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An Alternative Testable Form of the Consumption CAPM

HOSSEIN B. KAZEMI*

ABSTRACT

This paper develops a consumption-oriented model of asset prices in a multigood economy that is, in principle, testable even when aggregate consumption of goods and their market prices are only partially observable. Previous studies show that, when there are m consumption goods, equilibrium expected excess returns on securities are functions of their covariances with $m + 1$ variables—aggregate consumption expenditure and market prices of consumption goods. Without making any further assumptions, the present model shows that a similar equilibrium relationship can be expressed in terms of covariances of asset returns with the following $m + 1$ variables: market prices of k consumption goods and aggregate consumption of $m + 1 - k$ goods. Because the author's result provides researchers with some flexibility in choosing the set of $m + 1$ variables that measure riskiness of securities, it should lead to more powerful tests of the model.

THIS PAPER DEVELOPS AN equilibrium model of asset prices in a multigood economy that is, in principle, testable even when aggregate consumption of goods and their market prices are only partially observable. If there are m consumption goods, then, according to previous studies (e.g., see Breeden [4]), the stochastic properties of a *specific* set of $m + 1$ variables must be estimated to test or to apply the consumption CAPM (CCAPM); i.e., the movement of aggregate consumption expenditure and market prices of m consumption goods must be observed. This paper, on the other hand, shows that the CCAPM can be empirically tested using any combination of $m + 1$ variables from the consumption opportunity set. In particular, if aggregate consumption of $m - k$ goods can be observed, then market prices of only $k + 1$ goods have to be observed to test the model.

To illustrate the results of this paper, we will consider three specific examples. First, if aggregate consumption of only one good can be observed, market prices of all goods will provide enough information to test or to apply the CCAPM. Second, if aggregate consumption of all goods can be observed, the market price of only one good has to be observed to test the model. Finally, we show that, when investors' direct utilities are separable, a smaller set of variables from the consumption opportunity set has to be observed to test the CCAPM.[1]

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[1] Hansen and Singleton [17] and Breeden, Gibbons, and Litzenberger [7] also discuss the effects of separable utility functions on the consumption asset-pricing model.

Rubinstein [23] demonstrates that, in a single-good economy, an appropriate measure of risk for an uncertain cash flow is its covariance with the level of aggregate consumption. Breeden [4], using a continuous-time framework, shows that Merton's [20] multibeta asset-pricing model can be collapsed into a single-beta model where the riskiness of a security is measured in terms of the covariance of its rate of return with the rate of change in aggregate consumption. Breeden [4] also extends this model to a multigood economy and demonstrates that the riskiness of a security can be expressed in terms of the covariance of its real rate of return with the real rate of change in aggregate consumption expenditure. To determine these real rates, one has to form two price indices, which consist of weighted averages of market prices of all consumption goods.[2]

To test the CCAPM, one would face two basic difficulties.[3] The first difficulty, which is addressed by Cornell [8], has to do with nonstationarity of consumption betas. Cornell discusses that, though Merton's multibeta asset-pricing model collapses into a single-beta model, the resulting beta is likely to change through time and that, to estimate its stochastic properties, the state variables still have to be observed. Consequently, empirical verifications of the two models would give rise to the same problem—identifying the relevant state variables.

The second problem, which is the focus of this paper, has to do with the amount and type of data required to test the CCAPM; in particular, the following difficulties may arise:

1. The theoretical definition of aggregate consumption requires us to account for consumption expenditures on nondurable goods and consumption flows derived from durable goods. The U.S. government, for example, reports only consumption expenditures on all consumption goods, and hence, consumption flows have to be estimated from expenditures data.
2. Consumption expenditures are reported at discrete points in time, while the theoretical model is developed within a continuous-time framework. (Rubinstein [23] employs a discrete-time model.)
3. The reported expenditures data are the integral of consumption expenditures over the reported time interval, rather than expenditures at discrete points in time.

It is evident that there are several difficulties in estimating the stochastic properties of aggregate consumption. On the other hand, data on market prices of consumption goods are more readily available, and, compared with aggregate consumption figures, they are less likely to be corrupted by measurement errors. Hence, more powerful tests of CCAPM should result from using the information generated by prices of consumption goods.

The plan of the paper is as follows. Section I presents the assumptions of the paper and offers a brief derivation of Breeden's [4] CCAPM. Section II derives

[2] Other relevant studies are Breeden and Litzenberger [6], Bhattacharya [3], Grossman and Shiller [15], and Cox, Ingersoll, and Ross [9].

[3] For empirical tests of the CCAPM, see Nanisetty [21], Breeden, Gibbons, and Litzenberger [7], Wheately [25], and Grossman, Melino, and Shiller [16]. Hansen and Singleton [17, 18], Ferson [14], and Dunn and Singleton [12, 13] test the implications of the CCAPM for bond returns and the term structure.

the main results. Section III presents some examples, and Section IV offers some concluding comments. Some details are presented in the Appendix.

I. Assumptions and Definitions

Because the structure of our model is identical to that of Breeden [4], we present only a brief description of its major assumptions.

(A1) Perfect markets: All markets are perfectly competitive. There are no transaction costs and no personal tax. Short selling is allowed with the full use of the proceeds.

(A2) State-independent utility: Each investor's direct utility of consumption does not explicitly depend upon the state of the economy.

(A3) Consumption opportunities: There are m consumption goods available at all times. Prices of consumption goods are stochastic, and their dynamics can be presented by a system of Itô processes (e.g., see Arnold [1]) of the following form:

$$dP_j(t) = \alpha_j(t)dt + \xi_j(t)dZ_j(t), \qquad j = 1, \cdots, m,$$

where $\alpha_j(t)$ is the expected change in $P_j(t)$, $\xi_j(t)$ is its standard deviation, and $Z_j(t)$ is a standard Wiener process. These parameters may be functions of other Itô processes.

(A4) Investment opportunities: There are n risky securities available to investors (n need not be constant). The movement of security prices can be expressed through the following system of Itô processes:

$$dS_i(t) = S_i(t)\mu_i(t)dt + S_i(t)\sigma_i(t)dY_i(t), \qquad i = 1, \cdots, n,$$

where $S_i(t)$ is the price of security i, $\mu_i(t)$ is its expected rate of return, $\sigma_i(t)$ is its standard deviation, and $Y_i(t)$ is a standard Wiener process. These parameters need not be constant through time and may be functions of other Itô processes.

Security $i = 0$ is chosen as the risk-free asset, and its rate of return is denoted $r(t)$. If there are no opportunities for risk-free borrowing and lending, $r(t)$ will denote the rate of return on the "zero-beta" asset.

(A5) Investor's choice: There are L investors who agree that the investment and the consumption opportunity sets are as described. An investor seeks to maximize a utility function of the following form (investors are not identical) subject to a wealth constraint:

$$J(W(t), X(t), t) = \max \mathrm{E}_t \int_t^T V(E, P, s)\, ds, \tag{P1}$$

where $J(\cdot)$ is the maximum of the utility of lifetime consumption, $X(t)$ is a vector of relevant state variables, E_t is the conditional expectation operator, $V(\cdot)$ is the investor's indirect utility of consumption, and E denotes his or her optimal consumption expenditure.

The indirect utility of consumption is obtained through the following optimization problem:

$$V(E, P, s) = \max_{C_j}\left\{U(C_1, \cdots, C_m, s) + \lambda\left[E - \textstyle\sum_{j=1}^{m} P_j C_j\right]\right\}, \tag{P2}$$

where $U(\cdot)$ is the investor's direct utility function, which is assumed to be strictly concave and twice differentiable, and C_j denotes the amount of consumption good j.

Here, we present a brief derivation of the CCAPM to provide a better perspective on the contribution of this paper.

The first-order conditions for optimization problem (P1) are as follows (e.g., see Merton [20] and Breeden [4]):

$$J_w(t) = V_e(t), \tag{1}$$

$$\mu_i(t) - r(t) = -\text{cov}[J_w(t), S_i(t)], \qquad i = 1, \cdots, n, \tag{2}$$

where $J_w(t)$ is the partial derivative of $J(t)$ with respect to wealth, $V_e(t)$ is the partial derivative of $V(t)$ with respect to consumption expenditure, and cov$[x, y]$ denotes the covariance between the *rate* of change in x and *rate* of change in y.

Equation (1) represents the envelope condition, and equation (2) is the most general way to express risk premiums on security returns. By applying Itô's Lemma (e.g., see Arnold [1]) to the marginal utility of wealth, equation (2) will lead to Merton's multibeta asset-pricing model, while, by using the envelope condition and substituting the marginal utility of consumption expenditure for the marginal utility of wealth, Breeden's CCAPM will be obtained. That is,

$$\mu_i - r = R\ \text{cov}[E, S_i] + \textstyle\sum_{j=1}^{m} R_j \text{cov}[P_j, S_i], \qquad i = 1, \cdots, n, \tag{3}$$

where R is the measure of relative risk aversion, $-\partial \ln V_e/\partial \ln E$, and R_j denotes the degree of risk aversion toward changes in the price of consumption good j, $-\partial \ln V_e/\partial \ln P_j$.

Breeden [4] rearranges the last m terms on the right-hand side of this expression and develops two price indices, which are needed to determine real returns on securities and real change in aggregate consumption expenditure. Note that we have not aggregated equation (3) but that the aggregated pricing relationship will have the same structure (e.g., see Breeden [4]).

According to equation (3), the expected excess rate of return on asset i is a function of its covariances with changes in aggregate consumption expenditure and changes in prices of consumption goods. Market prices of these measures of risk are denoted R and R_j, $j = 1, \cdots, m$.

To test this pricing relationship, one has to determine the stochastic properties of prices of consumption goods and aggregate consumption expenditure. Furthermore, if the theoretical model is adjusted to account for consumption flows derived from durable goods, their stochastic properties must be determined as well, which can be estimated from data on aggregate consumption *expenditures* on durable goods, but this will introduce measurement errors into the data, leading to less powerful tests of the CCAPM.

II. The Model

The first-order conditions for optimization problem (P2) are

$$\frac{U_i}{P_i} - \frac{U_m}{P_m} = 0, \qquad i = 1, \cdots, m - 1, \tag{4a}$$

$$E - \textstyle\sum_{i=1}^{m} P_i C_i = 0, \tag{4b}$$

where U_i denotes $\partial U/\partial C_i$. Equations (4) represent a system of m equations and m unknowns (i.e., $C_1, \cdots, C_m$). If the second-order conditions are satisfied, the resulting demand functions will be unique (e.g., see Varian [24]).

PROPOSITION 1: *If the demand functions obtained from the system of m equations and m unknowns (i.e., (4a) and (4b)) are unique and differentiable, then any k equations of (4a) can be solved for optimal quantities of $k \leq m - 1$ consumption goods in terms of optimal quantities of the remaining $m - k$ goods and market prices of $k + 1$ consumption goods. The resulting k functions will also be unique and differentiable. For instance, the first k equations of (4a) can be solved for*

$$C_i \equiv g_i(C^{m-k}, P^{k+1}), \qquad i = 1, \cdots, k \leq m - 1, \tag{5}$$

where

$$C^{m-k} \equiv \{C_{k+1}, \cdots, C_m\} \quad and \quad P^{k+1} \equiv \{P_1, \cdots, P_k, P_m\}.$$

Proof: See the Appendix.

Equation (5) plays a central role in this paper. It states that data on market prices of consumption goods can be substituted for data on aggregate consumption of goods to describe investors' consumption decisions. For example, if optimal consumption of only one good is known (i.e., $m - k = 1$), then the information content of market prices of all consumption goods will be enough to determine optimal quantities of the remaining goods. Furthermore, given this information, investors' marginal utilities of consumption (i.e., the shadow price of one unit of wealth) will be determined as well.

This result leads to the following version of Theorems 1 and 2 in Breeden [5] and especially Theorem 2 in Breeden and Litzenberger [6].[4]

PROPOSITION 2: *If assumptions (A1) through (A5) hold and allocation of state-contingent claims are unconstrained Pareto optimal, then any two states of the economy that have (a) the same aggregate consumption of $k - m$ goods, (b) the same market prices of $k + 1$ goods, and (c) the same probability of occurrence will also command the same price for claims contingent on their occurrence.*

Proof: See the Appendix.

Breeden [4, 5] and Breeden and Litzenberger [6] show that, given the probability distribution of the states of the economy, cross-sectional differences in prices of state-contingent claims can be explained by cross-sectional differences

[4] I would like to thank the referee for suggesting this proposition.

in aggregate consumption expenditure and prices of consumption goods. Proposition 2 states that differences in prices of state-contingent claims can also be explained in terms of cross-sectional differences in aggregate consumption of $m - k$ goods and market prices of $k + 1$ goods. We are now prepared to present the major result of the paper.

PROPOSITION 3: *If assumptions* (A1) *through* (A5) *hold, then expected excess rates of return on assets can be explained in terms of their covariances with changes in aggregate consumption of* $m - k$ *goods and market prices of* $k + 1$ *goods, where* $0 \leq k \leq m - 1$.

Proof: By substituting equation (5) into the marginal utility of consumption for good j, we will have

$$U_j \equiv U_j(g_1(C^{m-k}, P^{k+1}), \cdots, g_k(C^{m-k}, P^{k+1}), C^{m-k}, t), \qquad 0 \leq k \leq m - 1. \tag{6}$$

This expression indicates that, when the investor is following an optimal consumption policy, the value of his or her marginal utility of consumption for good $j = 1, \cdots, m$ can be expressed in terms of consumption of $m - k$ goods and prices of $k + 1$ goods.

According to the first-order conditions of problems (P1) and (P2), we have

$$J_w = V_e = \frac{U_j}{P_j}, \qquad j = 1, \cdots, m. \tag{7}$$

Assuming that consumption good m is the numeraire and that its price is equal to one, we can substitute U_m for J_w in equation (2):[5]

$$\mu_i(t) - r(t) = -\operatorname{cov}[U_m(t), S_i(t)], \qquad i = 1, \cdots, n. \tag{8}$$

Applying Itô's Lemma to U_m and using the results of equation (6), we will have

$$\mu_i - r = \textstyle\sum_{j=m-k}^{m} H_j \operatorname{cov}[C_j, S_i] + \sum_{j=1}^{k} G_j \operatorname{cov}[P_j, S_i], \qquad i = 1, \cdots, n, \tag{9}$$

where

$$H_j = -\textstyle\sum_{i=1}^{k} \left[\dfrac{\partial \ln U_m}{\partial \ln g_i} \dfrac{\partial \ln g_i}{\partial \ln C_j}\right] - \dfrac{\partial \ln U_m}{\partial \ln C_j}$$

and

$$G_j = -\textstyle\sum_{i=1}^{k} \dfrac{\partial \ln U_m}{\partial \ln g_i} \dfrac{\partial \ln g_i}{\partial \ln P_j}.$$

Because we have assumed that $P_m = 1$, its covariance with security i is zero.

Equation (9) is not an aggregate equilibrium relationship, but it can be aggregated using the same procedure employed by Merton [20] and Breeden [4], and the aggregated expression will have the same structure as equation (9). For brevity we assume that equation (9) is already in aggregate form. Q.E.D.

[5] If P_m is different from one, equation (8) must be expressed as follows:

$$\mu_i - r = -\operatorname{cov}[U_m, S_i] + \operatorname{cov}[P_m, S_i].$$

Equation (9) represents the pricing relationship of this paper; it is rather similar to Merton's [20] multibeta asset-pricing model except that it measures riskiness of securities in terms of covariances of their returns with random variables that are, in principle, observable. In this model, market prices of risk are denoted by H_j and G_j. Because we have not made any assumption about investors' direct utility functions and because these market prices of risk depend on cross-derivatives of investors' direct utility functions, we cannot prove whether H_j and G_j are positive or negative.

III. Examples

To illustrate the implications of the pricing relationship discussed in Proposition 3, we present three specific examples of the model.

Example 1: There is a representative investor whose preference can be expressed by a Cobb-Douglas direct utility function of the following form:[6]

$$U = (\textstyle\prod_{i=1}^{m} C_i^{\beta_i})^{\gamma},$$

where $\sum_{i=1}^{m} \beta_i = 1$, $0 \leq \beta_i \leq 1$, and $\gamma < 1$.

Using this form of utility function, the pricing relationship will be as follows:

$$\mu_i - r = (1 - \gamma \textstyle\sum_{j=1}^{k} \beta_j)\mathrm{cov}[C_m, S_i] - \gamma \textstyle\sum_{j=m-k}^{m} \beta_j \mathrm{cov}[C_j, S_i] + \gamma \textstyle\sum_{j=1}^{k} \beta_j \mathrm{cov}[P_j, S_i].$$

For this type of utility function, β_j denotes the budget share of consumption good j and $(1 - \gamma)$ denotes the measure of relative risk aversion with respect to consumption expenditure.

The market price of risk for covariability with the numeraire, C_m, is always positive, while those for covariability with aggregate consumption and prices of other goods may be positive or negative depending on whether the investor's degree of relative risk aversion is greater or smaller than one.[7]

Example 2: Aggregate consumption of only one good and market prices of all goods can be observed. Without loss of generality, we assume that only aggregate consumption of the numeraire can be observed. Then, the pricing relationship will be as follows:

$$\mu_i - r = \beta_{im}(\delta_m - r) + \textstyle\sum_{j=1}^{m-1} \beta_{ij}(\delta_j - r), \qquad i = 1, \cdots, n, \tag{10}$$

where β_{im} is the beta of asset i with respect to aggregate consumption of the numeraire, β_{ij} is the beta of the same asset with respect to changes in the price of consumption good j, and δ_m and δ_j are, respectively, rates of return on portfolios that are highly correlated with changes in C_m and P_j.[8]

[6] The assumption that there is a representative investor can be relaxed. I would like to thank the referee for suggesting this example.

[7] Similar to other models that deal with many consumption goods, logarithmic utility function is the border line in the present model.

[8] Possible candidates for $\delta_j, j = 1, \cdots, m - 1$, are returns on commodity futures.

Example 3: In this example, we discuss the effects of separable direct utility functions on the pricing relationship. Let there be $s \leq m$ groups, written $m = m_1 + \cdots + m_s$ in such a way that each consumption good belongs to exactly one group (i.e., there are m_i goods in group i). Let investors' direct utility functions be weakly separable in the following sense (e.g., see Varian [24]):

$$U(C_1, \cdots, C_m, t) = F_1(Q_1, t) + \cdots + F_s(Q_s, t),$$

where Q_i is the vector of consumption goods belonging to group i.

Under this condition, if market prices of $k_i \leq m_i - 1$ goods can be observed, then aggregate consumption of only $m_i - k_i$ has to be observed to test the CCAPM. For example, if aggregate consumption of C_i can be observed, only the stochastic properties of prices of those goods that belong to the group to which C_i belongs have to be estimated to test or to apply the CCAPM.

IV. Summary

This paper develops an alternative form of the consumption CAPM in a multi-good economy that is, in principle, testable even if aggregate consumption of goods and their market prices are only partially observable. Our principal result is that the information content of market prices of goods can be used instead of the information content of aggregate consumption of goods to test or to apply the CCAPM.

Under the same set of assumptions used in previous studies, this paper shows that, when there are m consumption goods and aggregate consumption of only $m - k$ goods can be observed (where $k \leq m - 1$), then one need observe the movement of market prices of only $k + 1$ consumption goods to test or to apply the model.

Because data on prices of goods can be substituted for data on aggregate consumption of goods, the results of this paper introduce some flexibility into the type of data that researchers can use to test the CCAPM, and, because data on market prices of goods are less likely to be affected by measurement errors, our results should lead to more powerful tests of the CCAPM.

Appendix

Proof of Proposition 1: According to the Implicit Function Theorem, the system of equation (4a) can be solved for, say, $C_1, \cdots, C_k$ in terms of the remaining variables if and only if the determinant of the following matrix is different from zero (e.g., see Nikolsky [22], p. 270).

$$\begin{bmatrix} P_1^{-1}U_{11} - P_m^{-1}U_{m1} & \cdots & P_1^{-1}U_{1k} - P_m^{-1}U_{mk} \\ \vdots & & \vdots \\ P_k^{-1}U_{k1} - P_m^{-1}U_{m1} & \cdots & P_k^{-1}U_{kk} - P_m^{-1}U_{mk} \end{bmatrix}. \tag{A1}$$

Consider the following two matrices:

$$\begin{bmatrix} U_{11} & \cdots & U_{1m} \\ \vdots & & \vdots \\ U_{m1} & \cdots & U_{mm} \end{bmatrix} \tag{A2}$$

and

$$\begin{bmatrix} P_1^{-1} & \cdots & 0 \\ \vdots & P_i^{-1} & \vdots \\ 0 & \cdots & P_m^{-1} \end{bmatrix} \tag{A3}$$

Matrix (A2) is a submatrix of the second-order conditions to maximization in problem (P2) (e.g., see Intriligator [19]), while (A3) is the inverse of a diagonal matrix of prices of consumption goods.

Since we have assumed that the demand functions obtained from the original system are single valued, matrix (A2) must be nonsingular and its minor determinants different from zero. If this matrix is multiplied by matrix (A3), the result will be a nonsingular matrix with nonzero minor determinants (e.g., see Dhrymes [10]).

Matrix (A1) is a *submatrix* of a *linear* transformation of the matrix that will result from multiplying matrices (A2) and (A3). Hence, its determinant must be different from zero. Q.E.D.

Proof of Proposition 2: Our proof has two parts.

1. Following Breeden [5], if aggregate consumption of $m - k$ (note, $k \leq m - 1$) goods are the same for two states, then an investor's consumption of these goods will also be the same in the two states. If prices of $k + 1$ goods are also the same in these two states, then, according to Proposition 2, the investor's consumption of the remaining k consumption goods and, hence, his or her marginal utility of consumption, $U_i(s)$, $i = 1, \cdots, m$, will also be the same in the two states.

2. In an Arrow-Debreu economy, the price of a state-contingent claim can be expressed as follows (e.g., see Arrow [2] and especially Section 1.4 of Dreze [11]):

$$\lambda_s = \frac{U_m(s)}{P_m(s)} \times \frac{\pi_s}{\lambda_0}, \tag{A4}$$

where λ_s is the current price of an Arrow-Debreu security that will pay one "dollar" contingent on state s, π_s is the probability that state s will occur, and λ_0 is the shadow price of one "dollar" received today.

According to equation (A4), prices of Arrow-Debreu securities are the same for any two states that have the same probability and in which an investor has the same normalized marginal utility of consumption. By combining the two parts, the proof will be completed. Q.E.D.

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