)\varepsilon_t = -\lambda_i(L)\eta_{it}. \tag{13$'$}$$

Thus, the CCAPM delivers strong time-series restrictions linking the univariate (Wold) representations of the consumption growth rate and the log of real returns. In particular, the correlograms of real returns to all assets should be identical and equal (up to a factor of proportionality) to the correlogram of the consumption growth rate.

The empirical work in the preceding section indicates that the short nominal and real interest rates have unit roots; that is, the $\lambda_i(L)$ terms do not "decay" for high-order terms in the lag polynomial. Can the same be said of the $\psi(L)$ polynomial? If the growth rate of consumption does not have a unit root, the $\psi(L)$ coefficients decay and (13$'$) (and therefore (11)) does not hold. This would constitute a simple and direct rejection of the CCAPM. Succinctly, if the correlograms for the consumption growth rate and the log of real returns are grossly different, then conditions for the CCAPM are violated.

It is simple to test whether the growth rate of consumption is stationary. When the difference in the log of real consumption (CITIBASE mnemonic GC82) is tested for a unit root with the Dickey-Fuller procedure (setting $P = 4$), the hypothesis of a unit root in the growth rate of consumption is rejected at any conventional confidence level (e.g., $T\hat{\beta} = -131$ for 48M3–86M3; 0.01 critical value $\cong -20$). This rejection is quite robust and is also consistent with the existing literature.

The fact that the growth rate of consumption does not appear to have the same number of unit roots as the ex ante real bill return is broadly inconsistent with (13$'$). If the real short interest rate does have a unit root but the growth rate of consumption does not, it is hard to see how the CCAPM can be even approximately valid if utility is constant relative risk aversion.

However, this does not provide a statistical test of the CCAPM; rather, time-series characteristics of the ex ante real interest rate have been informally compared with those of the consumption growth rate. To derive a statistical test, consider the analogue to (10) expressed in nominal terms:

$$\mathrm{E}_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{\alpha} q_t/q_{t+1}R_{it+1}\right] = 1, \tag{10$'$}$$

$$\begin{aligned} \mathrm{E}_t(\log(\beta(c_{t+1}/c_t)^{\alpha}q_t/q_{t+1}R_{it+1})) &= 0 \\ &= \log\beta + \alpha\mathrm{E}_t(\log(c_{t+1}/c_t)) \\ &\quad + \mathrm{E}_t(\log(q_t/q_{t+1})) + \mathrm{E}_t(\log(R_{it+1})), \end{aligned} \tag{11$'$}$$

where q_t is the nominal price of goods at t and R_{it} is the nominal return on asset i analogous to the real return r_{it}. The analysis above indicates that $\log(c_{t+1}/c_t)$ and $\log(q_t/q_{t+1})$ do not have unit roots but the $\log(R_{it})$ does have a unit root (if one considers the short nominal interest rate). Thus, (11′) is violated in exactly the same way as (11). Further, note that for any two risky assets i and j, (11′) implies that

$$E_t(\log(R_{it+1})) = E_t(\log(R_{jt+1})). \tag{14}$$

Again, suppose that the nominal returns can be modeled as infinite-order moving-average processes:

$$\log R_{it} = \theta_{i0}\nu_{it} + \theta_{il}\nu_{it-1} + \cdots, \tag{15a}$$

$$\log R_{jt} = \theta_{j0}\nu_{jt} + \theta_{jl}\nu_{jt-1} + \cdots. \tag{15b}$$

Combining (14) and (15) yields

$$\theta_{i1}\nu_{it-1} + \theta_{i2}\nu_{it-2} + \cdots = \theta_{j1}\nu_{jt-1} + \theta_{j2}\nu_{jt-2} + \cdots \tag{16}$$

or

$$\theta_{ij1}(\nu_{it-1} - \nu_{jt-1}) + \theta_{ij2}(\nu_{it-2} - \nu_{jt-2}) + \cdots = 0. \tag{16′}$$

Unless the correlograms for all risky returns are equal to each other, the innovations to all returns should be equal. Clearly the latter condition is not true. The former can be tested statistically by examining the correlogram of $\log(R_{it}) - \log(R_{jt})$, i.e., the time-series characteristics of the differences in the logs of all nominal returns. A simple Box-Pierce Q-test for the correlogram of $\log(R_{it}) - \log(R_{jt})$ provides a rigorous test.

Table VI tabulates the correlograms for the assets considered by Ibbotson and Sinquefeld. Treasury bill returns seem to have a unit root, but other returns closely resemble white-noise processes. Thus, the correlograms for all nominally risky assets bear little resemblance to the Treasury bill correlogram; one is led to believe that the correlograms of the differences between the bill return and nominally risky returns will be statistically distinguishable from zero. This intuition is borne out by the Q-statistics, which statistically test the hypothesis

Table VI

Correlograms for the Ibbotson-Sinquefeld Asset-Return Data[a]

	$\hat{\rho}1$	$\hat{\rho}2$	$\hat{\rho}3$	$\hat{\rho}4$	$\hat{\rho}5$	(S.E.)
M	0.10	−0.02	−0.13	0.04	0.07	(0.04)
S	0.15	0.03	−0.10	−0.07	−0.01	(0.04)
I	0.96	0.95	0.94	0.92	0.91	(0.04)
J	0.21	−0.01	−0.13	−0.10	0.05	(0.04)
B	0.14	−0.01	−0.11	−0.01	0.10	(0.04)
G	0.01	−0.01	−0.15	0.01	0.07	(0.04)

[a] Sample: 1926.01 to 1983.12. $T = 696$ observations.

Key: M—S&P 500 return.
S—small stocks return.
I—Treasury bill return.
J—below Baa bond return.
B—long-term corporate bond return.
G—long-term Government bond return.

of flat correlograms for differences in returns between Treasury bills and nominally risky assets. The correlograms are twelfth order, but the conclusions do not depend on the precise order chosen. Under the null hypothesis of flat correlograms, these statistics are distributed as chi-squared with eleven degrees of freedom. The correlograms are tabulated in Table VII. They have been calculated using the monthly Ibbotson-Sinquefeld data from 1926M1 to 1983M12.

All of the statistics indicate rejection of the 0.01 confidence level of the hypothesis that the correlograms of the differences between nominally risky returns and the Treasury bill return are zero; thus, (16′) is rejected by the data.

To summarize, the CCAPM implies that the time-series characteristics of the growth rate of consumption and the real interest rate should be similar. The time-series characteristics of all nominal (and real) returns should also be similar. However, American data indicate that neither of these conditions is, in fact, valid.

V. Caveats and Conclusions

A. Caveats

The analysis in this paper does not take into account any empirical complications that arise from nonlinearities in the data-generation process. While preliminary theoretical work indicates that nonlinearities are not likely to be the source of the results, further research is certainly warranted. Time-varying parameters may also be of importance; although the results seem robust to choice of sample, more explicit treatment of models with time-varying parameters is desirable.

This paper does not analyze the fundamental source of the unit root in real interest rates. West [26] demonstrates that a monetary policy that smooths interest rates leads to unit-root behavior in both real output and real interest rates in the context of an overlapping wage-contract model. Alternatively, nonstationary shocks to aggregate demand can also lead to unit-root behavior in real interest rates, as in, e.g., DeLong and Summers [2]. However, neither of these papers explicitly tests the time-series properties of the theoretical models; further work along these lines is also appropriate.

Table VII

Q-Statistics of Twelfth-Order Correlograms of Differences between Nominally Risky Return and Treasury Bill Return

S&P 500 Stocks	36.5
Small Stocks	41.5
Corporate Bonds	58.9
Long-Term Corporate Bonds	42.3
Long-Term Government Bonds	84.3
.01 Critical Value	24.7

B. Conclusions

For many samples of interest, American data are consistent with the hypothesis that nominal rates and prices seem to be processes with a single unit root (i.e., they are both $I(1)$ processes). If inflation-forecasting errors are stationary, then real rates must be nonstationary. This can be deduced without analyzing ex post real rates, which are a mix of real rates and inflation-forecasting errors. These results seem very robust and do not depend on many restrictive assumptions.

Though there seems to be strong evidence that real rates are nonstationary, this fact is puzzling from a number of perspectives. First, the consumption capital asset pricing model is inconsistent with the stylized facts on consumption growth and real interest rates. Second, the nominal (and therefore real) time-series structures of the returns on various risky assets seem to be quite different; this is also difficult to rationalize from a theoretical perspective.

Data Appendix

Annual Data

$P1$: GNP Deflator from Nelson and Plosser (NP) [19], 1889–1970. 1900 = 24.7.
$P2$: Consumer Prices from NP, 1860–1970. 1900 = 25.0.
$P3$: Implicit Price Deflator from Friedman and Schwartz (FS) [7], 1869–1975. 1900 = 49.6.
$P4$: Wholesale Prices, All Commodities (BLS Index) from *Long Term Economic Growth*, 1960–1970. 1900 = 28.9.
$I1$: Yield on High-Grade Long-Term Corporate Bonds from FS, 1867–1975. 1900 = 3.30%.
$I2$: Short-Term Commercial Paper Rate from FS, 1867–1975. 1900 = 4.38.

Monthly Data: All Data from CITIBASE

$P1$: Producer Price Index (PWFSA) 1947M1–1986M6. 1970M1 = 109.7.
$P2$: Producer Price Index NSA (PWF) 1947M1–1986M6. 1970M1 = 109.8.
$P3$: CPI, Urban Consumers (PUNEW)1947M1–1986M6. 1970M1 = 113.5.
$P4$: CPI, Urban Consumers NSA (PZUNEW)1947M1–1986M6. 1970M1 = 113.3.
$I1$: One-Month Interest Rate on Finance Paper Placed Directly, % p.a., NSA. 1980M1 = 13.01.
$I2$: One-Month Interest Rate on Certificates of Deposit, Secondary Market, % p.a., NSA. 1980M1 = 13.26.
$I3$: One-Month Interest Rate on Euro Dollar Deposits, London, % p.a., NSA. 1980M1 = 14.247.
I: One-Month T-Bill Return from Ibbotson and Sinquefeld [14], 1926M1–1983M12. 1980M1 = 0.80.

Appendix: Decomposing the Conditional Variance of Nominal Rates

Suppose the real interest rate is nonstationary. Consider the following model:

$$i_t = r_t^e + \dot{p}_t^e, \tag{A1}$$

$$(1 - L)r_t = \varepsilon_t, \tag{A2}$$

$$\dot{p}_t = n_t, \tag{A3}$$

where ε_t and n_t are mutually independent stochastic processes, both i.i.d. with finite covariances. This model seems broadly consistent with most aspects of the data considered thus far. However, if $V(n) > V(\varepsilon)$, then the unit-root component in the real interest rate is not very important in explaining innovations in i_t. The question arises: how important is the nonstationary part of the series? Do nominal interest rates fluctuate mainly because of the real interest rate or because of innovations in inflationary expectations? In this appendix, I assess quantitatively the importance of real interest rate volatility.

Cochrane [1] has recently addressed the question of measuring the importance of a unit-root component of a nonstationary time series. In the context of (A1) to (A3), Cochrane's technique is to take the kth difference of the nominal interest rate and divide the sample variance of this series by k. This number, denoted σ_k^2, is approximately the variance of the unit-root component of the nominal interest rate (σ_ε^2) for large values of k. Thus, $\hat{\sigma}_k^2/\hat{\sigma}_1^2$; that is, the variance of the kth difference (if k is large) of i_t divided by k, over the variance of the first difference of i_t, provides an estimate of the proportion of the innovation variance of i_t, explained by the unit-root component. (Clearly, the unit-root component ε_t explains almost *all* of the *unconditional* variance of i, which is infinite.) If σ_k^2/σ_1^2 is low (e.g., 0.2), then the unit-root part of i_t is responsible for only a small (i.e., twenty percent) proportion of the variance in the *change* of i_t. Thus, a low value of σ_k^2/σ_1^2 for the nominal interest rate would mean that fluctuations in inflationary expectations rather than in real interest rates are the prime source of conditional uncertainty in the nominal interest rate.

Cochrane suggests two methods of estimating σ_k^2/σ_1^2: calculating the statistics directly and computing $1 + 2\sum_{j=0}^{k} \hat{P}_i i$, where $\hat{P}_i$ is the autocorrelation coefficient of $(1 - L)x_t$. Others suggest using $1 + 2\sum_{j=0}^{k}\left(1 - \dfrac{j}{k+1}\right)\hat{P}_j$, which is guaranteed to be non-negative in sample, has a smaller asymptotic standard error than $1 + 2\sum_{j=0}^{k} \hat{P}_i$, and has an associated t-statistic of $\sqrt{3/4 \times T/(k+1)}$. I concentrate below on the last statistic but also report other results (which are typically similar).

In Table VIII, I report estimates of σ_k^2/σ_1^2 for both the levels and the logs of annual interest rates from 1920 to 1970 ($T = 51$). Two sets of statistics are reported: the ratio of $\hat{\sigma}_k^2$ to $\hat{\sigma}_1^2$ and $1 + 2\sum_{j=0}^{30}\left(1 - \dfrac{j}{31}\right)\hat{P}_i$, along with the standard error for the latter. Again, the fraction represents the percentage of the innovation variance of the first difference of the nominal rate attributable to the unit-root component.

The natural log transformation seems to be relatively important for reasons that remain unclear. Further, the two statistics give very different point estimates; small-sample properties may be very important here. Both estimates suggest that a high proportion of the conditional variance of the nominal rate can be attributed to the unit-root component (which, it has been argued above, can be associated with the real rate). However, the confidence intervals are also wide. These results seem robust to changes in the sample size.

In Table IX, I report the monthly analogues to the results of Table IV, calculated with 168 monthly observations (1972M7 to 1986M11). The results for recent data in Table IX indicate that a high proportion of the innovation variance of nominal rates is due to the unit-root part. Even if one attributes all the conditional variance of the stationary part of the nominal interest rate to inflationary expectations, it seems that variations in the real interest rate will account for over half the conditional variance in nominal short rates. This estimate is higher than most of those in the extant literature (e.g., Fama and Gibbons [6]).

To check this conclusion for robustness with respect to the sample size, I recalculated the statistics for a variety of samples for the T-bill return of Ibboston and Sinquefeld (I). The results are in Table X. The estimates do not seem robust to the sample chosen and are uniformly smaller than the results of Table IX. However, for most of the samples, the unit-root component of the nominal interest rate is of importance in explaining the conditional variance of the nominal rate. If the nonstationary part can be associated with the real rate, as seems reasonable if inflation is stationary, then the real rate is not only volatile but is also responsible in explaining a sizable part of nominal-rate volatility.

Table VIII

Annual Estimates of the Unit-Root Component of Nominal Interest Rates

	$I1$	$I2$	log ($I1$)	log ($I2$)
From $\hat{\sigma}_k^2/\hat{\sigma}_1^2$:				
$k = 10$	1.28	0.75	1.70	1.43
$k = 20$	1.36	0.78	1.88	1.47
$k = 30$	0.46	0.49	0.89	0.81
From $1 + 2\sum\left(1 - \frac{j}{k+1}\right)\hat{P}_i$:				
$k = 30$	3.34	1.61	3.60	1.36
Standard error	(3.01)	(1.45)	(3.24)	(1.23)

Table IX

Monthly Estimates of the Unit-Root Component of Nominal Interest Rates: 1972M7 to 1986M11

	$I1$	$I2$	$I3$	$LI1$	$LI2$	$LI3$
From $\hat{\sigma}_k^2/\hat{\sigma}_1^2$:						
$k = 10$	0.91	0.97	0.89	1.35	1.46	1.30
$k = 20$	0.95	1.00	0.89	1.53	1.66	1.37
$k = 30$	0.79	0.84	0.77	1.36	1.47	1.22
From $1 + 2\sum\left(1 - \frac{j}{k+1}\right)\hat{P}_i$:						
$k = 30$	0.89	0.93	0.72	1.38	1.52	0.95
t-Statistic	(2.21)	(2.21)	(2.21)	(2.21)	(2.21)	(2.21)

Table X

Monthly Estimates of the Unit-Root Component of Nominal Interest Rates: *I*

	1930M1 1983M12	1951M4 1973M12	1934M1 1951M2	1951M4 1958M12	1959M1 1979M9
From $\hat{\sigma}_k^2/\hat{\sigma}_1^2$:					
$k = 10$	0.44	0.48	0.24	0.39	0.44
$k = 20$	0.38	0.34	0.17	0.22	0.41
$k = 30$	0.29	0.23	0.33	0.16	0.30
$k = 40$	0.23	0.12	0.37	0.08	0.17
From $1 + 2\Sigma\left(1 - \frac{j}{k+1}\right)\hat{P}_t$:					
$k = 40$	0.18	0.21	0.13	0.11	0.40
t-Statistic	(3.57)	(2.23)	(1.94)	(1.30)	(2.13)

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