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Source: *The Journal of Finance*, Vol. 43, No. 2 (Jun., 1988), pp. 357-373
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2328465

THE JOURNAL OF FINANCE • VOL. XLIII, NO. 2 • JUNE 1988

The Effect of Taxes and Depreciation on Corporate Investment and Financial Leverage

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ABSTRACT

This paper provides an analysis of the effect of corporate and personal taxes on the firm's optimal investment and financing decisions under uncertainty. It extends the DeAngelo and Masulis capital structure model by endogenizing the firm's investment decision. The authors' results indicate that, when investment is allowed to adjust optimally, the existing predictions about the relationship between investment-related and debt-related tax shields must be modified. In particular, the authors show that increases in investment-related tax shields due to changes in the corporate tax code are not necessarily associated with reductions in leverage at the individual firm level. In cross-sectional analysis, firms with higher investment-related tax shields (normalized by expected earnings) need not have lower debt-related tax shields (normalized by expected earnings) unless all firms utilize the same production technology. Differences in production technologies across firms may thus explain why the empirical results of recent cross-sectional studies have not conformed to the predictions of DeAngelo and Masulis.

ONE OF THE MOST fundamental results in corporate finance is the Modigliani-Miller [17] theorem that, in a perfect and competitive capital market, the financing decisions of the firm are irrelevant.[1] An immediate corollary to this result is that, under these same conditions, the real and financial decisions of the firm are independent and therefore can be made separately. However, when market imperfections such as taxes are introduced, the irrelevance result may no longer hold. Under these conditions, treating the real and financial decisions of the firm as independent may not be justified. Nevertheless, it remains a common practice for financial economists in such situations to hold the firm's investment decisions fixed when analyzing its financing decisions.[2]

Although the assumed independence between production and finance has led to the development of intuitively appealing and insightful results, there still

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[1] See Stiglitz [22] and Fama [10] for a discussion of the general conditions under which the M-M [17] theorem holds.

[2] See, for example, Modigliani and Miller [18], Kraus and Litzenberger [13], Scott [20], Brennan and Schwartz [4], and DeAngelo and Masulis [8].

remains a question as to whether simultaneous optimization in both the real and financial sectors of the economy will generate new insights regarding firm behavior or perhaps overturn or modify existing results. Hite [12], for example, modifies the tax-adjusted valuation model of Modigliani and Miller [18] by taking into account the effect of leverage on the firm's optimal production decision. He shows that an increase in financial leverage will reduce the "user cost of capital" and thus lead to an increase in the optimal output level of the firm. He concludes, therefore, that "the increment to the NPV of the firm due to the M-M 'pure leverage effect' which allows only debt to vary is strictly less than the 'total leverage effect' which requires all inputs and outputs to adjust optimally" (p. 191, Proposition 4). Hite's [12] model, however, places implicit limits on the amount of debt financing the firm can obtain by assuming away default and thus falls short of fully integrating the real and financial decisions of the firm.

More recently, the literature on capital structure and taxes has been largely influenced by the equilibrium analysis of Miller [16].[3] DeAngelo and Masulis [8], for example, show that the existence of non-debt tax shields, such as depreciation deductions and investment tax credits, together with an asymmetric corporate tax code that does not rebate losses, is sufficient to overturn the leverage-irrelevance proposition of Miller [16]. Contrary to Hite [12], however, the DeAngelo-Masulis [8] model predicts an inverse relationship between the level of investment-related tax-shield substitutes for debt (a proxy for Hite's [12] degree of capital intensity) and the degree of financial leverage employed by the firm. While the DeAngelo-Masulis [8] analysis may be viewed as a partial recognition of the interaction between the real and financial variables of the firm, it falls short of fully incorporating the productive side of the economy since the firm's investment decisions and non-debt tax shields are exogenous to the model.

The purpose of this paper is to analyze the effect of corporate and personal taxes on the firm's optimal investment and financing decisions under uncertainty, thereby extending the DeAngelo-Masulis [8] capital structure model. By incorporating investment, the optimal level of non-debt tax shields is determined endogenously, thus providing a more realistic look at the interaction between production and finance. We also conduct a comparative-statics analysis that examines the relationship between the level of investment-related tax shields and the degree of financial leverage employed by the firm. Of particular interest are the extensions and modifications to the predictions of DeAngelo and Masulis [8] that one can make when investment is not held fixed.

In a recent paper, Dotan and Ravid [9] endogenize the capital structure and production decisions of the firm in a model that includes corporate taxes and bankruptcy costs. While their model does not include personal taxes, they obtained results supporting the predictions of DeAngelo and Masulis [8]. In particular, they found that the optimal investment level and corresponding depreciation tax shields were negatively related to an *exogenous* change in the level of debt. Their results, however, are driven by the fact that an increase in

[3] See, for example, DeAngelo and Masulis [8], Taggart [23], Barnea, Haugen, and Senbet [1], Dammon [7], and Ross [19].

investment, while increasing the level of non-debt tax shields, has little or no effect on the level of the firm's output in the relevant states of nature. Thus, it is not surprising that their results are similar to those of DeAngelo and Masulis [8], where output is simply held fixed.

The results of our model are substantially different from those of Dotan and Ravid [9] and include the results of DeAngelo and Masulis [8] and Hite [12] as special cases. First, we show that increases in allowable investment-related tax shields due to changes in the corporate tax code are not necessarily associated with reductions in leverage at the individual firm level when investment is allowed to adjust optimally. The effect of an increase in allowable investment-related tax shields on firm leverage depends critically on the tradeoff between the "substitution effect" advanced by DeAngelo and Masulis [8] and the "income effect" associated with an increase in optimal investment. Second, in cross-sectional analysis, firms with higher investment-related tax shields (normalized by expected earnings) need not have lower debt-related tax shields (normalized by expected earnings) if firms employ different production technologies. However, for those firms with identical production technologies and, therefore, perfectly correlated pre-tax income, the relationship between investment-related tax shields and financial leverage is strictly negative. These results modify some of the predictions of DeAngelo and Masulis [8].

The paper is organized as follows. In Section I, we introduce investment into the DeAngelo-Masulis [8] capital structure model and describe the basic economic environment. In Section II, we derive the conditions for an optimal investment and financing plan and examine the effect of leverage on firm value. In Section III, we conduct a comparative-statics analysis and compare our results with those of DeAngelo and Masulis [8]. Section IV summarizes our findings and concludes the paper. All proofs appear in the Appendix.

I. The Model

To analyze the investment and financing decisions of firms, we employ a simple two-date, state-preference model similar to that of DeAngelo and Masulis [8]. In this model, firms make their investment and financing decisions today, at time zero, and all uncertainty is resolved tomorrow, at time one. To introduce uncertainty, we assume that the pre-tax output of the firm at time one, $x(\theta)$, is a function of the firm's capital investment at time zero, K, and the random state of nature, θ, prevailing at time one. Formally,

$$x(\theta) = f(K, \theta), \tag{1}$$

where f is twice-continuously differentiable with $f_K > 0$ and $f_{KK} < 0$, $f(0, \cdot) = 0$, and $\theta \in [\underline{\theta}, \overline{\theta}]$. In the DeAngelo-Masulis [8] framework, the firm's investment decision, K, is assumed to be given *exogenously*. Here, however, K is treated as an *endogenous* choice variable.

The firm finances its capital investment, K, at time zero by selling corporate debt and equity claims to the output it produces at time one, $x(\theta)$. Debt issued by the firm carries a face value of F (the firm's time-zero capital structure decision variable). Assuming that debt payments are fully deductible in computing taxable corporate income,[4] the total pre-personal-tax payoffs to bondholders and stockholders of the firm are

$$y_b(\theta) \equiv \min[F, x(\theta)] \tag{2}$$

and

$$y_e(\theta) \equiv \max[0, x(\theta) - F - \max[0, x(\theta) - F - \gamma K]t^c], \tag{3}$$

respectively, where t^c is the corporate tax rate, $\gamma \geq 0$ is the firm-specific rate of capital depreciation used for tax purposes,[5] and $\max[0, x(\theta) - F - \gamma K]$ is the firm's taxable income. Note that the corporate tax code we employ gives asymmetric treatment to gains and losses. While gains are assumed to be taxed at a constant rate of t^c, all tax losses are assumed to provide a tax rebate of zero.[6]

Figure 1 illustrates graphically how the pre-tax output of the firm, $x(\theta)$, is distributed to its three classes of claimants: bondholders, $y_b(\theta)$, stockholders, $y_e(\theta)$, and the government, $y_g(\theta)$. The pre-personal-tax payoff to bondholders is given by equation (2) and is represented by the concave function $y_b(\theta)$ in Figure 1. For a given level of the capital input, θ_1 represents the state of nature where the firm's output exactly equals the face value of its debt. That is, for a given value of K, θ_1 satisfies

$$x(\theta_1) \equiv f(K, \theta_1) = F. \tag{4}$$

At output levels below $x(\theta_1)$, the firm is obviously bankrupt, while, at output levels above $x(\theta_1)$, the firm remains solvent.

The equityholders of the firm hold a residual claim on the firm's assets and, therefore, receive nothing in the event of bankruptcy. As the firm's pre-tax

[4] Throughout the analysis, we will treat both principal and interest as taxable, or deductible, in order to remain consistent with the assumptions of DeAngelo and Masulis [8]. The major conclusions of the paper would survive, however, under the more realistic assumption that principal repayments are nontaxable to the investor and nondeductible to the issuer.

[5] Without loss of generality, we assume that the economic rate of capital depreciation is unity for all firms. The tax depreciation rate, γ, may differ across firms, however, because of differences in asset composition (i.e., depreciable versus nondepreciable assets) and depreciation methods (i.e., accelerated versus straight-line depreciation). While investment tax credits could also be added to the model, we do not do so because investment tax credits have the same qualitative implications for the investment and financing decisions of the firm as depreciation. Furthermore, the investment tax credit has been eliminated by the Tax Reform Act of 1986.

[6] This asymmetry, while it captures the essential features of the current U.S. corporate tax code, abstracts from it in one important respect. In reality, carry-back and carry-forward provisions make it possible for the firm to utilize current losses to recapture taxes paid in several past years or to reduce taxes to be paid in several future years. As a result, the probability that the firm's excessive tax shields in a given year will go unutilized may be substantially reduced. Unfortunately, by using a one-period model, we are unable to adequately incorporate these carry-back and carry-forward provisions. However, the fact that carry-forwards involve a time-value loss and expire after a fixed number of years if unutilized suggests that the existence of excessive tax shields is not costless. Consequently, the assumption that excessive tax shields produce no tax rebates may not be as distorting as one may have first thought. Moreover, our treatment here is consistent with the treatment given gains and losses in DeAngelo and Masulis [8]. For a discussion of the effect of tax-loss carry-forwards on the firm's future investment and financing decisions, see Cooper and Franks [6].

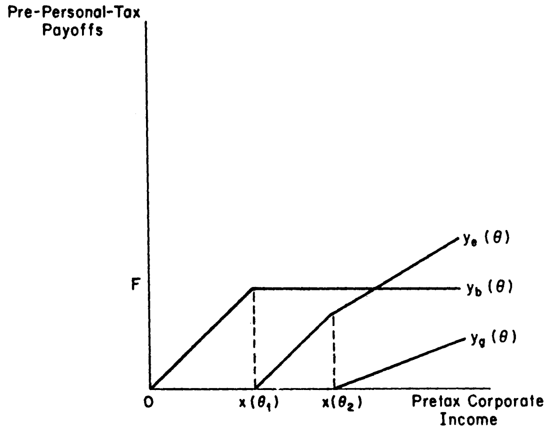

Figure 1. Pre-personal-tax payoffs to stockholders, $y_e(\theta)$, bondholders, $y_b(\theta)$, and the government, $y_g(\theta)$, as a function of the firm's pre-tax income, $x(\theta)$. The bankruptcy point is $x(\theta_1) = F$, where F is the face value of debt, and the zero-taxable-income point is $x(\theta_2) = F + \gamma K$, where γK is the value of non-debt tax shields.

output, $x(\theta)$, rises above the bankruptcy point, however, the payoff to shareholders increases monotonically. Initially the increase is one for one with increases in $x(\theta)$, reflecting the fact that corporate taxes are zero over the range of nonpositive taxable income, $x(\theta) \leq x(\theta_2)$. Here θ_2 represents the state of nature, given the level of the capital input, where the firm's taxable income is identically zero. That is, for a given value of K, θ_2 satisfies

$$x(\theta_2) \equiv f(K, \theta_2) = F + \gamma K. \tag{5}$$

At output levels above $x(\theta_2)$, the firm is liable for paying corporate taxes equal to the corporate tax rate, t^c, times the firm's taxable income level, $x(\theta) - x(\theta_2)$. This tax-liability claim on the firm can be expressed as

$$y_g(\theta) \equiv \max[0, x(\theta) - F - \gamma K]t^c \tag{6}$$

and is depicted in Figure 1 as a convex function reminiscent of the payoff on a portfolio of t^c call options written on the firm's equity with an exercise price of $x(\theta_2)$.[7] Since the corporate tax liability in equation (6) increases linearly at the rate of t^c for output levels greater than $x(\theta_2)$, the payment to equityholders will also increase linearly over this range, but at the rate of $(1 - t^c)$. The equity stream just described is given by equation (3) and is depicted as the piecewise-linear function labeled $y_e(\theta)$ in Figure 1.

At the personal level, we will assume that debt and equity income are differentially taxed, with the former being fully taxable and the latter being tax

[7] This option characteristic of the government's tax claim has previously been used in the finance literature to help explain a variety of financial phenomena. Green and Talmor [11] use the option characteristics of the government's tax claim to analyze its effect on the risk-incentive investment choices of the firm. Madj and Myers [15] have attempted to apply the concepts of option pricing to the valuation of the government's tax claim on risky assets when tax-loss carry-backs and carry-forwards are explicitly taken into account. Finally, Smith and Stulz [21] use the option characteristics of the government's tax claim to rationalize corporate hedging.

exempt. While it is not crucial to the results that equity income be strictly tax exempt, it is crucial that equity income be preferentially taxed relative to debt.[8] If not, the optimal capital structure for all firms in the economy, absent any other market imperfections, would be one hundred percent debt financing.[9] The same result, of course, would obtain in both the Miller [16] and DeAngelo-Masulis [8] models.

Following DeAngelo and Masulis [8], we assume that the capital markets are complete enough to uniquely value or to span the state-contingent before-personal-tax cash flows to debt, $y_b(\theta)$, and equity, $y_e(\theta)$. Letting $q(\theta)$ and $p(\theta)$ denote the current market prices of a unit of before-personal-tax debt and equity income, respectively, to be delivered in state θ, the net market value of the firm, V, can be written as

$$\begin{aligned} V &= B + S - K \\ &= \int_{\underline{\theta}}^{\overline{\theta}} q(\theta) y_b(\theta)\, d\theta + \int_{\underline{\theta}}^{\overline{\theta}} p(\theta) y_e(\theta)\, d\theta - K \\ &= \int_{\underline{\theta}}^{\overline{\theta}} p(\theta)[y_b(\theta)(1 - t^P(\theta)) + y_e(\theta)]\, d\theta - K, \end{aligned} \tag{7}$$

[8] The Tax Reform Act of 1986 reduces but does not eliminate the preferential treatment of equity income relative to interest income. Under the Tax Reform Act of 1986, capital gains, dividends, and interest are all taxed as ordinary income. Previously, long-term capital gains had been taxed at preferential rates. The major advantage of equity income remains intact, however, in that capital gains and losses are not taxed until they are realized by the investor. As pointed out by Constantinides [5], this is equivalent to the government conferring upon equityholders a timing option—to realize losses immediately and defer gains indefinitely.

Regarding the taxation of dividends and interest, Litzenberger [14] has argued that the Miller [16] equilibrium does not hold in a multiperiod framework if markets are complete and dividends and interest are taxed at the same rate for the marginal investor in both bonds and stocks, even when capital gains are taxed preferentially. Holding investment fixed, an increase in leverage requires the firm to use the corporate tax savings either to increase its distribution of ordinary income (i.e., dividends plus interest) or to increase its share repurchases. Litzenberger [14] argues that in either case the "gain from leverage" is positive and cannot be eliminated by competitive supply adjustments alone. However, an increase in leverage increases the firm's interest expense by more than the corporate tax savings it generates. Therefore, if share repurchases are held constant or increased (as in Litzenberger's [14] analysis), dividend payments must be *reduced* to satisfy the sources-and-uses-of-funds constraint. Since dividends are taxed more heavily than capital gains, firm value will rise. An interesting question is whether Litzenberger's [14] results would continue to hold if the firm held its dividend policy fixed as it changed its financial leverage. In any case, the problem described by Litzenberger [14] would not arise in a single-period model such as ours and, with the existence of non-debt tax shields that limit the deductibility of interest, would not prevent our results from holding in a multiperiod context.

[9] Actually, if debt and equity were taxed identically at the personal level, the firm would find it optimal to issue enough debt to drive its tax liability to zero in all states. According to equation (6), this would occur as long as the promised face payment on the firm's debt, F, were greater than or equal to $x(\overline{\theta}) - \gamma K$, where $x(\overline{\theta})$ is the maximum possible output of the firm across all states. However, as long as $F < x(\overline{\theta})$, the payoff to equityholders will not be zero in all states (see equation (3)), and, therefore, the value of equity will not be zero. As a result, any promised face payment, F, that satisfies

$$x(\overline{\theta}) - \gamma K \leq F < x(\overline{\theta})$$

will maximize firm value and, at the same time, leave the firm with less than one hundred percent where B and S denote the current market values of the firm's debt and equity securities, respectively, and $t^p(\theta) \equiv 1 - q(\theta)/p(\theta)$ is the marginal tax rate of the investor who, at the margin, is indifferent between debt and equity income in state θ. The possibility that the critical personal tax rate, $t^p(\theta)$, is state dependent arises because the aggregate levels of debt and equity income may differ across states in equilibrium.[10]

II. The Optimal Financing and Investment Decisions

To determine the optimal financing and investment decisions, value-maximizing firms differentiate equation (7) with respect to F (the face value of debt) and K (the capital investment), yielding[11]

$$V_F = t^c \int_{\theta_2}^{\bar{\theta}} p(\theta)\, d\theta - \int_{\theta_1}^{\bar{\theta}} p(\theta) t^p(\theta)\, d\theta \tag{8}$$

and

$$V_K = \int_{\underline{\theta}}^{\bar{\theta}} p(\theta) f_K(\theta)\, d\theta - \int_{\underline{\theta}}^{\theta_1} p(\theta) t^p(\theta) f_K(\theta)\, d\theta$$
$$- t^c \int_{\theta_2}^{\bar{\theta}} p(\theta)[f_K(\theta) - \gamma]\, d\theta - 1, \tag{9}$$

where $f_K(\theta)$ is the marginal product of capital in state θ and γ is the firm-specific rate of capital depreciation used for tax purposes. For the optimal values of F and K, equations (8) and (9) are identically zero.

The marginal value of debt, given by equation (8), is equal to the present value of the corporate tax savings that occurs over the firm's taxpaying states of nature (the first term), less the present value of the additional personal taxes paid over the firm's nondefault states of nature (the second term). The second term in equation (8) does not include the firm's default states of nature because additional promised payments to bondholders have no cash flow impact and, therefore, no tax effects over these default states. At a capital structure optimum, equation (8) will be identically zero, and, at the margin, the present value of the corporate tax savings will exactly offset the present value of the personal tax dissavings. If debt and equity income were taxed identically, however, the second term in

debt in its capital structure. If the corporate tax code fully rebated losses, however, or if the firm had no tax-shield substitutes for debt, as in the Miller [16] framework, the firm's optimal capital structure would indeed involve one hundred percent debt financing.

[10] If investors are risk neutral and face constant marginal tax rates, as opposed to progressive tax schedules, then the critical personal tax rate, $t^p(\theta)$, will be the same across all states. This is the special case considered by DeAngelo and Masulis [8]. Our model accommodates this situation but is more general, allowing for the possibility that investors are risk averse and face progressive tax schedules. The results we obtain are invariant to these assumptions, however.

[11] For simplicity, we will assume that $x(\theta)$ is monotonically increasing in θ over the set of possible states $[\underline{\theta}, \bar{\theta}]$ with $0 \le x(\underline{\theta}) \le x(\bar{\theta}) < \infty$.

equation (8) would vanish and, as predicted by Modigliani and Miller [18], all firms would have an optimal capital structure composed entirely of debt. (See footnote 9.)

As indicated by equation (8), the capital structure decision is not irrelevant in determining firm value. To be irrelevant, V_F would have to be identically zero for all possible values of the promised face payment, F. However, because the firm has investment-related tax shields (e.g., depreciation deductions) that reduce the firm's ability to utilize its interest deductions, it is impossible for V_F to be independent of the value of F. Consequently, there will exist some capital structure choices that are strictly preferred to others. This result, initially formulated by DeAngelo and Masulis [8], implies that the real and financial decisions of the firm are inseparable and, therefore, must be made simultaneously.[12]

Equation (9) represents the marginal value of the capital input and is composed of four terms. The first term measures the present value, over all states, of the pre-tax marginal product of capital. Over the firm's default states of nature, the marginal product of capital accrues to bondholders and, therefore, is taxed at the personal level. The present value of this personal tax liability is given by the second term in equation (9). When the firm is solvent but not in a taxpaying position, the marginal product of capital escapes taxation at both the corporate and personal levels. In its taxpaying states of nature, however, the firm is liable for paying taxes on the marginal product of capital less a depreciation deduction. The present value of this incremental corporate tax liability is given by the third term in equation (9). The last term in equation (9) simply represents the one-unit marginal cost of the capital input at time zero.

Note that the marginal value of capital, given by equation (9), is a function of the firm's bankruptcy and taxpaying states of nature (i.e., θ_1 and θ_2) and, therefore, is not independent of the firm's degree of financial leverage. The optimal investment and capital structure decisions are clearly interrelated and must be made such that equations (8) and (9) are simultaneously zero. In

[12] Unlike the DeAngelo-Masulis [8] model, however, the current model implies an optimal capital structure for the firm even without any tax-shield substitutes for debt or other debt-related costs. Under these conditions, the firm's taxpaying states coincide with its nonbankruptcy states (i.e., $\theta_1 = \theta_2$), and equation (8) becomes

$$V_F = \int_{\theta_1}^{\bar{\theta}} p(\theta)[t^c - t^p(\theta)]\, d\theta.$$

In order for capital structure to be irrelevant, V_F must be identically zero for any arbitrary choice of the promised face payment, F. However, this would require that $V_{FF} = 0$ for all F or, equivalently, that $t^c = t^p(\theta)$ for all θ. However, with progressive personal taxes and uncertainty, the critical personal tax rate, $t^p(\theta)$, is state dependent and may be greater than, equal to, or less than the corporate tax rate, t^c, on a state-by-state basis. Consequently, in equilibrium, optimal debt-equity ratios will emerge at the individual firm level. These optimal debt-equity ratios will differ across firms and will depend upon the covariability between the firm's pre-tax cash flows, $x(\theta)$, and the critical personal tax rate, $t^p(\theta)$. This result, which relates the firm's optimal capital structure to characteristics of its operating cash flows, was first formulated by Dammon [7] and is in direct opposition to the predictions of the Miller [16] equilibrium. A similar result was subsequently obtained by Ross [19].

equilibrium, a firm that employs at least some leverage will in general not employ the same level of investment that would be optimal if the firm were unlevered. Consequently, the increase in the net market value of the firm due to the DeAngelo-Masulis [8] "pure leverage effect", which allows only debt to vary, will be strictly less than the "total leverage effect", which requires both the real and financial variables of the firm to adjust optimally. Hite [12] has made this same argument in the context of the Modigliani-Miller [18] tax-correction model.

III. Comparative Statics

In this section, we examine the relationship between the optimal investment and financing decisions in more detail. We begin with a proposition first suggested by DeAngelo and Masulis [8]. In their well-known paper, DeAngelo and Masulis [8] hypothesize a negative relationship between the (exogenous) level of investment-related tax shields and the amount of debt employed by the firm (as measured by the optimal promised face payment, F^*). We formalize this hypothesis in the following proposition. All proofs appear in the Appendix.

PROPOSITION 1: *Ceteris paribus, increases in allowable investment-related tax shields due to changes in the corporate tax code, such as an increase in the depreciation rate, γ, will reduce the total amount of debt employed by the firm (as measured by the optimal promised face payment, F^*) (DeAngelo and Masulis [8]).*

The intuition behind Proposition 1 is quite appealing. An increase in the rate of capital depreciation, γ, holding investment fixed, increases the probability that the firm's interest deduction, F^*, will be partially or totally redundant in sheltering taxable corporate income. As a result, the after-tax cost of debt will increase relative to equity at the margin, thus providing an incentive for the firm to substitute equity for debt in its capital structure until condition (8) is again equal to zero.

Proposition 1, of course, is based on the assumption that the firm holds fixed its production (i.e., capital investment) decision while changing its capital structure decision. Yet, according to our earlier analysis, the real and financial decisions of the firm are interrelated. This raises the possibility that interaction effects may overturn or modify the DeAngelo-Masulis [8] hypothesis (i.e., Proposition 1). Therefore, we now want to provide the appropriate comparative statics. In particular, we want to examine the effect that an increase in the depreciation rate has on the firm's optimal production and capital structure decisions. To do so, however, requires that we be more specific about the form of the firm's production function, $f(K, \theta)$. To be as realistic as possible and to maintain tractability, we will assume that the firm's output, $x(\theta)$, is given by

$$x(\theta) = K^{\alpha} g(\theta), \tag{10}$$

where $\theta \in [\underline{\theta}, \overline{\theta}]$, $0 < \alpha < 1$, and $g' > 0$. Equation (10) implies a stochastic decreasing-returns-to-scale technology, where a change in the level of the capital input alters the scale of output but not the pattern of output across states. This helps facilitate the idea that each firm represents a separate production technology for which an optimal investment and financing plan must be determined.

PROPOSITION 2: *An increase in the depreciation rate, γ, will lead to an unambiguous increase in the optimal level of the capital input, K^*, and corresponding output of the firm, $x(\theta)$.*

Proposition 2 supports the common notion that more liberal depreciation schedules encourage investment, both at the individual firm level and in the aggregate. This, in turn, leads to an increase in production and a higher market value for the corporate sector. As a result, the optimal level of investment-related tax shields will increase. However, because output is also expanding, it is not immediately apparent how the level of investment-related tax shields will change relative to corporate earnings. The following proposition formally shows that an increase in the depreciation rate, γ, leads to an increase in the level of investment-related tax shields, both as a percentage of expected earnings and in absolute magnitude.

PROPOSITION 3: *An increase in the depreciation rate, γ, will lead to an unambiguous increase in the optimal level of investment-related tax shields, γK^*, both as a percentage of expected earnings and in absolute magnitude.*

Now that we have examined the effects of a change in the depreciation rate on the firm's investment decisions, we are ready to analyze its effect on the financing decisions. The change in the value of the optimal promised face payment, F^*, following an increase in the depreciation rate, γ, depends upon the relative magnitudes of two countervailing effects. The first effect is due to the fact that, as investment-related tax shields increase, other things held constant, interest deductions become less valuable in sheltering taxable corporate income. This is the "substitution effect" analyzed by DeAngelo and Masulis [8]. It measures the change in the value of the optimal promised face payment, F^*, following an increase in the depreciation rate, γ, holding output fixed as in Proposition 1. Clearly, the "substitution effect" is negative.

The second effect, which the DeAngelo-Masulis [8] model does not capture, arises because of the increase in firm earnings due to higher capital investment. (See Proposition 2.) As the output of the firm increases, interest deductions become more valuable as an offset to taxable corporate income at the margin. This produces an offsetting "income effect". It measures the change in the value of the optimal promised face payment, F^*, following an increase in the depreciation rate, γ, exclusively due to higher corporate earnings. Clearly, the "income effect" is positive. The net effect of an increase in the depreciation rate, γ, on the optimal promised face payment, F^*, is ambiguous, however, and depends upon the relative magnitudes of the "income and substitution effects".

The following proposition, stated in terms of elasticities, identifies the "income and substitution effects" and shows how the elasticity of F^* to γ, $e_{F\gamma}$, can be expressed as a weighted average of two other important elasticity measures: (a) $e_{x\gamma}$, the elasticity of pre-tax corporate earnings, x, to γ and (b) $e_{d\gamma}$, the elasticity of the firm's depreciation tax shields, γK^*, to γ. It implies that increases in investment-related tax shields may not be associated with decreases in interest tax shields at the corporate level, contrary to the predictions of DeAngelo and Masulis [8].[13]

PROPOSITION 4: *An increase in the depreciation rate, γ, will lead to an increase (decrease) in the value of the optimal face payment, F^*, if the "income effect", $(1-z)e_{x\gamma} \geq 0$, is greater (less) than the "substitution effect", $ze_{d\gamma} \leq 0$, in absolute value. That is,*

$$e_{F\gamma} = (1 - z)e_{x\gamma} + ze_{d\gamma}, \tag{11}$$

where $e_{x\gamma} = \alpha e_{K\gamma} \geq 0$, $e_{d\gamma} = (1 + e_{K\gamma}) \geq 0$, $z = -[\gamma/F^*]V_{F\gamma}/V_{FF} \leq 0$, and $e_{K\gamma} = [\gamma/K^*]dK^*/d\gamma \geq 0$ is the elasticity of the optimal capital investment, K^*, to γ.

Proposition 4 has some very interesting features. First, note that $e_{F\gamma} = z \leq 0$ if $e_{K\gamma} = 0$. This, of course, corresponds to the special case considered by DeAngelo and Masulis [8] where corporate investment, K, is assumed to be fixed. (See Proposition 1.) The interpretation of z, therefore, is that it is the elasticity of F^* to γ when the firm's production decisions are taken as given.[14] The smaller z is in absolute value, the larger will be the value of $e_{F\gamma}$ (holding $e_{K\gamma}$ fixed) since $e_{x\gamma} < e_{d\gamma}$. In the limit, as z approaches zero, $e_{F\gamma} = e_{x\gamma} \geq 0$. This corresponds to the special case in which the firm's tax shields, if unutilized, are fully marketable. In this case, there is no loss associated with having excessive tax shields, and, therefore, the "substitution effect" of DeAngelo and Masulis [8] vanishes completely. What remains is a pure "income effect" that is non-negative and thus consistent with the predictions of Hite's [12] model, where investment, output, and leverage are all positively related. Unlike Hite [12], however, we have allowed for the existence of personal taxes and risky debt. Proposition 4 thus generalizes the results of both DeAngelo and Masulis [8] and Hite [12] and reconciles the conflicting predictions of the two models.

While the value of the optimal promised face payment, F^*, may either rise or fall following an increase in the depreciation rate, γ, its value as a percentage of expected earnings is strictly decreasing in γ. We formalize this claim in the following proposition.

PROPOSITION 5: *An increase in the depreciation rate, γ, will lead to an unambiguous decline in the ratio of the optimal promised face payment, F^*, to expected earnings.*

Together, Propositions 3 and 5 imply that an increase in the depreciation rate, γ, will lead to an unambiguous increase in the ratio of investment-related tax shields to expected earnings and to an unambiguous decline in the ratio of debt-related tax shields to expected earnings. Similar results can also be shown to hold for a decrease in the corporate tax rate. This negative relationship emerges because, as a percentage of expected earnings, interest tax shields are substitutes

[13] Indeed, DeAngelo and Masulis [8] recognize that "potential interactions of investment and financing decisions can technically overturn [Proposition 1] when investment is not held fixed" (p. 21, footnote 23).

[14] In fact, the value of z can easily be obtained from Proposition 1 by simply multiplying both sides of equation (A1) by γ/F^*.

for non-debt tax shields at the individual firm level. The intuition behind this result is that the normalization of investment-related and debt-related tax shields by expected earnings essentially neutralizes the "income effect" described in Proposition 4, leaving only the "substitution effect" to influence the tax-shield tradeoff. While Proposition 5 is consistent with the predictions of DeAngelo and Masulis [8], one should not lose sight of the fact that their original hypothesis (as given by Proposition 1) may still be overturned by interactions of investment and financing decisions when investment is not held fixed. (See Proposition 4.)

Propositions 3 and 5 also suggest some cross-sectional predictions. For those firms that utilize the same production technology (i.e., those firms with perfectly correlated pre-tax output), the cross-sectional relationship between investment-related tax shields (normalized by expected earnings) and debt-related tax shields (normalized by expected earnings) will be strictly negative. A similar prediction has been made by DeAngelo and Masulis [8] (p. 21, H3), but their hypothesis was derived by holding the firm's investment decisions fixed and has been incorrectly interpreted by many (including DeAngelo and Masulis themselves) as an explanation for why firms in *different* industries have vastly different capital structures. However, firms in different industries are likely to have different production technologies and, therefore, different patterns of pre-tax output across states. For these firms, the relationship between investment-related tax shields (normalized by expected earnings) and debt-related tax shields (normalized by expected earnings) need not be negative. Firms with higher investment-related tax shields (as a percentage of expected earnings) may also employ a higher degree of financial leverage if the interest payments generate tax rebates for the firm in those states of nature where aggregate consumption and/or personal tax rates are low (i.e., state prices are high). This distinction is important because it has been documented empirically that the cross-sectional relationship between investment-related tax shields and financial leverage is not negative.[15] However, unless the technological differences across firms have been adequately controlled for in these empirical studies, there is no reason to believe that the relationship should be negative.[16]

IV. Summary and Conclusions

This paper has analyzed the effect of corporate and personal taxes on the firm's optimal investment and financing decisions under uncertainty. It extends the

[15] See, for example, Bradley, Jarrell, and Kim [3] and Boquist and Moore [2].

[16] To verify this, we constructed a simple numerical example in which a sample of ten firms with different production technologies (each consistent with equation (10)) and different depreciation rates was assumed to make investment and financing decisions to maximize firm value, subject to a set of exogenously determined state prices. This exercise yielded a system of nonlinear equations that was solved numerically. The cross-sectional relationship that emerged exhibited a *positive* correlation between investment-related and debt-related tax shields (normalized by expected earnings). However, for those firms with identical production technologies (i.e., perfectly correlated pre-tax earnings) but different depreciation rates, the relationship was indeed negative. Copies of the numerical example are available from the authors upon request.

DeAngelo and Masulis [8] capital structure model by endogenizing investment. It has been shown that the impact of leverage on firm value due to the DeAngelo-Masulis [8] "pure leverage effect", which allows only debt to vary, will be strictly less than the "total leverage effect", which requires the firm to adjust all of its decision variables optimally. It has also been shown that increases in allowable investment-related tax shields due to changes in the corporate tax code are not necessarily associated with reductions in leverage at the individual firm level when investment is allowed to adjust optimally. The effect of an increase in allowable investment-related tax shields on the firm's optimal level of debt depends critically on the tradeoff between the "substitution effect" advanced by DeAngelo and Masulis [38] and the "income effect" associated with an increase in optimal investment. In cross-sectional analysis, firms with higher investment-related tax shields (normalized by expected earnings) need not have lower debt-related tax shields (normalized by expected earnings) if firms utilize different production technologies and have less than perfectly correlated pre-tax output. This latter result suggests that empirical studies of the cross-sectional relationship between investment-related tax shields and financial leverage may be misspecified if they do not adequately control for differences in production technologies across firms.

Appendix

Proof of Proposition 1: To prove the proposition, consider an exogenous increase in the rate of capital depreciation, γ. Taking the total derivative of equation (8) with respect to γ and holding the investment level fixed yields

$$\frac{dV_F}{d\gamma} = V_{FF}\frac{dF^*}{d\gamma} + V_{F\gamma} = 0, \tag{A1}$$

where an asterisk denotes an optimal value satisfying the first-order condition (8). Assuming that the second-order condition $V_{FF} < 0$ is satisfied, it follows that $\operatorname{sgn}(dF^*/d\gamma) = \operatorname{sgn}(V_{F\gamma})$. However, from equation (8) we know that

$$V_{F\gamma} = -t^c p(\theta_2)\frac{d\theta_2}{d\gamma} < 0 \quad (\text{since } d\theta_2/d\gamma > 0), \tag{A2}$$

and, therefore, $dF^*/d\gamma < 0$. Q.E.D.

Proof of Proposition 2: Using Cramer's Rule, the value of $dK^*/d\gamma$ is given by

$$\frac{dK^*}{d\gamma} = \frac{V_{F\gamma}V_{KF} - V_{K\gamma}V_{FF}}{|H|}, \tag{A3}$$

where

$$|H| = \begin{vmatrix} V_{KK} & V_{KF} \\ V_{KF} & V_{FF} \end{vmatrix}.$$

If a maximum exists, then $V_{KK} < 0$, $V_{FF} < 0$, and $|H| > 0$. Using equations (8) to (10) then yields

$$V_{F\gamma} = -t^c p(\theta_2)K^* \frac{d\theta_2}{dF} < 0, \tag{A4}$$

$$V_{K\gamma} = t^c \left\{ \int_{\theta_2}^{\bar{\theta}} p(\theta)\, d\theta + p(\theta_2)[f_K(\theta_2) - \gamma]K^* \frac{d\theta_2}{dF} \right\} > 0, \tag{A5}$$

$$V_{FF} = p(\theta_1)t^p(\theta_1) \frac{d\theta_1}{dF} - t^c p(\theta_2) \frac{d\theta_2}{dF} < 0, \tag{A6}$$

$$V_{KF} = t^c p(\theta_2)[f_K(\theta_2) - \gamma] \frac{d\theta_2}{dF} - p(\theta_1)t^p(\theta_1) f_K(\theta_1) \frac{d\theta_1}{dF} \gtreqless 0, \tag{A7}$$

where we have used the fact that $f_K(\theta) = \alpha K^{-1} x(\theta)$, $x(\theta_1) = F^*$, $x(\theta_2) = F^* + \gamma K^*$, $\theta_1 = g^{-1}[F^*(K^*)^{-\alpha}]$, $\theta_2 = g^{-1}[(F^* + \gamma K^*)(K^*)^{-\alpha}]$, and $d\theta_2/d\gamma = K^* d\theta_2/dF$. Substituting equations (A4) to (A7) into the numerator of equation (A3) then yields

$$\frac{dK^*}{d\gamma} |H| = -V_{FF} t^c \int_{\theta_2}^{\bar{\theta}} p(\theta)\, d\theta - V_{F\gamma} p(\theta_1) t^p(\theta_1) \gamma (1 - \alpha) \frac{d\theta_1}{dF} > 0. \tag{A8}$$

Thus, $dK^*/d\gamma$ is unambiguously positive, and, since $f_K(\theta) > 0$, the output of the firm, $x(\theta)$, also increases. Q.E.D.

Proof of Proposition 3: To prove the claim that an increase in the depreciation rate, γ, leads to an unambiguous increase in the absolute magnitude of the investment-related tax shields, differentiate γK^* with respect to γ, yielding

$$\frac{d(\gamma K^*)}{d\gamma} = K^* + \gamma \frac{dK^*}{d\gamma}. \tag{A9}$$

According to Proposition 2, $dK^*/d\gamma$ is strictly positive. Thus, $d(\gamma K^*)/d\gamma$ is also strictly positive.

To prove the claim that an increase in the depreciation rate, γ, leads to an unambiguous increase in the level of investment-related tax shields as a percentage of expected earnings, differentiate $\gamma K^*/\mathrm{E}(x)$ with respect to γ, yielding

$$\frac{d[\gamma K^*/\mathrm{E}(x)]}{d\gamma} = \frac{\left[K^* + \gamma \dfrac{dK^*}{d\gamma}\right]\mathrm{E}(x) - \gamma K^* \dfrac{d\mathrm{E}(x)}{d\gamma}}{[\mathrm{E}(x)]^2}, \tag{A10}$$

where

$$\mathrm{E}(x) = \int_{\underline{\theta}}^{\bar{\theta}} x(\theta) h(\theta)\, d\theta \tag{A11}$$

and

$$\frac{d\mathrm{E}(x)}{d\gamma} = \int_{\underline{\theta}}^{\bar{\theta}} \left[f_K(\theta) \frac{dK^*}{d\gamma}\right] h(\theta)\, d\theta = \frac{dK^*}{d\gamma} \mathrm{E}(f_K). \tag{A12}$$

Substituting (A12) into (A10) then yields

$$\frac{d[\gamma K^*/\mathrm{E}(x)]}{d\gamma} = \frac{K^*\mathrm{E}(x) + \gamma \dfrac{dK^*}{d\gamma}[\mathrm{E}(x) - K^*\mathrm{E}(f_K)]}{[\mathrm{E}(x)]^2}. \tag{A13}$$

However, by the concavity of f, $\mathrm{E}(x) > K^*\mathrm{E}(f_K)$. Therefore, equation (A13) is strictly positive. Q.E.D.

Proof of Proposition 4: To prove the proposition, first use Cramer's Rule to show that $dF^*/d\gamma$ is given by

$$\frac{dF^*}{d\gamma} = \frac{V_{K\gamma}V_{KF} - V_{F\gamma}V_{KK}}{|H|}. \tag{A14}$$

Using equations (9) and (10), V_{KK} can be written as

$$\begin{aligned} V_{KK} = (\alpha - 1)(K^*)^{-1}\left\{1 - \gamma t^c \int_{\theta_2}^{\bar{\theta}} p(\theta)\, d\theta\right\} \\ + p(\theta_1)t^p(\theta_1)[f_K(\theta_1)]^2 \frac{d\theta_1}{dF} \\ - t^c p(\theta_2)[f_K(\theta_2) - \gamma]^2 \frac{d\theta_2}{dF} < 0, \end{aligned} \tag{A15}$$

where we have used the fact that $f_{KK}(\theta) = (\alpha - 1)K^{-1}f_K(\theta)$, $d\theta_1/dK = -f_K(\theta_1)d\theta_1/dF$, and $d\theta_2/dK = -[f_K(\theta_2) - \gamma]d\theta_2/dF$. Substituting equations (A6), (A7), and (A15) into $|H| = V_{KK}V_{FF} - V_{FK}^2$ and using equation (A8) to simplify yields

$$|H| = \frac{(\alpha - 1)(K^*)^{-1}V_{FF}}{\left[1 - (\alpha - 1)\gamma(K^*)^{-1}\dfrac{dK^*}{d\gamma}\right]}. \tag{A16}$$

Now substitute equations (A4), (A5), (A7), (A15), and (A16) into equation (A14) and simplify to obtain

$$\frac{dF^*}{d\gamma} = [\alpha(K^*)^{-1}F^*]\frac{dK^*}{d\gamma} - \frac{\left[1 - (\alpha - 1)\gamma(K^*)^{-1}\dfrac{dK^*}{d\gamma}\right]}{V_{FF}} V_{F\gamma}. \tag{A17}$$

Next, multiply both sides of equation (A17) by γ/F^* and let $e_{F\gamma} \equiv [\gamma/F^*]dF^*/d\gamma$, $e_{K\gamma} \equiv [\gamma/K^*]dK^*/d\gamma$, and $z \equiv -[\gamma/F^*]V_{F\gamma}/V_{FF}$. This reduces to

$$e_{F\gamma} = \alpha e_{K\gamma}(1 - z) + z(1 + e_{K\gamma}). \tag{A18}$$

Finally, note that $e_{d\gamma} \equiv K^{-1}d(\gamma K)/d\gamma = (1 + e_{K\gamma})$ and, from equation (10), that $e_{x\gamma} \equiv [\gamma/x(\theta)]dx(\theta)/d\gamma = \alpha e_{K\gamma}$ for all θ. Consequently, equation (A18) can be rewritten as

$$e_{F\gamma} = (1 - z)e_{x\gamma} + ze_{d\gamma}, \tag{A19}$$

thus yielding the desired result. Q.E.D.

Proof of Proposition 5: To prove the proposition, differentiate $F^*/\mathrm{E}(x)$ with respect to γ, yielding

$$\frac{d[F^*/\mathrm{E}(x)]}{d\gamma} = \frac{\mathrm{E}(x)\,\dfrac{dF^*}{d\gamma} - F^*\,\dfrac{d\,\mathrm{E}(x)}{d\gamma}}{[\mathrm{E}(x)]^2}. \tag{A20}$$

Substituting equations (A12) and (A17) into (A20) and using the fact that $\mathrm{E}(f_K) = \alpha(K^*)^{-1}\mathrm{E}(x)$ then yields

$$\frac{d[F^*/\mathrm{E}(x)]}{d\gamma} = -\frac{\left[1 - (\alpha - 1)\gamma(K^*)^{-1}\,\dfrac{dK}{d\gamma}\right]}{V_{FF}}\,V_{F\gamma}\mathrm{E}(x) < 0, \tag{A21}$$

thus completing the proof. Q.E.D.

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