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Warrant Exercise, Dividends, and Reinvestment Policy

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ABSTRACT

In this paper, we examine sequential exercise strategies by warrantholders and the gain from hoarding warrants. We analyze several obstacles to acquiring large blocks in order to exploit sequential strategies. First, we identify several reinvestment policies for which sequential exercise is not advantageous, thereby eliminating the gain from hoarding. However, sequential exercise strategies may be advantageous for monopoly or oligopoly warrantholders, even absent dividends, because using exercise proceeds to repurchase stock or to expand the firm's scale increases the riskiness of an equity share. Second, oligopoly warrantholders can receive a smaller warrant value than perfectly competitive warrantholders, suggesting a potential cost to unsuccessful hoarding.

THE POTENTIAL ADVANTAGE OF sequential exercise strategies by holders of warrants and convertible securities (demonstrated by Emanuel [12] and expanded upon by Constantinides [4] and Cox and Rubinstein [6]) suggests a basic puzzle.[1] Any advantage to sequential exercise is largest for a monopoly warrantholder, raising the issue of why these securities are often widely dispersed. One impediment to the acquisition of monopoly power is the ability of the firm to eliminate the advantage of sequential exercise (and monopolization). A second obstacle to the acquisition of monopoly power is rival investors. We show in some circumstances that oligopoly warrantholders cannot even achieve the maximal competitive valuation, resulting in a potential cost to incomplete hoarding.

The examples developed in the literature to illustrate the potential optimality of sequential exercise decisions are based upon differing assumptions about the firm's policy regarding the use of warrant-exercise proceeds. For example, Emanuel [12] includes an example in which the total amount paid as dividends to shareholders is fixed (rather than a fixed dividend per share) and not altered by the exercise of warrants. Another basis for sequential exercise strategies arises

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[1] A warrant issued by a firm allows the holder the option of obtaining a specified number of shares of the firm (fixed at one throughout our analysis) at a contractually guaranteed exercise price. We take the form of the firm's securities as given. Moral-hazard and adverse-selection considerations can explain the optimality of warrants and convertible securities in some environments (e.g., Green [13], Williams [18], and Brennan and Kraus [2]).

when the value of unexercised warrants increases with the early exercise of other warrants. The increase in the value of the unexercised warrants can exceed the cost of early exercise of the other warrants[2] if the warrant proceeds are used to repurchase common equity of the firm or to increase the scale of the firm's investment. Both alternatives increase the riskiness of equity. Under these assumptions, we show that a gain to sequential exercise can arise for any finite number of warrantholders. For some other reinvestment policies, we show that sequential exercise is not beneficial, even to a monopoly warrantholder.[3] These policies are methods for eliminating the gain from sequential exercise. Although warrants are subject to some antidilution protection in practice, it appears that the firm enjoys considerable latitude.[4]

The distribution of warrant ownership plays an important role in understanding the nature of warrant-exercise strategies. For example, the payoff to early exercise depends upon the extent to which the reinvestment impact is internalized by the warrantholder. Constantinides [4] showed that there can exist multiple competitive equilibria and that, among these equilibria, the one with the largest warrant valuation has the same value as the block-constrained monopoly value. We show some circumstances in which oligopoly warrantholders cannot achieve this maximal competitive valuation (yet in most game-theoretic contexts, oligopolists outperform competitors). In our example, the oligopolists rush to exercise because of a unilateral gain to early exercise. Therefore, simultaneous hoarding of warrants by rivals may lower the value of warrants. The resulting cost of unsuccessful hoarding could deter market cornering despite the potential gains, thus providing a possible explanation for the observed lack of hoarding. In contrast, Constantinides [4, pp. 393–394] motivates the observed lack of hoarding by citing the inability of a potential monopolist to acquire a substantial fraction of the outstanding warrants at a uniform price.[5] Our analysis of the obstacles to sequential exercise helps to justify the frequent simplifying restriction that warrants or convertible securities are valued as if exercised as a block[6] and

[2] A premium above parity results from the time value of delaying payment of the exercise price and the possibility that eventual exercise would not be optimal.

[3] In the case of convertible bonds and convertible preferred stock, there is no infusion of funds to the firm at the exercise (i.e., conversion) time. Instead, subsequent bond coupons (or preferred stock dividends) are eliminated. The reinvestment policy of the firm must reflect this.

[4] For example, citing the American Bar Foundation's Commentaries [1], Constantinides [4, p. 375] argues that the firm is typically unrestricted by bond covenants in the use of warrant proceeds. Kalay [15] presents empirical evidence suggesting that bond covenants do not constrain actual firm dividend policy. Firms can engage in a variety of tactics to transfer value from the warrantholders (for example, to the shareholders). Indeed, Value Line criticizes some of these methods, pointing to an example of a firm being acquired for cash at a price below the exercise price of the acquired firm's warrants. The warrants were then transformed to ones convertible at the exercise price into the per-share cash acquisition price, thereby eliminating the entire value of the warrant (*Value Line Convertible Survey*, April 15, 1974 [17], p. 268).

[5] Dunn and Spatt [10, pp. 417–418] suggest that this is a consequence of the nonoptimality of unconditional tender offers.

[6] Applications in which the exercise strategies are constrained to result in simultaneous exercise (conversion) include the analyses of convertible-debt call policy (e.g., Brennan and Schwartz [3] and Ingersoll [14]) and warrants with "usable bonds" in which either cash or bonds evaluated at par can be used to exercise warrants (Dunn, Heston, and Spatt [9]). We assume that the warrant can only be exercised in cash.

identifies some alternatives to the Constantinides [4] competitive warrantholding to sustain much of the long-standing theory of the valuation of warrants (i.e., the theory developed prior to Emanuel [12]).

I. The Model

We assume throughout the paper that there are no taxes or transactions costs, and no arbitrage opportunities in the project market. The restriction to zero-net-present-value projects fixes the total value of claims issued by the firm. If the firm invested in negative-net-present-value activities (such as destroying resources), these investments could deter warrant exercise. For expositional simplicity, we assume that common equity and one type of warrant are the only claims issued by the firm and that there are two possible exercise points (the early date and the maturity). We assume that there are a finite number of warrantholders who each hold an identical number of warrants and no equity.

To separate the role of the firm's reinvestment policy from the role of the firm's dividend policy on the exercise strategies of investors, we explicitly examine the case of no periodic dividends under several plausible policies for use of the proceeds of warrant exercise. Most firms with warrants outstanding pay low dividends or no dividends at all.[7]

We suppose that the capital structure of the firm is initially comprised of n shares of stock and m (infinitely divisible) warrants with an exercise price of E. There are two terminal states, and the value of the firm's project at the expiration of the warrant is either V_1 or V_2. We assume that $\frac{V_1}{n} > E \geq \frac{V_2}{n}$, so that without early exercise the warrant is in the money in the high state and out of the money in the low state. We denote the current state prices as q_1 and q_2, implying an interest rate of $r = (q_1 + q_2)^{-1} - 1$. The value of the firm's project is then $V = q_1 V_1 + q_2 V_2$ (so $V_1 > V$). Though we restrict attention to a stochastic process with two terminal states, our results can be generalized for other processes. The ith warrantholder holds m_i warrants such that $\sum_i m_i = m$. The number of warrants exercised in the initial period by the ith warrantholder is k_i such that $\sum_i k_i = k$. Though we assume that each warrantholder owns the same number of warrants, we do not assume that each exercises the same number. We will use L_i to denote $\sum_{j \neq i} k_j$, or equivalently $k - k_i$. The stock price rationally reflects anticipation of the exercise of k warrants and the assumed use of the exercise proceeds.

We first examine a reinvestment policy in which the effect of exercise is completely neutralized.

[7] We examined all firms included in the 1984 Moody's Industrial Manual list of companies with outstanding warrants. We were able to identify the dividend on 176 of these 189 firms from the Standard and Poor's Standard Stock Reports (revised through November 7, 1984). Only sixty-four of the 176 firms (36.4 percent) had a positive dividend rate (112 firms paid no dividends). Further, the median dividend yield on the subset of firms with a positive dividend rate was just below 2½ percent.

Proposition 1: *The firm's capital structure will remain unchanged if, when warrants are exercised early, the firm issues new warrants with the same exercise price and maturity as those just exercised and uses the proceeds of the exercise and the new issue to repurchase the corresponding number of equity shares, paying any excess funds to shareholders as an extraordinary dividend. Through this policy, the firm can always prevent the warrantholders from using sequential exercise strategies to increase the warrant valuation above that in the maximal competitive solution.*

Proof: The above reinvestment policy leaves both the wealth of the firm and its capital structure unchanged.[8] The value of unexercised warrants is uninfluenced by this activity, so the only effect on the ith warrantholder is related to the k_i warrants exercised early. The warrantholder gives up θk_i, where θ denotes the premium above parity ($\theta \geq 0$). The dividend collected by the ith warrantholder is $\frac{k_i}{n+k}[\theta k]$. The net effect on the ith warrantholder of early exercise of k_i warrants is $\left[\frac{k}{n+k} - 1\right]\theta k_i \leq 0$, so that there is no advantage to early exercise. Q.E.D.

This neutralization policy allows firms to eliminate any advantage from sequential strategies.[9] If firms engaged in this policy, the incentive to hoard warrants would be eliminated. However, since sequential exercise does not occur under this policy (or under the policy described in Proposition 2), the firm does not actually need to carry out the hypothesized response to sequential exercise.

Proposition 2: *If the firm invests the proceeds from any warrant exercise in a riskless zero-coupon bond that matures at the warrant's expiration, then holding all warrants until maturity is the unique Nash equilibrium and maximizes the total value of the warrants.*

Proof: The restriction $\frac{V_1}{n} > E$ implies that the warrant will be in the money if the good state occurs, even if warrants are exercised early. The terminal value of a share of stock in the good state is

$$\frac{V_1 + mE + (L_i + k_i)Er}{m+n}.$$

[8] The analogy between warrants under the neutralization policy and call options is potentially instructive. The firm issues additional shares to fulfill its obligation under the warrants, but the exercise of call options (side bets on the value of the firm) does not result in the issuance of additional shares. It is well known that there is no advantage to sequential exercise strategies for holders of call options, and, in fact, absent dividends, the optimal exercise date is the expiration of the option. Since the call option is not issued by the firm, there is no reinvestment decision to be made by the firm, and the capital structure and stochastic process generating the value of the firm are unaltered by the exercise decision. The valuations of the call option and the warrant under the neutralization policy are based upon the same exercise strategy and differ only to reflect the dilution effect at the exercise point in the case of the warrant.

[9] An alternative approach is given by Emanuel [12, pp. 233–234], who adjusts the dividend rate to form a "neutral dividend policy" in order to make simultaneous exercise optimal.

The ex post payoff to the warrantholder is

$$m_i\left[\frac{V_1 + mE + (L_i + k_i)Er}{m + n} - E\right] - k_iEr,$$

which is maximized at $k_i = 0$. If the warrant were in the money in the low state, the same reasoning would imply that the ex post payoff is also maximized at $k_i = 0$. Instead, if the warrant were out of the money in the low state, then $k_i > 0$ would result in a negative payoff while $k_i = 0$ (the optimum) would produce a zero payoff. Therefore, the warrantholder's optimal strategy must be $k_i = 0$ ex ante (prior to revelation of the state). Hence, $k_i = 0$ for all i is the unique Nash equilibrium and, in fact, a dominant strategy equilibrium. Q.E.D.

The firm policies described by Propositions 1 and 2 provide simple methods for the firm to eliminate both the gain from sequential exercise and the advantage of monopolization. The threat of such policies could deter hoarding.

In a model with debt and without warrants, shareholders are indifferent between using a fixed sum to repurchase equity or pay dividends. Because of the differential protection against dilution, warrantholders prefer that the firm engages in a stock repurchase rather than payment of a dividend (whether or not sequential strategies can be employed). If warrant-exercise proceeds are used to repurchase shares, then the warrantholder forfeits the premium above parity on those warrants exercised. However, this increases the payoff on the outstanding warrants because the stock price spreads out due to the change in capitalization. This may result in a net benefit to the warrantholder. The classic solution to the warrant-valuation problem without dividends, i.e., exercising the warrant only at the warrant's expiration, is not correct in general even if the firm's investments are not changed by warrant exercise. Another use of the funds for which a sequential strategy could be optimal even without periodic dividends is to expand the firm's investment project. This increases the riskiness of the firm, and, unlike each of the uses of funds discussed earlier, it is not purely a financial transaction. Under both of these reinvestment assumptions, the distribution of the stock price spreads out due to the early exercise.

If the proceeds are used to repurchase shares, then the price, $P(k)$, satisfies

$$P(k) = q_1\left(\frac{V_1 + (m - k)E}{n + m - \dfrac{Ek}{P(k)}}\right) + q_2\left(\frac{V_2}{n + k - \dfrac{Ek}{P(k)}}\right), \tag{1}$$

where k is the total quantity of warrants exercised early so that $Ek/P(k)$ shares are repurchased. If $P(k) > E$, the number of shares increases as a result of the exercise so that the warrant cannot be in the money in the low terminal state. (Since $P(k) > E$, the warrant must be in the money in the high state.) At the end of the period, exercise of the remaining $m - k$ warrants occurs in the high state. The value of the stock in each terminal state is the value of the project plus any final-period exercise proceeds divided by the number of shares outstanding in that state. Similarly, if the proceeds are used to increase the investment in the project, then the price, $P(k)$, is

$$P(k) = q_1\left(\frac{V_1 + \dfrac{kEV_1}{V} + (m-k)E}{n+m}\right) + q_2\left(\frac{V_2 + kE\dfrac{V_2}{V}}{n+k}\right). \tag{2}$$

Under either of these reinvestment assumptions, k_i is chosen to maximize

$$w_i = (m_i - k_i)q_1(P_H(k) - E) + (P(k) - E)k_i, \tag{3}$$

where $P_H(k)$ is the terminal price of the stock in the high state. The above analysis assumes that the warrant is out of the money or at the money if the low state occurs. The effect of a change in k_i is given by

$$\frac{\partial w_i}{\partial k_i} = -q_1(P_H(k) - E) + (m_i - k_i)q_1P_H'(k) + (P(k) - E) + P'(k)k_i, \tag{4}$$

where primes denote derivatives. Equivalently,

$$\frac{\partial w_i}{\partial k_i} = q_2P_L(k) - (1 - q_1)E + m_iq_1P_H'(k) + k_iq_2P_L'(k), \tag{5}$$

where $P_L(k)$ is the terminal price of the stock in the low state. If we assume that $P(k) > E$, then the equilibrium is always symmetric.[10]

Proposition 3: *There exist parameter values such that, for any number of rivals, there will be sequential exercise if the warrant proceeds are used either to repurchase equity or to rescale the firm's investment.*

Proof: In general, $P_H'(k) > 0$ for either repurchase or expanding the scale of the firm. In the case of rescaling, $P_H(k)$ is an increasing linear function of k. For repurchase of shares, $P_H(k)$ is an increasing nonlinear function of k. The early exercise results in repurchasing shares at the ex ante price $P(k)$. Since shares are repurchased (before it is known which state occurs) at $P(k)$ and are subsequently worth $P_H(k)$, the value of $P_H(k)$ increases in k.

The benefit to the warrantholders of repurchase or rescaling is the increased volatility; however, by exercising the entire position, a warrantholder loses the benefit of the higher volatility. Thus, with rescaling or repurchase, it is never optimal to exercise the entire position early.

If $q_1 + q_2 = 1$ and $E = P_L(0)$, then $\left.\dfrac{\partial w_i}{\partial k_i}\right|_{k=0} = m_iq_1P_H'(0) > 0$. These parameter values imply that $0 < k < m$; i.e., sequential exercise is optimal. Q.E.D.

In the Appendix, we show for a monopoly warrantholder that, if the parameters are such that sequential exercise is optimal if the firm repurchases shares with

[10] Since the first three terms of equation (5) are identical for all warrantholders, each must choose an identical k_i when $P_L'(k) < 0$. When the exercise proceeds are used to rescale the firm's project, then differentiating the final term of (2) shows that $V/n > E$ implies that $P_L'(k) < 0$. In the repurchase case, we suppose $P_L'(k) \geq 0$ at some k and obtain a contradiction, hence establishing that $P_L'(k) < 0$ for all k. Because $P_H'(k) > 0$, the maintained assumption that $P_L'(k) \geq 0$ implies that $P'(k) > 0$. If we assume that $P(k) > E$, it is easy to verify that $P'(k) > 0$ implies that $P_L'(k) < 0$, yielding a contradiction. Therefore, $P_L'(k) < 0$, and the equilibrium is symmetric.

exercise proceeds, then sequential exercise also is optimal if exercise proceeds are used to expand the scale of the firm.

Stock repurchase or investment in the firm can lead to sequential exercise even without regular dividends because of the increase in the variance of the stock price and the convexity of the warrant payoff function. For example, in the Cox and Rubinstein [6, pp. 397–399] monopoly example of expanding the firm's scale, the increase in the value of the unexercised warrants exceeds the forfeiture of the premium above parity on those exercised and there is a gain to hoarding of the warrant.

We next assume that the exercise proceeds are used to pay an extraordinary dividend so that they are divided among the stockholders, including new stockholders who obtained stock by exercising warrants early. First, we focus on a monopoly warrantholder.

Proposition 4: *If the firm does not pay a regular periodic dividend and pays an extraordinary dividend to equityholders with the proceeds of any warrant exercise, then, for a monopoly warrantholder, the optimal exercise strategy is to hold the warrants until maturity.*

Proof: Since $\frac{V_2}{n} \le E$, the warrant will be out of the money for any value of k if the low state occurs. The payoff to the warrantholder in the low state is then

$$k\,\frac{V_2}{n+k} - kE(1+r) + k\,\frac{k}{n+k}\,E(1+r), \quad \text{or} \quad k\left(\frac{V_2}{n+k} - \frac{n}{n+k}\,E(1+r)\right).$$

Since $\frac{V_2}{n} \le E$, this expression is maximized at $k = 0$.

If the high state occurs, then, for $k = 0$, the terminal payoff to the warrantholder is

$$W(0) = m\left(\frac{V_1 + mE}{m+n} - E\right). \tag{6}$$

If the warrant is exercised early, then the warrant may be out of the money or in the money depending upon k. If the warrant is out of the money, then the payoff is

$$k\left(\frac{V_1}{n+k} - \frac{n}{n+k}\,E(1+r)\right),$$

which is equivalent to

$$W(k) = k\left(\frac{V_1 + kE}{n+k} - E\right) - \frac{kn}{n+k}\,Er. \tag{7}$$

If the warrant is in the money, then the payoff is

$$m\left(\frac{V_1 + (m-k)E}{m+n} - E\right) - krE + k(1+r)E\,\frac{k}{n+k},$$

which is equivalent to

$$W(k) = m\left(\frac{V_1 + mE}{m+n} - E\right) - krE - \frac{mkE}{m+n} + \frac{k(1+r)}{n+k}Ek. \tag{8}$$

The assumption that $V_1/n > E$ implies that the value $W(0)$ from equation (6) is greater than or equal to $W(k)$ from either equations (7) or (8) provided that $0 < k \leq m$. Hence, $k = 0$ is optimal. Q.E.D.

Interestingly, in some circumstances for oligopoly, there is a unique Nash equilibrium in which the warrant value is below the highest value competitive solution.[11] Limited hoarding actually hurts the warrantholders in some situations.

We assume that there is a single early exercise date before the maturity of the warrant. We change our assumption regarding $\frac{V_2}{n} \leq E$ and instead assume that, at the early exercise point, it is known that the warrant will subsequently expire in the money, making optimal subsequent exercise of all warrants not exercised immediately. Therefore, the net cost of early exercise is the time value of the exercise price less the share of dividends, and, for expositional simplicity, we assume that no dividends are paid other than the extraordinary dividend from the exercise proceeds (and the liquidation dividend). Because all warrants are eventually exercised, after the warrants have expired there are $m + n$ shares of equity outstanding. The payoff function to the ith warrantholder is then

$$\pi^i(k_i, L_i) = \frac{k_i}{n + k_i + L_i}(k_i + L_i)E(1+r) - k_iEr - (k_i + L_i)\frac{E}{n+m}m_i. \tag{9}$$

This payoff is defined relative to none of the warrantholders exercising early. The first term of the payoff is the ith warrantholder's share of the terminal value of the extraordinary dividend; the second term is the warrantholder's interest cost of early exercise; the final term is the warrantholder's share of the reduction in the final-period exercise proceeds. In a Nash equilibrium, each warrantholder optimizes given the decisions of the others so that an equilibrium choice of k_i ($k_i \in [0, m_i]$) maximizes $\pi^i(k_i, L_i)$, where L_i is taken as fixed. The partial derivative is

$$\frac{\partial \pi^i}{\partial k_i} = \left(1 - \frac{n^2 + nL_i}{(n + k_i + L_i)^2}\right)E(1+r) - Er - \frac{E}{n+m}m_i. \tag{10}$$

The strict convexity of $\pi^i(k_i)$ implies that the ith warrantholder's best reply to any specified vector of rival strategies will be exercising either all or none of his or her warrants early. Assuming that the warrants are divided equally, then it is impossible to have an asymmetric equilibrium. This follows since, if the expression in (9) is maximized at $k_i = m_i$ (not $k_i = 0$) for some L_i^*, then $k_i = m_i$ is also maximal for $L_i > L_i^*$. The strategies $k_i = 0$ for all i form an equilibrium if

[11] Constantinides and Rosenthal [5] formally demonstrate existence of a competitive-equilibrium valuation.

and only if

$$m_i \frac{E(1 + r)}{n + m_i} - Er - \frac{E}{n + m} m_i \leq 0, \tag{11}$$

while the strategies $k_i = m_i$ for all i form an equilibrium if and only if

$$\frac{m}{n + m} E(1 + r) - Er - \frac{E}{n + m} m_i \geq 0. \tag{12}$$

There always exists an equilibrium to this Nash game, and, when both (11) and (12) are satisfied, there are multiple equilibria. The solution $k_i = m_i$ is the unique Nash equilibrium[12] provided that

$$r < \frac{(m - m_i)\, m_i}{n(n + m)}. \tag{13}$$

To further examine the features of the oligopoly equilibrium in this setting, we consider the monopoly and competitive outcomes. The monopolist's best strategy is not to exercise any warrants early.[13] Similarly, the highest value competitive equilibrium is for all of the warrants to be exercised at the expiration date.[14] In this case, there is no extraordinary dividend to be shared by those who exercise early, so that the interest cost of early exercise implies that there is no early exercise by competitors who perceive that the dividend pool is fixed at zero. The

[12] In fact, under condition (13), the unique Nash equilibrium of $k_i = m_i$ for all i is also a dominant strategy equilibrium because each player's strategy is optimal for every vector of fixed strategies (not just the equilibrium strategy vector) of his or her rivals. Furthermore, under equation (13), randomized exercise decisions cannot be optimal, ruling out the possibility of mixed-strategy equilibria (in contrast to the case in which both (11) and (12) hold).

[13] Because the monopolist's payoff function (like (9)) is strictly convex in k, the optimal solution is an extreme point ($\{0, m\}$). Since

$$\frac{m^2}{n + m} E(1 + r) - mEr - \frac{m^2 E}{n + m} < 0,$$

it follows that $\pi(0) > \pi(m)$ and that $k = 0$ is optimal. Of course, this is also the block-constrained monopoly solution. (The block constraint is not binding.)

[14] This example also illustrates the possibility of "panic equilibria" and multiple competitive equilibria. The individual warrantholder is small and does not influence the total level of early exercise, k. This competitive warrantholder does not influence the level of the final-period payoff, so that the final term of (9) is eliminated. Therefore, the incremental payoff for early exercise is proportional to $\frac{k}{n + k} E(1 + r) - Er$, i.e., the terminal value of the extraordinary dividend paid for the early warrant exercise proceeds less the interest cost of early exercise. Since this expression is negative at $k = 0$, one competitive equilibrium (and the most profitable one) is always no early exercise. The expression is non-negative at $k = m$ provided that $m \geq rn$. Therefore, another competitive equilibrium is for all warrantholders to exercise early provided that the extent of potential dilution is high enough. Finally, the incremental payoff is zero when $k = rn$. An equilibrium in which only some warrantholders exercise early exists provided that $m > rn$. Multiple equilibria (either two or three) exist if and only if $m \geq rn$. The equilibria with partial or complete early exercise illustrate the Constantinides "panic equilibria." Analogues to panic equilibria occur in a variety of other market contexts including bank runs (Diamond and Dybvig [7]), capital budgeting in settings with preemption and learning (Spatt and Sterbenz [16]), and adoption of new technology (Dybvig and Spatt [11]).

strategies inherent in the monopoly outcome can be sustained in a competitive market.

To summarize, for values of m that satisfy condition (13), we have shown that the unique Nash equilibrium is for all to exercise early. In contrast, the monopoly and highest value competitive outcome is no early exercise. The oligopoly warrant valuation is below the highest value competitive solution,[15] and the *maximal* competitive valuation cannot be achieved by the oligopoly warrantholders.[16]

When the firm uses all exercise proceeds to pay an extraordinary dividend (as in the example described in (9) through (13)), Proposition 4 guarantees that it is impossible to construct an example in which there is a gain to using a sequential strategy by a monopolist or a gain above competition to monopoly hoarding. To obtain an example in which the oligopolists do not achieve the maximal competitive value, with the further property that there is a gain to monopoly hoarding above competition, we assume that an initial amount of the warrant proceeds is reinvested in the firm's activity and that any excess above that level is used to pay an extraordinary dividend. The project reinvestment can lead to an advantage to monopoly hoarding, while the extraordinary dividend can produce the desired comparison between the oligopoly and maximal competitive values.

We now permit dispersed ownership of the warrants but do not allow the warrantholders to own any initial equity. We set $V_1 = 2500$, $V_2 = 1500$, $m = 100$, $n = 100$, $E = 15$, and $q_1 = q_2 = 1/2.002$. Therefore, the interest rate is 0.1 percent. The proceeds of the first (infinitely divisible) warrant exercised expand the scale of the firm, while any additional proceeds are used to pay an extraordinary dividend. If one warrant is exercised early, the \$15 is invested in a project that has an expected payoff in present-discounted-value terms of \$15. To match the original risky project, we assume that the payoff in the high state is $5/3$ times the payoff in the low state. If one warrant is exercised in the initial period and the high state occurs, then the present discounted value of the payoff to a stockholder is $[3985 + 0.75(25)\ 1.001]/[200(1.001)]$, and, if the low state occurs, then it is $[1500 + 0.75(15)\ 1.001]/[101(1.001)]$.

The monopolist's optimal strategy is to exercise one warrant early. Then the value of this warrant is 2.47344 (in present-value terms), and the value of each of the remaining ninety-nine warrants is 2.50691. The average value of a warrant is 2.50658.

If the warrants are competitively held, then there are three equilibria. The highest value equilibrium entails exercising no warrants in the initial period (the

[15] Constantinides and Rosenthal [5] include an example in which block-constrained oligopolists do strictly worse than competitors (unconstrained). This finding seems natural in light of the result that the block-constrained monopoly valuation equals the competitive (unconstrained) one. In contrast, our example allows unconstrained behavior by the oligopolists.

[16] Condition (13) implies that $rn < m$, so that there are three competitive equilibria in this example. (See footnote 14.) It can be shown using (9) that the interior solution ($k = rn$) has the same warrant value as the equilibrium in which all warrants are exercised early ($k_i = m_i$). Of course, the equilibrium with no warrants exercised early is the highest payoff competitive solution.

Because the value of the warrants of oligopoly warrantholders is lower under condition (13) than the maximal competitive equilibrium valuation, the shareholders lack an incentive to use the capital structure neutralization policy in this oligopoly setting, which would cause the warrant value to equal the maximal competitive value.

same as a monopolist's optimal block-constrained strategy). Then the warrants are worth 2.49750, which is less than the unconstrained monopoly value. Like the earlier example, the interior competitive equilibrium (in which only 1.45125 of the one hundred warrants are exercised in the initial period) has the same warrant valuation (2.49001) as when all of the warrants are exercised early. (See footnote 16, first paragraph.)

If each of two duopolists holds fifty warrants, then the unique Nash equilibrium is for both to exercise all warrants in the initial period. The warrants are worth 2.49001, which is less than the maximal competitive valuation. Despite the presence of a gain to monopolization, hoarding may not arise since incomplete hoarding (i.e., oligopoly) can result in a lower warrant value than competitive ownership. The fear of rival hoarders can prevent an attempt to corner the market in the warrants.

II. Conclusion

Early exercise of some warrants creates an externality that is internalized by a monopoly warrantholder. In contrast, oligopolists do not completely internalize this externality. This is analogous to the situation in which a group of vessels share a common fishing ground and each vessel ignores the impact that its fishing has on the profits of other vessels.

In the presence of a single large warrantholder and a competitive fringe, Constantinides [4] argues that the dominant player obtains a lower value than competitors because the competitors free ride on the dominant player's strategy by delaying exercise. For example, if the exercise proceeds are used to repurchase stock or reinvest in the firm's project (as in Proposition 3), then the dominant player bears a disproportionate share of the cost of early exercise of some warrants as the competitors free ride. However, it is not advantageous for any investor to split his or her position given the distribution of holdings of other investors.

We have examined the role of both the firm's response to sequential exercise and the rivalry among hoarders in limiting the attractiveness of hoarding warrants, but our research points to a variety of unresolved questions. For example, a rational model of trading in which hoarding can arise endogenously is needed to understand more fully both the valuation of warrants, convertible securities, and other assets, such as sinking-fund bonds (see Dunn and Spatt [10]), and the determination of the distribution of ownership of such assets. In an interesting study of callable convertible preferred stock, Dunn and Eades [8] document that conversion occurs over a long period of time but argue that the observed conversion paths are not consistent with the types of sequential strategies examined in the literature. The empirical importance of sequential exercise strategies by holders of warrants and convertible securities and the underlying observed ownership distributions need to be systematically examined.

Appendix

In this Appendix, we further explore the case of a monopoly warrantholder under the alternative assumptions that exercise proceeds are used to repurchase equity or rescale the firm's project. The valuation of the warrants is maximized by a strategy that minimizes the stock price because any gain on the warrants comes at the expense of equity.

If we assume that the exercise proceeds from the exercise of k warrants are reinvested in the project (proportionately rescaling it upward), $P(k)$ satisfies (2) so that

$$P'(k) = q_1\left(\frac{E}{n+m}\,\frac{V_1 - V}{V}\right) + q_2\left(\frac{V_2\left(\frac{nE}{V} - 1\right)}{(n+k)^2}\right). \tag{A1}$$

In this example, the block solution of exercising no warrants dominates exercising all the warrants early. ($V_2/n < E$ implies that $P(0) > P(m)$.) If $\frac{V}{n} \leq E$, then $P(k)$ is strictly increasing and the optimal sequential exercise strategy is the corner $k = 0$. If $\frac{V}{n} > E$, then $P'(k)$ is increasing and a necessary and sufficient condition for $P(k)$ to have an interior minimum (i.e., a sequential exercise strategy) is $P'(0) < 0$. Therefore, sequential strategies are optimal if and only if

$$\frac{q_1 E}{n+m}\left(\frac{V_1}{V} - 1\right) + \frac{q_2\left(\frac{V_2}{V}E - \frac{V_2}{n}\right)}{n} < 0. \tag{A2}$$

If (A2) holds, the optimal extent of early exercise is the k that solves $P'(k) = 0$, i.e.,

$$k = -n + \left[\frac{q_2 V_2\left(1 - \frac{nE}{V}\right)}{q_1 \frac{E}{n+m}\left(\frac{V_1}{V} - 1\right)}\right]^{1/2}. \tag{A3}$$

Condition (A2) simplifies to $V/n > E$ provided that the degree of potential dilution is sufficiently great, i.e., $m > m^*$. In fact, large enough m implies that $P(k) < E$, illustrating that, in some circumstances, the exercise of some of the warrants can be optimal under a sequential exercise strategy when the warrant is out of the money. We examine the price of a share of equity assuming that k warrants are exercised early. Immediately prior to using the exercise proceeds, the firm's capitalization is composed of $n + k$ shares and $m - k$ warrants. Suppose the exercise proceeds, kE, are used either to rescale the firm's project or to repurchase both warrants and equity so as to leave the ratio of shares to warrants fixed at $(n + k)/(m - k)$. Clearly, these two alternatives result in an identical price for the stock given k. This is used to prove the following proposition.

PROPOSITION A1: *When the firm uses the warrant exercise proceeds to rescale the firm's project, the payoff to the monopoly warrantholder is at least as large as when the firm uses the exercise proceeds to repurchase equity.*

Proof: We let $P_A(k)$ be the stock price associated with repurchase as defined in (1) and let $P_B(k)$ be the stock price associated with expanding the firm's scale as defined in (2). Since the total warrant and equity payoffs sum to a fixed amount, we can prove the result by assuming that $P_A(k) < P_B(k)$ and obtaining a contradiction. If the exercise proceeds are used to rescale the firm's project, then the price of a share from equation (2) is identical to the price that would result from using the proceeds to repurchase equity and warrants in the ratio of $n + k$ shares to $m - k$ warrants (the capitalization after the exercise of k warrants). If α and β are the proportions of the exercise proceeds used to repurchase warrants and equity, respectively, then

$$P_B(k) = q_1 \left(\frac{V_1 + \left(m - k - \dfrac{\alpha k E}{Z(k)}\right)E}{n + m - \dfrac{\alpha k E}{Z(k)} - \dfrac{\beta k E}{P_B(k)}} \right) + q_2 \left(\frac{V_2}{n + k - \dfrac{\beta k E}{P_B(k)}} \right), \tag{A4}$$

where $Z(k)$ is the price of the warrant for the exercise policy k. Using equations (1) and (A4), as well as $P_A(k) < P_B(k)$ and

$$\frac{V_2}{n + k - \dfrac{kE}{P_A(k)}} > \frac{V_2}{n + k - \dfrac{\beta k E}{P_B(k)}}$$

(which is implied by $\beta < 1$), it follows that

$$\frac{V_1 + \left(m - k - \dfrac{\alpha k E}{Z(k)}\right)E}{n + m - \dfrac{\alpha k E}{Z(k)} - \dfrac{\beta k E}{P_B(k)}} > \frac{V_1 + (m - k)\,E}{n + m - \dfrac{kE}{P_A(k)}}.$$

This implies that the warrantholder is better off in state one (and thus overall) when the exercise proceeds are used to rescale the firm's project than when the proceeds are used only to repurchase equity. However, $P_B(k) > P_A(k)$ implies that shareholders are also better off under the rescaling assumption. This is a contradiction since the equity and warrants are the only claims issued by the firm. Q.E.D.

This leads to the following corollary.

COROLLARY A1: *If sequential exercise is optimal for a monopolist when the exercise proceeds are used to repurchase shares of equity, then it is also optimal when the exercise proceeds are reinvested in the firm's project.*

Proof: Sequential exercise is optimal for policy i ($i = A$ or $i = B$) if and only if there exists a k such that $P_i(k) < \min[P_i(0), P_i(m)]$, i.e., there exists an exercise path that has a lower stock price (and larger average warrant value) than any block-exercise policy. However, the proof of Proposition A1 implies that $P_A(0) = P_B(0)$, $P_A(m) = P_B(m)$, and $P_B(k) \leq P_A(k)$. The corollary then follows directly. Q.E.D.

However, the converse of the corollary is false. For some parameter values, sequential exercise is optimal if the exercise proceeds are reinvested in the firm's project, but not if the proceeds are used to repurchase stock. An example that illustrates this point occurs when $q_1 = q_2 = 0.5$, $V_1 = 2500$, $V_2 = 1500$, $m = 100$, $n = 100$, and $E = 16.3$. Then the optimal exercise policy for the repurchase case is $k = 0$ ($P_A(0) = 17.825$) but for the rescaling case is approximately $k = 17$ ($P_B(17) = 17.797$).

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