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A Multiperiod Asset-Pricing Model with Unobservable Market Portfolio: A Note

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THIS PAPER DEVELOPS A multiperiod general equilibrium theory of asset prices that is simple and yet can be empirically tested without observing the market portfolio or aggregate consumption. We demonstrate that the systematic risk of an asset can be measured through the conditional covariance of its rate of return with the next-period risk-free rate of interest. The theory is developed using the minimum set of assumptions needed to produce the Intertemporal Sharpe-Lintner (ISL) model. (See Constantinides [5, 6].) Thus, if the present model holds, then the ISL model will also hold. However, to test or apply our results, the market portfolio does not have to be observed, and, thus, the empirical problems discussed in Roll [17] can be avoided.

Several authors have developed multiperiod asset-pricing models (e.g., Fama [8], Merton [16], Long [14], Rubinstein [20], Stapleton and Subrahmanyam [21], Lucas [15], Breeden [2], Constantinides [5, 6], Bhattacharya [1], Grossman and Shiller [11], and Cox, Ingersoll, and Ross [7]).[1] Some of these studies present risk-return relationships that measure riskiness of securities through covariances of asset returns with the rate of return on the market portfolio (e.g., Merton [16], Long [14], Stapleton and Subrahmanyam [21], and Constantinides [5, 6]). On the other hand, other studies use covariances of asset returns with changes in aggregate consumption to measure systematic risk of securities.[2]

The market portfolio and aggregate consumption are, however, not readily observable. Therefore, empirical tests of asset-pricing models have been rather inconclusive.[3] The market portfolio includes some assets with unobservable returns; therefore, the return on the true market portfolio must be estimated using the return on only a subset of assets. Aggregate consumption, on the other hand, must include aggregate consumption expenditures on nondurable goods as well as aggregate consumption flows derived from durable goods. Presently, no

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[1] We have ignored the APT because it is not directly comparable to models that are based on expected utility maximization.

[2] The most general measure of risk is the covariance of an asset's return with changes in the representative investor's marginal utility of wealth. Only under certain assumptions will changes in aggregate wealth or aggregate consumption be perfectly correlated with changes in the marginal utility of wealth; e.g., see Breeden [2].

[3] E.g., see Roll [17], Wheately [23], Breeden, Gibbons, and Litzenberger [3], and Grossman, Melino, and Shiller [12].

data on the latter are available, and this component of aggregate consumption must somehow be estimated.

This paper shows that expected excess returns on risky assets are proportional to their covariances with the *next-period* riskless rate of interest. This equilibrium relationship can be interpreted as follows. The fundamental result of the ISL model is that the market portfolio is mean-variance efficient, and, thus, changes in the value of the market portfolio provide the same amount of information about the aggregate risk of an economy that changes in investors' marginal utilities of wealth provide. Hence, the systematic risk of a security can be measured through the covariance of its return with changes in the value of aggregate wealth. If we can find an observable economic variable the intertemporal random behavior of which is completely explained by changes in aggregate wealth, then the market portfolio will no longer have to be observed, and, instead, this variable can be used to measure riskiness of securities. This paper shows that the one-period risk-free rate of interest is indeed such a variable.

The implications of the model are interesting because it can be applied to the same problems that the ISL model has been applied to, while it avoids the type of empirical difficulties that are discussed in Roll [17].

The plan of the paper is as follows. Section I will discuss the paper's assumptions and definitions. Section II obtains the pricing equation, Section III discusses the equilibrium relationship, and Section IV offers some concluding comments.

I. Assumptions and Definitions

We use the following notation: t = time subscript; ϕ_t = state of the economy; $\tilde{R}_i(\phi_{t+1})$ = one plus the rate of return on asset $i = 1, \cdots, N_t$ over $(t, t+1)$; $R_f(\phi_t)$ = one plus the riskless rate of interest available at time t; $W(\phi_t)$ = investor's wealth at time t prior to consumption at time t.

We also make the following assumptions:

(A1) Perfect markets: Investment opportunities are traded in perfect markets; riskless borrowing and lending rates are equal, and short sales of all assets are allowed.

(A2) Aggregation: The aggregation problem is solved, and, hence, equilibrium prices are determined as if all investors were identical.[4] Alternatively, we can assume that there is a composite investor (see Constantinides [6]) and use his or her utility of lifetime consumption to obtain our results. However, in this case we also must assume that markets are complete.

[4] Assume that investor i maximizes $U_i(C_0) + \rho_i E_0 V_i(C_1)$, where $-U_i'/U_i'' = A_i + B_i C_0$ and $-V_i'/V_i'' = A_i + B_i C_1$. Rubinstein [19] presents the following conditions for aggregations: (i) All individuals are homogeneous; (ii) All individuals have the same beliefs, $\rho_i := \rho$, and $B_i = B \neq 0$; (iii) All individuals have the same beliefs and $B_i = 0$; (iv) All individuals have the same beliefs and resources and $B_i = 0$; (v) Markets are complete and $B_i = 0$; (vi) Markets are complete, $\rho_i = \rho$, $A_i = 0$, $B_i = 1$, and all individuals have the same resources. Because returns on real assets are normally distributed, the representative investor's utility function must be defined for normally distributed wealth. Also, see Constantinides [5], footnote 8.

(A3) State-independent utility: Each investor's utility of lifetime consumption does not explicitly depend upon the state of the economy. Because, with heterogeneous investors, aggregation requires additive separable utility functions, we assume that the utility of lifetime consumption, $U(C_s, \cdots, C_T)$, can be expressed as $\sum_{t=s}^{T} U(C_t, t)$. Note that, if investors are identical, then the utility function need not be additively separable.

(A4) Rational expectations: Investors have rational expectations; i.e., when making optimal investment-consumption decisions, they take all available information into account. In the context of Proposition 1, this assumption implies that optimal investments in financial securities are zero.

(A5) Consumption opportunities: Prices of consumption goods are not random through time.

(A6) Investment opportunities: At time t there are n_t competitive value-maximizing firms that invest in real assets. The rate of return on the equity of firm i over $(t, t+1)$ is denoted by $\tilde{R}_i(\phi_{t+1})$, and, conditional on ϕ_t, it is normally distributed with mean $\bar{R}_i(W(\phi_t), t)$ and standard deviation $\sigma_i(W(\phi_t), t)$. In other words, returns on real assets can have nonstationary conditional distributions provided that the nonstationarity is only a function of aggregate wealth and time. Hence, given $W(\phi_t)$, returns on real assets over $(t, t+1)$ cannot depend on ϕ_t.[5] This situation can arise if, for example, production technology of firm i is

$$\tilde{K}_i(\phi_{t+1}) = K_i(\phi_t)(\mu_i(W(\phi_t), t) + \eta_i(W(\phi_t), t)\tilde{\xi}_i(t+1)),$$

where $K_i(\phi_t) \geq 0$ is the amount of capital invested in technology i, μ_i and σ_i are functions of $W(\phi_t)$ and time, and $\tilde{\xi}_i(t+1)$ is a stationary, normally distributed random variable. There are also $N_t - n_t$ financial securities (e.g., bonds, options, etc.) that have zero net supply, and their returns can have general nonstationary probability distributions.

Assumption (A6) of this paper is less restrictive than assumption (A4) of Constantinides [5] because we allow returns on real assets to follow nonstationary random processes.[6]

A representative investor will solve the following optimization problem:

$$J(W(\phi_s), \phi_s, s) = \max_{x_{it}, C_t} \mathrm{E}_s[\textstyle\sum_{t=s}^{T} U(C_t, t)],$$

subject to

$$W(\phi_t) = \{W(\phi_{t-1}) - C_t\}\{\textstyle\sum_{i=1}^{N_t} X_{it}[R_i(\phi_t) - R_f(\phi_t)] + R_f(\phi_t)\}.$$

The investor's wealth at the beginning of time, after the state of the economy is revealed, is $W(\phi_t)$; then he or she chooses the optimal values of C_t and X_{it}. This optimization problem has been extensively discussed in the literature. (E.g.,

[5] Firms issue only equity.

[6] Constantinides [5] assumes that asset returns have joint stationary distributions that belong to the family of two-fund separating distributions. (See Ross [18].) Constantinides [6] allows for nonstationary returns on real assets but assumes that markets are complete.

see Fama [8], Long [13, 14], and Constantinides [5].) Thus, we have presented only its essential aspects.

II. An Interest-Rate-Based Asset-Pricing Model

The first-order conditions for the representative investor's expected-utility-maximization problem are

$$J'(W(\phi_t), \phi_t, t) - R_f(\phi_t)\mathrm{E}_t[\tilde{J}'(W(\phi_{t+1}), \phi_{t+1}, t+1)] = 0, \tag{1a}$$

$$\mathrm{E}_t[\tilde{J}'(W(\phi_{t+1}), \phi_{t+1}, t+1)(\tilde{R}_i(\phi_{t+1}) - R_f(\phi_{t+1}))] = 0, \quad i = 1, \cdots, N_t, \tag{1b}$$

where $J'(\cdot)$ is the marginal utility of wealth. Throughout this paper, derivatives are denoted by primes.

Equation (1a) states that, when optimal policies are followed, one plus the expected rate of decline in the marginal utility of wealth is equal to one plus the riskless rate of interest; equation (1b) is the most basic equilibrium relationship that holds for returns on securities, and it will play a crucial role in the derivation of our results. Using the definition of covariance, this equation can be expressed as follows ($J'(\phi_t)$ represents $J'(W(\phi_t), \phi_t, t)$):

$$\mathrm{E}_t\tilde{R}_i(\phi_{t+1}) = R_f(\phi_t) - [\mathrm{E}_t\tilde{J}'(\phi_{t+1})]^{-1}\mathrm{cov}_t[\tilde{J}'(\phi_{t+1}), \tilde{R}_i(\phi_{t+1})]. \tag{2}$$

We now present a proposition that is central to the results of this paper.

PROPOSITION 1: *Assume that conditions specified in* (A1) *through* (A6) *hold. Then, the representative investor's derived utility of lifetime consumption will be state independent.*

Proof: See Fama [8] and Constantinides [5].

Two important implications of Proposition 1 are

(i) $J'(W(\phi_t), t)$, the current marginal utility of wealth, is only a function of current aggregate wealth and time;
(ii) $\mathrm{E}_t[J'(W(\phi_{t+1})\ \phi_{t+1}, t+1)] = g(W(\phi_t), t)$, the conditional expected value of the next-period marginal utility of wealth, is only a function of current aggregate wealth and time.

The following two properties of normally distributed random variables will be used throughout the paper. Assume that, conditional on y, random variables $\tilde{x}$ and $\tilde{z}$ are bivariate normally distributed. Also assume that $h(\tilde{x})$ is a continuous real function with continuous first derivative such that $\mathrm{E}_y[\,|\,h'(\tilde{x})\,|\,] < \infty$.

LEMMA 1:

$$\mathrm{cov}_y[h(\tilde{x}), \tilde{z}] = \mathrm{E}_y[h'(\tilde{x})]\mathrm{cov}_y[\tilde{x}, \tilde{z}]. \tag{3}$$

Proof: See Stein [22] and Rubinstein [20].

LEMMA 2:

$$\frac{\partial}{\partial y}\mathrm{E}_y[h(\tilde{x})] = \mu'\mathrm{E}_y[h'(\tilde{x})] + \frac{\sigma'}{\sigma}\mathrm{E}_y[h'(\tilde{x})(\tilde{x} - \mu)], \tag{4}$$

where, conditional on y, $\tilde{x} \sim N(\mu, \sigma^2)$ and μ' and σ' are, respectively, partial derivatives of μ and σ with respect to y. Because y and x may not be bivariate normal random variables, both μ and σ may be functions of y.

Proof: Set $x = \mu(y) = \sigma(y)\tilde{\varepsilon}$, where $\varepsilon \sim N(0, 1)$. The proof follows directly from differentiating the left-hand side of equation (4).[7]

Because the aggregation problem is solved, equilibrium prices are determined as if investors were identical. The representative investor's wealth, therefore, includes only assets that have positive aggregate supplies. (I.e., no financial securities are held by identical investors.) Since, conditional on the state of the economy at t, returns on real assets and hence the next-period level of aggregate wealth are normally distributed, we can apply Lemma 1 to the conditional covariance between the marginal utility of wealth and return on real asset i.[8] The result is

$$\mathrm{cov}_t[\tilde{J}'(W(\phi_{t+1}), t+1), \tilde{R}_i(\phi_{t+1})] = \mathrm{E}_t[\tilde{J}''(\cdot)]\mathrm{cov}_t[\tilde{W}(\phi_{t+1}), \tilde{R}_i(\phi_{t+1})].$$

Substituting the above expression into equation (2), we will have

$$\mathrm{E}_t\tilde{R}_i(\phi_{t+1}) = R_f(\phi_t) + \lambda(\phi_t)\mathrm{cov}_t[\tilde{W}(\phi_{t+1}), \tilde{R}_i(\phi_{t+1})], \tag{5}$$

where

$$\lambda(t) = [\mathrm{E}_t\tilde{J}''(W(\phi_{t+1}), t+1)][\mathrm{E}_t\tilde{J}'(W(\phi_{t+1}), t+1)]^{-1}.$$

Equation (5) is, of course, the multiperiod version of the Sharpe-Lintner model. As was discussed before, the main difficulty in any empirical testing of equation (5) is that the market portfolio, $\tilde{W}(\phi_{t+1})$, is not observable. The objective of this paper is to present the last term on the right-hand side of equation (5) in terms of the riskless rate of interest.

We use equation (1a) to characterize the intertemporal behavior of the riskless rate of interest. The current riskless rate of interest is

$$R_f(\phi_t) = \frac{J'(W(\phi_t), t)}{\mathrm{E}_t\tilde{J}'(W(\phi_{t+1}), t+1)} = R_f(W(\phi_t), t), \tag{6}$$

where results of Proposition 1 are used to obtain the last term of this expression. Hence, the current riskless rate of interest is a function of current wealth and time. Given all available information, the next-period riskless rate is a random variable that can be expressed as follows:

$$\tilde{R}_f(\phi_{t+1}) = \frac{\tilde{J}'(W(\phi_{t+1}), t+1)}{\tilde{\mathrm{E}}_{t+1}J'(W(\phi_{t+2}), t+2)} = \tilde{R}_f(W(\phi_{t+1}), t+1). \tag{7}$$

The next-period riskless rate is not known at time t only because the next-period value of aggregate wealth is not known. In this economy, random variations in

[7] I wish to thank the referee for suggesting this short proof.

[8] We need to assume that $\mathrm{E}_t\,|J''(\cdot)| < \infty$.

the value of aggregate wealth represent the aggregate risk of the economy. Hence, random behavior of all endogenous aggregate economic variables (e.g., the riskless rate, returns on default-free bonds, and the market price of risk) is driven by changes in the value of aggregate wealth.

If we can apply Lemma 1 to the conditional covariance between the next-period riskless rate and the rate of return on asset i, we will be able to show that this covariance is proportional to the conditional covariance of the same asset with aggregate wealth. That is,

$$\mathrm{cov}_t[\tilde{R}_f(W(\phi_{t+1})),\ \tilde{R}_i(\phi_{t+1})] = \psi(\phi_t)\mathrm{cov}_t[\tilde{W}(\phi_{t+1});\ \tilde{R}_i(\phi_{t+1})], \tag{8}$$

where $\psi(\phi_t) = \mathrm{E}_t[\tilde{R}_f'(W(\phi_{t+1}))]$ and is independent of the risky asset.

According to Lemma 1, this expression is correct if $\mathrm{E}_t[|\tilde{R}_f'(W(\phi_{t+1}))|] < \infty$. Using equation (7) and the results of Lemma 2, we have (dependence on $W(\phi_{t+1})$ or $W(\phi_{t+2})$ is suppressed)

$$R_f'(t+1) = -R_f(t+1) \times \left\{A(t+1) - \frac{\sigma(t+1)\mu'(t+1) - \sigma'(t+1)\mu(t+1)}{\sigma(t+1)} \times B(t+1) - \frac{\sigma'(t+1)}{\sigma(t+1)} D(t+1)\right\}. \tag{9}$$

$A(t+1) = -[J''(t+1)]/[J'(t+1)]$, a measure of absolute risk aversion at $(t+1)$;
$B(t+1) = -\mathrm{E}_{t+1}[J''(t+2)]/\mathrm{E}_{t+1}[J'(t+2)]$, a measure of the expected value of absolute risk aversion at $(t+2)$;
$D(t+1) = -\mathrm{E}_{t+1}[J''(t+2)W(t+2)]/\mathrm{E}_{t+1}[J'(t+2)]$, a measure of the expected value of relative risk aversion at $(t+2)$;
$\mu(t+1) = \mathrm{E}_{t+1}W(t+2)$, and μ' denotes its derivative with respect to $W(t+1)$;
$\sigma(t+1) = \mathrm{Std}_{t+1}W(t+2)$, and σ' denotes it derivative with respect to $W(t+1)$.

The requirement, $\mathrm{E}_t[|\tilde{R}_f'(W(\phi_{t+1}))|] < \infty$, does not appear to be very restrictive. For example, if $\tilde{W}(\phi_{t+1})$ and $\tilde{W}(\phi_{t+2})$ are independent, then $\tilde{R}_f'(W(\phi_{t+1})) = -\tilde{R}_f(W(\phi_{t+1}))\tilde{A}(t+1)$, or, if they are bivariate normal random variables, then

$$\tilde{R}_f'(W(\phi_{t+1})) = -\tilde{R}_f(W(\phi_{t+1}))\left\{\tilde{A}(t+1) - \frac{\mathrm{cov}[\tilde{W}(\phi_{t+1}),\ \tilde{W}(\phi_{t+2})]}{\mathrm{var}[\tilde{W}(\phi_{t+1})]}\tilde{B}(t+1)\right\}. \tag{10}$$

In both instances, the necessary condition is satisfied if the degrees of risk aversion of the representative investor are bounded. We will assume that $\mathrm{E}_t[|\tilde{R}_f'(W(\phi_{t+1}))|] < \infty$ and denote this assumption by (A7).

PROPOSITION 2: *Assume that the conditions specified in* (A1) *through* (A7) *are satisfied. Then the following equilibrium relationship will hold for all assets the returns of which, conditional on* $W(\phi_t)$, *are normally distributed over* $(t, t+1)$:

$$\mathrm{E}_t\tilde{R}_i(\phi_{t+1}) = R_f(W(\phi_t)) + \gamma(\phi_t)\mathrm{cov}_t[\tilde{R}_i(\phi_{t+1}), \tilde{R}_f(W(\phi_{t+1}))], \tag{11}$$

where $\gamma(\phi_t)$ *is defined as* $\lambda(\phi_t)\mathrm{E}_t[\tilde{R}_f'(W(\phi_{t+1}))]^{-1}$.

Proof: From equation (8), we can solve for the conditional covariance of $\tilde{W}(\phi_{t+1})$ with $\tilde{R}_i(\phi_{t+1})$. Equation (11) is obtained by substituting this result into equation (5).

In this expression, the market price of risk is denoted by $\gamma(\phi_t)$. To test this equilibrium relationship, we do not need to observe it directly. Similar to the approach often employed to present the ISL model in a testable form, we can write equation (11) for another real risky asset or a portfolio of real assets and thereby eliminate $\gamma(\phi_t)$.

III. Discussion

Fama and Schwert [9] and Ferson [10] have provided empirical evidence of a negative relationship between expected returns on portfolios of common stocks and the short-term riskless rate of interest. Though their studies were not concerned with testing the equilibrium relationship of this paper, their results are consistent with our theoretical model.[9]

If expected excess returns on risky assets are to be positive, then the market price of risk, $\gamma(\phi_t)$, and the measure of risk, $\mathrm{cov}_t[\tilde{R}_i, \tilde{R}_f]$, must have the same sign. Hence, if one can show that $\mathrm{E}_t[\tilde{R}_f'(W(\phi_{t+1}))] < 0$, it follows that both $\gamma(\phi_t)$ (see equation (11)) and $\mathrm{cov}_t[\tilde{R}_i, \tilde{R}_f]$ must be negative. This indicates that, if the return on a risky asset is regressed against the short-term risk-free rate, the slope coefficient will be negative, and this is what Fama and Schwert [9] and Ferson [10] have reported. If $\tilde{W}(\phi_{t+1})$ and $\tilde{W}(\phi_{t+2})$ are independent, $\mathrm{E}_t[\tilde{R}_f'(W(\phi_{t+1}))]$ will always be negative, while, if $\tilde{W}(\phi_{t+1})$ and $\tilde{W}(\phi_{t+2})$ are bivariate normal random variables, $\mathrm{E}_t[\tilde{R}_f'(W(\phi_{t+1}))]$ will be negative if $\tilde{W}(\phi_{t+1})$ and $\tilde{W}(\phi_{t+2})$ are not perfectly correlated and the investor's measure of absolute risk aversion does not increase over time. (See equation (10).)

In developing the pricing equation, we implicitly assumed that $\mathrm{cov}_t[\tilde{R}_f, \tilde{R}_i]$ is different from zero. This covariance can be zero under two circumstances:

(i) $\mathrm{E}_t[\tilde{R}_f'(W(\phi_{t+1}))] \neq 0$, but the return on the risky asset is uncorrelated with the return on aggregate wealth;

(ii) $\mathrm{E}_t[\tilde{R}_f'(W(\phi_{t+1}))] = 0$.

If condition (i) holds, then the pricing equation will correctly identify zero-beta securities or portfolios. Condition (ii) will hold when investors are risk

[9] Fama and Schwert were interested in the relationship between asset returns and expected and unexpected inflation. Ferson interprets the results as an indication of the behavior of the conditional covariances of asset returns with the marginal utility of wealth. They deal with nominal rates of returns, while our results deal with real returns.

neutral, but, under this condition, all financial securities will earn the risk-free rate of return, and only the firm with the largest expected return will remain active. However, condition (ii) can hold under other circumstances. For example, assume that the representative investor's measure of absolute risk aversion is constant. (I.e., both $A(t+1)$ and $B(t+1)$ are constants.) Also, assume that $\tilde{W}(\phi_{t+1})$ and $\tilde{W}(\phi_{t+2})$ are bivariate normal random variables and that the beta between them is equal to $A(t+1)/B(t+1)$. Then, from equation (10), we will have $\mathrm{E}_t[\tilde{R}_f'(W(\phi_{t+1}))] = 0$. This appears to be the only plausible condition under which (ii) will hold.[10] For instance, if the representative investor does not have exponential utility function, then $B(t+1)$ will be a nonlinear function of wealth. If $\mathrm{E}_t[\tilde{R}_f'(W(\phi_{t+1}))] = 0$ is to hold, the covariance between $\tilde{W}(\phi_{t+1})$ and $\tilde{W}(\phi_{t+2})$ must also be a nonlinear function of wealth. It should be noted that, when $\mathrm{E}_t[\tilde{R}_f'(W(\phi_{t+1}))] = 0$, $\mathrm{cov}_t[\tilde{R}_i, \tilde{R}_f] = 0$ for all risky assets, and this can be tested empirically.

In contrast to the arbitrage- and consumption-pricing models, and similar to the ISL model, the interest-rate-based model is a direct result of the mean-variance efficiency of the market portfolio. Hence, from a theoretical point of view, both models can be criticized for their rather restrictive assumptions. The present model should, therefore, be viewed as an attempt to introduce some new insights into the ISL model and to present an alternative equilibrium relationship that is in principle testable. We must, however, note that the conditional covariances of asset returns with the next-period riskless rate are likely to be nonstationary, and this could make any empirical testing of the model rather complicated.

IV. Conclusions

This paper develops a multiperiod asset-pricing model that does not require any estimation of the stochastic properties of the rate of return on the market portfolio or the rate of change in aggregate consumption. The results are developed using the framework of Fama [8] and Constantinides [5] and consequently will hold in capital markets that satisfy the minimum set of assumptions needed to produce the multiperiod version of the Sharpe-Lintner model.

We demonstrate that intertemporal changes in the riskless rate of interest can be used to measure the systematic risk of securities. In particular, the covariances of assets returns with the return on the market portfolio are shown to be proportional to the covariances of the same assets with the next-period riskless

[10] Clearly, if the riskless rate of interest is not a function of aggregate wealth, then $\mathrm{E}_t[\tilde{R}_f'(\phi_{t+1})] = 0$. This can happen if $R_f(\cdot) = \Pi(t)$, which is only a function of time; i.e.,

$$J'(W(\phi_t), t) = \Pi(t) \int [J'(W(\phi_{t+1}), t+1) f(W(\phi_{t+1}) \mid W(\phi_t))] \, dW(\phi_{t+1}).$$

This expression is a homogeneous Fredholm integral equation of the second kind, and it may be solved for $J'(\cdot)$ once $f(\cdot \mid \cdot)$ is specified. (E.g., see Bronshtein and Semendyayev [4].) The resulting solution may not, however, satisfy all the restrictions that we have imposed on $J'(\cdot)$.

rate of interest; hence, we can present equilibrium relationships that do not require data on the rate of return on the market portfolio.

Similar to the Sharpe-Lintner model, the pricing model of this paper is expressed in real terms; observed data on asset returns and the short-term riskless rate are, on the other hand, expressed in nominal terms. Hence, an interesting extension of our results is to relax the assumption that prices of consumption goods are nonrandom.[11]

[11] As discussed in Fama [8] and Constantinides [5], once this assumption is relaxed, the indirect utility function will become state dependent.

REFERENCES

1. S. Bhattacharya. "Notes on Multiperiod Valuation and the Pricing of Options." *Journal of Finance* 36 (March 1981), 163–80.
2. D. T. Breeden. "An Intertemporal Asset Pricing Model with Stochastic Investment and Consumption Opportunities." *Journal of Financial Economics* 7 (September 1979), 273–96.
3. ———, M. R. Gibbons, and R. H. Litzenberger. "Empirical Tests of the Consumption-Oriented CAPM." Research Paper No. 879, Graduate School of Business, Stanford University.
4. I. N. Bronshtein and K. A. Semendyayev. *Handbook of Mathematics.* New York: Van Nostrand Reinhold Co., 1985.
5. G. M. Constantinides. "Admissible Uncertainty in the Intertemporal Asset Pricing Model." *Journal of Financial Economics* 8 (March 1980), 71–86.
6. ———. "Intertemporal Asset Pricing with Heterogeneous Consumers and without Demand Aggregation." *Journal of Business* 55 (April 1982), 253–67.
7. J. C. Cox, J. Ingersoll, and S. A. Ross. "An Intertemporal General Equilibrium Model of Asset Prices." *Econometrica* 53 (March 1985), 363–84.
8. E. F. Fama. "Multiperiod Consumption-Investment Decision." *American Economic Review* 60 (March 1970), 163–70.
9. ——— and G. W. Schwert. "Asset Returns and Inflation." *Journal of Financial Economics* 5 (November 1977), 115–46.
10. W. E. Ferson. "Changes in Expected Risk Premiums and Security Risk Measures." Working Paper No. 177, CRSP, Graduate School of Business University of Chicago, 1986.
11. S. J. Grossman and R. J. Shiller. "Consumption Correlatedness and Risk Measurement in Economies with Non-traded Assets and Heterogeneous Information." *Journal of Financial Economics* 10 (June 1982), 195–210.
12. S. J. Grossman, A. Melino, and R. J. Shiller. "Estimating the Continuous-Time Consumption Based Asset Pricing Model." Working Paper No. 1643, NBER, Cambridge, MA, 1986.
13. J. B. Long. "Consumption-Investment Decisions and Equilibrium in the Securities Market." In M. C. Jensen (ed.), *Studies in the Theory of Capital Markets.* New York: Prager Publishers, 1972, 146–222.
14. ———. "Stock Prices, Inflation, and the Term Structure of Interest Rates." *Journal of Financial Economics* 1 (July 1974), 131–70.
15. R. E. Lucas. "Asset Prices in an Exchange Economy." *Econometrica* 46 (November 1978), 1429–46.
16. R. C. Merton. "An Intertemporal Asset Pricing Model." *Econometrica* 41 (September 1973), 867–87.
17. R. Roll. "A Critique of the Asset Pricing Model's Tests, Part I: On Past and Potential Testability of the Theory." *Journal of Financial Economics* 4 (March 1977), 129–76.
18. S. A. Ross. "Mutual Fund Separation in Financial Theory—the Separating Distributions." *Journal of Economic Theory* 17 (December 1978), 254–86.
19. M. Rubinstein. "An Aggregation Theorem for Securities Markets." *Journal of Financial Economics* 1 (September 1974), 225–44.

20. ———. "The Valuation of Uncertain Income Streams and the Pricing of Options." *Bell Journal of Economics* 7 (Autumn 1976), 407–25.
21. R. C. Stapleton and M. G. Subrahmanyam. "A Multiperiod Equilibrium Asset Pricing Model." *Econometrica* 46 (September 1978), 1077–96.
22. C. Stein. "Estimation of the Mean of a Multivariate Normal Distribution." In J. Hajek (ed.), *Proceedings of the Prague Symposium on Asymptotic Statistics*, vol. 2. Prague, Czechoslovakia: Universita Karlov Praha, September 1973, 345–83.
23. S. Wheately. "Some Tests of Consumption Based Asset Pricing Model." Working Paper No. 43, Center for the Study of Banking and Financial Centers, University of Washington, Seattle, WA, 1985.