Debt Management Under Corporate and Personal Taxation

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# Debt Management under Corporate and Personal Taxation

DAVID C. MAUER and WILBUR G. LEWELLEN*

**ABSTRACT**

The presence of long-term debt in a corporation's capital structure is shown to give rise to a valuable tax-timing option that can be exercised by the firm on behalf of its shareholders. This option, which is not available if the firm is fully equity financed, implies that leverage will have a positive tax effect on total firm value even if there is no such effect associated with the tax deductibility of the coupon interest payments on debt. The more volatile interest rates and bond prices are, the more valuable the tax-timing option and the larger the favorable impact of debt on shareholder wealth.

THE QUESTION OF WHETHER debt, when included as a component of a corporation's capital structure, will favorably affect the market value of the firm is a central theoretical issue in the financial economics literature. The implications of debt financing for agency costs (Jensen and Meckling [20]), investment incentives (Chang and Linn [7]; Myers [33]), signalling (Heinkel [17]; Ross [35]), and asymmetric-information problems (Miller and Rock [29]; Myers and Majluf [34]) have been among the factors considered. By far the most attention, however, has been directed toward the possible tax effects of leverage. Modigliani and Miller [31, 32] set the stage for the debate with their argument that debt financing results in an increase in the total distributable cash flows to security-holders, due to the tax deductibility of corporate interest payments, therebyenhancing firm value. Pursuant to that view, subsequent researchers proposed "balancing" theories of optimal capital structure that typically involved tradeoffs between tax advantages and prospective financial-distress costs (Brennan and Schwartz [5]; Kim [21]; Kraus and Litzenberger [24]; Scott [36]). Since Miller's [28] more recent equilibrium model, wherein the corporate tax gain from debt is posited as being neutralized by a commensurate tax disadvantage to bondholders, the debate has intensified both theoretically and empirically.[1]

The purpose of the present paper is to examine the effect on firm value of a tax feature of debt financing that is separate from and independent of the tax treatment of coupon interest payments and receipts. It is also independent of any agency, incentive, signalling, or informational implications of leverage. We show that, for a firm that "manages" its debt position properly, there will in fact be at least one positive tax impact of debt that will raise aggregate firm value

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[1] Of particular interest in the former category are the papers by Barnea, Haugen and Senbet [1], DeAngelo and Masulis [14], Kim, Lewellen, and McConnell [23], Kim [22], Modigliani [30], and Shelton [37]. Among the key empirical studies are those by Bradley, Jarrell, and Kim [2], Buser and Hess [6], Cordes and Sheffrin [11], Masulis [26, 27], Trczinka [38], and Trczinka and Kamma [39].

and augment shareholder wealth. The origin of the effect is the ability of a firm to deduct for tax purposes the premiums it pays to repurchase its long-term debt in the secondary market after that debt has been issued. Such repurchases are shown to comprise a tax-advantaged debt-management policy and to convey a valuable tax-timing option to the firm to exercise on behalf of its shareholders. We model the option and provide estimates of its value.

## I. Debt Management and Tax Options

When a firm issues long-term debt, whatever may be the underlying initial valuation motivation for doing so, one of the inevitable consequences in an environment with stochastic interest rates is to open up a valuation "side game" between bondholders and stockholders. If interest rates rise subsequent to issuance and if bond prices fall, stockholders win that side game; of the then-total value of the firm, their proportionate share is incrementally larger than it would have been had rates remained unchanged. If rates decline, shareholders lose the side game for symmetrical reasons.

While, in an efficient capital market, the game will be a fair one ex ante, it nonetheless *must* be played any time long-term debt is issued. As Brennan and Schwartz [3, 4] and Ingersoll [19] have pointed out in a no-tax setting, the way to control stockholders' ex post losses is to call and retire callable debt as soon as it trades at its call price in the secondary market. This, in effect, minimizes the evolving value (burden on stockholders) of the firm's debt and thereby, ceteris paribus, maximizes the contemporaneous market value of equity. Emery and Lewellen [15] have shown that the same principle applies even more strongly when corporations are taxed and that it applies to noncallable as well as callable debt.

A firm that repurchases either type of debt in the secondary market when that debt is trading above par can claim as a tax-deductible expense the difference between the repurchase price and par value.[2] Because the firm can finance the repurchase by simultaneously issuing new debt with exactly the same features (coupon rate, maturity, convenants) as the retired debt, the proceeds from the new issue will match the repurchase cost. All future cash-debt service obligations will also be maintained intact—i.e., "parity" in those obligations will be preserved (Lewellen and Emery [25]). The only net cash flow consequence for the firm will be the immediate tax saving on the repurchase premium, and this saving can be distributed to stockholders without altering the firm's investment plan, degree of leverage, or residual total market value.

The recommended corporate debt-management policy therefore would be to repurchase and replace long-term debt on those occasions when it trades above par, subject only to the condition that the available tax saving exceed the after-tax transactions costs incurred to execute the operation. The corollary, of course, is that no action be taken when a firm's debt ends up trading *below* par in the secondary market. The attendant valuation gain for stockholders should simply be allowed to ride rather than be recognized for tax purposes.

The parallel between such a policy and the appropriate securities tax-trading

[2] The applicable tax codes are 163-3C and 249.

strategy for investors managing their personal investment portfolios is apparent and has been drawn conceptually by Emery, Lewellen, and Mauer [16]. Constantinides [8] and Constantinides and Ingersoll [10] show that, if the tax rate on capital gains and losses is equal (as it is for a firm repurchasing its debt), optimal behavior involves realizing all losses immediately, to take the associated tax deductions, and deferring all gains indefinitely. A key implication of that result is the fact that the opportunity to pursue the recommended strategy means that a valuable tax-timing *option* is automatically acquired when an investor purchases a security. Moreover, this option can often account for a significant fraction of the total value of the position in the security when purchased.

We use a corresponding framework here to model the corporate debt-management problem and show that the indicated repurchase policy can also convey significant tax-timing valuation benefits to stockholders when long-term debt is issued as part of a firm's financing mix. Those benefits are in addition to, and do not depend on judgments about, any other possible valuation effects that corporate leverage may provide since the repurchase actions in question merely involve the replacement of old debt with identical new debt in all cases. The spirit of the analysis is the same as that adopted by Constantinides [8, 9], Constantinides and Ingersoll [10], and Williams [40]: establishing and assessing the worth of a strategy that minimizes the value of governmental tax claims—in the present instance, to maximize the market value of shareholder equity. Importantly, we recognize in the process the fact that tax-motivated debt repurchase actions by the firm will to some extent subject bondholders to taxable events that they would not otherwise elect and for which they must be compensated. Thus, the context is one of minimizing the value of the *combined* tax claim on the parties involved.

## II. Securityholder Cash Flows and Debt Repurchases

A decision by a firm to repurchase on the secondary market some portion of its previously issued long-term debt has, at the time of the repurchase, several simultaneous cash flow consequences occasioned both by transactions costs and by the tax treatment of corporate and securityholder income. To capture those consequences, we employ the following notation, where time $t$ refers to the repurchase date:

$T$ = pretax transactions costs incurred to repurchase the debt at time $t$, expressed as a *fraction* of the amount repurchased, where $T \in (0, 1)$;
$\tau_C$ = the time-$t$ tax rate on corporate income;
$\tau_S$ = the time-$t$ tax rate on income to stockholders;
$\tau_G$ = the time-$t$ tax rate on capital gains income to bondholders;
$D_F$ = the original issue price (face value) of the debt being repurchased;
$D$ = the time-$t$ secondary-market trading price of the debt.

Consider, then, the effect of repurchasing the debt at the price $D > D_F$, where the repurchase is accomplished as a "pure refunding" operation, a la Emery and Lewellen [15] and Lewellen and Emery [25], by issuing new debt having the same debt service schedule and other features, also at the price $D$.

The immediate incremental cash flows experienced by the firm, as compared with *not* repurchasing the debt, consist of an outflow in the amount $D$, a matching inflow of $D$, a tax saving of $\tau_C(D - D_F)$ on the deductible repurchase premium, and an after-tax transactions cost of $TD(1 - \tau_C)$. The net amount thereof can be distributed to shareholders without altering either the firm's established investment plan or the original division of its operating income between dividends and retentions, the result being an increase in shareholder after-tax income equal to

$$[\tau_C(D - D_F) - TD(1 - \tau_C)](1 - \tau_S). \tag{1}$$

On the other side of the transaction, bondholders as a group receive a cash payment of $D$, reinvest that amount in the firm's newly issued debt, and incur capital gains taxes of $\tau_G(D - D_F)$.[3] The net (of not repurchasing) incremental cash flow to both groups of securityholders therefore is

$$[\tau_C(D - D_F) - TD(1 - \tau_C)](1 - \tau_S) - \tau_G(D - D_F) \tag{2}$$

or

$$(D - D_F)[\tau_C(1 - \tau_S) - \tau_G] - TD(1 - \tau_C)(1 - \tau_S). \tag{3}$$

Thus, the desirability of debt repurchase depends in a complex way on the relative magnitudes of corporate and securityholder tax rates and transactions costs.

The motivation for and the nature of the payoffs from that repurchase policy are perhaps most transparent if we consider it in the equilibrium tax setting portrayed by Miller [28], where $\tau_S$ is equal to zero. Assume transactions costs to be zero as well. The net cash flow impact identified in (3) thereby reduces to

$$(D - D_F)(\tau_C - \tau_G), \tag{4}$$

which implies that debt repurchase will be worthwhile whenever $D > D_F$, as long as the corporate tax rate (which captures the tax benefit of repurchase) exceeds bondholders' capital gains tax rate (the penalty of repurchase). In Miller's equilibrium, the marginal bondholder is in the same *ordinary* tax bracket—for interest income—as the tax rate on corporate income. Hence, in that setting, repurchase will be desirable on balance when $D > D_F$ (and undesirable otherwise) if bondholders have owned the firm's debt long enough for its price increase over face value to qualify for long-term gains status ($\tau_C > \tau_G$). We consider the corresponding net benefits under the 1986 tax reform legislation below.

Indeed, the necessary condition is somewhat less stringent still. Not all the selling bondholders would be the "marginal" holder for which $\tau_C = \tau_G$, even for short-term capital gains. (In Miller's model, none will be in a *higher* bracket.) Moreover, as Constantinides [8, 9] and Constantinides and Ingersoll [10] have pointed out, certain bondholders would, in the normal course of affairs, be liquidating their holdings anyway—for consumption needs, for portfolio rebal-

[3] Note that our analysis assumes that the bondholders do not trade the bonds over the time interval between their issuance and the firm's repurchase decision. Therefore, the capital gains tax liability incurred by the bondholders upon selling the bonds back to the firm is computed as the difference between the repurchase price, $D$, and the original issuance price of the bonds, $D_F$. This assumption was made in order to focus exclusively on the *firm's* debt-repurchase policy.

ancing, or to establish a new price basis for possible subsequent short-term loss taking if their tax rates on short- and long-term capital gains differ. Such individuals therefore will simply be engaging in transactions with the firm rather than some other buyer, and for them there is *no* net tax burden associated with the debt repurchase. Finally, certain other bondholders who would not ordinarily be liquidating their positions immediately would be expecting to do so in the future, and the net tax burden on them due to the firm's repurchase action is only an extra *present-value* burden.

In essence, then, the appropriate interpretation of the tax rate, $\tau_G$, in the model is one of an "effective" (net) capital gains rate—lower than $\tau_C$ for many and perhaps most sellers and net of the capital gains taxes that would otherwise be paid. With positive transactions costs and positive taxes on stock income, the secondary-market debt price threshold $D$ for a worthwhile repurchase is subject to other influences as well, but the relationship in (3) permits these to be identified for the general case.

The valuation implications for the firm of the debt-repurchase opportunities thereby provided follow directly. Let $B_t$ denote the *burden* on the firm of its outstanding debt at time $t$, i.e., the then-current value of the claim $D$ that bondholders have on the firm (and will still have if the debt is repurchased and replaced with identical new debt) less the cash flow benefit available from a debt repurchase. Put differently, $D$ is the portion of total firm value that bondholders would own—and shareholders would not—if a repurchase opportunity did not exist or was ignored, and the debt simply let ride to maturity. $B_t$ is less than $D$ to the extent that the debt-service burden involved can be diminished by repurchases and the attendant cash flow payoffs. By the nature of the identified repurchase policy, we have that

$$B_t = D - (D - D_F)[\tau_C(1 - \tau_S) - \tau_G] + TD(1 - \tau_C)(1 - \tau_S) \qquad (5)$$

at all times when (3) is positive since repurchase will occur in order to realize a cash benefit. At all other times,

$$B_t < D - (D - D_F)[\tau_C(1 - \tau_S) - \tau_G] + TD(1 - \tau_C)(1 - \tau_S) \qquad (6)$$

since repurchase will be foregone because a net cash cost would be incurred. A specification of the functional form of $B_t$ thereby permits a quantification of the equity-valuation payoffs from "tax managing" the debt. Additionally, it will allow us to value the firm's debt-repurchase tax-timing option for shareholders.

## III. The Burden of Debt

As one approach to such a specification, we employ a continuous-time model similar to that developed by Constantinides and Ingersoll [10] to value investors' timing options to tax-trade debt securities. Like them, we assume the secondary-market price of the firm's debt to be a function of a stochastic market rate of interest, the movements of which are described by a stochastic differential equation. In particular, in order to obtain a closed-form solution for the debt burden $B$, we adopt the following assumptions about the economic environment:

(A1) The firm's debt is perfectly divisible and may be bought and sold with zero transactions cost by investors;
(A2) The debt has infinite maturity, and, therefore, its trading price is a function of the rate of interest $r$ and the debt's continuous dollar-denominated coupon rate $C$—i.e., $D = D(r; C)$;
(A3) The debt is default free and noncallable;
(A4) The term structure is specified by the riskless rate $r$. Its dynamics are given by

$$dr = \alpha r^2 dt + S r^{3/2} dZ(t),$$

where both $\alpha$ and $S$ are constants and $dZ(t)$ is the increment of a standard Wiener process $Z(t)$;
(A5) Long-term debt is priced relative to short-term instantaneous lending opportunities according to a riskless single-period bond with maturity $dt$ and before-tax yield $r$;
(A6) The after-tax version of the local expectations hypothesis of the term structure of interest rates holds.

Assumption (A1) describes the market setting. The characteristics of the debt embodied in assumptions (A2) and (A3) keep the problem tractable. By assuming that the debt is perpetual (as a surrogate for long-maturity corporate bonds), default free, and noncallable, we are able to derive a relatively simple closed-form solution for the burden of debt. The dynamics of the interest rate, (A4), have been employed previously by Cox, Ingersoll, and Ross [12] and by Constantinides and Ingersoll [10]. The specification implies that both the drift and the variance of the change in the interest rate are proportional to its level. Finally, assumptions (A5) and (A6) together state that the expected instantaneous after-tax holding-period return on default-free bonds of all maturities is equal to the after-tax single-period riskless rate of interest. That equilibrium condition was labeled the "local after-tax expectations hypothesis" by Cox, Ingersoll, and Ross [13].

For infinite-maturity debt, the secondary-market price $D(r; C)$ need not be time subscripted. Neither need be the burden of that debt, $B(r; C; \hat{D})$. This burden will depend upon the interest rate, $r$, the debt's coupon rate, $C$, and the secondary-market price, $\hat{D}$, at which the firm decides to repurchase the debt. With regard to the latter, the net repurchase cash flow identified in equation (3) above will be non-negative if the condition

$$D \geq D_F \Pi \tag{7}$$

is satisfied, where the constant $\Pi$ is defined by

$$\Pi = \frac{[\tau_C(1 - \tau_S) - \tau_G]}{[\tau_C(1 - \tau_S) - \tau_G] - [T(1 - \tau_C)(1 - \tau_S)]}. \tag{8}$$

For plausible values of the parameters, $[\tau_C(1 - \tau_S) - \tau_G] - [T(1 - \tau_C)(1 - \tau_S)] > 0$, and therefore, $\Pi > 1$. That is, with positive transactions costs, the firm can profitably repurchase its debt only when the secondary-market price $D(r; C)$ has risen above the original-issue face value $D_F$ by at least the fraction $\Pi$. Absent transactions costs, of course, $\Pi$ would be equal to one and repurchase would be both profitable and optimal as soon as $D \geq D_F$. Hence, rationally, $\hat{D} = D_F$ for that case.

With transactions costs present, on the other hand, a repurchase as soon as $D \geq D_F\Pi$ is, while profitable, not necessarily also *optimal*. Because those deadweight costs must be borne every time a repurchase occurs, it may be a better strategy for the firm to plan to wait each time for another instant $dt$ to repurchase, in hopes of capturing an even greater tax benefit. While fewer transactions will thereby occur, the long-run value of the debt tax-management timing option may be enhanced by the larger net gain realized per transaction. The specification of the optimal such strategy, however, does not lend itself readily to a generalizable analytical solution. For that reason, and to permit a closed-form solution here, we shall stipulate that a repurchase by the firm *is* undertaken whenever $D \geq D_F\Pi$ and, therefore, that $\hat{D} = D_F\Pi$. The implication is that the tax-timing option values estimated on this assumption will, if anything, *understate* the potential for shareholders to gain from the tax management of the firm's debt position. The actual payoffs in practice may be even greater.

On that basis, equations (5), (6), and (7), together with assumptions (A1) through (A6), permit a continuous-time formulation of the debt-management problem and the associated option value. The burden of the debt for the issuing firm will be defined over the feasible range of market interest rates according to

$$B(r; C; D_F\Pi) = D - (D - D_F)[\tau_C(1 - \tau_S) - \tau_G] + TD(1 - \tau_C)(1 - \tau_S), \qquad D(r; C) \geq D_F\Pi \tag{9a}$$

and

$$B(r; C; D_F\Pi) < D - (D - D_F)[\tau_C(1 - \tau_S) - \tau_G] + TD(1 - \tau_C)(1 - \tau_S), \qquad D(r; C) < D_F\Pi. \tag{9b}$$

Because (9a) is an exact specification, it remains only to determine the burden of debt when $D(r; C) < D_F\Pi$.

Given assumptions (A1) through (A6), for the range of interest rates for which the firm will *not* find it beneficial to repurchase its debt $[D(r; C) < D_F\Pi]$, the bondholders' after-tax rate of return over the interval $t$, $t + dt$ will be

$$[dB + (1 - \tau_B)Cdt]/B, \tag{10}$$

where $\tau_B$ is the tax rate on interest income to bondholders. This is the "continuation region", wherein the firm's burden of debt, $B(r; C; D_F\Pi)$, coincides with the secondary-market price of debt, $D$. Employing assumptions (A5) and (A6), we have that the expected return in the continuation region over the interval $t$, $t + dt$ is

$$\mathrm{E}[dB + (1 - \tau_B)Cdt]/B = (1 - \tau_B)rdt. \tag{11}$$

Because $B$ is a function of $r$, we can use Itô's Lemma to specify the instantaneous change in $dB$ due to the movement of the state variable $r$ as

$$dB = B_r dr + (1/2)[B_{rr}(dr)^2], \tag{12}$$

where the subscripts denote partial derivatives. By substituting (12) into (11), we find that the burden of debt in the continuation region must satisfy the differential equation

$$(S^2/2)r^3B_{rr} + \alpha r^2 B_r - (1 - \tau_B)rB + (1 - \tau_B)C = 0, \qquad D(r; C) < D_F\Pi. \tag{13}$$

The boundary conditions for equation (13) follow directly. From (9a), the continuity of $B$ at $D(r; C) = D_F\Pi$, requires that

$$B(r; C; D_F\Pi) = D_F\Pi. \tag{14}$$

Also from (9a), for the continuity of $B_r$ at $D(r; C) = D_F\Pi$, the "smooth-pasting" condition requires that

$$B_r(r; C; D_F\Pi) = \{1 - (\tau_C[1 - \tau_S] - \tau_G) + T(1 - \tau_C)(1 - \tau_S)\}[D_r(r; C)]. \tag{15}$$

Equation (13) can be simplified since, for debt with infinite maturity, the secondary-market price will be proportional to $C$ and inversely proportional to $r$, having the functional form

$$D(r; C) = Ca/r, \tag{16}$$

where $a$ will be a function of the parameters $\tau_B$, $\tau_C$, $\tau_G$, $\tau_S$, $\alpha$, $S$, and $T$. Consequently, we can eliminate $r$ and its derivatives from (13) and rewrite the equation as

$$(S^2/2)D^2B_{DD} + (S^2 - \alpha)DB_D - (1 - \tau_B)B + [(1 - \tau_B)D/a] = 0, \qquad D(r; C) < D_F\Pi. \tag{17}$$

The general solution to (17) is[4]

$$B(r; C; D_F\Pi) = \frac{(1 - \tau_B)D}{(1 - \tau_B + \alpha - S^2)a} + A_1(D_F\Pi)^{1-Z^-}D^{Z^-} + A_2(D_F\Pi)^{1-Z^+}D^{Z^+}, \qquad D(r; C) < D_F\Pi, \tag{18}$$

where $a$, $A_1$, and $A_2$ are constants to be determined and $Z^+$ and $Z^-$ are the positive and negative roots, respectively, of the quadratic equation

$$(S^2/2)(Z)(Z - 1) + (S^2 - \alpha)(Z) - (1 - \tau_B) = 0. \tag{19}$$

Since the firm's debt-management policy involves no debt repurchases if the secondary-market price of the debt is well below its face value (scaled up by $\Pi$), $D_F\Pi$ will have a negligible impact on $B(r; C; D_F\Pi)$ when $D \ll D_F\Pi$. This

[4] Constantinides and Ingersoll [10, p. 311, footnote 14] point out that, for a valid solution to (17), $(1 - \tau_B + \alpha - S^2) > 0$ or $(1 - \tau_B) > (S^2 - \alpha)$. As will be shown in equation (32) below, the expected rate of change in the secondary-market price of debt is $(S^2 - \alpha)r$. If $(1 - \tau_B) < (S^2 - \alpha)$, then the expectations hypothesis [assumption (A6)] cannot hold since the expected instantaneous rate of return on the debt—including the coupon—will exceed $(1 - \tau_B)r$.

stipulation will be met only if $A_1$ is equal to zero.[5] We may therefore use the boundary conditions (14) and (15) to solve in (18) for the remainaing two unknown constants $a$ and $A_2$.

Substituting (18) into (14) and setting $A_1 = 0$, we have

$$\frac{(1-\tau_B)D}{(1-\tau_B+\alpha-S^2)a} + A_2 D = D. \tag{20}$$

Similarly, substituting (18) into (15) and setting $A_1 = 0$ yields

$$\frac{(1-\tau_B)}{(1-\tau_B+\alpha-S^2)a} + Z^+ A_2 = 1 - (\tau_C[1-\tau_S] + \tau_G) + T(1-\tau_C)(1-\tau_S). \tag{21}$$

Solving equations (20) and (21) simultaneously for $a$ and $A_2$, we obtain

$$a = \frac{(1-\tau_B)(1-Z^+)}{(1-\tau_B+\alpha-S^2)[1-(\tau_C[1-\tau_S]-\tau_G)+T(1-\tau_C)(1-\tau_S)-Z^+]}$$

and

$$A_2 = \frac{[\tau_C(1-\tau_S)-\tau_G] - T(1-\tau_C)(1-\tau_S)}{(1-Z^+)}.$$

Given those values, we can specify the complete functional form of $B(r; C; D_F\Pi)$. At times when the firm finds it profitable to repurchase its debt, the burden of that debt is

$$B(r; C; D_F\Pi) = D - (D - D_F)[\tau_C(1-\tau_S) - \tau_G] + TD(1-\tau_C)(1-\tau_S), \qquad D(r; C) \geq D_F\Pi, \tag{22}$$

and, at all other times,

$$B(r; C; D_F\Pi) = \left(1 - \frac{[\tau_C(1-\tau_S)-\tau_G] - T(1-\tau_C)(1-\tau_S)}{1-Z^+}\right)D + \left(\frac{[\tau_C(1-\tau_S)-\tau_G] - T(1-\tau_C)(1-\tau_S)}{1-Z^+}\right)D^{Z^+}(D_F\Pi)^{1-Z^+}, \qquad D(r; C) < D_F\Pi. \tag{23}$$

With the magnitude of transactions costs, the various tax rates, and $Z^+$ as known parameters, there is then a determinate relationship between $B(r; C; D_F\Pi)$ and $D(r; C)$ for the range of market interest rates.

Equations (22) and (23) are graphed as lines 1 and 2, respectively, in Figure 1. The solid portion of each line is the applicable one for a tax-motivated debt repurchase policy, and it thereby captures the valuation burden on stockholders of having the debt outstanding. At point $X$, where $D(r; C) = D_F\Pi$, repurchase

[5] That is, we want $\partial B/\partial(D_F\Pi) = 0$ in the limit as $D/(D_F\Pi)$ approaches zero. That condition will be satisfied only if $A_1 = 0$ in (18). This argument parallels that by Constantinides and Ingersoll [10, p. 311].

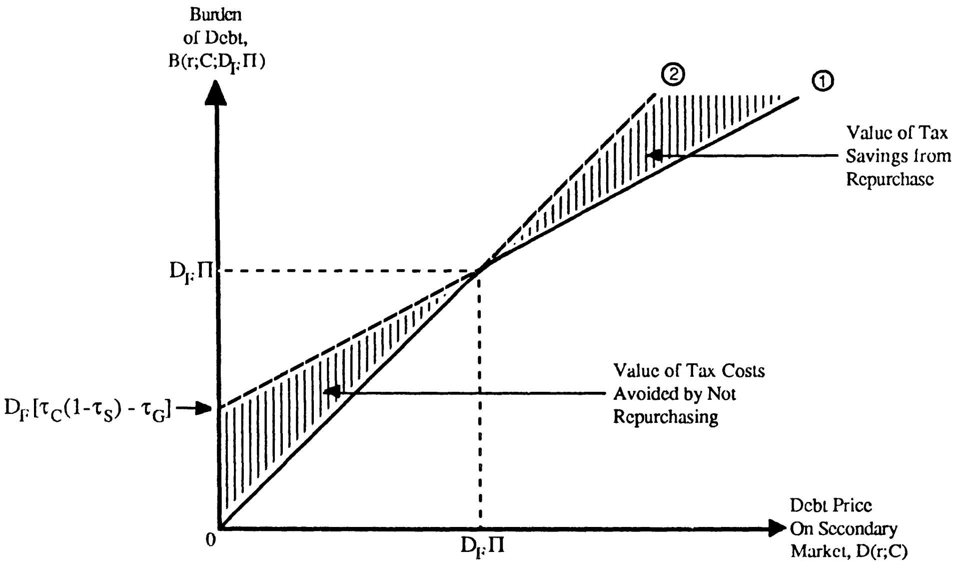


**Figure 1.** The Burden of Debt as a Function of the Secondary-Market Debt Price

would have a zero net payoff because transactions costs would exactly offset the available tax savings. For secondary-market debt prices above the price where that condition is met, from (22), repurchase will be profitable; at lower prices, repurchase should be avoided.[6] The opportunity for the firm to follow such a strategy in effect conveys to it a valuable tax-timing option when debt is included in its capital structure.[7]

## IV. Valuing the Debt-Repurchase Timing Option

It is possible from assumptions (A1) through (A6) to develop an expression to value that option. Consider first a firm that would never repurchase its debt. For a given firm asset value $V$ at any point in time, the value of equity will be

$$\hat{S} = V - \hat{B}, \tag{24}$$

where $\hat{B}$ is the (no-repurchase) valuation burden of debt. Consider then a firm with the same asset value but that engages in a tax-motivated debt-repurchase policy. We can write the value of equity in this firm as

$$S = V - B, \tag{25}$$

and, therefore, we have that

$$S = \hat{S} + (\hat{B} - B). \tag{26}$$

[6] If transactions costs ($T$) are zero, $\Pi = 1$, and the point at which repurchase would have a zero net payoff becomes $D(r; C) = D_F$.

[7] Note that this option has value even if it is only partially exercised—i.e., by repurchasing a portion of an outstanding debt issue rather than the full amount.

The value of equity in a firm that takes advantage of profitable debt-repurchase opportunities is equal to the value of the equity in a firm that does not, plus the difference in debt burdens. That difference measures the value of the debt tax-timing option.

Given the debt burden in equation (23), we can readily obtain a closed-form expression for $\hat{B} - B$. The burden of debt *absent* repurchase opportunities is the same as the burden of debt in an environment where $D$ never gets above $D_F\Pi$. Formally, we have

$$\hat{B} = \lim_{(D/D_F\Pi)\to 0} B = \left(1 - \frac{[\tau_C(1-\tau_S) - \tau_G] - T(1-\tau_C)(1-\tau_S)}{1 - Z^+}\right)D, \tag{27}$$

where $B$ is the valuation burden on stockholders [equation (23)] given a repurchase strategy. The difference between $\hat{B}$ and $B$ is

$$\hat{B} - B = \left(\frac{[\tau_C(1-\tau_S) - \tau_G] - T(1-\tau_C)(1-\tau_S)}{Z^+ - 1}\right)D^{Z^+}(D_F\Pi)^{1-Z^+}. \tag{28}$$

If we evaluate that difference at the time the bonds are issued (i.e., when $D = D_F$) and express the result as a fraction of the original issue price, in the form

$$\Lambda = \frac{\hat{B} - B}{D_F}, \tag{29}$$

we have a measure of the per-dollar (of debt) value of the timing option at issuance. Substituting (28) into (29) and evaluating the resulting expression at $D = D_F$, we obtain[8]

$$\Lambda = \left(\frac{[\tau_C(1-\tau_S) - \tau_G] - T(1-\tau_C)(1-\tau_S)}{Z^+ - 1}\right)(\Pi)^{1-Z^+}, \tag{30}$$

where $Z^+$ is the positive root of equation (19), i.e.,

$$Z^+ = -\,[(S^2/2) - \alpha - \{[(S^2/2) - \alpha]^2 + 2S^2(1-\tau_B)\}^{1/2}]/S^2. \tag{31}$$

For all plausible values of $S$ and $\alpha$, the variance and drift parameters of the underlying interest rate process, the corresponding value of $Z^+$ from (31) will be greater than one, causing the denominator of (30) to have a positive sign.[9] Similarly, for plausible values of $\tau_C$, $\tau_S$, $\tau_G$, and $T$, the numerator of (30) will also be positive. Finally, since $\Pi > 1$, $\Pi$ raised to the $1 - Z^+$ power will also be positive. Hence, $\Lambda$ will be positive as expected, and the debt-repurchase tax-timing option *will* be valuable to shareholders.

Specific estimates of $\Lambda$ can be derived in a manner similar to that employed by Constantinides and Ingersoll [10]. From assumption (A4) about the interest

[8] The corresponding option value in the absence of transactions costs is defined by setting $T = 0$ in this expression.

[9] It can be shown that the value for $Z^+$ must be greater than one for there to be a valid solution to the differential equation in (17). From Constantinides and Ingersoll [10], we know that $(1 - \tau_B + \alpha - S^2)$ must be greater than zero for the expectations hypothesis [assumption (A6)] to hold. Simplifying the equation for $Z^+$ in (31) reveals that $Z^+$ is greater than one if and only if $(1 - \tau_B + \alpha - S^2)$ is greater than zero.

rate process, and with perpetual debt, the dynamics of the market price of the debt are—using Itô's Lemma—specified by

$$dD = D[(S^2 - \alpha)rdt - (S)(r)^{1/2}dZ(t)]. \tag{32}$$

The most recent data from Ibbotson [18] studies of the American capital market indicate that the standard deviation of the annualized returns on a portfolio of long-term corporate bonds has been 7.6 percent historically. Adopting that figure as an estimate for the standard deviation of the annual return on perpetual debt and assuming $r$ to be ten percent as a long-run average, a mean value for $S$ of 0.240 can be inferred from equation (32).[10] We assume that $S$ ranges from 0.10 to 0.30 as a sensitivity analysis. Similarly, for the drift component $\alpha$, we consider three possible values: $\alpha = 0$ (no drift), $\alpha = S^2$ (which corresponds to a zero expected instantaneous return on the market value of the debt), and $\alpha = 2S^2$ (an arbitrarily chosen higher value). We estimate for the pre-1987 tax environment, following Miller [28], that $\tau_C = \tau_B = 0.50$ in equilibrium.[11] On the notion—also following Miller—that stock income is taxed less heavily than interest income, we assume $\tau_S = 0.20$ as a possible figure. Finally, the proportionate transactions costs for debt repurchase, $T$, are assumed to come to one half of one percent of the face amount of the debt involved.

On that basis, Table I lists the tax-timing debt-repurchase option values ($\Lambda$), implied by equation (30), for two alternative specifications of the effective tax rate on bondholder capital gains income: zero and the same overall rate as applies to stock income.[12] A characteristic figure for $\Lambda$ would be in the range of five to ten percent of the market value of the debt issued, which seems to us an eminently worthwhile valuation benefit from leverage. In addition, as noted above, if our stipulated repurchase policy of taking action whenever $D \geq D_F\Pi$ is not *quite* optimal in the presence of transactions costs, the figures as tabulated will somewhat understate the valuation gains that can in fact be realized for shareholders.

In accordance with intuition, the repurchase option value is an increasing function of the variance of the underlying interest rate process (which enhances the likelihood of large debt-price increases and the associated tax savings from repurchase) and a decreasing function of the drift of the process. (The more likely interest rates are to increase with time, the less likely debt is to trade above face value in the secondary market.) Further, a comparison of the valuation estimations in Parts A and B of the table indicates that an increase in the effective capital gains tax rate on bondholders from zero to twenty percent

[10] Alternatively, since the debt is assumed here to be default free, we could use the standard deviation of the annualized returns on a portfolio of long-term *government* bonds. Ibbotson [18] reports that the standard deviation of such a portfolio was only 0.1 percent less than the standard deviation for corporate bonds.

[11] Rounding up to a fifty-percent corporate and bondholder tax rate from the most recent forty-six-percent statutory rate implicitly allows for state as well as federal taxes on the two parties.

[12] Recall that the appropriate interpretation of this rate *is* as an "effective" rate—only the *difference* between the capital gains tax burden incurred by bondholders because of the firm's repurchase action and the burden that would be borne if the firm did not repurchase. As noted earlier, some bondholders would, in the normal course of affairs, be voluntarily or involuntarily liquidating their positions independently of the firm's actions either immediately or in the future (Constantinides and Ingersoll [10]), and many will be entities in tax brackets well below the marginal interest-income bracket $\tau_B$.

**Table I**

The Value of a Firm's Debt-Repurchase Tax-Timing Option, at the Time of the Issuance of Debt, Expressed as a Percentage of the Amount of the Debt[a]

A. Assuming that $\tau_G = 0$, for values of $S$ Equal to

| 0.10 | | | 0.20 | | | 0.30 | | |
|---|---|---|---|---|---|---|---|---|
| $\alpha = 0$ | $\alpha = S^2$ | $\alpha = 2S^2$ | $\alpha = 0$ | $\alpha = S^2$ | $\alpha = 2S^2$ | $\alpha = 0$ | $\alpha = S^2$ | $\alpha = 2S^2$ |
| 4.5% | 4.0% | 3.6% | 11.1% | 8.6% | 6.8% | 21.1% | 13.7% | 9.4% |

B. Assuming that $\tau_G = 0.20$, for Values of $S$ Equal to

| 0.10 | | | 0.20 | | | 0.30 | | |
|---|---|---|---|---|---|---|---|---|
| $\alpha = 0$ | $\alpha = S^2$ | $\alpha = 2S^2$ | $\alpha = 0$ | $\alpha = S^2$ | $\alpha = 2S^2$ | $\alpha = 0$ | $\alpha = S^2$ | $\alpha = 2S^2$ |
| 2.1% | 1.9% | 1.7% | 5.4% | 4.2% | 3.3% | 10.4% | 6.7% | 4.6% |

[a] For all cases, $\tau_C = 0.50$; $\tau_B = 0.50$; $\tau_S = 0.20$; $T = 0.005$.

approximately halves the value of the timing option. This tax represents the compensation that bondholders must receive when repurchase occurs, as a partial offset to the tax savings realized by the firm and passed on to shareholders.

The impact of changes in the other parameters can be determined from equation (30). In particular, it will be the case that

$$\frac{\partial \Lambda}{\partial \tau_C} > 0, \quad \frac{\partial \Lambda}{\partial \tau_S} < 0, \quad \text{and} \quad \frac{\partial \Lambda}{\partial T} < 0,$$

as intuition also would suggest. The higher the corporate tax rate, the greater the firm's tax payoff from repurchase. On the other hand, the higher the tax rate on stock income, the smaller the *net* payoff from the corporate tax savings distributed to shareholders. Thus, if we assume, as per Miller [28], that stock income can be *fully* shielded from taxation, the valuation estimates in Table I are commensurately increased, by approximately twenty-five percent in Part A and fifty percent in Part B. Larger transactions costs, of course—as a "dead-weight" drain—clearly will diminish the available benefits from debt-repurchase actions. The effect of a higher bondholder tax rate on interest income is perhaps less obvious but determinable through its impact on $Z^+$ from equation (31); the partial derivative $\partial \Lambda / \partial \tau_B$ is positive. The logic is that, since the value of debt to bondholders depends fundamentally on the after-tax coupon payments they receive, a higher tax burden on such payments tends to accentuate the magnitude of debt price fluctuations on the secondary market in response to interest rate changes and thereby expands the scope of the firm's profitable repurchase opportunities.

## V. Tax Reform and Debt Tax Management

Because the revisions in the federal tax code brought about by the 1986 tax reform legislation alter the valuation parameters that apply, those revisions will of course have an impact on the potential benefits from corporate debt repurchases of the sort described. The general effect of the legislation was to reduce the marginal tax rates on both corporate and personal ordinary income and to raise the marginal rates on certain capital gains. The deferral-until-realization feature of the tax on capital gains, however, remains as before. If the new rates and rules become permanent—which is by no means assured, given the ongoing federal budget situation—the likely consequence would be somewhat lower values for the corporate debt-repurchase tax-timing option.

Table II provides estimates of these option values, for cases parallel to the ones shown in Table I. The assumptions therein are that (a) the new statutory thirty-four-percent tax rate applies to corporate income, (b) bondholders are in the new top twenty-eight-percent personal tax bracket for interest income, (c) the overall effective tax rate on stock income is fifteen percent, and (d) debt-repurchase transactions costs continue to run at one half of one percent of the amount repurchased.[13] Again, estimates are derived under two alternative scenarios for the effective (net) capital gains tax burden on bondholders occasioned by a repurchase: zero and the tax rate on stock income. The values of the interest-rate-process environmental parameters $\alpha$ and $S$ selected for the sensitivity analysis are the same as in Table I.

A comparison of the two tabulations indicates that, for matching stipulations of $\alpha$ and $S$, the impact of the tax reform legislation would be approximately to halve the value of the firm's debt-management tax-timing option. This would be consistent with the expressed intent of the legislation to diminish incentives for purely tax-motivated transactions. Nonetheless, the option values remain positive and run typically in the range of three to five percent of the at-issue face amount of the debt involved. Once more, if we assume that $\tau_S = 0$, each of the tabulated estimates increases by between twenty-five and fifty percent. Thus, there is still a detectable valuation payoff from leverage to be realized even if the tax changes are in fact fully implemented.

## VI. Summary and Comments

Investor tax-timing options to trade the securities in their portfolios have been shown to make a potentially significant contribution to the value of the respective investment positions in those securities. We show here that a similar valuable timing option is created when a firm inserts long-term debt into its capital structure. When the debt appreciates in price on the secondary market, the firm can realize a tax saving by repurchasing the debt and recognizing the real economic loss that has been experienced by shareholders, for tax purposes. This tax saving can be passed on directly to shareholders without altering either the firm's investment plan or its degree of leverage, by replacing the repurchased debt with identical newly issued debt. Hence, any other valuation benefits from leverage are fully preserved in the process. Conveniently, if the debt declines in price in the secondary market, the consequent real economic gain to shareholders need *not* be recognized as a taxable gain.

This asymmetry, which parallels that which defines an optimal portfolio-management strategy for investors (defer gains and realizes losses), offers an opportunity for a profitable tax-motivated debt-management policy for the firm.

[13] Slightly different marginal tax rates apply in 1987 as a transitional year before the new code is scheduled to become fully effective.

**Table II**

The Value of a Firm's Debt-Repurchase Tax-Timing Option under Tax Reform, Expressed as a Percentage of the Amount of Debt Issued[a]

A. Assuming that $\tau_G = 0$, for values of $S$ Equal to

| 0.10 | | | 0.20 | | | 0.30 | | |
|---|---|---|---|---|---|---|---|---|
| $\alpha = 0$ | $\alpha = S^2$ | $\alpha = 2S^2$ | $\alpha = 0$ | $\alpha = S^2$ | $\alpha = 2S^2$ | $\alpha = 0$ | $\alpha = S^2$ | $\alpha = 2S^2$ |
| 2.5% | 2.1% | 1.8% | 5.7% | 4.7% | 3.9% | 10.4% | 7.5% | 5.5% |

B. Assuming that $\tau_G = 0.15$, for Values of $S$ Equal to

| 0.10 | | | 0.20 | | | 0.30 | | |
|---|---|---|---|---|---|---|---|---|
| $\alpha = 0$ | $\alpha = S^2$ | $\alpha = 2S^2$ | $\alpha = 0$ | $\alpha = S^2$ | $\alpha = 2S^2$ | $\alpha = 0$ | $\alpha = S^2$ | $\alpha = 2S^2$ |
| 1.1% | 0.9% | 0.8% | 2.6% | 2.1% | 1.7% | 4.8% | 3.4% | 2.5% |

[a] For all cases, $\tau_C = 0.34$; $\tau_B = 0.28$; $\tau_S = 0.15$; $T = 0.005$.

We have modeled that policy in continuous time, using consol bonds as a proxy for long-term corporate debt instruments, and have derived estimates of the associated debt-tax-timing option. The burden of the transactions costs of debt repurchase and the compensation to bondholders necessary to offset the extra capital gains taxes they bear when they liquidate their holdings are included formally in the model. For a tax environment like that which prevailed through 1986, the resulting estimates of option value average between five and ten percent of the amount of the debt issued. If the tax changes enacted in 1986 define the future environment, these estimates are reduced by roughly one half.

While we have treated repurchase opportunities here in relation only to a single issue of debt, an announced decision by a firm to maintain a levered capital structure as a matter of ongoing policy could lead to an enhanced market valuation of equity that may be more substantial in practice than the estimates we have provided. Thus, investors should recognize that a *series* of future debt-tax-timing options will become available as the firm grows over time and should impound that expectation immediately in share prices. The aggregate benefit to shareholders from leverage therefore may well be significant.

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