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Efficient Signalling with Dividends, Investment, and Stock Repurchases

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ABSTRACT

The efficient mix of dissipative dividends, investments in real and financial assets, and repurchases of stock is computed for a continuum of firms with inside information about the return on risky real assets. In the efficient signalling equilibrium, the representative firm optimally distributes dividends, invests in risky real assets to maximize net present value, holds no financial securities, and sells new stock in the market. This firm finances its value-maximizing investment first from internal funds and second from stock sold to new investors.

RECENT RESEARCH ON DIVIDENDS has included single-period signalling models with dissipative costs like adverse taxes on dividends relative to capital gains and deviations from value-maximizing corporate investment.¹ These models have provided partial answers to the following questions. Why should firms distribute dividends rather than repurchase shares? Why do firms sometimes distribute dividends and simultaneously sell new shares in the market? Under what conditions do firms signal with dividends rather than foregone investment? More generally, what properties characterize the efficient mix of corporate signals, including dividends and investment or, equivalently, dividends and repurchases of stock?²

Despite these recent results, many interesting questions are unanswered. For example, in an efficient signalling equilibrium, do firms finance their real investments first from internal funds and second from new securities sold to outside investors? Although this property is familiar from pooling equilibria, the latter equilibria are inconsistent with observed effects on stock prices from announced earnings, dividends, investments, and new issues.³ Can the insiders' maximand in signalling models be derived from the portfolio problem facing their firm's representative stockholder? This is important if properties of signalling equilibria are dependent on the maximand. Finally, can models of multiple corporate signals be generalized from two types of firms to a more realistic model with a continuum of firms possessing different private attributes?

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¹ Single-period models of dissipative signalling with either dividends or investment include Bhattacharya [3], Miller and Rock [13], and John and Williams [9]. These models are based on Riley [16].

² Single-period models with dividends, investment, and stock repurchases include Ambarish, John, and Williams [2]; Offer and Thakor [15]; and Vishwanathan [17]. Also, see Milgrom and Roberts [12].

³ In Myers and Majluf [14], firms finance investment first from internal funds.

In this paper an efficient signalling equilibrium is derived for a continuum of firms with private information about the return on risky real assets. These firms can distribute dissipative dividends, invest in both real and financial assets, and sell or repurchase stock in the capital market. In equilibrium each firm invests to maximize the net present value of its risky real assets. To finance its optimal investment, the firm sells stock in the market. In turn, to support its sale of stock, the firm distributes a dissipative dividend. This dividend is increasing in the value of insiders' information. Also, the firm optimally retains no financial securities. In this sense, firms finance real investments first from internal funds and second from stock sold externally in the capital market.

The model is specified in Section I. As part of the specification, the insiders' maximand is derived from stockholders' preferences about corporate actions. The result is similar to the maximand in Miller and Rock [13]. Properties of the efficient equilibrium are identified in Section II and summarized in Section III. All derivations appear in the Appendix.

I. Model

A. Preliminaries

The representative firm is operated for one period and then liquidated. This firm is financed solely with stock and internal cash. The internal cash C , possibly negative, equals the initial contribution of capital by the firm's founders minus any start-up costs. To its stockholders the firm distributes the total dividend D . Simultaneously, it sells or repurchases in the capital market at the ex-dividend price P_e the net new shares Q , measured as a fraction of all outstanding shares. The aggregate dividend D must be nonnegative, whereas the fractional new issue Q can be negative with repurchases. For each stockholder, dividends produce personal costs at the constant rate γ . In practice, these costs represent adverse personal taxes on dividends relative to capital gains or losses. As a result, stockholders realize the total dissipative costs γD and receive the net cash inflow $(1 - \gamma)D$. To preclude arbitrage around the ex-dividend date, the aggregate cum-dividend price P must exceed the aggregate ex-dividend price P_e by the aggregate dividend net of dissipative costs $(1 - \gamma)D$:

$$P = P_e + (1 - \gamma)D. \quad (1)$$

These prices measure total market values, not values per share.

The firm's actions are constrained by its sources and uses of funds. The firm distributes the dividend D , invests I in risky real assets, and then sells (buys) the stock $P_e Q$. Next, it uses any residual capital to purchase financial securities. To prevent complications with risky debt, this residual capital is constrained to be nonnegative:

$$0 \leq C - D - I + P_e Q. \quad (2)$$

All investments in financial securities earn interest at the riskless rate. Solely for notational simplicity, this riskless rate is set equal to zero.

Corporate insiders have private information about their firm's return on risky real assets. Conditional on all public and private information, insiders identify the present value of their firm's risky real assets as $F(I)x$. Specifically, they calculate the future value of the firm's risky assets as $F(I)\epsilon$ and assess the distribution of the random variable ϵ . Next, they identify in the capital market a security or portfolio of securities with the random future value ϵ . This security has the current price x . In this sense, the statistic x summarizes insiders' private information about the return on the firm's aggregate investment I in its portfolio of risky projects. Also, the function F is continuously differentiable, strictly increasing, and strictly concave, with the corner conditions: $F(0) \geq 0$, $F'(0) = \infty$, and $F'(\infty) = 0$. As specified, the inside information x is multiplicative; the ratio of the average and marginal returns to investment is independent of x . Because outside investors cannot observe the private value x , they must rely upon insiders' report y of the private value x . In general, this report need not be truthful.

Outside investors can observe the firm's sources and uses of funds. Specifically, outsiders observe the corporate dividend D , investment I , and net sale of new stock $P_e Q$. Combining this public information with insiders' report y , outsiders then price the firm's stock in the competitive capital market, at $P = P(D, I, Q, y)$, measured cum-dividend. Initially, insiders anticipate this stock price when selecting their corporate dividend, investment, new stock, and report to outsiders. For notational convenience the constant C is excluded as an argument of the cum-dividend pricing function P .

B. Unanimity

By assumption, corporate insiders act to maximize the welfare of their current stockholders. To identify the insiders' maximand, consider a representative stockholder i who has the utility function U^i defined over current consumption and subsequent wealth. Solely for notational simplicity, suppose that stockholder i initially holds all his or her personal wealth in the representative firm's stock, owning a fraction n_i of all outstanding shares. The stockholder then sells (buys) a fraction a_i of his or her personal shares at the total cum-dividend price P and buys (sells short) b_i riskless bonds maturing in one period. In addition, this investor buys (sells short) z_i shares of a risky security with the current price x and random future value ϵ . In this case, stockholder i spends on current consumption the personal cash $c_i \equiv n_i[(1 - a_i)(1 - \gamma)D + a_i P] - b_i - xz_i$ and realizes subsequently the personal wealth $w_i \equiv n_i(1 - a_i)[C - D + P_e Q + F(I)\epsilon - I]/(1 + Q) + b_i + \epsilon z_i$. In the latter definition, $C - D + P_e Q - I$ and $F(I)\epsilon$ represent respectively the future values of the firm's financial and real assets.

Suppose that the firm's insiders act to maximize the welfare of their representative stockholder i . In this case, insiders choose the corporate controls D , I , and Q , and then advise stockholder i to select the personal variables a_i , b_i , and z_i that solve the portfolio problem:

$$\max_{D, I, Q, a_i, b_i, z_i} E[U^i(c_i, w_i)]. \quad (3)$$

This expectation is computed using the insiders' distribution for the uncertain future value ϵ . By assumption, investor i is sufficiently small so that the price P is independent of his or her personal actions a_i , b_i , and z_i . If the pricing function P is weakly concave in D , I , and Q , then problem (3) has a unique solution, completely characterized by its first-order conditions.⁴ Denote this optimal solution by D_i^* , I_i^* , Q_i^* , a_i^* , b_i^* , and z_i^* .

The solution to this portfolio problem also solves a simpler problem. Specifically, the optimal dividend D_i^* , investment I_i^* , and new issue Q_i^* maximize the present value:

$$\max_{D, I, Q} \{ (1 - a_i^*)(1 - \gamma)D + a_i^*P + \frac{1 - a_i^*}{1 + Q} [C - D + P_e Q + F(I)x - I] \}, \quad (4)$$

computed conditional on the i th stockholder's optimal net sales of shares a_i^* . This equivalence follows immediately from the first-order conditions for (3) and (4). Finally, suppose that the firm has a single clientele of stockholders, each of whom optimally sells the fraction of shares: $a_i^* = \alpha$. In this case, insiders maximize the present value in (4) with $a_i^* = \alpha$, and then recommend to outsiders the optimal adjustments in their personal portfolios from (3). Alternatively, in a signalling equilibrium insiders convey their private information x through their solution to (4), so that each stockholder i can then complete the solution of (3). In either case, stockholders unanimously support the maximization of present value in (4).

C. Equilibrium

Events occur in the following sequence. First, insiders observe their corporate cash C and acquire the private information x about the return on risky real assets. Next, insiders report to outsiders some feasible value y not necessarily equal to the true value x . To be credible, this report y must fall within a region $R(x)$ containing an open interval surrounding the true value x . In other words, outsiders detect false reports $y \neq x$ with probability 0 whenever $y \in R(x)$ and with probability 1 otherwise. Next, insiders support their report with the dividend D , investment I , and fractional new shares Q . The residual capital then satisfies the sources and uses of funds (2). If outsiders are to believe the report y , then the three signals must be optimal for firms with private value y . Denote the optimal dividend, investment, and new issue by $D(y)$, $I(y)$, and $Q(y)$. Conditional on these credible signals, outsiders then pay for the firm's stock the total cum-dividend price $P[D(y), I(y), Q(y), y]$.

In a signalling equilibrium, insiders in the representative firm solve the following problem. Given the pricing function P , they report truthfully $y = x$ and then support their report with the credible signals $D(x)$, $I(x)$, and $Q(x)$ if and only if

$$x = \arg \max_{y \in R(x)} V[D(y), I(y), Q(y), x, y], \quad (5)$$

⁴ Problem (3) has a unique solution under the conditions specified subsequently.

where

$$[D(y), I(y), Q(y)] \equiv \arg \max_{D, I, Q} V(D, I, Q, y, y), \quad (6)$$

In (5) and (6), the present-value function V is

$$V(D, I, Q, x, y) \equiv (1 - \alpha)(1 - \gamma)D + \alpha P(D, I, Q, y) \\ + \frac{1 - \alpha}{1 + Q} \{C - D + [P(D, I, Q, y) - (1 - \gamma)D]Q + F(I)x - I\}. \quad (7)$$

In (5) and (6) insiders maximize their representative stockholder's present value, given a pricing function P . Also, in (6) any credible lie $y \neq x$ must be supported by signals that are optimal for firms with the private value y . In (7) this present value is computed exactly as in (4).

The capital market is perfectly competitive. Given any credible report y , supported by the optimal signals $D(y)$, $I(y)$, and $Q(y)$, outside investors must pay for the firm's stock a total price, measured cum-dividend, equal to its current value:

$$P[D(y), I(y), Q(y), y] = V[D(y), I(y), Q(y), y, y]. \quad (8)$$

Collectively, (5) through (8) define a signalling equilibrium.

Many signalling equilibria may exist. Among all such equilibria, an efficient (Pareto-optimal) equilibrium has a nonnegative pricing function P that maximizes a weighted average of present values:

$$\max_{P \geq 0} \int_{\bar{x}}^{\bar{x}} V[D(x), I(x), Q(x), x, x] dG(x). \quad (9)$$

The distribution G has the support $[\bar{x}, \bar{x}]$ satisfying $0 < \bar{x} < \bar{x} < \infty$. As shown subsequently, the efficient signalling equilibrium is unique, given an additional constraint on the parameters.

II. Solution

The problem specified in the previous section is solved in the Appendix. There, (5) through (10) is restated as a standard control problem, in part by rewriting the truth-telling condition in a technically more convenient form. One constraint in the standard problem is omitted; the optimality conditions for the resulting, relaxed problem are identified; and the solution is then derived. Whenever the parameters satisfy two inequalities, the solution also satisfies the omitted constraint. These inequalities are

$$\gamma > 1 - \alpha, \quad C \leq I(\bar{x}) + \frac{\alpha F[I(\bar{x})]\bar{x}}{1 - \alpha - \gamma}. \quad (10)$$

The first inequality, $\gamma > 1 - \alpha$, bounds below the proportional, dissipative costs of dividends. If the second inequality holds, then each firm must raise external capital to finance its optimal investment.

Given (10), the unique solution to problem (5) through (9) can be characterized as follows. The optimal investment I^* maximizes net present value:

$$I^*(x) = \arg \max_{I \geq 0} \{F(I)x - I\}, \quad (11)$$

for all $\underline{x} \leq x \leq \bar{x}$. Given this optimal investment I^* , the optimal dividend D^* satisfies

$$D^*(x) = \frac{1}{\gamma} x^{(1-\alpha)/\gamma} \int_{\underline{x}}^x \{\alpha F[I^*(z)]z + (1-\alpha)[I^*(z) - C]\} z^{-1-(1-\alpha)/\gamma} dz, \quad (12)$$

for all $\underline{x} \leq x \leq \bar{x}$. To finance this dividend and investment, the firm optimally sells the new stock $S^* = P_e Q^*$ satisfying

$$S^*(x) = D^*(x) + I^*(x) - C, \quad (13)$$

for all $\underline{x} \leq x \leq \bar{x}$.

Insiders invest in (11) to maximize their firm's true value, computed conditional on their private information. This value-maximizing or first-best investment I^* is uniquely characterized by the first-order condition: $F'[I^*(x)]x = 1$. Thus, it is increasing in the private value x : $I^{*'} = -F'(I^*)/F''(I^*)x > 0$. As in Ambarish, John, and Williams [2], the optimal investment is value-maximizing because the present value of risky real assets is multiplicatively separable in the investment I and private value x : $F(I)x$. With this technology, firms do not forego investments with positive net present value, as in Miller and Rock [13]. That is, insiders do not signal with investment, even though (7) matches Miller and Rock's maximand. Instead, they signal solely with dividends and thereby minimize the dissipative costs incurred in the efficient equilibrium by both their firm and its stockholders.

Almost all firms distribute dividends in (12). Only firms with the most unfavorable inside information $x = \underline{x}$ optimally distribute no dividend: $D^*(\underline{x}) = 0$. In addition, the optimal dividend D^* is increasing in the value of insiders' information x : $D^{*'} = [\alpha F(I^*)x + (1-\alpha)(D^* + I^* - C)]/\gamma > 0$. As a result, dividends separate all firms in the efficient equilibrium. This separation entails dissipative dividends even though the optimal investment I^* increases in the inside information x . If dividends are not distributed and investment is determined by (11), then insiders are not induced to report truthfully $y = x$ in (5). In other words, value-maximizing investment cannot alone support a separating equilibrium.

Finally, each firm sells only enough new stock in (13) to raise the capital required for its optimal dividend and investment in real assets. Thereby, the firm exhausts all corporate cash $C > 0$ and acquires no riskless financial securities. This has a simple explanation. Riskless securities earn only the competitive, riskless rate of return—set equal to zero in this model. However, stock sold to finance these securities must be supported by dissipative dividends. To minimize the dissipative cost of dividends, the firm then sells only enough stock to finance its projects with positive net present value in (11). In this sense, the firm finances its real investments first from internal funds invested in marketable securities and second from stock sold to outside investors.

The firm's optimal dividend, investment, and new stock depend on both its corporate cash and the dissipative cost of dividends. With additional cash C , the firm optimally distributes a smaller dividend in (12) and sells less stock in (13). Because the firm optimally finances its investment first from internal funds, additional cash C induces it to sell less stock and then support this smaller sale S^* with a smaller dissipative dividend D^* . An increase in the fractional cost γ incurred by the firm's stockholders on its dividends has a similar effect. With a larger cost γ , the firm distributes a smaller dividend in (12) and sells less stock in (13). In this case, the larger dissipative cost induces a smaller dividend D^* , which then permits a smaller sale of stock S^* .

Given this optimal dividend, investment, and sale of new stock, the firm's market value follows from the competitive-pricing condition. This competitive price is

$$\begin{aligned} P^*(x) &= P[D^*(x), I^*(x), Q^*(x), x] \\ &= C - \gamma D^*(x) + F[I^*(x)]x - I^*(x), \end{aligned} \quad (14)$$

for all $\bar{x} \leq x \leq \bar{x}$. In (14) the firm's total market price, measured cum-dividend, equals its corporate cash minus the dissipative cost of its dividend plus the net present value of its real assets. This price is greater for firms with more favorable private information x : $P^{*'} = (1 - \alpha)[F(I^*) + S^*] > 0$. In other words, the announcement effect is positive for more favorable, truthful reports $y = x$ when these reports are supported by optimal dividends D^* , investment I^* , and sales of stock $S^* = [P^* - (1 - \gamma)D^*]Q^*$. In the efficient equilibrium, the announcement effect cannot be attributed solely to the report, dividend, investment, or sale of stock.

This equilibrium has two implications for empirical work. First, empirical studies cannot distinguish between the impact on stock prices of reported earnings and concurrently announced dividends, investment, and sales of stock. For example, announced changes in dividends that shortly follow after reported changes in earnings may have little incremental impact on stock prices if dissipative dividends serve mainly to make reported earnings credible.⁵ Second, deleting from data sets observations with multiple announcements—a standard practice in event studies—can bias estimates of impacts on stock prices. In event studies, data are often deleted because the estimated impact of announced changes in one variable is thought to be contaminated by concurrently announced changes in other variables.

III. Conclusion

In this paper the efficient mix of dissipative dividends, investments in real and financial assets, and sales of stock is derived for firms with private information about the return on real assets. The problem differs on several dimensions from previous signalling models with dissipative dividends and investment. It includes

⁵ See, for example, Aharony and Swary [1].

financial assets, a continuum of firms with different private information, and an endogenous derivation of the corporate insiders' maximand. The unique, efficient signalling equilibrium is then characterized for a technology in which each firm's present value from risky real investments is multiplicatively separable from its inside information.

In equilibrium firms invest to maximize net present value, sell stock in the capital market to finance the investment, and distribute dissipative dividends to support the sale of stock. Under the assumed, separable technology characterizing the return on risky real assets, firms invest at the first-best, value-maximizing level and thereby signal solely with dividends. Other things equal, firms with more valuable inside information distribute larger dividends that distinguish all firms in the efficient equilibrium. Also, each firm optimally retains no financial securities and sells sufficient shares only to finance its optimal investment in real assets. In this sense, the optimal investment is financed first from internal funds and second from the sale of new stock to outside investors.

Models with multiple financial signals have yet to answer many important questions. For example, why do firms distribute dissipative dividends if the repurchase of stock and simultaneous sale of debt can support credible communication without dissipative costs?⁶ What properties do separating equilibria exhibit when firms have multiple private attributes?⁷ Do sequential equilibria have additional properties, including the empirically evident intertemporal smoothing of dividends?⁸

Appendix

The problem specified in Section I is solved in several steps. First, (5) through (9) is transformed into a standard control problem, in part by rewriting the truth-telling condition (5) and omitting one constraint. For this relaxed problem, the optimality conditions are then identified. Next, the relaxed problem is solved, and the solution is shown to satisfy the omitted constraint when (10) is true.

A. Relaxed Problem

For notational convenience define the net sale of stock $S \equiv P_e Q$ and the function H :

$$H(x, y) \equiv \{C - D(y) + F[I(y)]y - I(y)\} \frac{C - D(y) + S(y) + F[I(y)]x - I(y)}{C - D(y) + S(y) + F[I(y)]y - I(y)}$$

for all feasible $\bar{x} \leq x, y < \bar{x}$. From (7) and (8) the function H satisfies

$$V[D(y), I(y), S(y), x, y] = (1 - \gamma)D(y) + \alpha H(y, y) + (1 - \alpha)H(x, y).$$

⁶ Financial models of nondissipative signalling include Brennan and Kraus [4], and Constantinides and Grundy [5].

⁷ See Engers [6].

⁸ Sequential signalling models of dividends include John and Nachman [7, 8], and Kumar and Spatt [11].

Using this notation, the truth-telling constraint (5) is equivalent to

$$V[D(x), I(x), S(x), x, x] \\ = V[D(\underline{x}), I(\underline{x}), S(\underline{x}), \underline{x}, \underline{x}] + (1 - \alpha) \int_{\underline{x}}^x H_x(y, y) dy, \quad (A1)$$

when

$$H_x \text{ is nondecreasing in } y \text{ at } y = x \quad (A2)$$

for all $\underline{x} \leq x \leq \bar{x}$.

Integrate the maximand in (9) by parts and apply the envelope condition (A1). Also, temporarily ignore the global nonmimicry condition (A2). This produces the relaxed problem:

$$\max_{D, I, S} \{V[D(\underline{x}), I(\underline{x}), S(\underline{x}), \underline{x}, \underline{x}] + (1 - \alpha) \int_{\underline{x}}^{\bar{x}} H_x(x, x)[1 - G(x)] dx\}, \quad (A3)$$

subject to the envelope condition (A1) and the inequalities $D, I \geq 0$. If the solution to this relaxed problem also satisfies the omitted constraint (A2), then it solves the original problem (5) through (9).

B. Optimality Conditions

From (A1) the function V is differentiable almost everywhere with respect to x along the truth-telling path $y = x$. Accordingly, denote the points of nondifferentiability by $x_j, j = 1, \dots, J$, when $x_1 = \underline{x}, x_J = \bar{x}$, and $J \leq \infty$. On each interval $x_j \leq x < x_{j+1}$, associate with the envelope condition (A1) the multiplier function Λ^j with the derivative $\lambda^j \equiv d\Lambda^j/dx > 0$. Also, associate with the inequality constraint (2) the constant multiplier $\kappa \geq 0$. Next, construct the Hamiltonian:

$$(1 - G + \Lambda^j)V_x + \lambda^j V + \kappa(C - D + S - I),$$

evaluated at $y = x$, with $\kappa(C - D + S - I) = 0$. On this interval the Euler-Lagrange conditions are

$$0 \geq (1 - G + \Lambda^j)V_{Dx} + \lambda^j V_D - \kappa, \quad (A4)$$

with an equality whenever $D > 0$,

$$0 = (1 - G + \Lambda^j)V_{Ix} + \lambda^j V_I - \kappa, \quad (A5)$$

and

$$0 = (1 - G + \Lambda^j)V_{Sx} + \lambda^j V_S + \kappa, \quad (A6)$$

evaluated at $y = x$. The corner condition $F'(0) = \infty$ precludes a corner solution $I = 0$ in (A5). Also, from (A1) the function V is continuous everywhere; hence, from the Euler-Lagrange conditions the partial derivatives V_d, V_s , and V_y must be continuous in x . In addition, the transversality conditions at $x_j, j = 1, \dots, J$, require that D', I' , and S' be continuous everywhere in x . Finally, the initial values $D(\underline{x}), I(\underline{x})$, and $S(\underline{x})$ must maximize the first term in (A3).

C. Solution

To show that the financial constraint (2) holds with equality at the optimal solution, suppose otherwise. In this case, $\kappa = 0$, so that (A6) and $V_S = 0$ imply $V_{Sx} = 0$ and thereby $D = C + Fx - I$. Hence, if (2) holds with a strict inequality, then (A5) ensures (11). In turn, the envelope condition (A1) requires that $0 = V_D D' + V_I I' + V_S S' + V_y = (1 - \gamma)F$, an impossibility. Thus, (2) is binding or, equivalently, (13) is optimal.

Next, exploit (13) and simplify the function H as follows:

$$H(x, y) = \{C - D(y) + F[I(y)]y - I(y)\} \frac{x}{y}. \quad (\text{A7})$$

In this case, (A4) through (A6) hold with $\kappa = 0$. As a result, (A5) implies (11). In turn, the envelope condition (A1) requires that

$$\gamma x D' - (1 - \alpha)D = \alpha F(I^*)x + (1 - \alpha)(I^* - C).$$

Because the first term in (A3) decreases in $D(x)$, the optimal dividend D^* satisfies $D^*(x) = 0$. Subject to this boundary condition, the above differential equation has the unique solution (12).

To complete the proof, the solution (11) through (13) must be shown to satisfy the omitted constraint (A2). To show this, insert (11) through (13) into (A7) to yield

$$0 \leq H_{xy}(x, x) = -\frac{1}{\gamma x^2} \{\alpha F[I^*(x)]x + (1 - \alpha - \gamma)S^*(x)\}. \quad (\text{A8})$$

Also, note from (11) through (13) that

$$\frac{d}{dx} [\alpha F(I^*)x + (1 - \alpha - \gamma)S^*] = \frac{1 - \alpha}{\gamma x} [\alpha F(I^*)x + (1 - \alpha - \gamma)S^*] + (1 - \gamma)I^{*'}.$$

Since $I^{*'} = -F'/xF'' > 0$, the slope of (A8) is negative whenever (A8) is zero. Also, (A8) is continuous in x from (11) through (13). In this case, if (A8) is violated at $\xi \in [x, \bar{x}]$, then it is violated on $[\xi, \bar{x}]$. Thus, (A8) holds everywhere on $[x, \bar{x}]$ if and only if it holds at $\bar{x}$. If $\gamma > 1 - \alpha$, then the latter condition is equivalent to the second inequality in (10).

REFERENCES

1. J. Aharony and I. Swary. "Quarterly Dividend and Earnings Announcement and Stockholders' Returns: An Empirical Analysis." *Journal of Finance* 35 (March 1980), 1-12.
2. R. Ambarish, K. John, and J. Williams. "Efficient Signalling with Dividends and Investment." *Journal of Finance* 42 (June 1987), 321-44.
3. S. Bhattacharya. "Imperfect Information, Dividend Policy, and 'The Bird in the Hand Fallacy'." *Bell Journal of Economics* 10 (Spring 1979), 259-70.
4. M. Brennan and A. Kraus. "Efficient Financing under Asymmetric Information." *Journal of Finance* 42 (December 1987), 1225-43.
5. G. Constantinides and B. Grundy. "Optimal Investment with Stock Repurchase and Financing as Signals." Working paper, University of Chicago and Stanford University, 1987.
6. M. Engers. "Signalling with Many Signals." *Econometrica* 55 (May 1987), 663-46.
7. K. John and D. Nachman. "Investment Incentives, Risky Debt, and Reputation in a Sequential Equilibrium." *Journal of Finance* 40 (July 1985), 863-78.

8. K. John and D. Nachman. "On the Optimality of Intertemporal Smoothing of Dividends." Working paper, New York University and Georgia Institute of Technology, 1987.
9. K. John and J. Williams. "Dividends, Dilution, and Taxes: A Signalling Equilibrium." *Journal of Finance* 40 (September 1985), 1053–70.
10. P. Kumar. "Shareholder-Manager Conflict, Costly Monitoring, and the Information Content of Dividends." Working paper, Carnegie-Mellon University, 1985.
11. ——— and G. Spatt. "Asymmetric Information and the Smoothing of Dividends." Working paper, Carnegie-Mellon University, 1987.
12. P. Milgrom and J. Roberts. "Price and Advertising as Signals of Product Quality." *Journal of Political Economy* 94 (August 1986), 280–312.
13. M. Miller and K. Rock. "Dividend Policy under Asymmetric Information." *Journal of Finance* 40 (September 1985), 133–52.
14. S. Myers and N. Majluf. "Corporate Financing and Investment Decisions When Firms Have Information that Investors Do Not Have." *Journal of Financial Economics* 13 (June 1984), 187–221.
15. A. Offer and A. Thakor. "A Theory of Stock Price Responses to Alternative Corporate Cash Disbursement Methods: Stock Repurchases and Dividends." *Journal of Finance* 42 (June 1987), 365–94.
16. J. Riley. "Informational Equilibrium." *Econometrica* 47 (March 1979), 331–59.
17. S. Vishwanathan. "A Multiple Signalling Model of Corporate Financial Policy." Working paper, Duke University, 1987.