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An Unconditional Asset-Pricing Test and the Role of Firm Size as an Instrumental Variable for Risk

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ABSTRACT

In an intertemporal economy where both risk (stock beta) and expected return are time varying, the authors derive a linear relation between the unconditional beta and the unconditional return under certain stationarity assumptions about the stochastic process of size-portfolio betas. The model suggests the use of long time periods to estimate the unconditional portfolio betas. The authors find that, after controlling for the betas thus estimated, a firm-size proxy, such as the logarithm of the firm size, does not have explanatory power for the averaged returns across the size-ranked portfolios.

THE CAPITAL ASSET PRICING Model (CAPM) can be generalized to describe the period-by-period risk-return relation in a multiperiod equilibrium.[1] The model predicts that an asset's conditional expected return (E_{it}) is linearly related to its conditional market risk (β_{it}), both being conditioned on the information available at $t-1$:[2]

$$E_{it} = \lambda_{ot} + \lambda_{1t}\beta_{it}. \tag{1}$$

If we can impose sufficient structures on the stochastic processes of the beta and the expected returns, the model can still be tested in the unconditional form. In this paper, we make assumptions that take into account recent empirical evidence on the movements of the expected returns and betas[3] and obtain a linear relation between the unconditional expected returns and the unconditional betas. Consequently, the test presented in this paper is a joint test of the conditional pricing equation and the assumption on the stochastic process of the beta.

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[1] See, e.g., Fama and MacBeth [16]. The additional assumptions are usually sufficient conditions ensuring that the investors *on the aggregate* do not hedge against stochastic shifts in the investment and consumption opportunity set—e.g., an economy where the stochastic elements cannot be hedged or the investors, consistent with their utility maximizations, behave as if they did not want to hedge on the aggregate. Alternatively, we can consider the conditional pricing equation (1) as a statement regarding the conditional mean-variance efficiency of the selected market index.

[2] In the Sharpe [35]–Lintner [24] model, $\lambda_0 = r_f$, the risk-free rate, and λ_1 = the expected market return over the risk-free rate. In Black's [2] model, λ_0 = the expected return of a zero-beta portfolio and λ_1 = the expected market return over λ_0.

[3] See, e.g., Keim and Stambaugh [22] and Chan [8].

Our alternative hypothesis is motivated by the size effect of Banz [11] and Reinganum [29]. They show that equity values have significant explanatory power for the average returns of financial assets even after controlling for the estimated betas. The size effect indicates either that (1) is misspecified or that the estimated portfolio betas contain substantial errors.

Theoretically, the precision of the estimated betas can be improved by simply using more observations.[4] Unfortunately, there is no straightforward way to introduce more observations. Daily data are plagued by the problems of nonsynchronous trading and bid-ask spreads. If we lengthen the estimation period instead, past studies by Blume [4], Gonedes [20], and Officer [28] indicate that the risk characteristics of portfolios with constant composition may drift over time.

However, if we can construct our portfolios systematically in such a way that their betas follow a stationary distribution (as opposed to being constant) even over a relatively long period, we can then estimate the unconditional model with more observations than most existing studies, with potential gains in estimation precision. We propose to use an instrument that is highly correlated with risk, period by period, so that portfolios formed consistently on the basis of this instrumental variable can be assumed to have the same relative characteristics over time. Thus, by keeping the relative characteristics of the portfolios the same while letting the compositions change, we can test the unconditional model against the size alternative with more precision. In this study, under the null hypothesis of a single-factor pricing model, any instrumental variable that is a good instrument for expected return will be a good instrument for the beta. From the size effect, we believe that firm size is such an instrument. Therefore, we shall form portfolios by equity market values and test whether the single-factor asset-pricing model can be rejected in favor of the size effect.

The rest of the paper is organized as follows. Section I discusses the general model and the conditions under which there is a meaningful relation between the unconditional return and beta. We assume that the betas of the size-ranked portfolios follow a stochastic process that is stationary and derive a linear relation between the unconditional returns and the unconditional betas. Section II presents the empirical results using estimated "time-averaged" betas. The results are very different from those without using average betas. We find that the instrumental variable, firm size, does not have additional explanatory power after controlling for the betas estimated over long time periods. Section III summarizes the findings.

I. The Model

A. The Time-Series Assumptions

In order to use time-series data to estimate the parameters in the pricing relation, we must first specify assumptions about the time evolution of asset return.

[4] Estimates of betas were used explicitly in the studies of Douglas [13], Miller and Scholes [26], Black, Jensen, and Scholes [3], and Fama and MacBeth [15], and estimates of betas were used implicitly in the Seemingly Unrelated Regression approach of Gibbons [18] and Stambaugh [36].

(A1) Let the realized return (in excess of the risk-free rate) of portfolio i at time period t be

$$\tilde{r}_{it} = \mathrm{E}(\tilde{r}_{it} \mid I_{t-1}) + \beta_{it-1}\tilde{\delta}_{mt} + \tilde{\varepsilon}_{it}, \tag{2}$$

where $\mathrm{E}(\tilde{r}_{it} \mid I_{t-1})$ is the expected excess return of portfolio i known at the beginning of period t based on the information I_{t-1} available at the end of period $t-1$, β_{it-1} is the sensitivity (known at the beginning of period t) of portfolio i's return to unanticipated return of the market factor $\tilde{\delta}_{mt}$, and $\tilde{\varepsilon}_{it}$ is portfolio i's unanticipated return given I_{t-1}. We assume that $\tilde{\varepsilon}_{it}$, $\tilde{\delta}_{mt}$, and I_{t-1} are mutually independent.

It is clear that, if the stochastic nature of the pricing equation parameters were completely arbitrary, it would be difficult to draw any inference from the data. Motivated by some recent empirical evidence,[5] we impose the following simple structure on the stochastic beta.

(A2) Assume that $\tilde{\beta}_{it-1}$ has a stationary distribution with mean $\bar{\beta}_i$ and that $\bar{\bar{\beta}}$ exists and is the cross-sectional mean of the $\bar{\beta}_i$'s. (If we take an equally weighted index of financial assets in our sample as the market, then $\bar{\bar{\beta}} = 1$ by construction.) We assume that $\tilde{\beta}_{it-1}$ can be described by

$$\tilde{\beta}_{it-1} = \bar{\beta}_i + \tilde{\theta}_{t-1}(\bar{\beta}_i - \bar{\bar{\beta}}) + \tilde{\eta}_{it-1}, \tag{3}$$

where $\tilde{\theta}_{t-1}$ has zero mean and $\tilde{\eta}_{it-1}$ is a random noise independent of all other parameters. Equation (3) models the changing spread of the size-portfolio betas (around the mean $\bar{\bar{\beta}}$): smaller ($\theta_{t-1} < 0$) in good times and larger ($\theta_{t-1} > 0$) in bad times.[6]

(A3) Let the realized excess (over T-bill) market return for period t be

$$\tilde{r}_{mt} = \mathrm{E}(\tilde{r}_{mt} \mid I_{t-1}) + \tilde{\delta}_{mt}, \tag{4}$$

[5] In Chan [8], the spectrum of size-portfolio betas is inversely related to past stock market index return.

[6] The stochastic beta process (equation (3)) models the "stylized fact" that the cross-sectional spectrum of betas of size-ranked portfolios expands and contracts with the indicators of business cycles. If we assume that the stochastic beta of a portfolio can be expressed as a linear function of the real activity as measured by, say, the growth of the industrial production rates in the previous two years, $(IP2Y_{t-1})$, then

$$\beta_t = \bar{\beta} + \phi(IP2Y_{t-1}) + u_t.$$

The market model incorporating this changing beta becomes

$$r_t = \alpha + \bar{\beta}r_{mt} + \phi(IP2Y_{t-1}r_{mt}) + e_t,$$

and the stochastic behavior of beta is reflected in the slope coefficient ϕ. We estimate this "market model" for the twenty size-ranked portfolios with the equally weighted NYSE market as the market proxy in the 1954 to 1983 period. The estimated portfolio ϕ's are negative and significant for small firms and positive and significant for large firms. At the same time, if we regress the market return on $IP2Y_{t-1}$ (which in itself is negatively autocorrelated), the slope coefficient has a t-statistic of -3.16. Together these regressions suggest that the expected market return and the size-portfolio betas are both affected by the business cycle. Of course, we can model the fluctuating spectrum of betas with a more general process. However, our simplifying assumptions are useful in reducing the problem to tractable form, and the following empirical tests are tests of the joint assumptions.

where $E(\tilde{r}_{mt} \mid I_{t-1})$ is the expected excess return of the market based on information at the end of time $t - 1$.

(A4) Assume the pricing equation:[7]

$$E(\tilde{r}_{it} \mid I_{t-1}) = E(\tilde{\lambda}_{ot} \mid I_{t-1}) + \beta_{it-1}[E(\tilde{r}_{mt} \mid I_{t-1}) - E(\tilde{\lambda}_{ot} \mid I_{t-1})], \quad (5)$$

where $E(\tilde{\lambda}_{ot} \mid I_{t-1})$ is the expected excess return of a zero-beta portfolio based on I_{t-1} known at the beginning of t. We assume that the parameters in this excess-return relation are jointly stationary and that all the relevant moments exist.

B. Estimation of the Model

In the system described above, there is a linear relation between the unconditional expected return and $\bar{\beta}_i$, the mean of the conditional beta. To show this, we substitute (3) into (5) and take the expectation, obtaining

$$E(\tilde{r}_i) = E(\tilde{\lambda}_0) - \bar{\bar{\beta}}Cv + \bar{\beta}_i[E(\tilde{r}_m) - E(\tilde{\lambda}_0) + Cv], \quad (6)$$

where $Cv = \text{cov}[\tilde{\theta}_{t-1}, E(\tilde{r}_{mt} \mid I_{t-1}) - E(\tilde{\lambda}_{ot} \mid I_{t-1})]$. The term Cv is positive if the market premium goes up as the cross-sectional spectrum of betas expands; the riskier firm becomes riskier and the safer firm becomes safer, relative to the cross-sectional average. The slope coefficient of $\bar{\beta}_i$ in (6) indicates that the average compensation is the sum of two terms, the unconditional market risk premium and the covariance term Cv. The first term is the compensation for bearing the average market risk. The second term gives additional compensation to high-beta (small) stocks, relative to low-beta stocks, for bearing more market risk when it is relatively undesirable (as reflected in the higher risk premium) and bearing less market risk when it is relatively desirable. Thus, the interpretation of our pricing equation (6) is different from the single-period CAPM, but the linearity between the unconditional returns and mean conditional betas is maintained.

Next, we shall show that the unconditional expected return is also linearly related to the unconditional beta. First, the unconditional beta can be expressed as

$$\begin{aligned}
\bar{\beta}_{ui} &= \text{cov}(\tilde{r}_{it}, \tilde{r}_{mt})/\sigma_m^2 \\
&= \bar{\beta}_i + (1 - \bar{\beta}_i)\text{cov}[E(\tilde{\lambda}_{0t} \mid I_{t-1}), \tilde{r}_{mt}]/\sigma_m^2 \\
&\quad + (\bar{\beta}_i - \bar{\bar{\beta}})\text{cov}[\theta_{t-1}E(\tilde{r}_{mt} - \tilde{\lambda}_{ot} \mid I_{t-1}), \tilde{r}_{mt}]/\sigma_m^2 \\
&= d_0 + d_1\bar{\beta}_i, \quad (7)
\end{aligned}$$

where σ_m^2 is the variance of $E(\tilde{r}_{mt} \mid I_{t-1}) + \tilde{\delta}_{mt}$, the two covariance terms are constant across portfolios, and d_0 and d_1 are constants. Thus, under the appropriate regularity conditions, the unconditional beta $\bar{\beta}_{ui}$ is a linear function of $\bar{\beta}_i$. Combining this relation and (6) gives a linear relation between the unconditional

[7] This assumption can be considered as a statement about the period-by-period mean-variance efficiency of the equally weighted NYSE index on the subset of size portfolios.

expected return $E(\tilde{r}_i)$ and $\bar{\beta}_{ui}$:

$$E(\tilde{r}_i) = c_0 + c_1\bar{\beta}_{ui}. \tag{8}$$

Our alternative hypothesis is that a measure of the equity value can explain the cross-sectional differences in average returns:

$$E(\tilde{r}_i) = c_0 + c_1\bar{\beta}_{ui} + \lambda_2\overline{\ln MV_i}, \tag{9}$$

where $\overline{\ln MV_i}$ is the time-series mean of the logarithm of equity value $\ln(MV_i)_t$.[8] Thus, the null hypothesis (equation (8)) is nested in the alternative hypothesis (equation (9)), and a test of the null hypothesis is equivalent to testing $\lambda_2 = 0$. If the reported size effect were partly due to estimation errors in betas, then the use of the more precisely estimated unconditional betas would attenuate the size effect.[9]

[8] Since using nominal rather than real firm size in the regressions will only induce an additive factor constant across i that does not affect the estimate or the inference about λ_2, we use the nominal firm sizes in the following empirical tests. Also, for the alternative hypothesis (9) to be meaningful in terms of the unconditional expectations, we assume that $\ln(MV_i)_t$ has a jointly stationary distribution with other parameters in I_{t-1}. If λ_2 is not a constant, we assume that $E(\tilde{\lambda}_{2t} \mid I_{t-1})$ also has a jointly stationary distribution and that $\mathrm{cov}(E(\tilde{\lambda}_{2t} \mid I_{t-1}), \ln(MV_i)_t) = 0$. The last assumption about the covariance is unnecessary if we use $\ln(MV_i)_t$ rather than their averages in the empirical tests. In the absence of estimation errors, the average estimate of λ_2 will be an unbiased (negatively biased) estimate of $E(\tilde{\lambda}_{2t})$ if $\ln(MV_i)_t$ is uncorrelated (negatively correlated) with errors in $\bar{\beta}_{ui}$. The empirical results using $\ln(MV_i)_t$ and using the averages are qualitatively the same.

[9] If we let T, the number of time-series observations used in the estimation of $\bar{\beta}_{ui}$, approach ∞, we do not have the errors-in-variable problem.

Suppose we use observations $t = 1, \cdots, T$ (finite number) to estimate $\bar{\beta}_{ui}$ and use observations $t = T+1, \cdots, T+T'$ to test the model; the estimated $\bar{\beta}_{ui}$ can be expressed as

$$\hat{\bar{\beta}}_{ui} = \bar{\beta}_{ui} + \bar{u}_{iT}.$$

Assume that the variance of $\bar{u}_{iT}$, $\bar{\beta}_{ui}$, and $\overline{\ln MV_i}$ exist and are equal to σ_u^2, σ_1^2, and σ_2^2, respectively, and the covariance between $\bar{\beta}_{ui}$ and $\overline{\ln MV_i}$ is σ_{12}. We can analyze the effect of estimation of errors of $\bar{\beta}_{ui}$ in the equation:

$$\tilde{r}_i = a_0 + a_1\hat{\bar{\beta}}_{ui} + a_2\overline{\ln MV_i} + \tilde{e}_i,$$

assuming that the true equation is

$$\tilde{r}_i = a_0^* + a_1^*\bar{\beta}_{ui} + a_2^*\overline{\ln MV_i} + \tilde{\omega}_i.$$

Asymptotically,

$$\mathrm{plim}\ \hat{a}_1 = \frac{\sigma_2^2\sigma_1^2 - \sigma_{12}^2}{\sigma_2^2\sigma_1^2 - \sigma_{12}^2 + \sigma_2^2\sigma_u^2}\, a_1^*.$$

Therefore, $\hat{a}_1$ will be asymptotically downward biased. The induced bias on $\hat{a}_2$ depends on the sign of σ_{12}. Empirically, small firms tend to have high betas, which implies that $\sigma_{12} < 0$. In this case,

$$\mathrm{plim}\ \hat{a}_2 = a_2^* + \frac{\sigma_{12}\sigma_u^2}{\sigma_2^2\sigma_1^2 - \sigma_{12}^2 + \sigma_2^2\sigma_u^2}\, a_1^*$$

will be negatively biased if $a_1^* > 0$ since $\sigma_2^2\sigma_1^2 - \sigma_{12}^2 + \sigma_2^2\sigma_u^2 > 0$. (See Levi [23] and Miller and Scholes [26] for a similar analysis.) From the above analysis, we note that, under the null hypothesis $a_2^* = 0$, (i) plim $\hat{a}_2$ will be negative whenever the market risk premium a_1^* is positive, (ii) the bias is a strictly increasing function of σ_u^2: the larger the estimation error variance, the more negative the estimated slope coefficient for the firm-size variable, and (iii) in the extreme case when $\sigma_u^2 \to \infty$, plim $\hat{a}_1 \to 0$ and plim $\hat{a}_2 \to \frac{\sigma_{12}}{\sigma_2^2}\, a_1^*$. (See Gibbons [17] and Shanken [33, 34] for related analyses.)

II. Empirical Results

The analysis above is based on the assumption that the size-ranked portfolio betas have a stationary distribution over the investigation period. Therefore, to implement the empirical tests, we must first decide on an appropriate time period and an appropriate market index for which the stationary-distribution assumption is reasonable. As the stock return volatility was much higher before World War II than after, it is unlikely that the betas have the same distributions over both periods. The time interval we choose is the recent thirty-five years in the postwar period from 1949 to 1983, which is considered by many to have been a fairly stable period for the economy. We will present the results for the 1926 to 1948 period in a separate section.

As for the market index, since we use only equally weighted portfolios of stocks in testing the pricing equation, the index chosen is an equally weighted index of NYSE stocks rather than an index that contains other types of investments such as bonds, real estate, and collectibles.[10] It is hoped that such betas, being the relative risk measures of the constituents against the aggregate, will have a stationary distribution, even though the nature of the economy aggregate may be slowly changing.

A. The Data

We gather return and firm-size data from the CRSP monthly file for the period 1949 to 1983. During each year, firms on the NYSE that have existed for at least the five previous years and that have price information on December of the previous year are chosen and ranked according to the market value of equity as of that December. The firms are then put into one of twenty portfolios arranged in order of increasing size, each containing (to within one) the same number of securities. For example, for the year 1954, firms in existence since January 1949 that have price information for December 1953 are ranked and put into an equally weighted portfolio based on equity market value as of December 1953. The composition of the portfolio is then held constant through 1954 except for missing returns or delisting of securities. Thus, the portfolio returns are monthly rebalanced equally weighted rates of return.[11] We call the portfolio returns during 1954 post-ranking returns and call the portfolio returns from January 1949 to December 1953 pre-ranking returns. As we repeat this process for each year in the sample, we have from 1954 to 1983 thirty nonoverlapping years of post-ranking returns that we use for testing the asset-pricing model.

[10] See Roll [30] for a discussion of problems in testing the CAPM when the true market is nonobservable. See Stambaugh [36] for an example of using investment returns other than stocks in constructing a market index.

[11] See Roll [31] and Blume and Stambaugh [5] for a discussion of the problems involved in computing the average returns of small-firm portfolios. Using a methodology suggested by Blume and Stambaugh [5] to correct for possible biases arising from the bid-ask effect, we find that the average returns thus computed and the average returns computed using monthly rebalance intervals are essentially indistinguishable for our portfolios over this period. The maximum difference in average monthly returns is 0.00078 (portfolio 3), and the average difference is 0.00013.

B. The Methodology

The method we use to test the pricing model is a variant of the two-step method.[12] In the first step, we estimate $\bar{\beta}_{ui}$, $i = 1, \cdots, 20$, with the equally weighted NYSE market index (EWNY) as the market proxy. In the second step, we perform cross-sectional regressions of the post-ranking realized returns of the twenty portfolios on the estimated beta (and a firm-size variable where appropriate) month by month over 1954 to 1983 using a generalized least-squares procedure that takes into account the residual heteroscedasticity. In each month, we obtain an estimate of the intercept, $\hat{a}_{ot}$, an estimate of the slope coefficient associated with beta, $\hat{a}_{1t}$, and an estimate of the slope coefficient associated with the size variable, $\hat{a}_{2t}$. If there is no "size effect", then $\hat{\bar{a}}_2 = \frac{1}{T} \sum_{t=1}^{T} \hat{a}_{2t}$ should be statistically indistinguishable from zero.

We use the two-step method here rather than the multivariate test as used by Gibbons [18], Stambaugh [36], and others for the following reason. Inspection of the conditional pricing equation (5) and the beta process (equation (3)) suggests that we have a random-coefficients model. If the multivariate test with constant coefficients is used, the standard errors computed from the residual covariance matrix, in particular for the coefficient of the size proxy, may not be appropriate. However, to check the sensitivity of our results with respect to test methodologies, the multivariate test results will also be reported.

C. Estimation of the Betas

We estimate portfolio betas using the post-ranking portfolio returns from 1954 to 1983. A summary of the portfolio betas and averaged returns is in Table I. In addition to the overall period from 1954 to 1983, we also present estimated betas from the first and second fifteen-year subperiods. While the range of the estimated betas fluctuates, the pattern is almost always monotonic from large to small firms.

The cross-sectional correlations of the average portfolio return and betas are presented in Table II. Here we also include statistics pertaining to ten-year and five-year subperiods. The stationary-distribution assumption and the time-series assumption (equation (3)) imply that (i) cross-sectional correlations of betas from nonoverlapping data should be highly positive and increase as the lengths of the estimation periods increase—a consequence of lower estimation-error variances and convergence to the mean of the beta distributions—and (ii) keeping the estimation-period length constant, the cross-sectional correlations from nonoverlapping data should not decrease as the estimation periods become further apart. From Table II, we observe that the pattern of correlations is consistent with (i). The average correlation between the five-year betas is less than the average correlation between the ten-year betas, which is less than the correlation between the fifteen-year betas. From Part 1 of Table II, we also observe that the average correlation between ten-year betas from adjacent periods is similar to

[12] Examples of the two-step method include studies of Black, Jensen, and Scholes [3] and Fama and MacBeth [15].

Table I

Unconditional Portfolio Betas and Returns[a]

Portfolio	Average Monthly Return (1954–1983)	Estimated Beta (1954–1983)	Estimated Beta (1954–1968)	Estimated Beta (1969–1983)
1	0.01919	1.274	1.183	1.315
2	0.01665	1.172	1.087	1.209
3	0.01674	1.120	1.105	1.127
4	0.01522	1.093	1.056	1.107
5	0.01545	1.066	1.116	1.047
6	0.01449	1.038	1.068	1.027
7	0.01434	1.022	1.090	0.995
8	0.01438	1.011	1.021	1.011
9	0.01276	0.950	1.028	0.920
10	0.01342	0.975	0.986	0.970
11	0.01191	0.941	0.972	0.928
12	0.01234	0.936	0.953	0.930
13	0.01181	0.929	0.944	0.922
14	0.01230	0.878	0.933	0.856
15	0.01116	0.878	0.919	0.861
16	0.01089	0.836	0.849	0.829
17	0.01015	0.822	0.832	0.815
18	0.00926	0.749	0.826	0.717
19	0.00824	0.749	0.823	0.718
20	0.00830	0.668	0.779	0.621

[a] Portfolio 1 contains the smallest five percent of NYSE firms. Portfolio 20 contains the largest five percent of NYSE firms. The unconditional betas were estimated using the equally weighted NYSE market index.

the one for nonadjacent periods, and, from Part 2 of Table II, we observe that the averages of the five-year correlations from successive subdiagonals of the correlation matrix are 0.848, 0.806, 0.804, 0.850, and 0.808, respectively, and do not show any particular pattern of decrease. Therefore, even though we cannot provide any direct test of the stationary-distribution assumption,[13] the observed pattern of correlations is not in conflict with the assumption. Thus, we adopt the stationary-distribution assumption as a reasonable working approximation and estimate betas accordingly.

The high cross-sectional correlation between the thirty-year betas and the average log(size) is also very interesting. Given the superior pricing ability of the size proxy, this is precisely what we should expect from the correlation between the size proxy and the "true" risk measure for which size is proxying. Therefore, under the null hypothesis that our model is correct, the correlation between $\overline{\beta}_{ui}$ and $\overline{\ln MV_i}$ should be very high, as we observe. However, the high correlation will likely increase the standard errors of the estimates when both variables are included in a regression, and this will make the results more difficult to interpret. Nevertheless, if the size proxy does not clearly dominate the estimated market beta in the pricing when both are included, we shall regard the evidence as consistent with the hypothesis that the size variable is a proxy for the beta.

[13] We thank George Tiao for helpful discussion on this issue.

Table II

Cross-Sectional Correlations (1954–1983)[a]

Part 1

	$\bar{R}$	$\overline{\ln MV}$	B(full)	B(1-15)	B(2-15)	B(1-10)	B(2-10)
$\overline{\ln MV}$	−0.972						
B(full)	0.986	−0.988					
B(1-15)	0.961	−0.966	0.960				
B(2-15)	0.980	−0.980	0.997	0.936			
B(1-10)	0.935	−0.909	0.915	0.970	0.889		
B(2-10)	0.940	−0.971	0.963	0.941	0.955	0.854	
B(3-10)	0.972	−0.959	0.985	0.918	0.990	0.889	0.907

Part 2

	$\bar{R}$	B(1-5)	B(2-5)	B(3-5)	B(4-5)	B(5-5)
B(1-5)	0.934					
B(2-5)	0.818	0.734				
B(3-5)	0.929	0.816	0.835			
B(4-5)	0.907	0.803	0.685	0.924		
B(5-5)	0.964	0.928	0.727	0.872	0.859	
B(6-5)	0.916	0.808	0.772	0.883	0.853	0.888

[a] $\bar{R}$ is the average monthly portfolio return; $\overline{\ln MV}$ is the average of the logarithm of firm size; B(full) is the estimated beta from the full period 1954 to 1983; B(1-15) and B(2-15) are the estimated betas from the first and second fifteen-year subperiods within the sample interval 1954 to 1983; in general, $B(n\text{-}m)$ is the estimated beta using m years from the nth period within the sample interval 1954 to 1983.

In the following subsections, we describe how the betas used in the cross-sectional tests are constructed.

C.1. Banz Beta (β_B)

In order to determine the difference between using a long time period and using a short time period for estimating betas, we first replicate the beta-estimation method used in Banz's [1] study. The portfolio betas are estimated over the past five years using post-ranking portfolio returns. Note that the portfolio composition may be changing from year to year during the estimation period, but it always corresponds to the size ranking based on December of the previous year. The estimated betas are then updated every year by adding on the most recent year and dropping off the earliest year. Thus, the cross-sectional regressions (1954 to 1983) are run on the betas estimated from the most recent five years of data.

C.2. Averaged Banz Beta ($\bar{\beta}_B$)

The simplest way to introduce more data into the estimation of beta is to compute the time average, $\bar{\beta}_B = 1/30 \sum_{t=1954}^{1983} \beta_{Bt}$, of the Banz portfolio betas. This average beta, $\bar{\beta}_B$, is the mean of the five-year betas and thus a consistent estimate of the unconditional beta. The estimated $\bar{\beta}_B$'s are held constant for each portfolio throughout the cross-sectional regressions (1954 to 1983).

C.3. The Thirty-Four-Year Beta ($\bar{\beta}_u$)

We have thirty-five years of post-ranking return data from 1949 to 1983. During each test year from 1954 to 1983, we leave out the data from the test year and use the other thirty-four years of data to estimate $\bar{\beta}_{ui}$. For example, in computing the cross-sectional regressions for 1964, we use data from 1949 to 1963 and 1965 to 1983 to compute the estimated betas.

D. Cross-Sectional Results

Table III describes the pricing evidence related to the market beta and to the role of firm size as an instrumental variable for risk. In Part 1, we estimate the risk-return relation of our model (equation (8)). The overall slope coefficient corresponding to the estimated market beta is positive and significant, indicating that higher average returns are associated with holding size portfolios with higher unconditional market risk. If this model is also capable of capturing the size effect, the time-series means of the portfolio residuals should be indistinguishable from zero. The monotonicity of the size effect suggests that violations are more likely to concentrate in the extreme size portfolios—positive for small firms and negative for large firms. The residual difference between the top and bottom portfolios, reported as $\hat{u}_1 - \hat{u}_{20}$ in Part 1 of Table III, is never significant.[14] This behavior of the residuals is consistent with the hypothesis that our model is well specified.[15] Therefore, in the test using the size-ranked portfolios, the linear risk-return relation of equation (8) is not rejected against unspecified alternatives.

The size variable in Part 2 of Table III is the logarithm of the equity market value as of December of the previous year. The size variable in Part 3 is the time average of the size variable in Part 2 from 1954 to 1983 (actually the average of $\ln MV_t$ from December 1953 to December 1982). Despite the difference in this definition, we observe in Table III that the pricing results for the overall period and for each of the subperiods are essentially the same for each of the size variables and the market beta.

The evidence suggests that $\bar{\beta}_{ui}$ and the firm-size variables might well be two different measures of the same underlying risk that is being priced. Which of the two is a "better" measure of risk depends on both the theoretical justification and the empirical regularity of the variables. At the theory level, the inclusion of $\bar{\beta}_{ui}$ may be justified by certain assumptions about the opportunity set (see Fama

[14] The same is true for the residual for each portfolio as well as the difference between the residuals for the largest quintile and smallest quintile portfolios.

[15] Notice that the implications of our model are different from those of the stationary Sharpe-Lintner CAPM examined by Gibbons, Ross, and Shanken [19]. The corresponding test (to our residual test) in their case would be the test of whether the intercept is different from zero in the regression of the return difference between the top and bottom portfolios on the excess market return, which is the same test presented in Table 3 of Banz [1]. As in Banz [1] the stationary Sharpe-Lintner CAPM is rejected using the intercept test (with the return difference between portfolios 1 and 20). The estimated intercept is 0.00585/month (t = 2.31) using the equally weighted NYSE index and 0.00970/month (t = 3.17) using the value-weighted NYSE index. The last result is similar to Banz's result of 0.0101/month (t = 3.07) using the smallest and largest fifty securities on NYSE for his overall period from 1931 to 1975.

Table III

The Pricing of the Stock Beta and the Size Proxy (1954–1983)[a]

	Part 1		
	$r_i = a_0 + a_1[\hat{\hat{\beta}}_u]_i + u_i$		
	$\hat{\hat{a}}_0$	$\hat{\hat{a}}_1$	$\hat{u}_1 - \hat{u}_{20}$
1954–1983	−0.00579	0.01940	−0.00060
	(−1.37)	(3.78)	(−0.55)
1954–1963	0.00787	0.00536	−0.00018
	(1.40)	(0.91)	(−0.11)
1964–1973	−0.00523	0.01313	−0.00025
	(−0.85)	(1.65)	(−0.13)
1974–1983	−0.02001	0.03971	−0.00136
	(−2.10)	(3.40)	(−0.66)

	Part 2	
	$r_i = a_0 + a_1 \ln MV_i + \varepsilon_i$	
	$\hat{\hat{a}}_0$	$\hat{\hat{a}}_1$
1954–1983	0.03362	−0.00176
	(4.63)	(−3.73)
1954–1963	0.01813	−0.00048
	(2.33)	(−0.90)
1964–1973	0.02043	−0.00115
	(1.59)	(−1.44)
1974–1983	0.06232	−0.00366
	(4.00)	(−3.55)

	Part 3	
	$r_i = a_0 + a_1[\overline{\ln MV}]_i + \eta_i$	
	$\hat{\hat{a}}_0$	$\hat{\hat{a}}_1$
1954–1983	0.03393	−0.00180
	(4.68)	(−3.77)
1954–1963	0.01895	−0.00051
	(2.28)	(−0.91)
1964–1973	0.02115	−0.00118
	(1.73)	(−1.55)
1974–1983	0.06170	−0.00372
	(3.90)	(−3.48)

[a] $\ln MV$ is the logarithm of the equity size; $\overline{\ln MV}$ is the time average of $\ln MV$ for each portfolio. $\hat{\hat{\beta}}_u$ is estimated using the post-ranking returns over 1949 to 1983 excluding the year in which the cross-sectional regressions are run. t-Statistics are in parentheses.

[14]) or about the aggregate preference of the investors, or the experiment can be motivated as a test of the mean-variance efficiency of the equally weighted NYSE index. We shall come back to this point later in Section III. On the other hand, the firm-size variable is included only to demonstrate that smaller firms tend to have higher ("risk-adjusted") average returns over time, and it is not justified by any theoretical model of risk-return equilibrium. Thus, if we can *also* show empiricallly that the size variable does not have marginal explanatory

power after controlling for $\bar{\beta}_{ui}$, then we are justified in choosing $\bar{\beta}_{ui}$ over the firm-size variable as a measure of risk.

Before reporting the regression results with both $\bar{\beta}_{ui}$ and a firm-size variable as regressors, let us first ascertain that our firm-size variable enjoys the same empirical regularities in our sample data and our sample period from 1954 to 1983. Part 1 of Table IV reports the results of an experiment similar to that in Banz [1]. In the cross-sectional regression with beta and size, the betas are estimated in the same way as in Banz's study using the past five years of data

Table IV

The Size Effect (1954–1983)[a]

Part 1

$r_i = a_0 + a_1[\hat{\beta}_B]_i + a_2 \ln MV_i + \omega_i$

	$\hat{a}_0$	$\hat{a}_1$	$\hat{a}_2$
1954–1983	0.03184	0.00194	−0.00175
	(3.37)	(0.52)	(−3.25)
1954–1963	0.01650	0.00093	−0.00041
	(1.71)	(0.20)	(−0.70)
1964–1973	0.01842	0.00231	−0.00115
	(1.19)	(0.37)	(−1.34)
1974–1983	0.06060	0.00259	−0.00368
	(2.80)	(0.31)	(−2.99)

Part 2

$r_i = a_0 + a_1[\hat{\hat{\beta}}_B]_i + a_2 \ln MV_i + v_i$

	$\hat{a}_0$	$\hat{a}_1$	$\hat{a}_2$
1954–1983	−0.00487	0.01866	−0.00001
	(−0.41)	(2.72)	(−0.03)
1954–1963	0.00959	0.00369	0.00001
	(0.56)	(0.37)	(0.01)
1964–1973	−0.03893	0.02804	0.00161
	(−1.99)	(2.38)	(1.94)
1974–1983	0.01474	0.02426	−0.00167
	(0.61)	(1.79)	(−1.57)

Part 3

$r_i = a_0 + a_1[\hat{\hat{\beta}}_u]_i + a_2[\overline{\ln MV}]_i + \xi_i$

	$\hat{a}_0$	$\hat{a}_1$	$\hat{a}_2$
1954–1983	−0.01278	0.02273	0.00032
	(−0.77)	(2.52)	(0.44)
1954–1963	0.00301	0.00760	0.00023
	(0.13)	(0.62)	(0.23)
1964–1973	−0.05847	0.03875	0.00245
	(−2.08)	(2.75)	(2.05)
1974–1983	0.01710	0.02184	−0.00170
	(0.50)	(1.11)	(−1.13)

[a] $\hat{\beta}_B$ is the market beta estimated over the past five years using the post-ranking returns, and $\hat{\hat{\beta}}_B$ is the time average of the $\hat{\beta}_B$; $\hat{\hat{\beta}}_u$ is estimated over 1949 to 1983 excluding the year in which the cross-sectional regressions are run. t-Statistics are in parentheses.

(see Subsection *C.1.* above), and the size variable is the logarithm of firm size as of the December previous to each of the test years. Not surprisingly, the size variable comes in significantly overall, consistent with the results of Banz [1].

Part 2 of Table IV reports the same regressions but with only one difference. Instead of using the betas estimated from the five years prior to each test year, we compute the time average of the five-year portfolio betas and hold those averaged betas (Subsection *C.2.*) constant throughout all the cross-sectional regressions. Thus, we hold all aspects of the experiment in Part 1 of Table IV unchanged other than replacing the five-year rolling estimated portfolio betas with their averages. The difference in results induced from using the time averages of betas is striking. As we compare Part 2 with Part 1 of Table IV, we observe that the slope coefficient of $\overline{\ln MV}$ is no longer significant, while the slope coefficient of $\bar{\beta}_i$ has now become significant. A possible explanation of the dramatic difference, as we suggested in Section I, is the reduction of estimation error of betas. Random errors of the five-year unconditional betas for each portfolio tend to cancel out each other in their time averages, and, therefore, the averaged betas are estimated with higher precision. If equation (8) is true, one would expect to observe regression results consistent with those in Part 2 of Table IV.

Finally, in Part 3 of Table IV, we report the results putting both $\bar{\beta}_{ui}$ (Subsection *C.3.*) and a size variable in the same regression. Again we observe that the *t*-statistics for the firm-size variable are not significant.[16] We have also estimated betas using slightly different procedures,[17] and the results are robust. Therefore, the main conclusion we draw from Table IV is that, after controlling for $\bar{\beta}_{ui}$, average returns across firms are not correlated with our firm-size variable. In other words, the implication of equation (8) cannot be rejected in favor of the alternative hypothesis (equation (9)).[18]

We have also examined equation (9) in a Seemingly Unrelated Regression framework.[19] The estimates for the intercept and the slope coefficients correspond to our equation (8) and (9) and have the same interpretation as those in Tables III and IV. The results are also qualitatively the same. Without the size proxy, the linearity restriction (equation (8)) cannot be rejected against unspecified alternatives, and the estimate for the slope coefficient for $\bar{\beta}$ is 0.01808 ($t = 4.72$). With the size proxy, the estimated coefficient corresponding to the size variable is 0.00069 ($t = 1.06$), and the coefficient corresponding to the beta is 0.02487 ($t = 3.36$). Thus, the results are not sensitive with respect to methodologies that we use.

[16] Similar results were also obtained if we used $\ln MV_t$ rather than $\overline{\ln MV}$. Also, the average estimated coefficient for the $\overline{\ln MV}$ variable in January is 0.00437 ($t = 1.52$).

[17] Other betas include the average of betas estimated using five years of pre-ranking returns as well as betas estimated with whitening state variables. See Subsection *E* of this section.

[18] However, the results are sensitive to the choice of the market index. If the value-weighted NYSE index is used instead of the equally weighted NYSE index, the size-portfolio betas are less stable over time and do not have the same stationary stochastic properties. Consequently, the use of the unconditional betas in the cross-sectional tests is not justified. When included in the same regression with the unconditional value-weighted market beta, the size proxy remains negative and significant.

[19] See Zellner [37] for a discussion of this procedure. A similar procedure is used in the studies of Gibbons [18] and Stambaugh [36] in investigating the CAPM.

After obtaining the empirical results described above, we examine another period, 1926 to 1948, to find out how robust the results are. In this earlier period, when we compare the average returns computed using monthly rebalancing with the returns computed using the methodology suggested in Blume and Stambaugh [5], which simulates a buy-and-hold strategy, we find that the two sets of returns are substantially different. The betas estimated on two different sets of returns are also different. In contrast, the average returns and estimated betas in the 1949 to 1983 period are not sensitive to how returns are computed. For this reason, plus the fact that return volatility was much higher in the earlier period, we keep the results for the 1926 to 1948 period separate from the more recent period.

Using the buy-and-hold returns, the pricing results over the 1931 to 1948 period are similar to the 1954 to 1983 period. The size coefficients corresponding to those reported in Parts 1, 2 and 3 in Table IV are −0.00159 ($t = -1.82$), 0.00131 ($t = 0.94$) and 0.00037 ($t = 0.35$), respectively. In testing the alternative hypothesis (equation (9)), the size variable again is insignificant. Therefore, we conclude that our model also applies to the earlier period.

E. Results with Whitening State Variables

We have also examined the robustness of our results using estimates of the mean conditional beta, $\bar{\beta}_i$, estimated with whitening state variables that other researchers have shown to be related to changing expected returns. The estimated mean conditional beta is the slope coefficient corresponding to the equally weighted market index as we regress the return of portfolio i on the market index and the linear and quadratic terms of the state variables (including the cross-products). The state variables that we include are (i) equally weighted previous share price inverse, (ii) yield of Baa bonds, (iii) T-bill rate, (iv) long-term government bond yield, and (v) equally weighted NYSE index return over the past two years. The results are qualitatively the same: (a) without the size proxy, the coefficient of beta is positive and significant while the residual difference between the top- and bottom-size portfolios is not (mean of $U1 - U20$ is −0.00047/month with $t = -0.42$), and (b) when the size proxy is included, its coefficient is 0.00043 ($t = 0.60$) for the 1954 to 1983 period and 0.00010 ($t = 0.09$) for the earlier period. We have also included lagged real consumption as another whitening variable for the period 1959 to 1983, and the results are qualitatively the same. We also replaced the Baa yield variable by a similar return variable (PREM) used in Chen, Roll, and Ross [11] and Chan, Chen, and Hsieh [10], and the results are qualitatively the same. Therefore, we conclude that the results are robust with respect to whether whitening variables are used.

III. Conclusion

In this paper, we derive a linear relation between the unconditional betas and expected returns implied by the conditional single-factor pricing equation under some assumptions about the time-series process of the size-portfolio market betas (of which the constant market beta is a special case). Under these assumptions, a test of the pricing equation is equivalent to a test of this linear relation, and any other variable that is cross-sectionally correlated with returns (e.g., firm size) should not have marginal explanatory power on the returns after controlling for the $\bar{\beta}_{ui}$.

The standard analysis of the errors-in-variable problem suggests the importance of using sufficient data to estimate $\bar{\beta}_{ui}$. We then empirically demonstrate this point by reconsidering the "firm-size" effect. We show that we can observe the "size effect" if we use only five years of data to estimate betas. However, if we use data from a long period of time to estimate the betas, the explanatory power of the firm-size variable disappears. We conclude that the observed "firm-size" effect is at least partly due to the imprecision with which betas were estimated in the past.

Although our results show that the pricing equation cannot be rejected in favor of the alternative pricing equation with the firm-size variable, theoretical reasoning (see Merton [25], Ross [32], and Cox, Ingersoll, and Ross [12]) suggests that we should have a multifactor asset-pricing model if risks corresponding to a changing investment opportunity set (e.g., changing expected returns, changing implicit market risk aversion, and changing market risk measures) can be hedged. Empirically, factors that are related to the changing investment opportunity set were shown to have explanatory power on cross-sectional differences in average returns. (See Chen, Roll, and Ross [11], Chan, Chen, and Hsied [10], and Chan and Chen [9].) Thus, we are not proposing here that our single-factor equation can provide a complete description of the risk-return tradeoff. The main point of our study is that, among the size-ranked portfolios, the proxy log(size) does not have additional explanatory power on the cross-sectional returns after controlling for the unconditional equally weighted market beta.

It is worth emphasizing again that the success of our estimated betas depends critically on how we construct the portfolios. Past empirical results (see, e.g., Miller and Scholes [27]) indicate that the inverse of price ($1/p$) is a good proxy for changing risk. In our portfolio formation, it turns out that beta and ($1/p$) are both monotonically related to size, making assumption (A2) a reasonable working approximation for our size portfolios. The size portfolios are also used here for the purposes of both stratifying risk characteristics and spreading out the expected returns since, after all, it is the extraordinary ability of the size variable to proxy for risk that has created the size effect.

In this study, we use an equally weighted market index consisting only of stocks because, once again, we believe that such betas, being the relative risk measures of the constituents against the aggregate, are more likely to satisfy the stationary-distribution assumption. We conjecture that the stationary-distribution assumption is also likely to be a good approximation if the index against which stock risk exposures (betas) are computed is also constructed from the stock market with fixed weights (e.g., the size-based factors used in Huberman and Kandel [21]). Thus, even though we do not believe that we can generalize the methodology in this study indiscriminately to the estimation of any stock risk exposures, if there are reasons to believe that the stationary-distribution assumption is a reasonable approximation, then using a longer time period can lead to more precise estimates of risk exposures and a more meaningful interpretation of the empirical results. Our results show the dramatic difference in inference a researcher can draw from the same data when betas are estimated over a longer time period.

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