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Joint Estimation of Factor Sensitivities and Risk Premia for the Arbitrage Pricing Theory

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ABSTRACT

The APT is represented as a multivariate regression model with across-equations restrictions. Both observed and unobserved (latent) macroeconomic factors are included, thus generalizing and unifying two previous strands of literature. Large portfolios representing unobserved factors are treated as endogenous, and nonlinear 3SLS estimates are shown to differ sharply from estimates that ignore this endogeneity. Using monthly stock returns and six factors, we cannot reject January effects. The following results are invariant with respect to the inclusion of January effects: we reject the CAPM in favor of the APT; however, we cannot reject the APT restrictions on the linear factor model.

MOST EMPIRICAL WORK ON the arbitrage pricing theory (APT) has followed a factor-analysis approach; papers by Burmeister and Wall [5], Chan, Chen, and Hsieh [7], Chen, Roll, and Ross [9], and McElroy and Burmeister [22] represent some exceptions in which measured macroeconomic factors such as unexpected inflation are used to explain asset prices in an APT or multifactor model.

Both the factor-analysis approach and the measured-macroeconomic-factor approach have their merits. The primary advantages of using measured economic factors are: (1) the factors and their APT prices in principle can be given economic interpretations, while with a factor-analysis approach it is unknown what factors are being priced; and (2) rather than using only asset prices to explain asset prices, measured macroeconomic factors introduce additional information, linking asset-price behavior to macroeconomic events.

In this exploratory paper we investigate an APT (or multifactor) model in which there are both measured macroeconomic factors and other nonobserved factors. We impose the nonlinear across-equation APT pricing restrictions exactly by using three multivariate estimation methods: iterated nonlinear weighted least squares (ITNLWLS), iterated nonlinear seemingly unrelated regressions (ITNLSUR), and iterated nonlinear three stage least squares (ITNL3SLS); see Gallant [13] and discussion below. The appropriate estimation method depends upon alternative assumptions about the error terms (nonsystematic risk) in the asset-returns equations. The primary objective of this work is to investigate whether or not certain APT tests are sensitive to these alternative error assumptions, the details of which are given in the text. Our focus is on the interrelation-

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ships of three nested tests: (1) the APT versus the linear factor model (LFM), (2) a "CAPM" versus the APT, and (3) the existence versus nonexistence of a "January effect."

The paper is organized as follows. In Section I we introduce notation and the LFM with both unobserved and measured factors. The APT pricing restrictions are given in Section II, while the data and model specifications are discussed in Section III. In Section IV we turn attention to the APT prices for the unobserved factors and the related issue of the January effect. Section V contains a very brief discussion of the restrictions a "CAPM" imposes on the APT, and then in Section VI we turn to important questions of econometric estimation.

The empirical results, presented in Section VII for ITNLWLS, ITNLSUR, and ITNL3SLS estimators, address four issues: (i) the APT risk prices and their statistical significance; (ii) the importance of a January effect; (iii) tests of the APT versus the LFM; and (iv) tests of the "CAPM" versus the APT. We conclude the paper with Section VIII containing a summary and suggestions for future work.

I. Linear Factor Model Representation with Measured and Unobserved Factors

Returns for the ith asset in period t are assumed to be generated by the linear factor model (LFM)

$$r_i^t = b_{0i}^t + \textstyle\sum_{j=1}^{K} b_{ij} F_j^t + \varepsilon_i^t \tag{1}$$

where

$$\mathrm{E}\varepsilon_i^t \varepsilon_j^s = \begin{cases} \sigma_{ii}, & \text{for } i = j \text{ and } t = s \\ 0, & \text{otherwise,} \end{cases}$$

where $\mathrm{E}(\varepsilon_i^t \mid F_i^t) = \mathrm{E}(F_i^t) = \mathrm{E}(\varepsilon_i^t) = 0$, and where the ε_i^t's are serially uncorrelated.[1] We assume that the K factors consist of J unobserved factors and $K - J$ observed (measured) factors, and we write

$$\textstyle\sum_{j=1}^{K} b_{ij} F_j^t \equiv \sum_{j=1}^{J} b_{ij} f_j^t + \sum_{j=J+1}^{K} c_{ij} g_j^t,$$

where the f_j^t's are unobserved factors and the g_j^t's are measured factors.

We assume that (1) holds for a sample of $i = 1, \cdots, N + J$ assets, and write the first N returns as the vector $r^t = (r_1^t, \cdots, r_N^t)'$ and the last J returns as $R^t = (r_{N+1}^t, \cdots, r_{N+J}^t)'$. The model then is

$$r^t = b_0^t + Bf^t + Cg^t + \varepsilon^t \tag{2}$$

$$R^t = b_{0J}^t + B_J f^t + C_J g^t + \varepsilon_J^t \tag{3}$$

where b_0^t is $N \times 1$, B is $N \times J$, f^t is $J \times 1$, C is $N \times (K - J)$, g^t is $(K - J) \times 1$, ε^t is $N \times 1$; and b_{0J}^t is $J \times 1$, B_J is $J \times J$, C_J is $J \times (K - J)$, and ε_J^t is $J \times 1$. B, C, B_J and C_J are assumed to be of full column rank. By the previous assumptions,

[1] These assumptions may be relaxed so that $\mathrm{E}\varepsilon_i^t \varepsilon_j^t = \sigma_{ij}$. The consequences for estimation will be tracked in subsequent footnotes.

$E\varepsilon^t = 0_N$, $E\varepsilon_J^t = 0_J$, $E\varepsilon^t(\varepsilon_J^t)' = 0$, and $E\varepsilon^t(\varepsilon^t)' = D_r$ and $E\varepsilon_J^t(\varepsilon_J^t)' = D_R$, where D_r and D_R are diagonal matrices of order N and J, respectively, each with typical element σ_{ij}.

By assumption B_J is nonsingular so that (3) may be solved for the unobserved factors:

$$f^t = B_J^{-1}(R^t - b_{0J}^t - C_J g^t - \varepsilon_J^t). \tag{4}$$

Then substituting (4) into (2) yields

$$r^t = \beta_0^t + \beta R^t + \gamma g^t + \eta^t \tag{5}$$

where:

$$\beta_0^t \equiv b_0^t - BB_J^{-1}b_{0J}^t \qquad (N \times 1) \tag{5a}$$

$$\beta \equiv BB_J^{-1} \qquad (N \times J) \tag{5b}$$

$$\gamma \equiv C - BB_J^{-1}C_J \qquad (N \times (K - J)) \tag{5c}$$

$$\eta^t \equiv \varepsilon^t - BB_J^{-1}\varepsilon_J^t. \qquad (N \times 1) \tag{5d}$$

with $E\eta^t = 0_N$, $E(\eta^t \mid R^t) \neq 0_N$, $\Omega \equiv E\eta^t(\eta^t)' = D_r + BB_J^{-1}D_R(B_J^{-1})'B'$, and the η^t are serially uncorrelated. Note that for the special case where $\varepsilon_J^t = 0_J$, then $\eta^t = \varepsilon^t$, $\Omega = D_r$ and $E(\eta^t \mid R^t) = 0_N$.[2]

Without loss of generality we can choose $B_J = I_J$, an identity matrix of size J. Such a normalization defines the factors f^t. The reasoning is as follows. Suppose that one (of the infinitely many) sets of factors that, apart from idiosyncratic noise, span the space of returns is (f_*^t, g^t) so that

$$R^t = b_{0J}^t + B_J^* f_*^t + C_J g^t + \varepsilon_J^t. \tag{6}$$

Following Burmeister and McElroy [4], we can replace f_*^t with f^t provided there exists a nonsingular $J \times J$ matrix L such that

$$Lf_*^t = f^t. \tag{7}$$

Letting

$$B_J = B_J^* L^{-1}, \tag{8}$$

for example, leads to (3). In particular, for any set of factors f^t that spans the same space as f_*^t, we can pick $L = (B_J^*)^{-1}$ so that $B_J = I_J$, and subsequently in Section III will specify the model for estimation with the choice $B_J = I_J$. In general, as discussed by Burmeister and McElroy [4], the appropriate choice of R^t and B_J is governed by our desire for economic interpretation of the factors and by econometric considerations.

[2] If the error covariance matrix of

$$[(\varepsilon^t)', (\varepsilon_J^t)']' = \begin{bmatrix} \Sigma_{rr} & \Sigma_{rR} \\ \Sigma_{Rr} & \Sigma_{RR} \end{bmatrix},$$

a positive-definite matrix, then

$$\Omega = \Sigma_{rr} - BB_J^{-1}\Sigma_{Rr} - \Sigma_{rR}(B_J^{-1})'B' + BB_J^{-1}\Sigma_{RR}(B_J^{-1})'B'.$$

Also note that no generality is lost by taking the unobserved factors f^t to be orthogonal to the measured factors g^t, and we shall do so.

II. APT Pricing Restrictions

The APT imposes well-known restrictions on (1), namely

$$E(r_i^t) = b_{0i}^t = \lambda_0^t + \textstyle\sum_{j=1}^K b_{ij}\lambda_j \tag{9}$$

where λ_0^t is the riskless return, provided at least one risk-averse investor holds a portfolio without any residual risk; see Ingersoll [19] (pp. 160–162). Moreover, unless $K = 1$, this condition does *not* imply that there exist K portfolios free of residual risk.

Letting $\lambda^J = (\lambda_1^J, \cdots, \lambda_J^J)'$ and $\lambda^K = (\lambda_{J+1}^K, \cdots, \lambda_K^K)'$, we write (9) as

$$E(r^t) = b_0^t = \lambda_0^t \iota_N + B\lambda^J + C\lambda^K, \tag{10}$$

where ι_x denotes an $x \times 1$ vector of ones. Similarly, from (3) we have

$$E(R^t) = b_{0J}^t = \lambda_0^t \iota_J + B_J\lambda^J + C_J\lambda^K, \tag{11}$$

while from (5)

$$E(r^t) = \beta_0^t + \beta E(R^t). \tag{12}$$

Substitution of (11) into (12) gives

$$E(r^t) = \beta_0^t + \beta(\lambda_0^t \iota_J + B_J\lambda^J + C_J\lambda^K). \tag{13}$$

Equating (10) and (13) and solving for β_0^t gives, using (5), the intercept

$$\begin{aligned}\beta_0^t &= (\iota_N - \beta\iota_J)\lambda_0^t + (B - \beta B_J)\lambda^J + (C - \beta C_J)\lambda^K \\ &= (\iota_N - \beta\iota_J)\lambda_0^t + 0 + \gamma\lambda^K. \end{aligned} \tag{14}$$

Thus APT pricing imposes the intercept restriction (14) on the unrestricted LFM (5). With no unobserved factors ($J = 0$), (14) collapses to the models of McElroy and Burmeister [22], Chan, Chen, and Hsieh [7], and Chen, Roll, and Ross [9]. In contrast, with no observed factors ($J = K$), it collapses to a generalization of the standard multibeta models with portfolios substituting for factors. Our (14) is a generalization of these multibeta models because here the assumption that $E(\eta^t \mid R^t) = 0$ is not made.

III. Model Specifications and Data

Substituting (14) into (5) gives the APT specification

$$r^t - \lambda_0^t \iota_N = \beta(R^t - \lambda_0^t \iota_J) + \gamma(\lambda^K + g^t) + \eta^t, \tag{15}$$

which is a special case of the LFM

$$r^t - \lambda_0^t \iota_N = \alpha_0 + \beta(R^t - \lambda_0^t \iota_J) + \gamma g^t + \eta^t. \tag{16}$$

In (16) the intercept α_0 is a $N \times 1$ vector. Thus the APT restriction on (16) is

$$\alpha_0 = \gamma\lambda^K. \tag{17}$$

Likewise, from (3) and (11) with $B_J = I_J$, the last J returns are given by

$$R^t - \lambda_0^t \iota_J = (\lambda^J + f^t) + C_J(\lambda^K + g^t) + \varepsilon_J^t$$
$$= \alpha + C_J g^t + u_J^t \tag{18}$$

where

$$\alpha \equiv \lambda^J + C_J \lambda^K \tag{19}$$

and

$$u_J^t \equiv f^t + \varepsilon_J^t. \tag{20}$$

With this specification (18) defines unobserved factors f^t that are orthogonal to the observed factors g^t, where C_J is a $J \times (K - J)$ matrix of population OLS regression coefficients. Thus, given the previous assumption that $\mathrm{E}(\varepsilon_J^t \mid f^t) = \mathrm{E}(\varepsilon_J^t \mid g^t) = 0$, by construction we also have that $\mathrm{E}(u^t \mid g^t) = 0$.

We use monthly data and assume that the riskless rate, λ_0^t, is measured by the 30-day Treasury-bill rate that is known at the beginning of each month. The dependent variables used here in (16) are described in McElroy and Burmeister [22]: total monthly returns, in excess of the 30-day Treasury-bill rate, for seventy randomly selected stocks from the CRSP data file for the time period 1972.01 to 1982.12 (132 months).

For the portfolios representing unobserved factors we took $J = 3$ with:

R_1 = the total monthly return on a portfolio of 20-year corporate bonds;
R_2 = the total monthly return on a portfolio of 20-year government bonds;
R_3 = the total monthly return on the S&P 500 index.

The data λ_0^t, R_1, R_2, and R_3 were provided by Ibbotson Associates.

The $J - K = 3$ measured factors are derived from:

$$G_1^t = \hat{\mathrm{E}}_t[\pi(t)] - \pi(t),$$

where $\pi(t)$ is the rate of inflation in month t and $\hat{\mathrm{E}}_t[\pi(t)]$ is an estimate of the unobserved expectation for inflation held by agents at the beginning of month t, a series taken from Burmeister, Wall, and Hamilton [6] who use Kalman-filtering estimation methods to estimate expected inflation.

$$G_2^t = \mathrm{E}_{t+1}[\mu(t)] - \mathrm{E}_t[\mu(t)],$$

where $\mu(t)$ is the monthly growth rate of real final sales (excluding services). This series was constructed from data in the *Survey of Current Business* using the methods of Chow and Lin [11] and Litterman [21] to first construct a monthly real final-sales series, and then estimating expectations of $\mu(t)$ from an autoregressive model with lagged values of $\mu(t)$ and lagged values of real disposable income. Finally,

$$G_3^t = \lambda_0^{t+1} - \lambda_0^t$$

is the change in the 30-day Treasury-bill rate.

The measured factors g_1^t, g_2^t, and g_3^t are then derived from G_1^t, G_2^t, and G_3^t by: $g_2^t \equiv G_2^t$; $g_1^t \equiv$ the residuals from a regression of G_1^t on a constant and g_2^t, adjusted to have mean $\overline{G}_1$; and $g_3^t \equiv$ the residuals from a regression of G_3^t on a constant, g_1^t, and g_2^t, adjusted to have mean $\overline{G}_3$. Consequently g_1^t, g_2^t, and g_3^t are orthogonal to each other by construction. Analysis of g_1^t, g_2^t, g_3^t shows that they have zero expectations at the beginning of each month, their autocorrelation functions and Q-statistics give no evidence that any one of them can be predicted from its own past, and none of them appears predictable from any variables known at the beginning of the month. Tests for G_1^t are reported in Burmeister, Wall, and Hamilton [6]; tests on g_1^t, g_2^t, and g_3^t yield similar results.

For ITNL3SLS estimations, the instruments were the exogenous variables (g_1^t, g_2^t, g_3^t, and, when appropriate, the dummy variable D^t defined below) and the excess returns $r_i^t - \lambda_0^t$ for twenty randomly selected firms not included in either our sample or the S&P 500 index.

IV. The APT Prices and the January Effect

Suppose we are given $\tilde{\lambda}^K$, some consistent estimator of λ^K. Let hats denote the (equation-by-equation) OLS estimators from (18), $\hat{\alpha}$ and $\hat{C}_J$. Then by (19)

$$\tilde{\lambda}^J = \hat{\alpha} - \hat{C}_J \tilde{\lambda}^K \tag{21}$$

is a consistent estimator for λ. Now the sampling error for $\tilde{\lambda}^J$ can be written as

$$\tilde{\lambda}^J - \lambda^J = C_J(\lambda^K - \tilde{\lambda}^K) + (C_J - \hat{C}_J)\bar{g} + \bar{f} + \bar{\varepsilon}_J. \tag{22}$$

Note in particular that the sampling error for $\tilde{\lambda}^J$ depends on $\bar{f}$.

The presence of $\bar{f}$ in the sampling error for $\tilde{\lambda}^J$ raises a basic difficulty in the estimation of λ^J. The historical data from 1926 on, as well as the data for our sample period, reveal that $\bar{f}$ is a serious source of error in the estimation of λ^J. For simplicity, assume that in a sample $\bar{g} = \bar{\varepsilon}_J = 0$, and also that it is known that $\lambda^K = 0$. Then

$$\tilde{\lambda}^J = \hat{\alpha} = \overline{R} - \overline{\lambda}_0 \iota_J, \tag{23}$$

where $\hat{\alpha}$ is the OLS estimator of α from (18). For our sample period $\overline{R}_i - \overline{\lambda}_0$ are -0.000904, -0.00124, and 0.000832, respectively, for $i = 1, 2, 3$. The only economically sensible interpretation of these results in an APT framework is that $\bar{f} < 0$. Even without the APT, the data suggest that people who held corporate bonds, government bonds, or even the S&P 500 on average must have held *ex ante* expectations that exceeded the *ex post* realizations.

Most importantly, this observation illustrates one advantage of using observed factor measures rather than portfolios for APT pricing. With measured factors, the g^t vector in our notation, the mean $\bar{g}$ is also observed and hence is accounted for in the estimate of λ^K. Moreover, λ^K is estimated by imposing exactly all of the APT across-equation restrictions. Thus there may be considerable econometric efficiency gains from substituting, whenever possible, observed factors for portfolios that represent unobserved factors.

We now turn to the January effect. To investigate whether or not there is a January effect in this model, we now assume that the intercepts in (2) and (3) are replaced by

$$b_0^t + \Phi D^t \tag{24}$$

and

$$b_{0J}^t + \Phi_J D^t, \tag{25}$$

respectively, where $D^t = 1$ in January and $D^t = 0$ otherwise, and where Φ and Φ_J are $N \times 1$ and $J \times 1$ vectors of asset sensitivities to the "January effect." The same calculations that lead to (15) and (18) now give the analogous results

$$r^t - \lambda_0^t \iota_N = \Psi D^t + \beta(R^t - \lambda_0^t \iota_J) + \gamma(\lambda^K + g^t) + \eta^t, \tag{26}$$

where Ψ is a $N \times 1$ vector defined by

$$\Psi \equiv \Phi - B\Phi_J, \tag{26a}$$

and

$$R^t - \lambda_0^t \iota_J = \Phi_J D^t + \alpha + C_J g^t + \eta^t. \tag{27}$$

There is no January effect if (26) satisfies the restriction $\Psi = 0$.

Note from (26a) that $\Psi = 0$ is possible for nonzero Φ and Φ_J. This observation shows that by picking an additional portfolio, say R_4^t, that exhibits a January effect, a "January factor" f_4^t is implicitly defined that will enable the APT "to explain" the January effect. However, as with all unobserved factors, this approach is unsatisfactory unless such a January factor can be linked to fundamental economic variables. Thus our test of $\Psi = 0$ merely investigates whether or not our particular choices of R^t and g^t are capable of explaining the January effect. Note also that similar remarks apply to *all* anomalies: e.g., if you discover that small firms earn larger than expected returns with your set of factors, add a small-firm portfolio to capture the unobserved "small-firm factor."

V. The "CAPM" and the APT

If returns are generated by a linear factor model, it is well known that the CAPM is nested in the APT. Since R_3^t is the total monthly return on the S&P 500 index, the "CAPM" with the S&P 500 index as a market proxy is nested within both APT specifications (15) and (26) (without and with a January effect). A test of whether or not the "CAPM" restrictions hold for (15) or (26), therefore, is merely one test of whether or not the S&P 500 index is mean-variance efficient, and with this understanding we now drop the qualifying quotation marks around "CAPM."

The CAPM predicts that

$$\mathrm{E}(r_i^t - \lambda_0^t) = \beta_i \lambda_M \tag{28}$$

where $\beta_i \equiv \mathrm{cov}(r_i^t, R_M^t)/\sigma_M^2$ is the CAPM beta, $\lambda_M \equiv \mathrm{E}(R_M^t - \lambda_0^t)$, and where for clarity we now use M to denote the S&P 500 index.

Under the usual assumption that var ε_M^t (the last element of the vector ε_J^t) is approximately zero, the well-known CAPM restrictions that imply (28) are

$$\lambda_j^J = \omega\ \mathrm{cov}(f_j^t, R_M^t), \qquad j = 1, 2, 3, \tag{29}$$

$$\lambda_j^K = \omega\ \mathrm{cov}(g_j^t, R_M^t), \qquad j = 1, 2, 3, \tag{30}$$

where $\omega \equiv \lambda_M/\sigma_M^2$. Let c_{M1}, c_{M2}, c_{M3} be the sensitivities of R_M^t to the observed factors g_1^t, g_2^t, and g_3^t, respectively, which without loss of generality were orthogonalized by construction; then $\text{cov}(g_i^t, R_M^t) = c_{Mj}\,\text{var}(g_j^t)$, and the last three restrictions are written as

$$\lambda_j^K = \omega c_{Mj}\,\text{var}(g_j^t), \qquad j = 1, 2, 3. \tag{31}$$

Our tests will consist of estimating (15) and (26) with and without the CAPM restrictions (31), with ω added as an additional parameter to be estimated in the restricted case. Since not all of the CAPM restrictions are imposed, this test is biased against rejection of the CAPM.

VI. Econometric Estimation

In a universe where returns are generated by a linear factor model, the CAPM is nested in the APT, which in turn is nested in the LFM. Using a combination of measured factors and portfolios to represent unmeasured factors, we estimate all three models and carry out tests of the nesting. We do this both assuming that the portfolio errors vanish and that they do not.

Suppose $\varepsilon_J^t = 0$ for all t; thus, $\eta^t = \varepsilon^t$ and $\text{E}(\eta^t \mid R^t) = 0$. Suppose we have a sample of T observations on r^t, R^t, g^t and λ_0^t. Then the returns system for the APT (15) or (26) is a system of T observations on the N nonlinear regressions with across-equation restrictions that force the λ^K to be the same for all N assets. Suppose the error structure is more general than that assumed below (1). Suppose the errors are correlated across assets (as in footnote 2) so that Ω is not diagonal. Then nonlinear seemingly unrelated regression techniques deliver estimators for (15) or (26) that are consistent and asymptotically normal. If, in addition, the errors are jointly normal, then iterating on the residual covariance matrix to convergence yields iterated nonlinear seemingly unrelated regression (ITNLSUR) estimators that are (tail-equivalent) maximum-likelihood estimators; see McElroy and Burmeister [22] and Gallant [13]. Now suppose the error structure in the specific diagonal structure assumed below (1) holds. It is trivial to impose this diagonal error structure on the ITNLSUR setup. Due to the across-equations restrictions, joint estimation improves efficiency and, in this special case, the (iterated) maximum-likelihood estimators are called ITNLWLS estimators.

Note that the corresponding LFM, (16), is a linear system with no across-equations restrictions and the same right-hand-side variables in every equation. Thus, even if the contemporaneous errors are correlated, (iterated or not) seemingly unrelated regressions estimators for the unrestricted system reduce to the equation-by-equation ordinary least square estimators.

So far in the discussion of estimation we have assumed that $\varepsilon_J^t = 0$. This assumption is potentially quite serious. For if $\varepsilon_J \neq 0$, all of the above estimation techniques for the APT and the LFM deliver inconsistent estimators.

Furthermore, for our choice of R^t there is no theoretical justification for assuming that $\varepsilon_J = 0$. Thus, it is essential to provide estimators for the case where $\text{E}(\eta^t \mid R^t) \neq 0$, that is when the portfolios on the right-hand sides of (15) and (16) are correlated with the errors. In this case we can use $H \geq J$ instruments for R^t. A natural choice is a set of H excess returns on assets other than the first $N + J$ assets. Referring to (1) and (9), we write the J portfolios and the H instrumental assets returns as

$$\begin{aligned} R^t - \mathrm{E}R^t &= [B_J \quad C_J]\begin{bmatrix} f^t \\ g^t \end{bmatrix} + \varepsilon_J^t \\ &\equiv B_R F^t + \varepsilon_J^t \end{aligned}$$

and in the analogous notation,

$$r_H^t - \mathrm{E}r_H^t = B_H F^t + \varepsilon_H^t.$$

Thus it is easy to see that these instruments are correlated with the right-hand-side portfolios, i.e.,

$$\mathrm{E}(R^t - \mathrm{E}R^t)(r_H^t - \mathrm{E}r_H^t)' = B_R \mathrm{E}[F^t(F^t)']B_H' \neq 0.$$

Moreover, since ε^t and ε_J^t are uncorrelated with the F_t (via $\mathrm{E}\varepsilon^t(\varepsilon_H^t)' = 0$ and $\mathrm{E}\varepsilon_J^t(\varepsilon_H^t)' = 0$), then the η^t are uncorrelated with the errors in (15): $\mathrm{E}\eta^t(r_H^t - \mathrm{E}r_H^t) = 0$. Hence r_H^t constitutes a valid set of instruments for R^t.[3] Thus multivariate instrumental-variable techniques will be used. One easy way to implement this procedure is to use nonlinear three stage least squares, using $(r_H^t - \lambda_0^t \iota_H)$ as instruments. This delivers joint estimators of β, γ, and λ^K that are consistent and asymptotically normal. Even if the errors $((\varepsilon^t)', (\varepsilon_J^t)')$ are jointly normal, iterating on the residual covariance matrix does not deliver maximum-likelihood estimators. We do iterate, however, to stabilize the estimate of Ω. With a moderate number of instruments the ITNL3SLS and ITNLSUR estimators achieve the analogue of the Cramer-Rao lower bound; see Tauchen [26].

Note that if $\varepsilon_J^t = 0$, so that the R^t themselves rather than r_H^t would be the optimal instruments for R^t, these ITNLSUR estimators would retain consistency but be inefficient relative to the ITNLWLS estimators discussed above. Thus, assuming that $\varepsilon_J^t \neq 0$ when $\varepsilon_J^t = 0$ is the case is not nearly as serious a mistake as the opposite error.

Letting θ be a vector whose elements are those of Ψ, β, γ, and λ^K, these ITNL3SLS estimators minimize the quadratic function $s(\theta, \Omega) = \eta'(\Omega^{-1} \otimes P_Z)\eta$, where η is the TN stacked vector of η^t's, Z is the $T \times H$ matrix of instruments, and $P_Z \equiv Z(Z'Z)^{-1}Z'$. Let $\hat{\theta}$, $\hat{\Omega}$ minimize $s(\theta, \Omega)$ under the alternative hypothesis and $\tilde{\theta}$ minimize $s(\theta, \hat{\Omega})$ under the null. Then $Q = s(\tilde{\theta}, \hat{\Omega}) - s(\hat{\theta}, \hat{\Omega})$ is asymptotically distributed as χ^2 with q degrees of freedom, where q is the number of restrictions. Using Bartlett's small-sample correction (see Gibbons [14], Amsler and Schmidt [1], and Jobson and Korkie [20]) for testing $\alpha_0 = \gamma\lambda^K$, for example, the number of restrictions is $q = 67$, and we reject the null hypothesis when $Q(T - N_1 - q/2 - 1)/T$ exceeds χ_α^2 for q degrees of freedom. In our sample this correction factor is $(132 - 3 - 67/2 - 1)/132 = .716$. For T and N not so different from these found here, both Jobson and Korkie [20] and Amsler and Schmidt [1] found this

[3] If the covariance matrix of $((\varepsilon^t)', (\varepsilon_J^t)')'$ is not (block) diagonal, then an arbitrary set of $H \geq J$ returns will not serve as instruments. In this case we could pick $H \geq J$ well-diversified portfolios (so that ε_J can be taken as zero) as valid instruments.

adjustment yielded reliable test statistics. For the χ^2 tests A and C in Table I, $N_1 = 0$ and $N_1 = 1$, respectively. While the appropriate value of N_1 for test A is unclear, the outcome of the test is robust.

VII. Empirical Results

Since we have no a priori reason to believe that the returns for the portfolios in (3) have errors that vanish ($\varepsilon_J^t = 0$), we present the results from multivariate instrumental-variable estimation first. The left panel of Table I presents some results from multivariate instrumental-variable estimation of (26), the APT inclusive of a January effect. With $N = 70$ firms, six factors, and with a January effect there are 70×3 β_{ij}'s, 70×3 γ_{ij}'s, and 70 Ψ_i's, for a total of 493 coefficients. The right-hand panel presents the corresponding results for (15), which excludes a January effect.

We observe that estimated risk premia in the left-hand column (when the effect of January on returns is controlled) differ from those in the right-hand column (January excluded): λ_3^J changes sign; λ_1^K goes from insignificant to significant. In the bottom portion of the table these data (the Bartlett-small-sample-adjusted) χ^2 statistics indicate overwhelming rejection of the null hypothesis of no January effect (A: $\Psi = 0$ in equation (26)).

Table I also shows results of the (small-sample-adjusted) tests that the APT restrictions hold vs. the LFM holds (B: $\alpha_0 = \gamma\lambda^K$ from equation (16)). Controlling

Table I

ITNL3SLS Estimates of Risk Premia and Tests of Hypotheses

	January Dummy Included, (5.6)	January Dummy Excluded, (4.1)
λ_1^J (CB)	−.003726	−.00404
λ_2^J (GB)	−.003004	−.00407
λ_3^J (S&P)	−.000558	.00476
λ_1^K (g_1)	−.000262	−.000631
	(1.13)[a]	(2.76)
λ_2^K (g_2)	−.0042314	−.00365
	(5.40)	(4.99)
λ_3^K (g_3)	−.00000104	.0000406
	(.01)	(.50)

Null Hypothesis	Test Statistic	df	Test Statistic	df
A: $\Psi = 0$ vs. $\Psi \neq 0$	186.38[b]	70	(not applicable)	
B: $\alpha_0 = \gamma\ \lambda^K$	66.47	67	67.90	67
C: $\lambda_j^K = \omega\ c_{Mj}\ \mathrm{var}(g_j^t)$	9.74	2	13.78	2

[a] Absolute value of asymptotic t-ratio.

[b] χ^2 (df) adjusted via Bartlett's small sample of $(T - K/2 - 1) = .727$. For 70, 60 and 2 df, the $\alpha = .05$ ($\alpha = .01$) cutoffs for the χ^2 distribution are 90.53, 79.08, and 5.99 (100.43, 88.38, and 9.21), respectively.

for the effect of January on returns does not make an appreciable difference in the test results. In both cases the APT pricing restrictions cannot be rejected for any reasonable size test.

Finally, maintaining a linear factor model and both including and excluding a January effect, we test the null hypothesis that the CAPM restrictions hold vs. the APT holds (C: $\lambda_j^K = \omega c_{Mj} \operatorname{var}(g_j^t)$). In both cases at an $\alpha = .01$ level, we reject the CAPM restrictions in favor of the APT, the rejection being stronger when the January dummy is excluded.

Table II presents estimates from ITNL3SLS, ITNLWLS, and ITNLSUR techniques for selected parameters: λ^K and, for the excess returns to Exxon, the coefficients on the portfolios (β's), on the observed factors (γ's), and on the January effect. As before, results in the left panel were obtained by controlling for January, those in the right panel by excluding January. The three nonlinear estimation techniques and required error assumptions are as follows:

	ITNL3SLS	ITNLWLS	ITNLSUR
$\operatorname{cov}\begin{bmatrix}\varepsilon^t \\ \varepsilon_J^t\end{bmatrix}$	diagonal	diagonal	nondiagonal
ε_J	$\neq 0$	$= 0$	$= 0$

In general the ITNLWLS and ITNLSUR estimates are similar to each other whether or not January is included. Thus given that $\varepsilon_J = 0$, the diagonality of the covariance structure appears not to be a key issue. In contrast, the assumption that $\varepsilon_J^t = 0$ has a serious impact on other results. For both the risk premia and the Exxon-specific coefficients, the ITNL3SLS estimates (based on $\varepsilon_J^t \neq 0$) appear quite different from both their ITNLSUR and ITNLWLS counterparts (based on $\varepsilon_J^t = 0$). In the left panel, for example, the ITNL3SLS estimate of λ_3^K is negative and insignificant; with ITNLWLS and ITNLSUR the estimates are positive and significant. Whether or not one uses ITNLSUR or the other techniques also changes the size, sign, and significance of the Exxon-specific coefficients. Similar differences between the simultaneous-equations and nonsi-

Table II

Comparison of Alternative Nonlinear Estimators

	With January Dummy Included			With January Dummy Excluded		
Parameter	ITNL3SLS	ITNLWLS	ITNLSUR	ITNL3SLS	ITNLWLS	ITNLSUR
λ_1^K	−.000262	−.0000984	−.000192	−.000631*	−.00126*	−.000652*
λ_2^K	−.000423*	−.00171	−.00359*	−.00365*	−.00394*	−.00379*
λ_3^K	−.00000104	.000268*	.000284*	.000406	.000266*	.000118
β_{i1}**	−1.857*	−.228	−.232	−1.615*	−.217	−.227
β_{i2}	1.966*	.211	.211	1.564*	.181	.183
β_{i3}	.862*	.853*	.854*	.888*	.858*	.861*
γ_{i1}	.408	−.189	−.201	.310	−.280	−.233
γ_{i2}	−.303	−.156	−.175	−.441	−.198	−.239
γ_{i3}	5.841	5.19	5.03	3.55	4.74	4.58
Φ_{i1}	0.0186	.00648	.00598	NA	NA	NA

* Significant at the five percent level.

** Asset i is Exxon.

multaneous-equations estimators arise in the right-hand panel when January effects are excluded. Since the consequences of erroneously assuming that $\varepsilon_J^t \neq 0$ are not nearly as serious as those of erroneously assuming that $\varepsilon_J^t = 0$, we prefer the ITNL3SLS estimates to either the ITNLWLS or ITNLSUR estimates.

As estimates of risk premia and factor sensitivities depend crucially on whether or not R^t is assumed exogenous, we attempted a Hausman [16]-Wu [27] test for exogeneity. Unfortunately, this attempt failed because the relevant difference in covariance matrices was not positive definite. More sophisticated tests will be carried out in future research.

VIII. Conclusions

In this study we maintained a linear factor model and used both measured and unmeasured factors to estimate a LFM, the APT, and a CAPM. We found that the January effect is an important determinant of expected returns. The existence of a January effect that is not explained by this set of factors is evident, but, as discussed above, it would be trivial to add a portfolio that exhibits a strong January effect and hence represents a "January factor." Including or excluding a January effect has, however, no appreciable effect on the following results from nested testing: the CAPM restrictions on the APT are rejected; the APT restrictions on the LFM are not rejected.[4] This result is in contrast to those found by Gultekin and Gultekin [15] and Cho and Taylor [10].

The similarity we found between ITNLWLS and ITNLSUR estimates probably indicates that the assumption of a diagonal covariance marix for the ε_i^t's is empirically tolerable, at least with this set of factors. In contrast, the marked difference of both the latter estimates from ITNL3SLS estimators is a significant empirical finding that merits further investigation.

In the asset-pricing literature it is common to assume *zero* residual risk for the portfolios used to represent factors ($\varepsilon_J = 0$ in this paper) so that these portfolios may be treated as exogenous variables in the estimation of asset returns and risk prices ($E(\eta^t \mid R^t) = 0$ in the notation of this paper). The great contrast of our ITNL3SLS estimators with those for ITNLWLS and ITNLSUR constitutes clear evidence that this common exogeneity assumption may not be innocuous. If, on the one hand, one is interested in economic structure (e.g., the allocation of total risk across types of risk, or the similar sensitivities of firms within industries), then these exogeneity assumptions are crucial. If, on the other hand, one is interested in the results of whether or not the LFM collapses to the APT and if, in turn, the APT collapses to the CAPM, then this exogeneity assumption appears to make little difference. The warning here is that the exogeneity of any vector of right-hand-side portfolios requires testing. The implementation of exogeneity tests should have high priority both in our own work and the work of others.

[4] In other work where we assumed that $\varepsilon_J = 0$, we obtained essentially the same results for these tests as found here; see McElroy and Burmeister [22] and Burmeister and McElroy [4].

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