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The Temporal Price Relationship between S&P 500 Futures and the S&P 500 Index

IRA G. KAWALLER, PAUL D. KOCH, and TIMOTHY W. KOCH*

ABSTRACT

This paper empirically examines the intraday price relationship between S&P 500 futures and the S&P 500 index using minute-to-minute data. Three-stage least-squares regression is used to estimate lead and lag relationships with estimates for expiration days of the S&P 500 futures compared with estimates for days prior to expiration. The results suggest that futures price movements consistently lead index movements by twenty to forty-five minutes while movements in the index rarely affect futures beyond one minute.

THE EVOLUTION OF STOCK index futures markets has had a profound effect on the nature of traditional common stock trading. Futures traders frequently take coincident positions in the cash market such that a substantial volume of equity transactions is tied to futures activity. Critics allege that futures trading unduly influences the underlying equity prices, especially on days when institutional investors implement program trading strategies. While this intermarket effect can appear at any time, it is commonly associated with the final hour of trading on the futures contract expiration days.[1] During 1984 and 1985, for example, the stock markets closed with prices either rising or falling dramatically during the last hour of trading for six out of the eight expiring contracts. Proponents of futures argue that these markets provide an important price discovery vehicle and offer an alternative marketplace for adjusting equity exposure. Moreover, they do not view the aforementioned price swings as a problem. Observed price changes have been temporary and nonsystematic as prices have risen on some expiration days and declined on others.

This study addresses two basic questions that have a fundamental bearing on the debate between critics and advocates of stock index futures markets. First, does the futures market serve as a price discovery vehicle for stock prices? Many analysts believe that the difference between the futures price and the index can be used as an indicator of forthcoming moves in the index. An extraordinarily large basis (futures price minus index price) would tend to be an indication of

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[1] The term "triple witching hour" is used to describe this trading period because the CME's S&P 500 futures, the CBOE's S&P 100 options, and contracts on individual stock options all expire on the third Friday of March, June, September, and December. After the March 1987 contract expired, the CME moved expiration of S&P 500 futures to the preceding Thursday to reduce expiration-day price pressures.

an impending bullish move, while an extraordinarily small or negative basis would be indicative of an impending market decline. Second, does the character of the spot/futures price relationship differ on expiration days from that on nonexpiration days?

We specifically investigate the empirical intraday price relationship between the S&P 500 futures and the S&P 500 index. The primary objective is to determine whether movements in the futures price provide predictive information regarding subsequent movements in the index and/or vice versa. We employ time-series regression analysis to identify the nature of this intraday dynamic relationship and test whether a systematic lead/lag relationship exists. The tests distinguish between the price relationship on expiration day versus days prior to expiration because market activity on expiration day may differ from that on nonexpiration days. Evidence regarding the existence, nature, and robustness of this relationship should prove valuable to both advocates and critics of futures markets, allowing them to exploit or avoid the markets as they see fit.

The paper is organized as follows. The general relationship between the nearby S&P 500 futures price and the S&P 500 index is described in Section I. Section II develops the model used to analyze the intraday price relationship and discusses rationale suggesting differences for various trading days prior to expiration of the futures contract. The empirical results are presented in Section III, and a final section summarizes the implications.

I. The Relationship between the S&P 500 Futures and Index Prices

The S&P 500 stock index represents the market value of all outstanding common shares of five hundred selected firms. The group consists of four hundred industrials, forty financial institutions, forty utilities, and twenty transportation firms. While all of the shares are not traded on the NYSE, the cumulative market value equals approximately eighty percent of the aggregate value of NYSE-listed stocks. The index changes whenever the price (and, thus, the cumulative market value) of any underlying stock changes.

A S&P 500 futures contract represents the purchase or sale of the five hundred stocks underlying the S&P 500 index in a proportion consistent with the weights set by the index, with the trade date specified for the expiration of each contract. Before accessing the market, traders must post an initial margin deposit or collateral equal to only a fraction of the stocks' market value. Futures prices change intermittently throughout each trading day. At day's end, there is a marking to market of the contract position, whereby traders must cover any losses when prices move against them or may withdraw any profit in excess of their initial margin requirement when prices move favorably. Following the date upon which the contract stops trading, a final marking-to-market adjustment is made, with the final settlement price set equal to the final settlement index calculation. Prior to June 1987, this spot market expiration value was the actual index value at 4:15 P.M. Eastern Standard Time on the third Friday of the month in which the futures stopped trading. Thereafter, trading has terminated on the third Thursday of the month, with the final settlement value determined the following morning with a special settlement quotation based upon opening trades for the stocks in the index. Given the final settlement value, one last marking-to-market adjustment and variation-margin adjustment suffice to close the contract.

The use of stock index futures typically falls into one of three categories: (a) hedging, which involves the purchase or sale of index futures in anticipation of an intended cash market trade, whereby the hedge provides compensation for adverse price moves prior to the cash transaction, (b) arbitrage, which involves the simultaneous purchase and sale of stocks and futures in order to capture realignments of relative prices following a perceived mispricing opportunity, and (c) trading, which involves the active use of futures to speculatively take advantage of anticipated broad market price movements. While arbitrage uses both cash and futures contracts, hedging and trading strategies normally incorporate only one type of instrument at any given point in time.

Considerable research has been directed at determining the "theoretically correct futures price". The most common model predicts that the price reflects the cost of carrying a S&P 500 look-alike basket of stocks until the expiration of the S&P 500 future.[2] This cost of carry incorporates the cost of financing the investment less the dividends that derive from the basket of stocks and any additional reinvestment income. If the actual futures price is higher than the predicted value, the futures contract is overvalued, justifying the purchase of the stocks and the simultaneous sale of the futures contract. If the actual futures price is below the predicted value, the futures is undervalued and the reverse trade is initiated. Upon the convergence of the futures price to the actual index value at the expiration of the contract, the arbitrager is assured of achieving some predetermined fixed-income return or fixed-rate cost of borrowing. It is possible that a liquidation opportunity may arise sooner if the futures price returns to its predicted value prior to expiration.

Numerous studies have explained the price relationship between stock index futures and the underlying stocks in terms of arbitrage behavior. In practice, futures prices normally vary relative to stock prices without triggering the arbitrage, so that arbitrage opportunities are available infrequently. Cornell and French [1], Figlewski [5], Modest and Sundaresan [15], and Stoll and Whaley [16] attempt to identify and measure the arbitrage trading boundaries. Results indicate that the futures-to-cash price differential, labeled the basis, normally falls within boundaries determined by financing costs adjusted for dividend uncertainty, transactions costs, and taxes. Because market interest rates have historically exceeded the dividend rate on common stocks, the stock index futures price normally exceeds the stock index value, and the basis is positive. We can characterize the normal relationship between the S&P 500 futures price at time t (F_t) and the index price at time t (I_t) as[3]

$$e_{L,t} < (F_t - I_t) < e_{U,t}, \tag{1}$$

where $e_{L,t}$ = lower bound of the no-arbitrage trading range at time t and $e_{U,t}$ = upper bound of the no-arbitrage trading range at time t.

[2] Kawaller [11] identifies a variety of shortcomings of this traditional model.

[3] The theoretical upper and lower bounds are discussed extensively in the literature. We use a general characterization for ease of exposition and brevity.

Stock index arbitragers compare the basis ($F_t - I_t$) with net financing and transactions costs. While the basis normally falls within the no-arbitrage boundaries, it occasionally moves outside that range. When it exceeds $e_{U,t}$, traders buy stocks and sell futures. This is labeled program buying in that institutional investors implement trades by computer in large volume as indicated by the temporary mispricing opportunity. Because the arbitrage generally involves at least three hundred stocks and a minimum $5 million investment, traders acting in concert bid cash equity prices relatively higher and futures prices relatively lower, thereby driving the basis back within the no-arbitrage range. The opposite occurs when the basis falls too low. Traders sell stocks and buy futures, labeled program selling, thereby driving the basis higher. In the first case, arbitragers earn a fixed-income return comprised of the basis adjustment plus dividends because they can reverse the transactions later when the basis approaches zero at expiration. The second case guarantees a fixed-income expense from selling stock, financed by security repurchase agreements, with the cost offset by the basis adjustment at expiration.

II. Intraday Leads and Lags between Futures and Index Prices

A. General Characteristics

Two phenomena, market sentiment and arbitrage trading, are the major determinants linking stock index futures and the stock market. The conventional wisdom among professional traders is that movements in the S&P 500 futures price reflect market expectations of subsequent movements in cash prices. The futures price presumably embodies all available information regarding events that will affect cash prices and responds quickly to new information. S&P 500 index price movements may similarly convey information regarding subsequent price variation in the futures contract. It is unlikely, however, that the relationships are symmetric. For the index to completely reflect new information, the underlying stocks must trade at prices different from their previous trade. Because most index stocks do not trade at different prices each minute, the index responds to new information with a lag.[4]

Consider a trader reacting to new information on the health of the economy. If the information is bullish, a trader has the choice of buying either S&P 500 futures or the underlying stocks. While the futures trade can be effected immediately with little up-front cash, actual stock purchases require a greater initial investment and may take longer to implement as they involve a subsequent stock selection and numerous individual stock transactions. This speculative transactions preference for futures explains why changes in futures prices may lead changes in stock prices and the S&P 500 index. Futures prices may thus provide a sentiment indicator of forthcoming cash prices, which follow when investors who are unwilling or unable to use futures incorporate the same information into their cash market transactions.

[4] It is possible that new information affects a subset of index stocks disproportionately, relative to the entire stock market. In such cases, not all index stocks must be traded each minute for the index to adjust completely and quickly to new information.

Changes in the S&P 500 index may also lead changes in the futures price as the value of the index represents part of the information that affects futures prices. Futures traders likely incorporate recent changes in the index in their pricing decisions. Put another way, if the index were to decline or rise for whatever reason, the price change might induce a change in sentiment that would be reflected in subsequent declines or increases in futures prices.

As long as the basis lies within the no-arbitrage trading range designated by (1), changes in market sentiment would affect both futures prices and the index in the same direction. If the basis varies outside the no-arbitrage range, however, arbitragers would take opposite positions in the two markets so that the basis would ultimately approach its predicted value. This adjustment could arise because both prices move in a common direction, with one price moving more rapidly than the other, or because the two prices move oppositely. Regardless of which occurs, the lead/lag relationship during periods when arbitrage activity is present might reasonably be expected to differ from the lead/lag relationships present when no arbitrage activity occurs.

B. The Model

The previous discussion suggests that the S&P 500 futures and index price movements may each convey predictive information regarding subsequent price variation in their own market and the other's market. That is, movements in the S&P 500 index are determined by its own past movements and the history of S&P 500 futures price movements, as well as other relevant market information. Likewise, movements in the S&P 500 futures price are determined by its own past history, the history of index movements, and other market information. We test these temporal price relationships by estimating distributed lags between the first differences of the index and nearby futures price, specified as follows:[5]

$$i_t = z_1 + \sum_{k=1}^{\infty} a_k i_{t-k} + \sum_{k=0}^{\infty} b_k f_{t-k} + e_{1t}, \tag{2}$$

$$f_t = z_2 + \sum_{k=0}^{\infty} c_k i_{t-k} + \sum_{k=1}^{\infty} d_k f_{t-k} + e_{2t}, \tag{3}$$

where z_1 and z_2 are intercept terms, i_t equals the change in I_t, f_t equals the change in F_t (i.e., $i_t = (1 - L)I_t$ and $f_t = (1 - L)F_t$, with L equal to the lag operator ($L^k I_t = I_{t-k}$, $L^k F_t = F_{t-k}$)), and other relevant market information affecting these prices is represented by random noise, e_{1t} and e_{2t}.

Equations (2) and (3) represent a simultaneous-equations model because futures and cash prices may affect each other contemporaneously through b_0 and c_0. If such is the case, ordinary least-squares estimation of (2) or (3) would yield biased and inconsistent estimates. Furthermore, any attempt to test restrictions on the coefficients of the distributed lags would be inappropriate because the Wald F-test would not have the usual desirable statistical properties. With this in mind, (2) and (3) may be rewritten as the following joint dynamic simultaneous-

[5] The use of first differences assumes that all relevant information is contained in the minute-to-minute changes in prices. Such an assumption is theoretically supported by efficient-markets arguments and empirically supported by time-series-analysis results presented later.

equations model:

$$\begin{bmatrix} i_t \\ f_t \end{bmatrix} = \begin{bmatrix} A(L) & B(L) \\ C(L) & D(L) \end{bmatrix} \begin{bmatrix} i_t \\ f_t \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}, \tag{4}$$

where the distributed lags specified in (2) and (3) appear as convergent rational polynomials in the lag operator ($A(L)$, $B(L)$, $C(L)$, and $D(L)$) and where e_{1t} and e_{2t} may be contemporaneously correlated.

In order to estimate (4), we must first select a finite lag parameterization. While longer lag lengths lessen the chance of misspecification, they also result in the loss of more degrees of freedom. Hence, it is desirable to choose the minimum lag length that specifies the relationship accurately. Geweke [8] argues that the number of lags on the dependent variable in each equation should be kept generous in order to minimize the chance of serially correlated errors, while the number of lags on the other variables should be set lower to retain power in the hypothesis tests. In the following estimation, the lag length is set at sixty minutes for the dependent variable in each equation ($A(L)$ and $D(L)$) and at forty-five minutes for the right-hand-side variable ($B(L)$ and $C(L)$).[6]

We estimate (4) using three-stage least squares. This provides two potential advantages over ordinary least squares applied to a single equation. First, because of the potential simultaneity, an instrumental-variables estimator is required to produce consistent estimates. Second, the "other relevant information" embodied in each error term, e_{1t} and e_{2t}, may affect both prices. This would imply contemporaneous correlation between the error terms, and, even in the absence of simultaneity (if $b_0 = c_0 = 0$), ordinary least squares would yield inefficient estimates.

The temporal interrelationships described above are most likely of one nature when the basis falls within the no-arbitrage boundaries and are of a different nature when the basis lies outside the boundaries and arbitrage activity occurs. In particular, the distributed lags specified in (2) and (3) may extend over relatively long periods for trading activity that occurs within the no-arbitrage range. When the basis penetrates the boundaries, however, new forces enter the market that bring the basis back in line and thus could disturb the "normal" lead/lag relationships.

Ideally, we would like to estimate (4) during periods when the basis traded within the no-arbitrage boundaries versus periods when the basis traded outside the boundaries and compare the results. Unfortunately, quantifying the no-arbitrage trading range is difficult, and one would not expect the basis to lie outside any reasonably quantified trading range long enough to collect sufficient observations for the analysis. We allow for differences in the frequency of arbitrage activity that may occur systematically over the life of a contract by investigating the distributed-lag relationships in (4) on different days prior to expiration. Because arbitrage activity is presumably the greatest on expiration

[6] We have also estimated (4) with lag lengths of thirty-one and sixty minutes on the right-hand-side variable. The results of the hypothesis tests and the patterns in distributed-lag coefficients are robust with respect to this choice. In most cases, the distributed-lag coefficients beyond forty-five minutes display no distinct pattern or significant coefficients. For this reason, as well as the gain in power of the tests, we present results for the lag length of forty-five minutes.

day when program traders close out earlier positions, estimates of the lead/lag relationship may differ on expiration day compared with estimates from days prior to expiration of the same contract.[7]

C. Data

Minute-to-minute data on the prices of nearby S&P 500 futures contracts (F_t) and the S&P 500 index (I_t) were obtained from the Chicago Mercantile Exchange for all trading days during 1984 and 1985. The futures prices are actual transactions prices for all trades during a day. The index, however, is available only once each minute. We match each index price with the last futures price quoted during the minute the index appears as the coincident futures quote. Each day represents six hours of trading and provides 360 pairs of observations on (F_t, I_t).[8]

Besides examining the nature of any distributed lags and the predictive information contained in such lags, we wish to investigate the hypothesis that the distributed lags change as expiration day approaches. For each nearby futures contract investigated, our sample consists of six trading days: eighty-eight days prior to expiration (Monday), sixty days prior to expiration (Monday), thirty days prior to expiration (Wednesday), fourteen days prior to expiration (Friday), one day prior to expiration (Thursday), and expiration day (Friday). Of particular interest is the question of whether the last day of trading exhibits a different character from earlier days. On expiration days, any open arbitrage positions require unwinding or a rolling of the existing futures positions into the next expiration month. Given the special requirements of that day, it remains an empirical question as to whether such days exhibit unique features. Importantly, our results can only identify uniqueness as it relates to the lead/lag relationships between the two price series.

III. Empirical Results

A. Preliminary Time-Series Analysis

Haugh [10], Geweke [8], and Koch and Yang [14] demonstrate that, in the early stages of developing a model such as (4), it is useful to compute the cross-correlation function between filtered data series to identify the empirical dynamic relationships. Such cross-correlation analysis revealed several important results. First, in filtering each price variable (F_t and I_t), the estimated autoregressive parameter at lag one was extremely close to one (above 0.99 in most cases). This implies that first differencing is appropriate and that changes in each variable

[7] Realizing that arbitrages are initiated when futures contracts are mispriced, subsequent adjustments to fair value might allow arbitragers to reverse their positions prior to the expiration day.

[8] Prior to the June 1984 contract, S&P 500 futures expired on Thursdays. We restrict our sample to the last three contracts in 1984 and all contracts that expire in 1985. It should also be noted that futures trade for fifteen minutes after the stock markets close. We ignore these quotes in our analysis. Finally, since September 30, 1985, quotes are available beginning at 8:30 A.M. We restrict our analysis to the six hours between 9:01 A.M. and 3:00 P.M. so that the results can be compared across quarters. Since June 1986, the S&P 500 index has been reported every fifteen seconds.

Table I

Contemporaneous Coefficients and Goodness-of-Fit Statistics from System (4)

Date	b_0	Standard Error	t-Ratio	c_0	Standard Error	t-Ratio	MSE[a]	R^{2b}
Second Quarter, 1984								
3/19/84	0.112	0.016	6.81	1.891	0.277	6.83	1.475	0.518
4/16/84	0.196	0.014	14.05	3.129	0.227	13.81	1.271	0.786
5/16/84	0.188	0.016	11.58	2.521	0.226	11.16	1.341	0.695
6/01/84	0.220	0.024	9.29	1.710	0.173	9.87	1.398	0.627
6/14/84	0.021	0.029	0.72	0.195	0.175	1.12	1.555	0.469
6/15/84	0.342	0.032	10.62	1.302	0.122	10.66	1.368	0.653
Third Quarter, 1984								
6/25/84	0.209	0.014	14.94	3.255	0.212	15.33	1.232	0.800
7/23/84	0.246	0.019	12.96	2.294	0.184	12.49	1.302	0.745
8/22/84	0.142	0.015	9.34	2.495	0.271	9.21	1.416	0.650
9/07/84	0.161	0.016	10.11	2.799	0.255	10.96	1.369	0.670
9/20/84	0.063	0.020	3.09	0.729	0.244	2.99	1.541	0.451
9/21/84	0.158	0.020	8.03	1.761	0.219	8.03	1.441	0.673
Fourth Quarter, 1984								
9/24/84	0.172	0.011	15.72	4.093	0.254	16.14	1.218	0.815
10/22/84	0.154	0.013	11.76	3.245	0.283	11.48	1.335	0.727
11/21/84	0.175	0.029	6.09	0.944	0.154	6.12	1.495	0.556
12/07/84	0.188	0.019	9.96	2.185	0.210	10.38	1.387	0.633
12/20/84	0.094	0.023	4.14	0.906	0.218	4.16	1.526	0.478
12/21/84	0.219	0.033	6.65	0.900	0.136	6.60	1.479	0.642
First Quarter, 1985								
12/17/84	0.080	0.032	2.50	0.413	0.151	2.73	1.543	0.555
1/14/85	0.214	0.017	12.89	2.626	0.203	12.97	1.305	0.746
2/13/85	0.184	0.019	9.66	2.070	0.218	9.50	1.401	0.705
3/01/85	0.195	0.022	9.04	1.763	0.192	9.20	1.430	0.695
3/14/85	0.194	0.025	7.70	1.377	0.174	7.90	1.446	0.578
3/15/85	0.179	0.038	4.72	0.588	0.133	4.42	1.518	0.566

Second Quarter, 1985								
3/25/85	0.088	0.018	4.93	1.375	0.266	5.17	1.508	0.483
4/22/85	0.176	0.021	8.26	1.901	0.213	8.92	1.420	0.649
5/22/85	0.086	0.030	2.87	0.499	0.170	2.94	1.545	0.423
6/07/85	0.025	0.023	1.09	0.376	0.224	1.68	1.553	0.389
6/20/85	0.259	0.054	4.76	0.515	0.096	5.36	1.512	0.539
6/21/85	0.115	0.032	3.62	0.582	0.156	3.73	1.530	0.562
Third Quarter, 1985								
6/24/85	0.127	0.027	4.77	0.864	0.185	4.68	1.513	0.450
7/22/85	0.234	0.022	10.84	1.951	0.177	11.03	1.360	0.641
8/21/85	0.204	0.024	8.55	1.594	0.185	8.60	1.424	0.564
9/06/85	0.277	0.025	11.06	1.723	0.155	11.13	1.367	0.670
9/19/85	0.203	0.022	9.17	1.698	0.178	9.57	1.430	0.611
9/20/85	0.192	0.027	7.05	1.181	0.166	7.11	1.478	0.517
Fourth Quarter, 1985								
9/23/85	0.175	0.019	9.11	1.928	0.213	9.07	1.418	0.605
10/21/85	0.190	0.023	8.16	1.522	0.177	8.59	1.450	0.570
11/20/85	0.165	0.020	8.42	1.919	0.230	8.37	1.425	0.551
12/06/85	0.247	0.024	10.26	1.700	0.166	10.22	1.383	0.668
12/19/85	0.129	0.026	5.02	0.892	0.188	4.75	1.519	0.461
12/20/85	0.174	0.025	6.87	1.239	0.178	6.94	1.480	0.609

[a] The weighted mean square error for the system is represented by MSE.

[b] The weighted R^2 for the system is represented by R^2 that corresponds to the approximate F-test on all non-intercept parameters in the system.

convey the relevant market information contained in each price.[9] Second, the futures and index price innovations displayed high contemporaneous correlation on virtually all trading days examined. Finally, the empirical distributed lag from f_t to i_t typically extended over many minutes, while the distributed lag from i_t to f_t appeared to be quite short, when it existed at all.[10]

B. Distributed-Lag Estimates

Three-stage least-squares estimation of (4) confirms the preliminary results discussed above. Table I presents the estimates of the contemporaneous coefficients, b_0 and c_0, for each trading day examined in 1984 and 1985. On all but two of these days, both coefficients are highly significant. A comparison of coefficient estimates across trading days in each quarter reveals no systematic patterns. Most importantly, neither contemporaneous coefficient consistently increases on expiration day. While b_0 takes its largest value twice on expiration day relative to other test days during each quarter, c_0 is always either the smallest or second smallest on expiration day. Evidently, contemporaneous price relationships on expiration days do not differ substantively from those on other days. Any differences in contemporaneous effects under arbitrage conditions are not sufficiently strong or pervasive throughout the course of the entire day to result in significantly different coefficients from other days tested.

The remaining coefficients in the distributed lags ($B(L)$ and $C(L)$) provide statistical information about the nature of the lead/lag relationships. Coefficient estimates for the six days examined in the fourth quarter of 1984 are presented in Figures 1 through 6. Figures 7 through 12 provide the comparable estimates for the fourth quarter of 1985.[11] The distributed lag from f_t to i_t ($B(L)$) extends for twenty to forty-five minutes, depending on the trading day under scrutiny, and typically includes numerous coefficients that are significantly different from zero. The distributed lag from i_t to f_t ($C(L)$) is short where it exists, rarely extending for more than one minute. These results are similar to the estimates for the corresponding days in the second and third quarters of 1984 and the first three quarters of 1985.

It is interesting to observe the manner in which the distributed lag from f_t to i_t ($B(L)$) changes as expiration day approaches. In Figure 1 (eighty-eight days prior to the December 1984 expiration date), the first thirty-five distributed-lag coefficients are mostly positive, whereas the subsequent coefficients appear to diminish in magnitude and deviate randomly about zero. The distributed lag in Figure 2 extends for approximately twenty minutes before falling off to random noise about zero. In Figures 3, 4, and 5, the lag coefficients are larger in magnitude and virtually all positive, indicating a substantive distributed lag from futures to spot prices for at least forty-five minutes. Figure 6 indicates that the lag from f_t

[9] This result implies that the price series are themselves nonstationary. First differencing is required to achieve stationarity (Granger [9]; Geweke [8]).

[10] The cross-correlation analysis results are available upon request.

[11] The estimated distributed lags beyond the contemporaneous coefficient are summarized in Figures 1 through 12. The contemporaneous coefficients, b_0 and c_0, are too large to fit into the scale for the plots and are therefore omitted from the figures. For brevity, results for only the last quarters of 1984 and 1985 are presented. The pattern of results is robust across all periods.

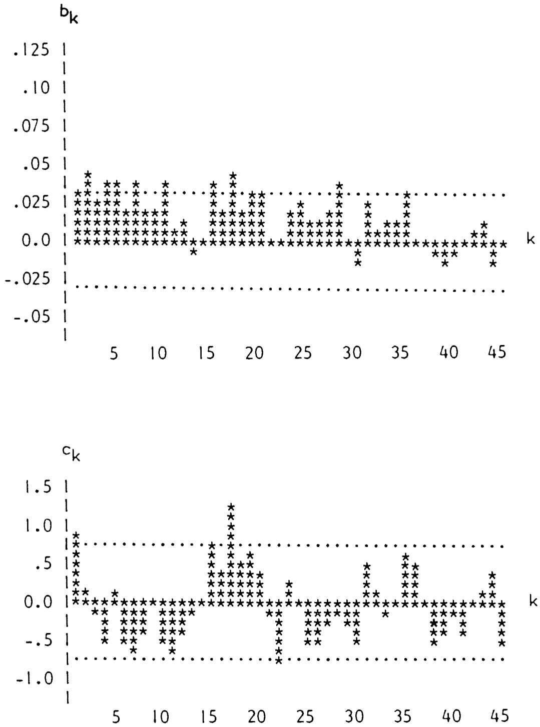

Figure 1. Distributed lags, $B(L)$ and $C(L)$, estimated in equation (4). 9/24/84: eighty-eight days prior to expiration. Ninety-five percent confidence intervals of two standard deviations are presented as "·" for all distributed-lag regression coefficients.

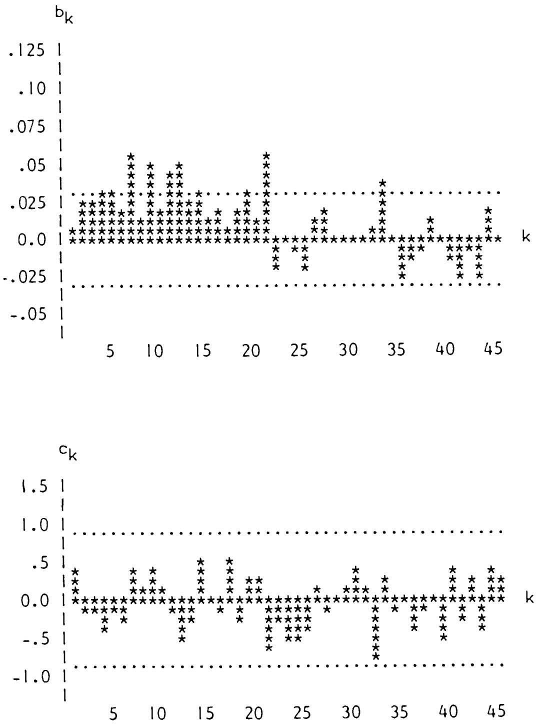

Figure 2. Distributed lags, $B(L)$ and $C(L)$, estimated in equation (4). 10/22/84: sixty days prior to expiration. Ninety-five percent confidence intervals of two standard deviations are presented as "·" for all distributed-lag regression coefficients.

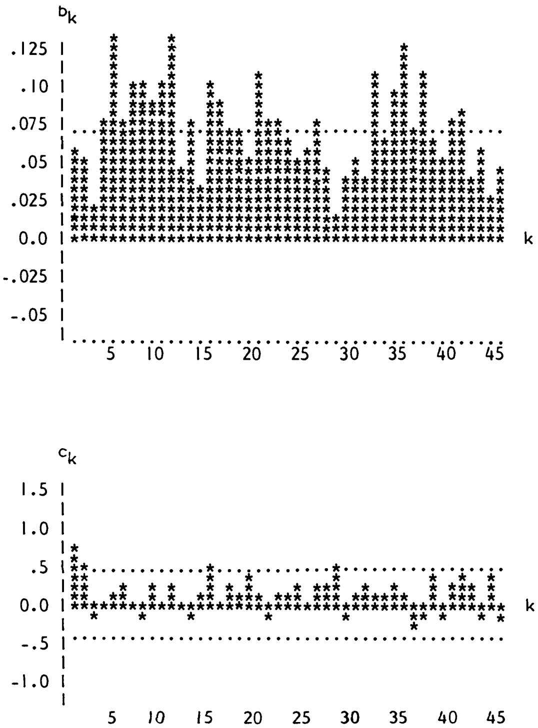

Figure 3. Distributed lags, $B(L)$ and $C(L)$, estimated in equation (4). 11/21/84: thirty days prior to expiration. Ninety-five percent confidence intervals of two standard deviations are presented as "·" for all distributed-lag regression coefficients.

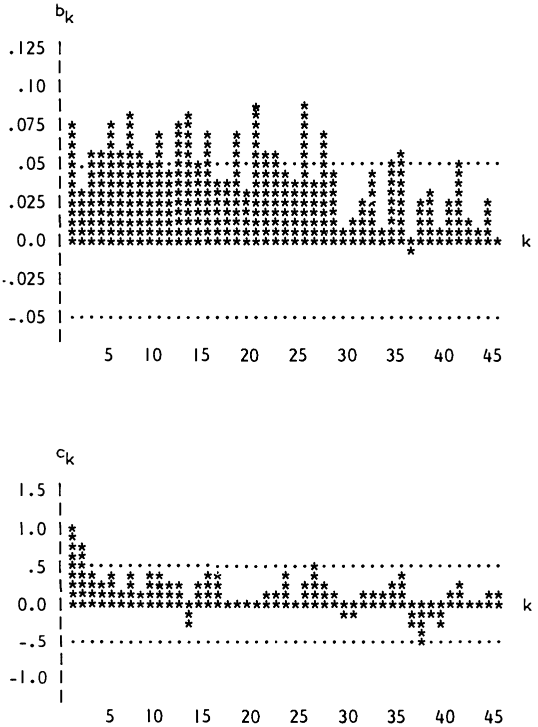

Figure 4. Distributed lags, $B(L)$ and $C(L)$, estimated in equation (4). 12/7/84: fourteen days prior to expiration. Ninety-five percent confidence intervals of two standard deviations are presented as "·" for all distributed-lag regression coefficients.

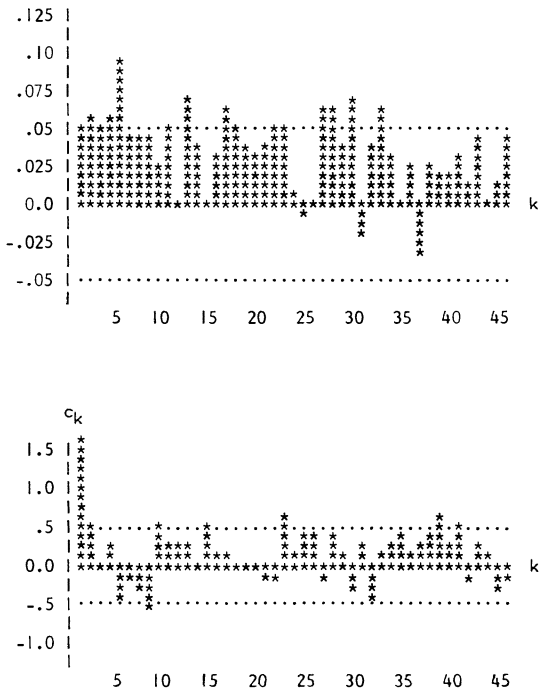

Figure 5. Distributed lags, $B(L)$ and $C(L)$, estimated in equation (4). 12/20/84: one day prior to expiration. Ninety-five percent confidence intervals of two standard deviations are presented as "·" for all distributed-lag regression coefficients.

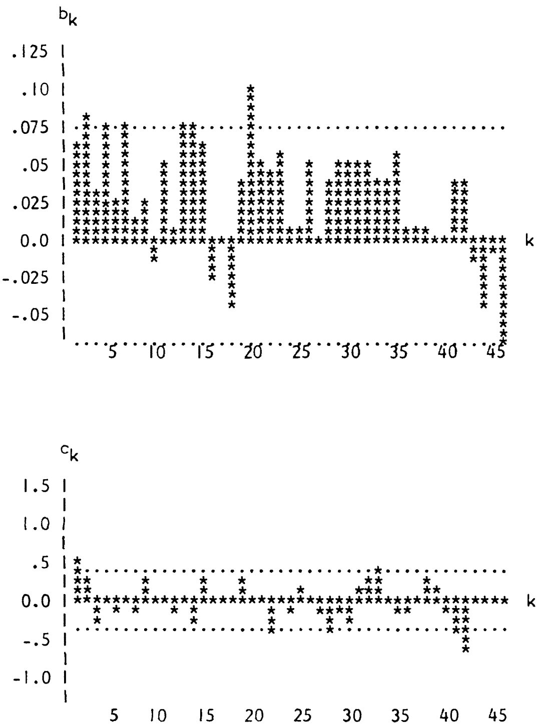

Figure 6. Distributed lags, $B(L)$ and $C(L)$, estimated in equation (4). 12/21/84: expiration day. Ninety-five percent confidence intervals of two standard deviations are presented as "·" for all distributed-lag regression coefficients.

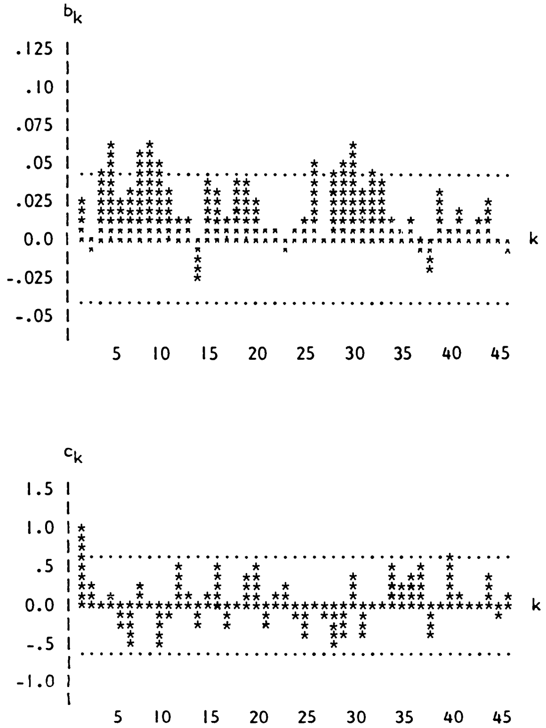

Figure 7. Distributed lags, $B(L)$ and $C(L)$, estimated in equation (4). 9/23/85: eighty-eight days prior to expiration. Ninety-five percent confidence intervals of two standard deviations are presented as "·" for all distributed-lag regression coefficients.

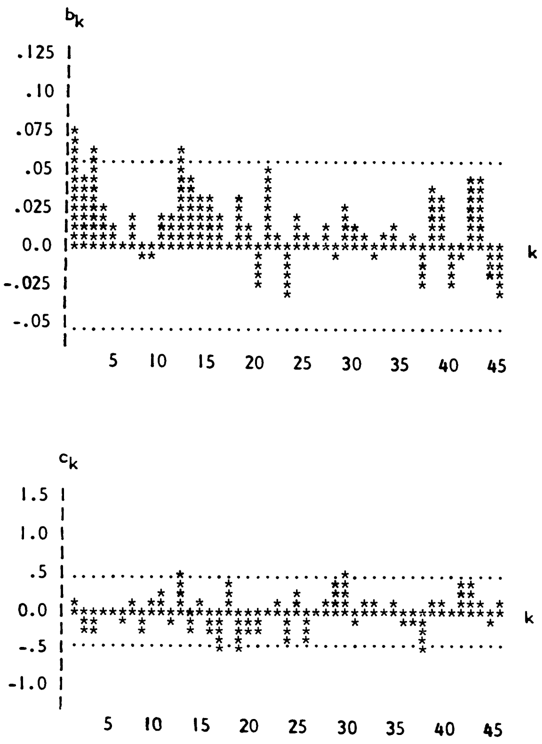

Figure 8. Distributed lags, $B(L)$ and $C(L)$, estimated in equation (4). 10/21/85: sixty days prior to expiration. Ninety-five percent confidence intervals of two standard deviations are presented as "·" for all distributed-lag regression coefficients.

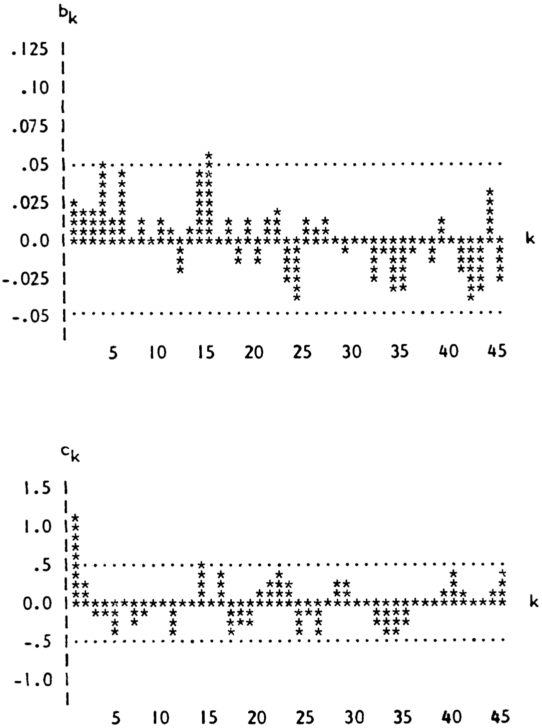

Figure 9. Distributed lags, $B(L)$ and $C(L)$, estimated in equation (4). 11/20/85: thirty days prior to expiration. Ninety-five percent confidence intervals of two standard deviations are presented as "·" for all distributed-lag regression coefficients.

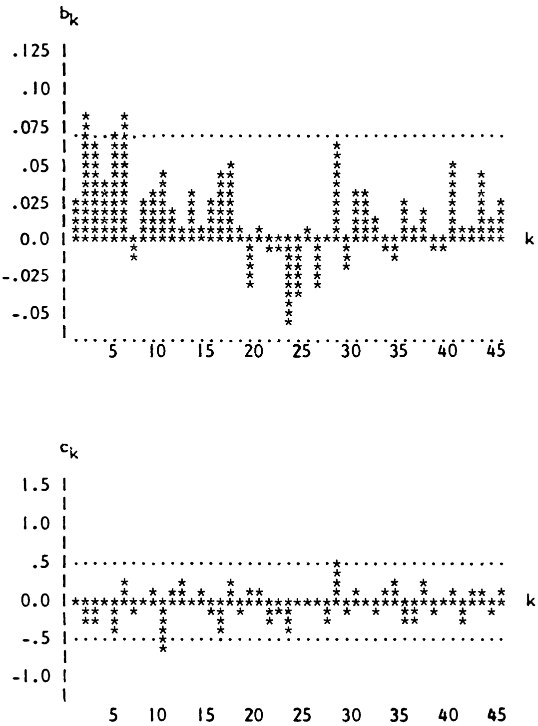

Figure 10. Distributed lags, $B(L)$ and $C(L)$, estimated in equation (4). 12/6/85: fourteen days prior to expiration. Ninety-five percent confidence intervals of two standard deviations are presented as "·" for all distributed-lag regression coefficients.

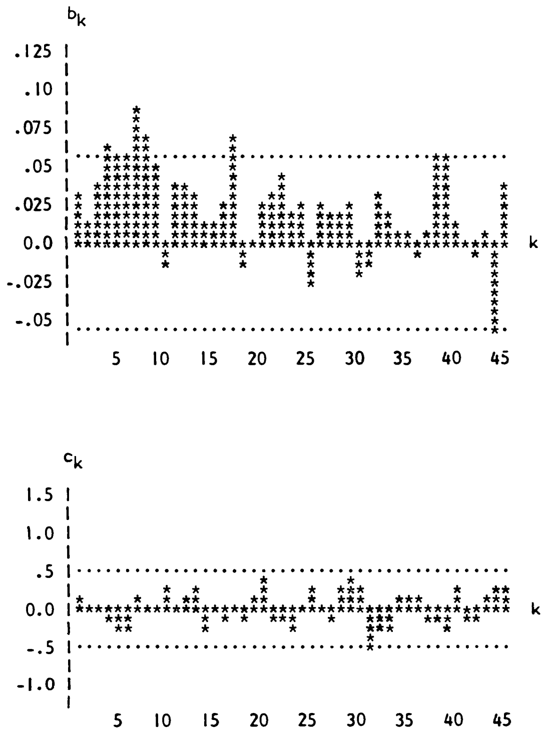

Figure 11. Distributed lags, $B(L)$ and $C(L)$, estimated in equation (4). 12/19/85: one day prior to expiration. Ninety-five percent confidence intervals of two standard deviations are presented as "·" for all distributed-lag regression coefficients.

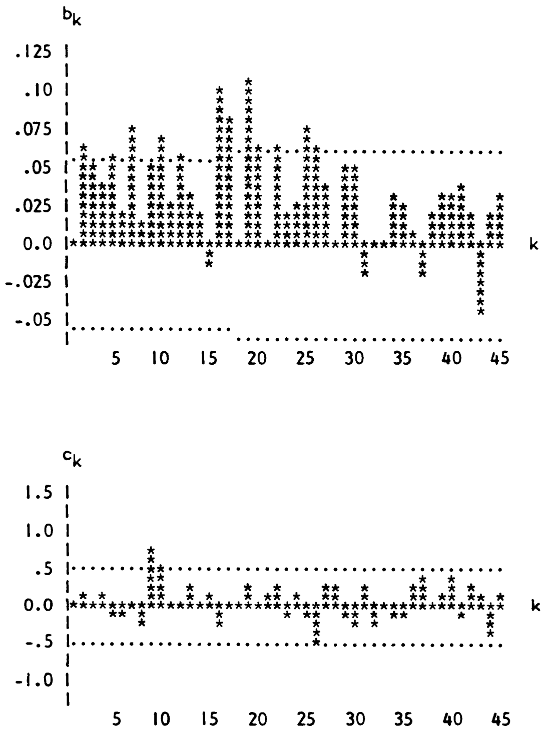

Figure 12. Distributed lags, $B(L)$ and $C(L)$, estimated in equation (4). 12/20/85: expiration day. Ninety-five percent confidence intervals of two standard deviations are presented as "·" for all distributed-lag regression coefficients.

to i_t extends for approximately thirty-five minutes on expiration day, although the pattern is less distinct and fewer coefficients are significantly different from zero. This mildly changing pattern in the distributed lag from f_t to i_t as expiration approaches is similar to the analogous results for the other periods tested until the fourth quarter of 1985.

Figures 7 through 12 reveal the lag patterns for fourth-quarter-1985 data. The lag from f_t to i_t extends for fifteen to forty-five minutes, while the lag from i_t to f_t remains very short at one to two minutes. Interestingly, the lags at sixty, thirty, and fourteen days prior to expiration shorten to fifteen to twenty minutes but then lengthen one day before and on expiration day. In general, the lag on expiration day is at least as long as the lag on earlier days. It is not meaningful to test whether differences among the estimated lags on different days are statistically significant.[12]

The robustness of these results across trading days and quarters is remarkable, given that unrestricted regression analysis is not a very powerful method for estimating distributed lags. For this type of problem, multicollinearity among the distributed-lag coefficients typically results in imprecise estimates and usually requires that a specific lag structure be imposed on the model to obtain statistically significant coefficients that follow a smooth pattern. Scrutiny of Figures 1 through 12 reveals that such a restriction is unnecessary here.

To further investigate the strength and nature of the distributed-lag relationships appearing in Figures 1 through 12, we can test zero restrictions on the distributed-lag coefficients, allowing the contemporaneous coefficient to remain unconstrained. Specifically, we test the following two hypotheses:

H_1: f_t does not lead i_t (i.e., $b_1 = b_2 = \cdots = b_{45} = 0$);

H_2: i_t does not lead f_t (i.e., $c_1 = c_2 = \cdots = c_{45} = 0$).

These hypotheses are similar to those that have been labeled "Granger causality" in the literature. They test whether each variable can be predicted more accurately using past values of both variables than using past values of each variable alone. These hypotheses differ, however, from those that test Granger causality, in that we allow the coefficients at lag zero to enter the model in (4), acknowledging the simultaneous nature of this relationship.[13]

Results are presented in Table II and support the discussion above. Under H_1 and H_2, each test statistic is distributed $F_{45,384}$. On fourteen of the forty-two days examined, H_2 is rejected at the one percent level of significance, and, on twelve

[12] Some form of the Chow test could be applied to test the null hypothesis that the distributed-lag relationship does not change across days in each contract period. However, such an application is impractical and of questionable value. The combined sample across trading days under the null hypothesis would treat up to six subsamples of 360 observations that occur up to eighty-eight days apart as if they were sequential observations drawn from the same population. This assumption is not necessarily justified. Furthermore, a rejection of the null would theoretically occur if just one of the forty-five distributed-lag coefficients varied substantively on different days. This is not inconsistent with the fundamental dynamic relationship revealed in Figures 1 through 12. For this reason, we prefer to present the distributed-lag estimates explicitly.

[13] The possibility of "instantaneous causality" is ruled out in tests of Granger causality. Granger [9], Geweke [8], and Koch and Ragan [13] provide detailed discussions of this approach.

Table II

Results of Testing Hypotheses 1 and 2[a]

H_1: f_t does not lead i_t (i.e., $b_1 = b_2 = \cdots = b_{45} = 0$)
H_2: i_t does not lead f_t (i.e., $c_1 = c_2 = \cdots = c_{45} = 0$)

Date	H_1	H_2
Second Quarter, 1984		
3/19/84	1.78 (0.0022)	2.04 (0.0002)
4/16/84	3.32 (0.0001)	1.79 (0.0021)
5/16/84	1.15 (0.2403)	0.99 (0.5019)
6/01/84	1.86 (0.0011)	1.86 (0.0011)
6/14/84	1.01 (0.4557)	2.05 (0.0002)
6/15/84	1.74 (0.0033)	1.64 (0.0080)
Third Quarter, 1984		
6/25/84	1.64 (0.0075)	1.11 (0.2976)
7/23/84	1.94 (0.0005)	1.21 (0.1766)
8/22/84	1.49 (0.0258)	1.36 (0.0668)
9/07/84	1.35 (0.0741)	1.36 (0.0664)
9/20/84	1.49 (0.0256)	1.64 (0.0079)
9/21/84	1.85 (0.0013)	1.85 (0.0012)
Fourth Quarter, 1984		
9/24/84	2.12 (0.0001)	1.65 (0.0069)
10/22/84	1.93 (0.0005)	0.77 (0.8570)
11/21/84	1.88 (0.0009)	1.60 (0.0107)
12/07/84	1.67 (0.0062)	1.35 (0.0718)
12/20/84	2.01 (0.0002)	2.28 (0.0001)
12/21/84	1.68 (0.0054)	1.75 (0.0031)
First Quarter, 1985		
12/17/84	1.49 (0.0268)	2.11 (0.0001)
1/14/85	1.81 (0.0017)	1.49 (0.0256)
2/13/85	1.67 (0.0058)	1.08 (0.3497)
3/01/85	2.01 (0.0003)	1.24 (0.1453)
3/14/85	1.82 (0.0015)	0.93 (0.6041)
3/15/85	1.60 (0.0109)	1.87 (0.0010)
Second Quarter, 1985		
3/25/85	1.61 (0.0096)	1.08 (0.3375)
4/22/85	1.89 (0.0008)	1.21 (0.1723)
5/22/85	1.43 (0.0421)	1.00 (0.4836)
6/07/85	1.15 (0.2469)	1.32 (0.0888)
6/20/85	3.18 (0.0001)	0.94 (0.5817)
6/21/85	1.63 (0.0082)	1.52 (0.0199)
Third Quarter, 1985		
6/24/85	0.98 (0.5178)	1.01 (0.4660)
7/22/85	1.45 (0.0348)	1.48 (0.0287)
8/21/85	1.04 (0.4062)	1.23 (0.1562)
9/05/85	1.74 (0.0032)	1.62 (0.0092)
9/19/85	1.51 (0.0217)	1.61 (0.0101)
9/20/85	1.26 (0.1310)	1.34 (0.0770)
Fourth Quarter, 1985		
9/23/85	1.49 (0.0254)	1.76 (0.0028)
10/21/85	1.61 (0.0103)	1.33 (0.0855)
11/20/85	1.15 (0.2463)	1.29 (0.1094)
12/06/85	1.41 (0.0490)	1.28 (0.1164)
12/19/85	1.40 (0.0525)	0.97 (0.5369)
12/20/85	1.66 (0.0068)	1.50 (0.0246)

[a] Under H_1 and H_2, each test statistic is distributed $F_{45,384}$. The marginal significance level associated with each test appears in parentheses.

more days, H_2 is rejected at the ten percent significance level. On the other hand, H_1 is rejected at the one percent level on twenty-three days and is rejected at the ten percent level on all but seven days. The results suggest that the lead from futures to spot prices is mildly more robust across days than that from spot to futures prices.

In this light, we recall that the lead from spot to futures prices rarely extends beyond one minute, while that from futures to spot prices typically extends for many minutes. We can statistically investigate the hypothesis that each distributed lag ($B(L)$ and $C(L)$) is effectively zero beyond one minute by allowing the first distributed-lag coefficient in each direction to be unconstrained and by testing the following hypotheses analogous to H_1 and H_2:

H_3: $b_2 = b_3 = \cdots = b_{45} = 0$;

H_4: $c_2 = c_3 = \cdots = c_{45} = 0$.

Results are presented in Table III and are quite revealing. They indicate that, when the first coefficient is unconstrained, there is often no significant further distributed lag from i_t to f_t, while a substantive portion of the distributed lag from f_t to i_t typically appears beyond one lag. This result is again consistent with our earlier scrutiny of Figures 1 through 12, supporting a relationship in which the futures price leads the spot price over twenty to forty-five minutes, while the spot price simultaneously affects the futures price with a brief lag rarely extending beyond one minute.

IV. Summary and Conclusions

This paper presents an empirical investigation of the intraday price relationship between S&P 500 futures and the S&P 500 index. We investigate the existence and nature of any predictive information contained in each price, regarding subsequent movements in the other price. The estimated price relationship on expiration days of the futures contract is compared with comparable estimates for days prior to expiration. This enables an investigation of whether the distributed lags vary systematically throughout the life of each futures contract and whether they differ on expiration days as opposed to nonexpiration days.

Results suggest that S&P 500 futures prices and the index are simultaneously related on a minute-to-minute basis throughout the trading day. Further, significant lag coefficients suggest that the lead from futures to cash prices extends for between twenty and forty-five minutes, while the lead from cash prices to futures prices, though significant, rarely extends beyond one minute. The length of the lead from futures to the index reflects, in part, inertia in the stock market. Stocks are not traded as readily as futures contracts.

The lead/lag relationships are remarkably stable across the different days and futures contracts examined in 1984 and 1985. Interestingly, the lead from futures to the index on expiration day is at least as long as other days prior to expiration, suggesting that expiration days do not demonstrate a temporal character substantially different from earlier days. The magnitudes of the contemporaneous coefficients are similarly comparable. Thus, while arbitrage activity may be presumed to be greatest at expiration, transactions under such arbitrage condi-

Table III

Results of Testing Hypotheses 3 and 4[a]

H_3: $b_2 = b_3 = \cdots = b_{45} = 0$

H_4: $c_2 = c_3 = \cdots = c_{45} = 0$

Date	H_3	H_4
Second Quarter, 1984		
3/19/84	1.75 (0.0031)	0.92 (0.6169)
4/16/84	3.39 (0.0001)	1.74 (0.0033)
5/16/84	1.18 (0.2142)	0.88 (0.6946)
6/01/84	1.55 (0.0166)	1.67 (0.0062)
6/14/84	0.99 (0.4929)	1.07 (0.3619)
6/15/84	1.59 (0.0120)	1.50 (0.0258)
Third Quarter, 1984		
6/25/84	1.68 (0.0058)	0.97 (0.5272)
7/23/84	1.63 (0.0090)	1.09 (0.3206)
8/22/84	1.52 (0.0220)	1.34 (0.0810)
9/07/84	1.36 (0.0695)	1.36 (0.0695)
9/20/84	1.49 (0.0272)	1.23 (0.1537)
9/21/84	1.82 (0.0017)	1.80 (0.0021)
Fourth Quarter, 1984		
9/24/84	2.04 (0.0002)	1.64 (0.0082)
10/22/84	1.98 (0.0004)	0.77 (0.8546)
11/21/84	1.87 (0.0011)	1.20 (0.1855)
12/07/84	1.46 (0.0330)	1.11 (0.3017)
12/20/84	2.00 (0.0003)	1.31 (0.0903)
12/21/84	1.64 (0.0079)	1.27 (0.1248)
First Quarter, 1985		
12/17/84	1.44 (0.0407)	1.32 (0.0922)
1/14/85	1.74 (0.0034)	1.48 (0.0305)
2/13/85	1.48 (0.0296)	0.89 (0.6657)
3/01/85	1.77 (0.0027)	1.21 (0.0780)
3/14/85	1.80 (0.0020)	0.92 (0.6279)
3/15/85	1.51 (0.0236)	1.23 (0.1583)
Second Quarter, 1985		
3/25/85	1.22 (0.1663)	0.98 (0.5114)
4/22/85	1.62 (0.0100)	1.20 (0.1909)
5/22/85	1.45 (0.0378)	0.82 (0.7919)
6/07/85	1.17 (0.2173)	1.12 (0.2864)
6/20/85	3.17 (0.0001)	0.91 (0.6307)
6/21/85	1.66 (0.0069)	1.32 (0.0898)
Third Quarter, 1985		
6/24/85	0.98 (0.5070)	0.71 (0.9144)
7/22/85	1.30 (0.1048)	1.44 (0.0387)
8/21/85	0.88 (0.6967)	1.05 (0.3884)
9/05/85	1.67 (0.0062)	1.19 (0.1972)
9/19/85	1.34 (0.0807)	1.20 (0.1825)
9/20/85	1.23 (0.1576)	1.12 (0.2789)
Fourth Quarter, 1985		
9/23/85	1.53 (0.0203)	1.15 (0.2433)
10/21/85	1.41 (0.0496)	1.26 (0.1314)
11/20/85	1.13 (0.2705)	0.89 (0.7449)
12/06/85	1.44 (0.0404)	1.31 (0.0994)
12/19/85	1.43 (0.0431)	0.96 (0.5532)
12/20/85	1.66 (0.0066)	1.52 (0.0216)

[a] Under H_3 and H_4, each test statistic is distributed $F_{44,384}$. The marginal significance level associated with each test appears in parentheses.

tions are not sufficiently strong or pervasive to alter the empirical price relationship for the entire day.

Despite the significance of the lag coefficients, the magnitudes of the contemporaneous coefficients typically swamp all lag impacts in both directions, providing strong evidence that futures and spot prices move largely in unison. Although the analysis does not directly address the question of market efficiency, the relative sizes of the contemporaneous coefficients compared with the lag coefficients do suggest that any indications of forthcoming price movements implied by the lag structure are unlikely to be exploitable for profit. Subsequent price adjustments due to the lag structure generally can be expected to be substantially smaller than subsequent price moves due to independent causal forces.

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