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Was the Tax-Exempt Bond Market Inefficient or Were Future Expected Tax Rates Negative?

LEVIS A. KOCHIN and RICHARD W. PARKS*

ABSTRACT

This paper shows that the sharp narrowing with maturity of the spread between taxable and tax-exempt yields leaves room for tax arbitrage. At times, tax-exempt forward rates have exceeded taxable forward rates. At such times, only expectations of higher taxes on Treasury than on municipal bonds would eliminate profit opportunities. The authors develop the idea of forward tax rates and compute forward tax rates for 1955 through 1984. They also outline tax-arbitrage mechanisms involving private forward sale of long municipal bonds or the use of the Municipal Bond Futures Contract and show the potential profits.

THE MUNICIPAL YIELD CURVE has been upward sloping for the entire postwar period, even at times when taxable yield curves were humped or downward sloping.[1] Thus, the spread in yields between taxable and tax-free bonds narrows dramatically with increasing maturity. This paper focuses on the term structure of the spread in yields between taxable and tax-exempt bonds. We show that the sharp narrowing of the spread with maturity leaves room for some apparently unexploited tax arbitrage. At times, the narrowing has been so dramatic that tax-exempt forward rates have exceeded taxable forward rates. In other words, only expectations of negative future tax rates on Treasury interest or the expectations of a reversal in the *relative* taxability of Treasury and municipal bonds would eliminate profit opportunities. This situation makes little sense if the markets are well arbitraged.

Three theories of the relationship between the yields on taxable and tax-exempt bonds have attracted attention.[2] Miller [8] asserts that the corporate tax rate determines the yield spread between taxable and tax-exempt debt. In the

* Associate Professor and Professor of Economics, respectively, University of Washington, Seattle. The authors would like to thank Cary Schumacher and Mark Thomson for their careful research assistance on this project. We would also like to thank an anonymous referee for comments that led us to develop the extension of the model referred to in footnote 9.

[1] See Salomon Brothers Inc. [13], Part III, Table 1. The table provides monthly yield data from January 1950 onward for various grades of municipal bonds at maturities of one, two, five, ten, fifteen, twenty, and thirty years. In every month, the yield of a longer maturity bond is never lower than that of any shorter maturity bond.

[2] Other attempts to explain the changing yield spread have pointed to factors such as changes in the riskiness and changes in the supply of municipal debt. Yawitz, Maloney, and Ederington [16] present measurements of the default risk for municipals based on the assumption that most of the difference represents risk. As Poterba points out, their estimates are implausibly high. Arak and Guentner [1] point out that long-term tax deferral represents a substitute for tax-free debt. A yield of ten percent per year tax deferred for twenty-five years is equivalent to an after-tax yield of eight percent per year to an individual with a fifty percent marginal tax rate.

simpler version of the Miller model, the supply of corporate debt and equity varies, so that, for a marginal investor choosing between tax-exempt and taxable debt, the effective tax rate on interest income is the *corporate* tax rate. There is no role for individual tax rates. Under an extension of the Miller models that allows for positive tax rates on equity income, individual tax rates do secure a role in the determination of the taxable versus tax-exempt yield spreads (cf. Buser and Hess [3]). Neither the original Miller model nor the extended version used by Buser and Hess for their test of the model considers variations in the spread across maturities.

Bank-arbitrage models rely on the fact that commercial banks can deduct from taxable income the interest on borrowing used to hold tax-exempt bonds (cf. Fama [6] and Haley, Higgins, and Howard [7]). These models imply that the marginal tax rate of the commercial banks determines the yield spread between taxable and nontaxable bonds.[3] There is no role in the bank-arbitrage model for the effects of the marginal tax rates of individuals. Although useful in explaining the behavior of the yield spread for bonds of short maturity, the bank-taxable model fails to provide an adequate explanation of the much narrower spreads at longer maturities.

The market-segmentation and preferred-habitat models view the municipal bond market as being segmented by maturities, with commercial banks' holdings concentrated on short maturities and with casualty insurance companies' and individuals' holdings concentrated on long maturities. In these models, the tax rates of commercial banks determine the yield spread for short maturities as in the bank-arbitrage model, but the tax rates of individuals help to determine the spread for long maturities. Mussa and Kormendi [10], chapter 4, devote some attention to the differences between the yield structures of taxable and tax-exempt bonds, including the term structure of the yield spread as measured by implied tax rates.[4] They explain the existence of the nonuniform implied tax rate by what they call a theory of equalizing differences together with some distinct clientele effects.

In light of these theories, Poterba [12] attempts to explain movements in the yield spread in terms of changes in expectations about tax rates. His results are consistent with the bank-arbitrage and Miller theories for short maturities. They give support to the market-segmentation theories and contradict the Miller implications, however, by showing that changes in personal tax rates affect the

[3] The tax code provisions were changed in 1982 to permit deduction of only eighty percent of the interest. As of March 1986, the House-passed tax reform bill HR 3838, approved December 17, 1985, would eliminate this deduction completely, effective retroactively to January 1, 1986. The municipal markets seem to have anticipated the adoption of this provision in the final bill. As of this writing, the implicit tax rates on short and long maturities are more nearly equal than they have been during the previous three decades.

[4] What has commonly been called the implied or implicit tax rate should properly be called the implied differential tax rate. In efficient markets, if the tax rate on municipal bond interest is zero and is expected to remain zero, then the implied tax rate can be viewed as the expected marginal tax rate for the municipal holder of taxable bonds. In a setting where municipal bond interest is taxed at positive rates, the implied tax rate captures the differential tax rate between municipal and government bonds. Parks and Kochin [11] develop the differential tax rate interpretation further. See also footnote 9.

yield spread at longer maturities. Poterba does not provide an adequate explanation for the considerable variation in yield spreads across maturities.

Unless there are legal or transaction-cost barriers, segmented markets offer potential profits from arbitrage between the segments. In this paper, we offer several riskless arbitrage schemes that were apparently available during much of the recent past. The presence of these arbitrages casts doubt on the adequacy of existing theories of equilibrium in bond markets.

Section I of this paper develops the idea of *forward* tax rates and derives measures of these forward tax rates for the thirty-year period 1955 through 1984. This analysis uncovers conditions that seem inconsistent with a fully equilibrated market. Section II outlines several possible tax-arbitrage mechanisms involving forward sale of long municipal bonds and shows the nature of potential profits during several key periods. The recently created Municipal Bond Futures Contract offers other means for possible arbitrage of the municipal market.

I. Implicit Tax Rates and Forward Tax Rates

A. Introduction

It has been common to measure the spread between yields on taxable and tax-exempt bonds of maturity T in terms of the implicit tax rate $\theta_T^I = 1 - y_{mT}/y_{tT}$, where y_{mT} and y_{tT} denote the yield to maturity on tax-exempt and taxable bonds of maturity T. In this section, we define a set of forward tax rates based on the implicit tax rates. Forward rates exhibit unusual behavior in the Salomon data employed by Poterba and others in their analysis of the yield spreads. There are a number of episodes where forward tax rates are negative and many more where they are smaller than would be expected under the various theories that have been advanced to explain the spread. These results seem to make no sense in a well-arbitraged bond market.

Table I gives the yearly average values for the implicit tax rate for various maturities during the period 1955 through 1984. These implicit tax rates were computed using the yield data from Salomon Brothers Inc. [13]. The Salomon data provide yields to maturity on par bonds for both taxable Treasury bonds and tax-exempt prime general-obligation municipal bonds. The yield data are available for various maturities as of the first of each month. The implicit tax rates were first calculated using the monthly data. The results were then averaged to produce a more manageable table.

Table I shows that the implicit tax rate typically declines with maturity from levels that average about forty-three percent at the one-year maturity to levels that average about 19.5 percent at the thirty-year maturity. There have been substantial movements in the implicit tax rate spreads over time, but these movements have not been uniform or proportional.

There have been some attempts to explain movements in the implicit tax rates or spreads over time. Poterba [12] attempts to test various theories of the spread between the yields on taxable and tax-exempt bonds and to explain variations in this spread in terms of changing expectations of tax rates. Using the dates of "news" regarding likely changes in the tax structure, he found that he could

Table I

Implicit Tax Rates, Based on the Yields for Treasury Bonds and for Prime General-Obligation Municipal Bonds[a]

	Maturities (in Years)					
Year	1	2	5	10	20	30
1955	41.4	44.2	40.6	34.1	24.4	19.9
1956	41.3	37.1	33.3	27.9	21.9	18.4
1957	38.0	35.9	29.6	22.2	15.1	12.2
1958	41.2	39.0	32.6	26.2	18.9	14.8
1959	43.3	43.0	37.6	29.0	22.2	18.5
1960	42.2	39.6	36.4	29.3	22.7	17.5
1961	47.6	46.0	39.7	28.4	19.0	14.9
1962	46.8	46.7	42.3	35.3	25.6	20.6
1963	46.5	45.0	41.2	35.1	26.1	21.9
1964	44.2	43.0	37.5	32.7	26.5	22.1
1965	42.6	39.9	34.6	31.6	26.4	21.9
1966	33.6	33.4	31.6	26.6	22.7	20.7
1967	37.0	35.7	32.5	28.6	23.9	20.8
1968	40.5	37.7	33.0	28.2	22.6	18.4
1969	34.4	32.3	27.8	21.4	13.3	8.8
1970	38.7	37.9	35.3	25.9	10.1	6.0
1971	40.5	40.6	39.0	29.2	13.2	7.4
1972	43.5	43.3	38.8	33.1	15.4	10.6
1973	45.3	40.7	37.4	33.9	28.2	25.5
1974	42.4	38.9	36.6	30.0	28.2	26.2
1975	40.8	41.2	36.4	26.6	21.7	19.7
1976	47.5	46.6	42.4	36.1	27.6	24.2
1977	50.7	48.9	43.9	40.6	32.2	29.0
1978	49.3	47.3	43.6	40.8	34.6	31.8
1979	49.7	46.7	42.9	41.7	35.5	32.8
1980	48.5	46.8	43.9	40.0	30.8	27.5
1981	46.3	44.1	39.5	32.3	23.0	19.6
1982	42.4	41.0	33.6	24.9	15.4	12.5
1983	44.5	43.5	37.2	28.1	20.6	18.6
1984	44.0	43.9	38.3	29.9	22.2	21.0
Average	43.2	41.7	37.3	31.0	23.0	19.5

[a] Source of yield data: Salomon Brothers Inc. [13]. Implied tax rate = $1 - y_m/y_t$. Rates are in percent.

explain some of the variation over time in the spread. Yawitz, Maloney, and Ederington [16] explain some of the variation in the spread in terms of changing default risk over time.

B. Computing Forward Tax Rates

Newly issued T-period bonds with a par value of one dollar and a tax-exempt coupon C_m will sell at par if

$$1 = \sum_{j=1}^{T} (1 + R_j)^{-j} C_m + (1 + R_T)^{-T}, \tag{1}$$

where R_j is the nominal after-tax interest rate for after-tax payments made in period j. In equilibrium, a taxable bond selling at par will satisfy

$$1 = \sum_{j=1}^{T} (1 + R_j)^{-j} C_t(1 - \theta_j^e) + (1 + R_T)^{-T}, \tag{2}$$

where C_t is the taxable coupon payment and θ_j^e is the forward tax rate for period j.[5]

Since these bonds are assumed to sell at par, their coupon payments are equal to their yields to maturity; hence, $y_{mT} = C_m$ and $y_{tT} = C_t$ for the bonds in (1) and (2). The implicit tax rate (used by Poterba as a measure of the yield spread between taxable and tax-exempt bonds) is then given by θ_T^I, where

$$(1 - \theta_T^I) = y_{mT}/y_{tT}. \tag{3}$$

If we subtract (2) from (1) and solve for C_M/C_T, we obtain

$$1 - \theta_T^I = \sum_{j=1}^{T} w_j(1 - \theta_j^e), \tag{4}$$

where the weights $w_j = \delta_j/\sum_{i=1}^{T} \delta_i$, with $\delta_j = (1 + R_j)^{-j}$, the discount factor for tax-free payments made in period j. Since the weights sum to unity, we can write

$$\theta_T^I = \sum_{j=1}^{T} w_j\theta_j^e. \tag{5}$$

The implicit tax rate is a weighted average of the forward tax rates where, for each future period, the weights are the normalized discount factors, i.e., the discount factor divided by the sum of the discount factors over the maturity of the bond.[6] Notice that, unless the forward tax rates θ_j^e are all equal, the implicit tax rates will be affected by changes in the term structure of interest rates as well as by changes in the expected tax rates.

Using (5), we can derive an expression for the forward tax rate for the period T as

$$\theta_T^e = \frac{\theta_T^I \sum_{i=1}^{T} \delta_i - \theta_{T-1}^I \sum_{i=1}^{T-1} \delta_i}{\delta_T} \tag{6}$$

in terms of the implicit tax rates and the discount factors. Expression (6) would be useful in cases where we had data on bonds with adjacent maturities.[7] In the more common case, where data for adjacent maturities are not available, we can use an expression for the average forward tax rate over the maturity interval from s to t:

$${}_s\theta_t^e = \sum_{j=s}^{t} w_j^* \theta_j^e = \frac{\sum_{j=s}^{t} \delta_j\theta_j^e}{\sum_{k=s+1}^{t} \delta_k} = \frac{\theta_t^I \sum_{i=1}^{t} \delta_i - \theta_s^I \sum_{i=1}^{s} \delta_i}{\sum_{k=s+1}^{t} \delta_i}. \tag{7}$$

The forward tax rate for the interval between periods s and t is a weighted average of the forward tax rates for that interval, with weights equal to the normalized discount factors for payments in that interval. It can also be computed using the implicit tax rates for bonds of maturities s and t.[8]

[5] Poterba called the θ_j^e "the expected marginal tax rate of the marginal holder of this bond in period j" (page 22). As we show below, this interpretation is valid only in a fully arbitraged market. Both Poterba's evidence and ours suggest that this condition is not met.

[6] This expression for the implicit tax rate is consistent with Poterba's expression (12) (using his (6)) and with his expression (13), the special case for consols.

[7] Our model involves discrete time, as does the tax code. The tax rate for any interest recipient is constant for all moments within a given tax year.

[8] Initially we thought that one minus the forward tax rate could be computed as the ratio of the conventional forward rates for tax-exempt and taxable bonds. That approach, however, does not seem to give useful results.

In a well-arbitraged market, the forward tax rates could be interpreted as expected future tax rates. Under this interpretation, we would expect them to be bounded between zero and one. In the next section, we show that the measured forward tax rates sometimes violate these bounds.[9]

C. Measuring the Forward Tax Rate

Our first measure of the forward tax rate uses the same data employed by Poterba [12], Yawitz, Maloney, and Ederington [16], and others. Table II shows the annual averages of the forward tax rate for various maturities from 1955 to 1984 computed using prime general-obligation municipals and Treasuries. Calculations were based on expression (7) using the monthly Salomon data. The monthly forward tax rates were then averaged for ease of presentation. This aggregation does not seriously distort the general pattern of forward rates shown in the monthly data. The entry given in each case is the average forward tax rate for the period from the prior listed maturity up to the given maturity.

Table II shows that, for the short maturities, from one to five years, the tax rates appear normal. During the early 1970's and 1980's, the forward tax rates for maturities from five to ten years are quite low. During the 1969 to 1972 period and again for 1981, 1982, and 1984, the forward tax rates for maturities from ten to twenty years are negative. The forward tax rates for maturities from twenty to thirty years are also negative from 1980 through 1983.

These results are interesting because the interpretation of the forward tax rate as the average tax rate expected over a future period makes negative values extremely unlikely. Alternatively, the negative forward tax rates represent dramatic evidence of market segmentation. Whenever there is market segmentation not imposed by law, there is a profit opportunity.[10]

Before we can assert the existence of profit opportunities based on this evidence of market segmentation, we need to consider several aspects of the data that could artificially give rise to unusual results. In comparing the yields on taxable and tax-exempt bonds, we should like to compare bonds that are matched in terms of all features other than taxability. This is not the case for a comparison between Treasury bonds and prime municipal bonds. There are important differences in the customary call features of the two classes of bonds, and there may be significant differences in risk as well.

[9] It is actually the differential rate of tax between the two classes of assets that matters. The bound on the forward tax rate interpreted as an expected future tax rate comes from the assumption that presently tax-free bonds will be no more heavily taxed than bonds that are presently fully taxable. In Parks and Kochin [11], we have extended the development of this section to incorporate the possibility of federal taxation of both kinds of interest. The resulting expression for the forward tax rate is similar to (6). If expression (6) is used with data generated by the more general context, a bias results, but we show in Parks and Kochin [11] that our finding of negative forward rates cannot be the result of this bias.

[10] Schaefer [15] shows that markets with assets and individuals subject to different rates of tax may fail to find an interior equilibrium in the absence of some market imperfections such as a prohibition on short sales. Short sales of tax-free bonds are generally impossible because payments made in lieu of interest would not be tax free. In Section II, we show several transactions using forward sales that effectively transfer tax-free interest while avoiding the prohibition on short sales. Schaefer seems not to have considered this kind of transaction.

Table II

Forward Tax Rates Implied by Comparison of Taxable and Tax-Exempt Bond Yields, 1955–1984: Treasury Bonds versus Prime Grade Municipals[a]

	Maturities (in Years)					
Year	0–1	1–2	2–5	5–10	10–20	20–30
1955	0.414	0.469	0.382	0.269	0.122	0.065
1956	0.413	0.329	0.305	0.218	0.142	0.079
1957	0.380	0.337	0.250	0.137	0.055	0.027
1958	0.412	0.368	0.282	0.189	0.090	0.014
1959	0.433	0.427	0.338	0.189	0.128	0.056
1960	0.422	0.370	0.341	0.210	0.135	−0.005
1961	0.476	0.443	0.352	0.153	0.059	0.005
1962	0.468	0.465	0.392	0.273	0.123	0.031
1963	0.465	0.435	0.386	0.281	0.137	0.073
1964	0.442	0.418	0.336	0.272	0.179	0.070
1965	0.426	0.371	0.308	0.282	0.192	0.065
1966	0.336	0.331	0.303	0.206	0.171	0.134
1967	0.370	0.343	0.301	0.240	0.169	0.093
1968	0.405	0.348	0.295	0.224	0.139	0.011
1969	0.344	0.300	0.245	0.130	−0.011	−0.141
1970	0.387	0.370	0.333	0.135	−0.200	−0.193
1971	0.405	0.406	0.379	0.168	−0.151	−0.251
1972	0.435	0.431	0.354	0.260	−0.148	−0.137
1973	0.453	0.359	0.350	0.294	0.187	0.125
1974	0.424	0.352	0.349	0.213	0.251	0.157
1975	0.408	0.415	0.329	0.136	0.120	0.060
1976	0.475	0.456	0.394	0.278	0.118	0.040
1977	0.507	0.471	0.402	0.364	0.175	0.120
1978	0.493	0.452	0.408	0.373	0.235	0.164
1979	0.497	0.436	0.400	0.400	0.242	0.170
1980	0.485	0.450	0.417	0.344	0.092	−0.038
1981	0.463	0.416	0.356	0.207	−0.068	−0.463
1982	0.424	0.395	0.275	0.102	−0.181	−0.574
1983	0.445	0.424	0.323	0.142	0.003	0.050
1984	0.440	0.437	0.339	0.167	−0.005	0.058

[a] Source of yield data: Salomon Brothers Inc. [13]. Calculations were based on expression (7) using the monthly Salomon data. The monthly forward tax rates were then averaged for ease of presentation.

D. Controls for Callability and Risk

Although Treasury bonds are not normally callable except at long maturities, tax-exempt issues of maturities longer than fifteen years are normally callable, with ten years to first call apparently the modal value for the period of call protection (cf. Cook [5]). This difference in the callability of Treasury and tax-exempt issues would tend to narrow the apparent yield spread between the two at the longer maturities in the Salomon data discussed above. This narrowing of the spread because of differences in callability could in turn produce a spurious measure of implicit tax rates. Thus, it is important to eliminate differences in callability in our comparisons.

Risk differences between the two classes of bonds could also have the effect of narrowing the spread at the higher maturities, particularly if the riskiness of municipal debt over long horizons is greater than the riskiness over short horizons. Yawitz, Maloney, and Ederington [16] measure a substantial default risk for tax-exempt bonds, but their measure of the default probability appears to be much higher than would be warranted by past experience in this market. Poterba cites data on the low historical rates of default and argues that the default probability, at least for general-obligation bonds, is sufficiently small to make comparisons with Treasury issues reasonable. We shall continue to treat the difference in default risk between the Treasuries and the general-obligation bonds as negligible.[11]

In an attempt to eliminate the effect of differences in callability and in default risk in generating the negative forward tax rates shown in Table II, we have sought data for some specific tax-exempt bonds that are both noncallable and essentially free of default risk. Using data from *The Wall Street Journal* tombstone advertisements during the periods showing negative forward tax rates, we were able to find a number of tax-exempt bond issues that involve very high-grade general-obligation issues of states and where the bonds are not callable. In addition, the serial nature of the issue provides enough detail on the term structure to permit the calculations outlined above. The bonds used for the comparisons include State of California bonds dated September 1, 1971 (maturities one to twenty years) and March 1, 1982 (maturities one to fifteen years), State of Maine bonds dated October 15, 1971 (maturities one to twenty years) and March 15, 1982 (maturities one to twenty years), and State of New Jersey bonds dated April 1, 1971 (maturities five to thirty years) and May 1, 1982 (maturities two to twenty-one years).[12]

Table III shows the results of the forward tax calculations based on a comparison of Treasury yields and the yields on selected noncallable general-obligation municipal bonds.[13] These bonds also show some unusually low values for the forward tax rates in the five- to ten-year range and negative values for the forward tax rate at maturities above ten years.

II. Tax Arbitrage in Municipal Markets

It appears that municipal bond markets with the characteristics shown above provide the opportunity for tax arbitrage. The basis for the tax arbitrage using

[11] Trzcinka [15] uses comparisons between the yields on corporate bonds and municipal bonds in his test of the Miller hypothesis. Unfortunately, the Salomon corporate data do not provide an accurate enough maturity match between the corporate and municipal yields to support the detailed forward rate calculations shown above. There are two other problems that limit the usefulness of the corporate-municipal yield comparison for dealing with riskiness. The comparison involves two unknown risks so that the standardization is not really achieved. In addition, there is a difference in the state tax treatment of the two kinds of bonds, a difference that does not enter into the comparison between Treasuries and municipals.

[12] The advertisements for these noncallable, general-obligation municipal bonds were found in *The Wall Street Journal* for the following dates: September 11, 1971, March 11, 1982, March 11, 1971, April 6, 1982, October 6, 1971, and March 4, 1982.

[13] The municipal data giving maturity date, coupon rate, and yield to maturity were used to compute the bond prices. Next the data were used to obtain a term structure and the yield to maturity of par bonds with maturities of one, two, five, ten (and, when possible), twenty, and thirty years. These tax-free yields were then used together with the Salomon Treasury yield data in expression (7) to obtain the forward tax rates. For the two bonds with midmonth issue dates, we used the average of the Treasury yields for the beginning and end of the month.

Table III

Forward Tax Rates Based on a Comparison of Treasury Yields and Yields on Noncallable General-Obligation Municipals[a]

	Maturities (in Years)					
State/Date	0–1	1–2	2–5	5–10	10–20	20–30
CA 9/71	0.461	0.461	0.409	0.232	−0.178	n/a
ME 10/71	0.432	0.414	0.435	0.224	−0.142	n/a
NJ 4/71	n/a	n/a	n/a	0.111	−0.101	−0.194
CA 3/82	0.475	0.411	0.259	0.136	n/a	n/a
ME 3/82	0.458	0.395	0.286	0.138	−0.209	n/a
NJ 5/82	n/a	n/a	0.202	0.055	−0.374	n/a

[a] Source of Treasury bond yield data: Salomon Brothers Inc. [13]. Sources of municipal bond yield data: advertisements from *The Wall Street Journal*, various issues—Maine, October 6, 1971 and March 4, 1982; California, September 10, 1971 and March 11, 1982; New Jersey, March 11, 1971 and April 6, 1982. Calculations were based on expression (7). The Salomon Treasury yields were interpolated to match the date of the municipal bonds that were issued midmonth.

forward sales is outlined. Then in the Appendix, we develop an example, and we show the size of the potential gains given the yields on taxable and tax-exempt bonds available during the recent period. Finally, we show that the newly developed Chicago Board of Trade Municipal Bond Futures Contract offers some additional arbitrage possibilities.[14]

A. Tax Arbitrage Using Forward Sales

Suppose that taxable and tax-exempt coupon bonds are issued in only two maturities, short term and long term. Further, assume that there are two classes of participants, with preferred habitats at short and long maturities, respectively. Assume that par bonds are issues in the taxable market with yields to maturity y_1 and y_2 for bonds of the short and long maturities. Similarly, par bonds are issued in the tax-exempt market with yields to maturity of y_{m1} and y_{m2}. In keeping with the evidence given in Section I above, assume that the implied tax rate for the short-term bonds, $\theta_1^I = 1 - y_{m1}/y_1$, exceeds that for the long-term bonds, $\theta_2^I = 1 - y_{m2}/y_2$.

Given the yield structure in the taxable and tax-exempt markets, there is a yield incentive for holders of short-term tax-exempt bonds to hold long-term bonds, but this incentive is presumably offset by the strength of their preference for short-maturity bonds. Holders of long-term tax-exempt bonds do not have any yield incentive to hold short-term bonds. In discussing arbitrage, we thus take seriously the maturity preference of the bondholders.

Suppose now that some of the short- and long-maturity tax-exempt bondholders (Short and Long) get together and agree to the following exchange. Short

[14] In this section, we discuss arbitrages available to bondholders. Over the last few years, borrowers have developed devices that offer the possibility of arbitrage on the supply side. Put bonds, for example, offer the issuer the same refunding guarantees as long bonds, while they are the equivalent of short-term debt for the holders. There is something to the notion that municipal finance officers are conservative. Private-purpose bond issuers were the dominant supplier of these instruments.

buys a long-term tax-exempt bond but sells it forward one year to Long. Long buys a short-term taxable bond that provides security for the forward purchase. This contract effectively leaves Short with a (higher yield) short-term bond and leaves Long with a long-term bond.

The price at which the forward transaction takes place is crucial for the *division* of any potential gains from the exchange, but for the moment let us assume that the price is set at par (one dollar). Short initially ($t = 0$) purchases a long-term tax-exempt bond at par (one dollar) and then, after one year ($t = 1$), receives its tax-free coupon payment of y_{m2} plus the one dollar proceeds from the forward sale. Long initially purchases a short-term taxable bond at par (one dollar) and received $(1 - \theta_2^I)y_1$ interest plus one dollar return of principal after tax at time one. The one dollar is used to pay for the forward purchase of the remainder of the long-term tax-exempt bond. At the end of two years, Long receives $y_2^m + 1$ after tax.

Under what conditions is there a *joint* profit from this exchange? After one year, Short clearly gains $y_{m2} - y_{m1}$, while Long "gains" $(1 - \theta_2^I)y_1 - y_{m2}$. All of the other cash flows duplicate those that would occur if the exchange had not taken place. The joint gain (after one year) is simply the sum of the gains to Short and to Long:

$$G_J = y_{m2} - y_{m1} + (1 - \theta_2^I)y_1 - y_{m2} = -(1 - \theta_1^I)y_1 + (1 - \theta_2^I)y_1. \tag{8}$$

The joint gain, G_J will be positive as long as $\theta_1^I > \theta_2^I$ as assumed above. Thus, there are gains from trade between Short and Long as long as the implied tax rates for the short- and long-term bonds differ.

Given that the tax-exempt yield curve is rising (which appears always to be the case), Short clearly gains from the exchange because $G_S = y_{m2} - y_{m1} > 0$. With the forward price set at par, however, Long's gain,

$$G_L = (1 - \theta_2^I)y_1 - y_{m2} = y_{m2}((y_1/y_2) - 1), \tag{9}$$

will be positive only if the taxable yield curve is inverted with $y_1 > y_2$. Although the yield data show this pattern is rising tax-exempt yields with inverted taxable yields during some periods, these conditions are necessary for gains to both parties only if the forward sale is at par. We have shown that there is a *joint* gain whenever $\theta_1^I > \theta_2^I$; therefore, it will always be possible to adjust the forward transaction price to give positive gains to both parties. These transactions are illustrated in the Appendix using data from the March 1982 Maine bonds.

B. Arbitrages Using the Municipal Bond Futures Contract

The arbitrages discussed in the previous sections do not explicitly consider transaction costs, including the costs of searching for a trading partner. These costs could be substantial and could explain the absence of the forward transactions described above, at least when interest rates were low or moderate.

The recent development of new financial instruments including the Chicago Board of Trade's Municipal Bond Futures Contract, which began trading in June 1985, makes it possible to take advantage of the arbitrage possibilities at much lower cost. Using the Municipal Bond Futures Contract, the participants do not have to be concerned about the identity of the person or institution on the other side of the contract. Explicit transaction costs are low. We show below that, since the inception of the futures contract, there has frequently been a profitable position available either to the short-maturity municipal bond buyer or to the long-maturity bond buyer, and sometimes to both.[15]

The Municipal Bond Futures Contract at the Chicago Board of Trade is based on the Bond Buyer Municipal Bond Index. The index contains forty (relatively) frequently traded, recently issued, investment-grade, long-term, callable, revenue, and general-obligation municipal bonds. The composition of the index is changed twice a month, with older bonds and bonds with small trading volumes being replaced by newer bonds. The bonds in the index are evaluated once each day (twice a day during the settlement month) using the average of the middle three prices obtained from five dealer-to-dealer brokers. The futures contract is settled in cash against the index.[16]

B.1. Profitability Conditions for a Short-Term Municipal Bond Buyer

Consider a high-tax-rate individual or institution such as a commercial bank that would normally be a buyer of short-term municipal bonds. We shall show that it has often been profitable to construct an arbitrage using the Municipal Bond Futures Contract in the following way. The investor purchases a *long* municipal bond portfolio that is equivalent to the portfolio underlying the Municipal Bond Index. The investor simultaneously sells the municipal futures contract short.

Let $P_f(t)$ be the price of the futures contract and $P_b(t)$ be the price of the bonds at time t. Let y_{mL} and y_{ms} represent the yield on long- and short-maturity municipal bonds, respectively, and let y_s be the yield on Treasury bills. Finally, in taking account of the taxation relevant for the transactions, let τ_s be the ordinary income tax rate for the short-term municipal investor. Short-term capital gains or losses on the bonds would be taxed at this rate. On the futures contract, forty percent of the gains or losses would be taxed at the ordinary income tax rate while the remaining sixty percent would be taxed at the capital gains rate, $0.4\tau_s$. Thus, the gain or loss on the futures contract would be taxed at $0.64\tau_s$.[17]

Finally, let $P_b(t + m)$ be the price of the long bonds at the expiration of the futures contract. Although this price is unknown at time t, the investor can

[15] The arbitrages discussed here are not zero-cash arbitrages but are arbitrages that provide a profit relative to the original investment position without altering the investment risk. The arbitrages based on the municipal bond futures contract also assume that the investor could construct a long-bond portfolio that essentially duplicates the behavior of the Bond Buyer Index portfolio. Unless this condition is met, there will remain an element of basis risk stemming from the differences between the portfolio underlying the Bond Buyer Index to which the Municipal Bond Futures Contract is tied and the investor's own bond portfolio.

[16] A more detailed description of the Municipal Bond Index Futures Contract is given in Chicago Board of Trade [14] and other CBT publications.

[17] Under some circumstances, the gains and losses associated with the futures contract could be considered trading profits and would be subject to tax as ordinary income. In can be shown that, while the details of the taxation determine the specific hedge ratio needed to make the arbitrage riskless, they do not affect the conditions for the existence of profitable opportunities.

eliminate the risk associated with this uncertainty by selecting the appropriate quantity of bonds relative to futures contracts, H. Suppose that the investor sells $1000P_f(t)$ worth of the futures contracts and buys $P_b(t) \times H$ worth of the bonds. The after-tax gain on the futures is $(1 - 0.64\tau_s)(P_f(t) - P_b(t + m)) \times 1000$. The after-tax gain on the bonds is $(1 - \tau_s)(P_b(t + m) - P_b(t)) \times H$. Setting the hedge ratio to $H = 1000(1 - 0.64\tau_s)/(1 - \tau_s)$ eliminates the bond price risk. The combined gain on the long bonds and the futures is then $1000 \times (1 - 0.64\tau_s)(P_f(t) - P_b(t))$. In addition, there would be interest income on the bonds equal to $m \times y_{mL} \times H \times P_b(t)$.

The arbitrage position is profitable if the after-tax income from the position exceeds the income from an equal investment in short-term municipal bonds. In yield terms, the condition for the arbitrage to be profitable is thus

$$(P_ft) - P_b(t))(1 - \tau_s)/P_b(t)m + y_{mL} - y_{ms} > 0. \tag{10}$$

If we consider a marginal investor for whom $y_{ms} = (1 - \tau_s)y_s$, then we can write the profit condition as

$$(P_f(t) - P_b(t))(1 - \tau_s)/P_b(t)m + y_{mL} - (1 - \tau_s)y_s > 0. \tag{11}$$

B.2. Condition for Profitability for a Long-Maturity Municipal Bond Buyer

The arbitrage for the long-maturity municipal bond buyer involves the postponement of the municipal bond purchase, with the originally intended investment used instead to purchase Treasury bills plus futures contracts. Instead of purchasing $H'P_b(t)$ worth of long-maturity municipal bonds, the investor invests the same amount in Treasury bills and simultaneously buys $1000P_f(t)$ futures contracts. At the expiration of the futures contract, the investor will actually acquire long bonds, and the saving (possibly negative) over the original acquisition price will be $H'(P_b(t) - P_b(t + m))$. The after-tax gain on the futures contract will be $1000(1 - 0.64\tau_L)(P_b(t + m) - P_f(t))$. When $H' = 1000(1 - 0.64\tau_L)$, the price risk associated with the bonds is eliminated, and the combined gain is $1000 \times (1 - 0.64\tau_s)(P_b(t) - P_f(t))$. In addition, there would be interest income on the Treasury bills in the after-tax amount, $my_s(1 - \tau_L)H'P_b(t)$. In comparison with the yield on the alternative strategy of holding the long municipal bonds, the condition for profitability is[18]

$$(P_b(t) - P_f(t))/mP_b(t) + (1 - \tau_L)y_s - y_{mL} > 0. \tag{12}$$

Essentially, conditions (11) and (12) indicate that, when the futures contract stands at a small discount relative to the spot index, there are profits available

[18] The analysis of investment strategies given in Chicago Board of Trade [4], pages 31–37, provides conditions for profitability that are identical to those given here for the short investor. The conditions given for the long investor differ from those given here because they assume that the holdings will be sold off after one year, whereas we consider a position that is to be maintained.

The Chicago Board of Trade's analysis is all done using a single tax rate. It does not exploit the fact that the implied tax rates for the marginal participant in the long and short municipal bond markets differ systematically. The marginal buyer of long municipal bonds would buy taxable Treasury bills and pay the tax rather than buy short-term municipals, given that the spread between Treasury bills and the short municipals is determined by investors with higher tax rates.

to the short-term investor and, when the discount is large, there are profits to long-term investors. There may also be intermediate conditions when profits are available to both.

B.3. Profits Using the Municipal Bond Index Futures Contract

Table IV shows the profit, ignoring transactions costs, to short- and long-term investors using the Municipal Bond Futures Contract from mid-1985, when trading began, through the end of 1986 on contracts expiring in 1986. The coverage is limited in this way because substantial tax code revisions affecting these assets were to take effect starting in 1987. The profit conditions (11) and (12) were used to calculate the profit, not including transaction costs, available to short- and long-term investors, respectively. The profit was scaled with a common investment of $100,000, and a tax rate of 0.45 was assumed for the short-term investor, 0.25 for the long-term investor.

When the futures contract began trading in June 1985, it was priced with large discounts that provided profit opportunities for long-term tax-exempt bond buyers. Starting in September 1985, there was a period of about two months with substantial profits to both long- and short-term investors. Starting in December 1985 and continuing until August 1986, prices were such that only short-term investors could use the futures contract profitably. The situation reversed again in September 1986, and, from then until the end of the year, there were some profits for long-term investors. The decline in the size of the profits at the end of 1986 is in part simply the result of the approaching expiration of the contract.

B.4. Transaction Costs Using Municipal Bond Futures

A long-term municipal bond buyer who temporarily substitutes the purchase of the Municipal Bond Index Futures Contract and Treasury bills for the actual purchase of long municipal bonds incurs no *additional* transactions costs for the municipal bonds. The transactions costs that would have been incurred for the purchase of the bonds are simply postponed. There are transaction costs for the futures contract and Treasury bills, but these costs are low. Through a discount broker, the investor would pay $27 in brokerage fees on the futures contract plus a bid-ask margin of $1/16$ or about $62.50. In addition, there would be a cost of about $34 to buy the Treasury bills. With $2000 of cash margin and using the Treasury bills as the additional margin, the foregone interest would be about $125 for nine months. Thus, the total additional transaction costs are about $248.50.

Based on the results in Table IV, an investor with a twenty-five percent tax rate who bought the March 1986 futures contract plus Treasury bills on July 26, 1985 would have had a gain before transaction costs of $5010 and a gain after transaction costs of $4762 when compared with simply holding long-term municipal bonds.

A short-term investor who wanted to construct a synthetic short-term position by combining a short position in the Municipal Bond Index Futures Contract together with a long position in municipal bonds would face much larger transaction costs and other technical difficulties as well.

Table IV

Profit Opportunities Using the Municipal Bond Futures Contract[a]

Date	Short	Long
March 1986 Contract		
6/21/85	85	4136
6/28/85	268	3730
7/05/85	626	2992
7/12/85	−62	4050
7/19/85	−385	4480
7/26/85	−704	5010
8/02/85	−153	3905
8/09/85	14	3524
8/16/85	835	1988
8/23/85	1132	1341
8/30/85	793	1801
9/06/85	125	2934
9/13/85	610	2017
June 1986 Contract		
9/20/85	1087	2590
9/26/85	1128	2404
September 1986 Contract		
10/04/85	1928	2299
10/11/85	1613	2651
10/18/85	2147	1524
10/25/85	2400	939
11/01/85	2666	255
11/08/85	2592	336
11/15/85	2417	421
11/22/85	2069	806
11/29/85	1994	884
12/06/85	2016	719
12/13/85	2518	−349
12/20/85	2972	−1354
12/27/85	2770	−1121
1/03/86	2641	−1113
1/10/86	4001	−3665
1/17/86	1770	226
1/24/86	1214	1089
1/31/86	997	1310
2/07/86	706	1778
2/14/86	1040	907
2/21/86	1482	−17
2/28/86	1766	−713
3/07/86	1714	−625
3/14/86	1538	−427
3/21/86	2117	−1497
3/27/86	2448	−2263
4/04/86	1796	−1071
4/11/86	1784	−1057
4/18/86	2830	−3136
4/25/86	1599	−955
5/02/86	2199	−2113
5/09/86	1666	−1255
5/16/86	1380	−750

Table IV—*Continued*

Profit Opportunities Using the Municipal Bond Futures Contract[a]

Date	Short	Long
September 1986 Contract		
5/23/86	1585	−1201
5/30/86	596	508
6/06/86	1155	−566
6/13/86	1246	−804
6/20/86	1046	−600
6/27/86	1068	−783
7/03/86	1325	−1362
7/11/86	751	−381
December 1986 Contract		
7/18/86	1502	−562
7/25/86	1163	6
8/01/86	1140	−97
8/08/86	1625	−1013
8/15/86	1368	−779
8/22/86	833	91
8/29/86	804	12
9/05/86	592	353
9/12/86	229	948
9/19/86	−260	1742
9/26/86	149	884
10/03/86	390	355
10/10/86	53	864
10/17/86	−189	1215
10/24/86	−151	1048
10/31/86	−18	700
11/07/86	947	−1143
11/14/86	−63	595
11/21/86	−117	596
11/28/86	−157	585
12/05/86	−91	377
12/12/86	17	96
12/19/86	65	−82

[a] On each trading day, the longest available 1986 contract was used. Profit conditions (10) and (11) were scaled to represent the profit on a $100,000 investment, ignoring transaction cost. The profit calculations assume tax rates of 0.45 for the short-term investor and 0.25 for the long-term investor.

The costs would certainly be prohibitive for a small-scale investor, and they would be large even for an investor with a $1 million portfolio of short-term assets. Bonds of the type included in the Municipal Bond Futures Contract have bid-ask spreads on small lots of two points; therefore, the transaction costs of the bid-ask spread alone would total about $2000 per $100,000. In addition, the Bond Buyer Index is frequently turned over. For example, in a typical three-month period, twenty-four of the forty bonds initially in the Index were replaced. The additional transaction costs associated with maintaining a match with the Bond Buyer list would have totaled another $1200 per $100,000. Thus, the total transaction costs to a retail, but fairly large, short-term investor of trying to match the Index would be $3200 per $100,000. (With a portfolio as large as $20 million, an investor could maintain a nearly matched portfolio while incurring only the wholesale bid-ask spread.[19])

Although the cost of completely eliminating the basis risk is extremely high, an investor willing to bear a small amount of basis risk could construct a portfolio with a performance that closely matched the Index at a more modest cost. The individual bonds in the Index are far more similar in their variations than are the equities of different companies. During a recent three-month period, the standard deviation of the price changes for the sixteen bonds that remained on the Index was 1.06 percent, or 2.12 percent annualized. This suggests that a four-bond portfolio would have a standard deviation (of the basis risk) of only 0.53 percent over a three-month period, or 1.06 percent over a year.

An investor with a $2 million portfolio who was willing to use the four-bond proxy for the Index would face more reasonable transaction costs. If he or she purchased $500,000 worth of each of the four issues, he or she would pay a wholesale bid-ask spread of ¼ of a point. Assuming that, during the course of a six-month holding period, the investor would have had to turn over the entire portfolio once in order to maintain a reasonable match with the Index, his or her transaction costs in the cash bond market would be $500 per $100,000. In addition, the transaction costs for the futures contract would again be about $214.50. On March 27, 1986, using the September 1986 bond contract, the expected gross profit would have been $2448 per contract and then expected net profit would have been $1525.50 per contract. This would represent an annualized return of about three percent more than would have been available on short-term tax-free assets.

For the period that we have examined, a long-term municipal investor would have found it profitable even on a single contract to substitute a long position in the futures market for actual purchase of the bonds roughly half the time. Transaction costs for this strategy are quite low. Small short-term municipal investors, using a short position in the futures market together with a long position in long bonds, would have faced prohibitive transaction costs, but a high-tax-rate, large investor, willing to tolerate modest basis risk, would have had higher returns than on short-term municipals over half the time.

III. Conclusions

In this paper, we define a forward tax rate using the relationship between the yields on taxable and tax-exempt bonds of various maturities. If the bond markets are fully arbitraged, then these forward tax rates can be interpreted as expected future tax rates or as expected future differential tax rates. Our empirical evidence, based first on a comparison between prime general-obligation munici-

[19] The fact that the Index is linked and based on an assumed standardized coupon of eight percent, together with the fact that newly issued municipal bonds are available only in units of $5000 or more, means that an *exact* match to the Index would require a portfolio of *more* than $20 million.

Table V

Interest Rates and Prices of State of Maine Bonds Issued March 15, 1982[a]

Date Due	Maturity	Coupon	Price	Yield	Forward Rate	Par Yield	Treasury Yield	Tax Rate
1983	1	11.25	103.31	7.75	7.75	7.75	14.29	45.77
1984	2	11.25	105.43	8.25	8.81	8.26	14.43	42.76
1985[b]	3	11.25	106.61	8.70	9.76	8.71	—	—
1986[b]	4	11.25	107.42	9.00	10.12	9.02	—	—
1987[b]	5	11.25	108.07	9.20	10.25	9.22	14.21	35.12
1988	6	11.25	108.34	9.40	10.79	9.42	—	—
1989	7	11.25	108.27	9.60	11.38	9.62	—	—
1990	8	11.25	107.91	9.60	12.02	9.82	—	—
1991	9	11.25	107.31	10.00	12.72	10.02	—	—
1992	10	10.25	100.00	10.25	13.93	10.25	14.08	27.20
1993	11	10.25	99.03	10.40	13.29	10.40	—	—
1994	12	10.50	99.33	10.60	15.43	10.60	—	—
1995	13	10.50	97.93	10.60	16.42	10.79	—	—
1996	14	10.50	96.47	11.00	17.80	10.98	—	—
1997[b]	15	9.25	85.98	11.20	16.79	11.11	14.00	20.64
1998[b]	16	9.25	84.34	11.40	20.94	11.29	—	—
1999[b]	17	9.25	83.36	11.50	17.58	11.39	—	—
2000[b]	18	9.25	82.40	11.60	18.93	11.48	—	—
2001[b]	19	9.25	81.47	11.70	20.66	11.57	—	—
2002[b]	20	9.25	81.21	11.70	12.93	11.59	13.96	16.95

[a] Coupon rates and yields were taken from *The Wall Street Journal* advertisement, March 4, 1982. The price is computed to be consistent with the published yields. Forward rates are computed assuming a flat yield curve between maturities. Par yields are the yields to maturity on comparable bonds issued at par with a forward rate structure consistent with the set of forward rates computed for the actual Maine bonds. Par Treasury yields are the average of the March 1 and April 1, 1982 yields given in Salomon Brothers Inc. [13].

[b] The ad indicates, "These maturities are not available from the syndicate. Securities of these maturities may or may not be available from syndicate members or others at the yield indicated." We do not use these bonds in constructing the examples.

pals and Treasuries, shows that the forward tax rates are generally very low and sometimes negative. These results are confirmed when a comparison is made using very high-grade municipals that are also noncallable. The finding of negative forward tax rates is clearly inconsistent with the interpretation of forward tax rates as expected future tax rates or as expected differential forwards. We interpret these results as strong evidence that these markets are segmented by maturity. Since the segmentation does not appear to be the result of legal limitation, it should signal the availability of profitable riskless transactions.

We show that, during much of the recent past, there have been a number of profitable arbitrages available to investors that apparently have not been fully used. These arbitrages have the effect of shifting tax-exempt interest from investors with low marginal tax rates to those with higher rates. In general, tax arbitrages of the sort considered here are likely to exist whenever there are differences in marginal tax rates among investors.[20] The cost of affecting these

[20] Miller and Scholes [9] discuss a similar issue in the context of corporate dividends. The problem is treated more generally by Auerbach and King [2] and by Schaefer [14].

Table VI

Cash Flows with Spot and Forward Transactions

A. Cash Flows with Spot Purchases			
Date:	3/15/82	9/15/82	3/15/83
Long-Maturity Bond Buyer:			
Purchase Bond	−9647.00		
Coupon Interest		525.00	525.00
Short-Maturity Bond Buyer:			
Purchase Bond	−10,331.00		
Coupon Interest		562.50	562.50
Redeem Bond			10,000.00
Joint Cash Flow:	−19,978.00	1087.50	11,087.50
B. Cash Flows with Forward Transactions			
Date:	3/15/82	9/15/82	3/15/83
Long-Maturity Bond Buyer	(Tax Rate 0.25):		
Purchase T-bill or CD	−9122.00		
Principal Plus Interest			10,472.10
Tax			−337.53
Purchase M-bond Forward			−9474.45
Net Cash Flow Long:	−9122.00		660.12
Short-Maturity Bond Buyer	(Tax Rate 0.46):		
Purchase M-bond	−9647.00		
Purchase T-bill or CD	−646.50		
Interest on M-bond		525.00	525.00
Redeem T-bill Including Interest			742.19
Tax on Interest			−44.02
Sell M-bond Forward			9474.45
Net Cash Flow Short:	−10,293.50	525.00	10,697.62
Joint Cash Flow:	−19,415.50	525.00	11,359.74

arbitrages has been lowered by developments such as the Chicago Board of Trade's Municipal Bond Index Futures Contract. We show for the period from mid-1985 through the end of 1986 the size of the potential gains to investors using the Municipal Bond Futures Contract.

Appendix: A Forward Sale Tax-Arbitrage Example

A tombstone advertisement from *The Wall Street Journal* of March 4, 1982 gives data for a series of noncallable AAA-rated general-obligation bonds issued by the State of Maine on March 15, 1982. Table V gives the published coupon rates and yields taken from the advertisement together with the corresponding prices, the forward rates implied by the prices and yields, and the calculated yields for a set of hypothetical par bonds having the same forward rate structure. The yields for a set of par Treasury bonds as of March 15, 1982 are estimated as the average of the Salomon March and April 1982 yields. The implicit marginal tax rates at which it would just pay to hold the municipal bonds instead of Treasuries appear as the last column of Table V.

An individual investor, Long, whose marginal tax rate is twenty-five percent, initially would have purchased the fourteen-year 1996 Maine bond. A commercial bank, Short, whose marginal tax rate is forty-six percent, initially would have purchased the one-year 1983 Maine bond. The cash flows during the first year for their normal purchases are shown in Table VI, Panel A.

The forward transaction involves Long's purchase of a short-term Treasury bill or certificate of deposit and the purchase of the long municipal bond forward one year. The bank, Short, buys the long municipal and sells it forward one year. The amount of the short-term investment by Long is adjusted for the absence of a coupon payment at six months. Short's initial investment is also increased (through purchase of a Treasury bill or CD) so that the amount of the original outlay adjusted for the difference between the coupon payments at six months does not exceed that which was originally made. The cash flows for these transactions are shown in Table VI, Panel B, where the price of the forward transactions is set in such a way that the gains are equally divided, with each participant gaining 135.12 at the end of one year.[21]

[21] We have neglected the fact that the bank has generated a loss on the purchase and forward sale of the municipal bond. If the bank is at risk, this loss is presumably deductible as a trading loss at ordinary rates. The individual has a capital gain that is realized at maturity. The bank's early tax saving will dominate the investor's delayed tax on gain, thus increasing the joint gain.

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