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Source: *The Journal of Finance*, Vol. 42, No. 5 (Dec., 1987), pp. 1225-1243
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2328524
Accessed: 26/11/2009 08:13

THE JOURNAL OF FINANCE • VOL. XLII, NO. 5 • DECEMBER 1987

Efficient Financing under Asymmetric Information

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ABSTRACT

This paper characterizes the conditions under which the adverse-selection problem, which may prevent a firm from issuing securities to finance an otherwise profitable investment, may be costlessly overcome by an appropriate choice of financing strategy. The conditions are specialized when the information asymmetry may be characterized by either a first-degree-stochastic-dominance or a mean-preserving-spread ordering across possible distributions of firm earnings. Possible financing strategies that resolve the information asymmetry are discussed, and the results are related to recent empirical findings concerning security issues.

IT HAS BEEN DEMONSTRATED by Myers and Majluf [17] that, to the extent that firms are unable to communicate their future prospects credibly to investors, the resulting adverse-selection problem may cause significant social welfare losses by inducing firms to forego investment opportunities that would otherwise be profitable. In this paper, we explore the possibility that, despite the information asymmetry, the investment opportunities may yet be efficiently financed by an appropriate choice of financing instruments that reveals the private information of corporate insiders to investors. In doing so, we are able to explain some of the complexities of corporate financings and to offer at least a partial interpretation for so far anomalous findings concerning the effects of corporate financings on security prices.

It is natural to consider the firm's choice of financing as a communication device since these choices are easily verifiable and, to the extent that the conditions of the Modigliani-Miller propositions are satisfied, they are costless. Therefore, if the adverse-selection problems due to information asymmetry in capital markets can be overcome by financial policy, it will be efficient to do so, and other more costly forms of communication will be eschewed.

Of course, we are not the first to see the communication possibilities of financial policy. Stiglitz [19] remarked that "changes in financial policy may be an important signal for the real prospects of the firm", and there is now extensive literature exploring the signalling aspects of financial policy. The models in most of the earlier papers rely either on an ad hoc managerial-compensation function

* UCLA and the University of British Columbia, respectively. An earlier version of this paper appeared as "Notes on Costless Financial Signalling" in *Risk and Capital*, edited by G. Bamberg and K. Spreman, Springer-Verlag, Berlin, 1984. Previous versions of the paper have been presented at the Western Finance Association Meetings (1982) and seminars at Yale University (1981) and Columbia University (1982). The authors thank J. Williams and the referee for suggestions that have improved the clarity and focus of the paper.

or on exogenously specified costs in order to achieve a signalling equilibrium. This paper, in contrast, considers only *costless* signalling possibilities so that, to the extent that the choice of cost functions is arbitrary, our analysis is more robust. This paper also differs from the foregoing in providing a general characterization of the conditions under which a fully revealing equilibrium can be obtained; these conditions involve joint restrictions on the nature of the information asymmetry and the set of financing strategies open to the firm. Since the equilibria we explore are efficient or costless and since any deviation of investment policy from the full-information optimum implies deadweight costs, we take the firm's investment policy as given and restrict the range of possible signals to the pure financing decisions of the firm.

The model we present is most closely related to that of Heinkel [11], in which the firm's choice of debt ratio serves as a costless signal of firm value since the distribution of firm earnings is characterized by a single unknown parameter. However, a significant distinction between Heinkel's analysis and our own is that, while he takes the security types as given and demonstrates the existence of a fully revealing equilibrium for a particular type of information asymmetry, we derive the properties the securities must have in order to be informative for general types of information asymmetry. In more recent work, Constantinides and Grundy [4] develop a financial signalling model in which the amount of investment by the firm and the stock-repurchase decision of inside equityholders act as signals about a single unknown parameter of the distribution of earnings.

There have also been extensive empirical investigations of the effects of announcements of corporate financings on security prices.[1] The first general finding is that financing announcements are associated with security price declines. This is usually interpreted as being consistent with a model such as that of Miller and Rock [16], in which the net payouts of the firm are related to cash flow realizations, which in turn are correlated with future cash flows. However, the Miller-Rock model would also predict a negative association between the size of the issue and the security price response; while such a relation has been detected for equity issues, there appears to be no such relation for debt issues.[2] The second general finding is that the more junior and equity-like the security issued, the more negative is the price response. This has been interpreted as being consistent with the spirit of the Myers-Majluf [17] model, in which insiders with privileged information about firm value tend to sell equity securities when they are overvalued. Such an interpretation may be premature, however, for, not only does the Myers-Majluf model not explicitly incorporate any securities except equity and riskless debt, but in addition there are several empirical findings that are difficult to reconcile with it. First, there appears to be no clear relation between bond rating and price response.[3] Second, low-rated convertibles are associated with a significantly *smaller* negative price response than are more

[1] For an excellent survey, see Smith [18], which is part of a symposium in the *Journal of Financial Economics*.

[2] Asquith and Mullins [2] and Masulis and Korwar [14] report finding an association for equity issues. Mikkelson and Partch [15] and Eckbo [7] could find no such association for debt issues.

[3] Mikkelson and Parch [15] and Eckbo [7] report a weak negative association.

highly rated convertibles.[4] Third, firms that issue sufficient equity to retire debt as well as finance their capital expenditures experience significantly *smaller* negative price reactions than do firms with equity issues that are used only for capital expenditures.[5] The examples we present below are consistent with any price response to convertible issues and offer an explanation for the simultaneous issue of equity and retirement of debt that is consistent with a smaller negative price reaction for these issues than for simple equity financings.

The remainder of the paper is organized as follows. In Section I, we develop a general characterization of a costless signalling equilibrium and give necessary and sufficient conditions for the existence of such an equilibrium. Section II analyzes the case in which the family of possible distributions of firm earnings is ordered by first-degree stochastic dominance and presents an example in which an equilibrium financing strategy consists of an equity issue combined with a debt retirement. Section III is concerned with the case in which the family of possible distributions is ordered by mean-preserving spread and shows that the convertible bond provides a possible resolution of the information asymmetry. Section IV offers some observations of the possibility of costless signalling of multiple attributes. Section V concludes.

I. Characterization and Existence of a Signalling Equilibrium

We consider initially a general situation in which a firm has private information about its "type", which refers to the joint probability distribution of earnings on its existing assets and on a new investment project. The new project is assumed to be indivisible and to require finance by the sale of securities to uninformed investors. Throughout the analysis, we make the following two assumptions:

(A1) The firm chooses its financing instruments so as to maximize the difference between the price it receives for the package of securities it sells, and the true, full-information, value of the securities, subject to the constraint that it raises K, the amount required for the investment, which is common knowledge.

(A2) Securities are traded in competitive markets, and investors possess rational expectations.

While (A2) is standard, (A1) merits some discussion. Combined with the assumption that original claimants are protected by appropriate me-first clauses, (A1) is tantamount to the assumption that the firm is maximizing the true, full-information, value of the securities held by its original shareholders. It can be shown that this criterion of true or intrinsic value maximization will be unanimously supported by the original shareholders, assuming competition in the capital markets and no redistributions to senior security holders, if the shareholders do not indulge in side bets about the characteristics of firms (either because the market is not rich enough to permit such side bets or because investors share common prior beliefs about firm characteristics so that they do

[4] See Mikkelson and Parch [15], Table 7.

[5] See Masulis and Korwar [14], Table 6.

not wish to make such bets).[6] (A1) implicitly ignores any agency problems and corresponds to the classical assumption that the board of directors, in selecting a financing strategy, identifies with the interests of all existing shareholders. This is consistent with the managerial objective implicit in Heinkel [11] but contrasts with that of Constantinides and Grundy [4], who assume that financing strategies are chosen to benefit a group of insider shareholders.

In what follows, we shall distinguish between a "security" and a "financing". Securities are the basic claims traded in capital markets: bonds, stocks, warrants, etc. A financing, on the other hand, refers to the complete set of financial decisions announced by a firm at a point in time; thus, a financing will typically include the issue of more than one type of security—for example, bonds and warrants—and may include retirements or repurchases of existing securities. The aggregate net claim issued in a financing is the difference between the payoffs due to security holders (excluding current shareholders) before and after the financing is announced. For example, if a firm announces a financing consisting of a stock issue and a debt retirement, the aggregate net claim consists of the future dividends on the stock and the immediate payment required to retire the debt, less the future payments that would have been due on the debt if it were not retired.

We then define a *value-revealing* costless signalling equilibrium as a choice of financing by each firm and an assignment of a market price to each traded security that is consistent with assumptions (A1) and (A2), such that the aggregate net claim issued under every financing is priced at its true, full-information, value. With these preliminaries, a value-revealing equilibrium can be characterized by the following theorem.

THEOREM 1 (Property of a Value-Revealing Equilibrium): *A value-revealing costless signalling equilibrium requires that the net claim issued under each financing be priced on the supposition that it was made by that firm (type) with characteristics that would cause the net claim to have the lowest true, full-information, value, and that supposition is correct.*

Proof: Define

- z: a vector of parameters describing the aggregate net claim issued under a financing.
- Z: the set of feasible financings; Z is common knowledge.
- $V(z, t)$: the market value of the aggregate net claim issued under financing, $z \in Z$, when the financing is made by a firm of type t and t is common knowledge. Thus, $V(z, t)$ is the true, full-information, value of financing z when made by firm t.
- T: the set of possible firm types, which is common knowledge.
- $P(z)$: the funds subscribed by investors under financing z when investors are unable to observe the characteristics of the issuing firm but have rational expectations about them.

[6] Compare Feltham and Christianson [8].

$z^*(t)$: the financing chosen by firm t from the set $Z = \{z \mid z \in \bar{Z}$ and $P(z) = K\}$, where K is the amount required for the investment.

$\tau(z) \equiv \{t \mid z^*(t) = z\}$: the set of firms that choose financing z.

We shall show that, if $\tau(z)$ is not empty (i.e., some firm chooses financing z), then

$$P(z) = \min_{\{t \in T\}} V(z, t). \tag{1}$$

Assumption (A1) implies that $z^*(t)$ satisfies

$$P(z^*(t)) - V(z^*(t), t) \geq P(z) - V(z, t), \quad \forall z \in \bar{Z}. \tag{2}$$

For every z for which $\tau(z)$ is not empty, a value-revealing equilibrium requires that $V(z, t)$ be constant over all $t \in \tau(z)$. Then, assumption (A2) implies that the market price of the financing be equal to its true, full-information value:

$$P(z) = V(z, t), \quad \text{where } t \in \tau(z). \tag{3}$$

Now suppose that (1) did not hold for some $\hat{z} \in \bar{Z}$ for which $\tau(\hat{z})$ were not empty. This would imply the existence of some set of firm characteristics, t_0, such that

$$V(\hat{z}, t_0) < P(\hat{z}) = K. \tag{4}$$

Then, (2) and (4) imply that

$$P(z^*(t_0)) - V(z^*(t_0), t_0) \geq P(\hat{z}) - V(\hat{z}, t_0) > 0, \tag{5}$$

and this contradicts (3). Therefore, (1) must hold for $z \in \bar{Z}$. Q.E.D.

If $\tau(z)$ is a singleton for all z, then any particular financing is chosen in equilibrium by only one firm type. In this case, it will be possible for investors to infer the type of any firm from the financing it chooses, and such an equilibrium may be described as *fully revealing.*

Theorem 1 establishes that value-revealing equilibria are characterized by a "lemons property"; each financing strategy is chosen by the worst possible type of firm for that financing strategy (from the investor's viewpoint), and investors anticipate this in pricing the aggregate net claim issued under the financing. If investors did not always expect the worst, there would be scope for cheating or misrepresentation by firms. Figure 1 illustrates a fully revealing equilibrium. By hypothesis, firm 1 selects the financing, $z^*(1)$, with a value that in equilibrium satisfies $V(z^*(1), 1) = P(z^*(1)) = K$. The lemons property states that there can exist no other firm t such that $V(z^*(1), t) < P(z^*(1))$. This is contradicted by the hypothetical $V(z, 2)$ schedule in the figure. However, recalling that, by assumption, firm 2 will select the financing that maximizes $P(z) - V(z, 2)$, it is apparent that, if $V(z, 2)$ is as drawn, firm 2 will not select financing $z^*(2)$ or any other financing that is properly priced. Hence, $V(z, 2)$ cannot lie below $V(z, 1)$ at $z = z^*(1)$ if there is to be a fully revealing equilibrium.

While Theorem 1 provides a general characterization of a value-revealing equilibrium, if one exists, we require a more specific framework within which to

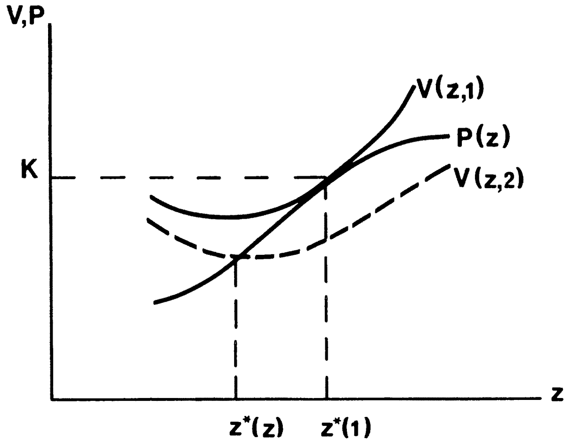

Figure 1. A Costless Signalling Equilibrium

analyze the question of existence itself.[7] Therefore, we consider a two-date world in which the firm arrives at time 0 with a predetermined capital structure and with private information about the time-1 returns on its existing assets and on a new investment opportunity. K, the amount required for this opportunity, is common knowledge.

If the firm decides to undertake the investment, it must raise K by a financing. Let x represent the time-1 return on the total assets (including the new project) of the firm, the probability distribution of which is private information, and let $y(x; z)$ represent the present value of the aggregate net claim issued in the financing as a function of the realized return x. Recall that z is a vector of parameters describing the financing—the terms and numbers of the constituent bonds, stocks, warrants, etc. We assume that, at the time of the financing, the firm is able to precommit to the investment.

The capital market is assumed to be complete, and the true, full-information, value of any claim is given by the expected value of its payoff.[8] Thus, the true value of the (aggregate net claim issued under) financing z when made by a firm of type t can be written as

$$V(z, t) = \int y(x; z) f(x; t)\, dx, \tag{6}$$

where $f(x; t)$ is the density function for the returns on a firm of type t. As we shall see, the possibility of a signalling equilibrium depends on the properties of the class of possible probability density functions governing the returns on the firm's assets. In order to describe the class of densities that do admit signalling equilibria, the following definitions will be useful.

[7] As the maximum net present value of new investment projects approaches zero, the only possible signalling equilibrium in which financings are carried out is a value-revealing equilibrium.

[8] This involves no loss of generality since, by an appropriate choice of numeraire, the value of any claim in a market that does not permit arbitrage can be expressed as the expected value of its payoff under an appropriately defined density function. Compare Harrison and Kreps [10].

Definition: Financing z is a *worst-case financing* for a firm of type $\hat{t}$ if $\hat{t} = \operatorname{argmin}_{\{t \in T\}} V(z, t)$. Thus, the payoff $y(x; z)$ of the aggregate net claim under a worst-case financing satisfies

$$\min_{\{t \in T\}} \int y(x; z) f(x; t)\, dx = \int y(x; z) f(x; \hat{t})\, dx. \tag{7}$$

Definition: A family of probability density functions $\{f(\cdot; t)\}$ and a set, Z, of feasible financings are *K-compatible* if, for each $\hat{t}$, there exists a worst-case financing $z^*(\hat{t})$ such that $z^* \in Z$ and

$$\int y(x; z^*(\hat{t})) f(x; \hat{t})\, dx = K. \tag{8}$$

If, in addition, the minimum in (7) is unique, then $\{f(\cdot, t)\}$ and z are *strongly K-compatible.*

Clearly, a worst-case financing is one for which the firm can be assured that the market price will not be below the true, full-information, value of the financing. The following lemma, the proof of which is immediate, states that compatible financings and probability densities ensure the existence of worst-case financings that raise the necessary investment amount.

LEMMA: *If a family of probability densities and a set of feasible financings are K-compatible, then, for each $\hat{t}$, there exists a worst-case financing $z^*(\hat{t})$ with a true, full-information, value of K.*

The link between worst-case financings and a revealing equilibrium is intuitively clear. If firms believe that investors will price financings at their worst-case values, then the best that a firm can achieve in seeking to maximize the difference between the price of its financing and the true, full-information, value of the financing is achieved by making a worst-case financing. This achieves a difference of zero, but any other choice of financing would produce a negative difference. Thus, if firms believe that investors will price financings at their worst-case values, firms will indeed be induced to make worst-case financings, confirming investors' beliefs. Furthermore, Theorem 1 shows that no other revealing equilibrium is possible. We formalize this intuition in the following:

THEOREM 2 (Existence of a Value-Revealing Equilibrium): *A necessary and sufficient condition for the existence of a value-revealing (fully revealing) equilibrium is that the family of possible risk-adjusted density functions of firm returns and the set of feasible financings be (strongly) K-compatible.*

Proof: We shall prove the theorem for the general case; extension to the fully revealing case is immediate.

Necessity: Combining equation (1) of Theorem 1 with the valuation function (6), a necessary condition for a value-revealing equilibrium is that, for each $\hat{t}$, there exist $z^*(\hat{t}) \in Z$ such that

$$P(z^*(\hat{t})) = \min_{\{t \in T\}} V(z^*(\hat{t}), t) = \min_{\{t \in T\}} \int y(x; z^*(\hat{t})) f(x, t)\, dx = K, \tag{9}$$

where $y(x; z)$ is the net payoff function for financing z. If $f(\cdot, t)$ and Z are not K-compatible, no solution can exist in (9) so that the property of a value-revealing equilibrium cannot be satisfied.

Sufficiency: If the family of possible distributions and the set of feasible financings are K-compatible, the lemma implies that there exists for each firm $\hat{t}$ a worst-case financing $z^*(\hat{t}) \in Z$ such that

$$\min_{\{t \in T\}} \int y(x; z^*(\hat{t})) f(x; t)\, dx = \int y(x; z^*(\hat{t})) f(x; \hat{t})\, dx \equiv V(z^*(\hat{t}), \hat{t}) = K. \tag{10}$$

Suppose now that investors price all financings according to the "lemons principle", expressed in (1). Then $P(z^*(\hat{t})) = K$, so that financing $z^*(t)$ raises the required amount. Moreover, since, from (10), $V(z^*(\hat{t}), t) \geq K$ for all t, assumption (A1) implies that no firm t for which $V(z^*(\hat{t}), t) \neq K$ will have an incentive to choose financing $z^*(\hat{t})$. Therefore, investor inferences about the value of the financing, as reflected in (1), are correct, and all financings are properly priced. Q.E.D.

Since, in a revealing equilibrium, the worst-case financing $z^*(\hat{t})$ is chosen by firm $\hat{t}$ and by no other firm that would cause its true value to be different, the financing is a signal of the characteristics of the firm. The existence of a revealing equilibrium then turns on the existence of a worst-case financing for each firm type, and that depends on the set of allowable financings, Z, and the set of possible firm types, T.

A significant implication of Theorems 1 and 2 is that, if the family of probability densities of firm returns and the set of feasible financings are K-compatible, then the investment inefficiency described by Myers and Majluf [17] may be eliminated. Myers and Majluf show that a firm that is restricted to financing through equity may fail to undertake a positive NPV investment project or may undertake a negative NPV project. If the density function of firm returns and the set of financing strategies are K-compatible, then we have shown that the firm can make a worst-case financing that raises K and is properly priced. Therefore, if the investment has a positive NPV, it will be undertaken. Similarly, a negative NPV investment will not be undertaken.

This demonstrates the fundamental importance of assumptions about the feasible set of financing strategies. In the following sections, we shall consider the type of securities that may be required to achieve a value-revealing equilibrium for particular assumptions about the firm's private information.

II. First-Degree Stochastic-Dominance Orderings

In this section, we specialize our general result in order to characterize the aggregate net payoff functions that are required for a value-revealing equilibrium under a first-degree stochastic-dominance ordering and show that these payoff functions may correspond to a financing consisting of an equity issue combined with a debt retirement.[9]

THEOREM 3: *If the firm's type, t, orders the family of possible probability density functions of the firm's total returns* $\{f(\cdot, t)\}$ *by first-degree stochastic dominance, then a value-revealing equilibrium requires that the first derivative of the net payoff function chosen by firm t in the interior of* $\{T\}$ *change sign over the allowed range of x.*

Proof: If $y(x; z)$ were monotone in x, (7) implies that $V(z, t)$ would be monotone in t, and no minimum would exist in (1) for interior values of t. Q.E.D.

If the first-degree stochastic dominance takes the form of a shift in the mean of the distribution holding the other central moments constant, then the net payoff function of the chosen financing must exhibit a minimum in order that a value-revealing equilibrium exist; a V-shaped payoff function is the simplest example. The intuition underlying this theorem is straightforward; if the possible probability densities differed, say, only in their means, the value of the financing would be least if it were made by the firm with the lowest mean. It was shown in Theorem 1 that, in a revealing equilibrium, investors (correctly) infer the issuer type as that which minimizes the value of the financing. Therefore, such a monotone payoff function would be inconsistent with a revealing equilibrium if it were the result of a financing by any firm except that with the lowest possible mean.

Whether there will exist a revealing equilibrium in which financings are properly priced will depend on both the family of possible probability densities of firm earnings and the set of feasible financing strategies; the latter may depend on the pre-existing capital structure of the firm. Some cases in which revealing equilibria can be constructed involve the following kinds of worst-case financing: a debt repurchase and the issue of equity and possibly warrants, the granting of a guarantee to the debt of a subsidiary combined with an equity issue by the subsidiary, and the retirement of one convertible bond or preferred and its replacement by another.

Figure 2 illustrates how a V-shaped net payoff function results from a debt repurchase combined with an equity issue. In this figure, *ABCD* is the payoff function to bondholders from the debt repurchase, and *OFG* is the payoff function to purchasers of the new stock issue. The two functions are added vertically to yield a V-shaped aggregate net payoff function *ABHI*. In this figure, *OE* is the original par value of debt outstanding, and the par amount *FE* was repurchased for a consideration *OA*,[10] which corresponds to a discount of *CE*. The schedule *ABCD* reflects the sure payment of *OA* that bondholders receive as a result of the repurchase; this gives them an incremental gain for low levels of earnings realizations.[11] On the other hand, for high levels of earnings realizations, they

[9] This type of financing is quite common. Masulis and Korwar [14] report that the proceeds of 179 out of the 372 equity issues they examined were used for both capital expenditures and debt retirement.

[10] By setting the interest rate equal to zero, we have been able to ignore issues of timing; *OA* is, in fact, the repurchase price compounded for one period.

[11] Note that, as a result of the repurchase, bondholders receive *OA* even if the subsequently realized earnings turn out to be zero.

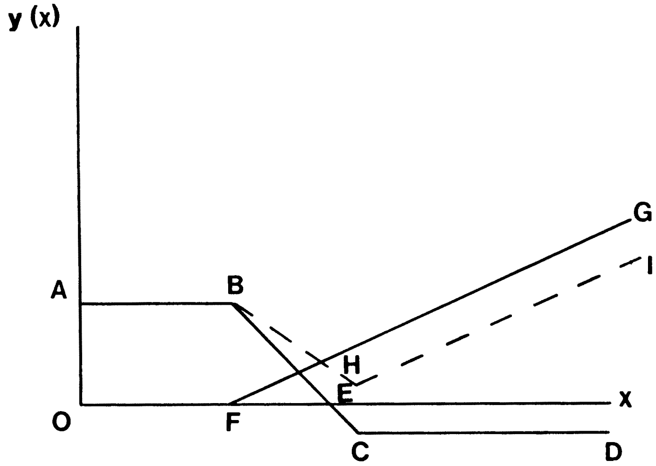

Figure 2. Payoff function for stock issue and debt repurchase. *ABHI*, aggregate net payoff; *ABCD*, net payoff to bondholders; *OFG*, net payoff to new stockholders; *OE*, par value of original debt; *FE*, par value of repurchased debt; *OA*, repurchase price of debt.

suffer an incremental loss corresponding to the discount, *CE*, at which the debt was repurchased. The payoff to the new stockholders, *OFG*, reflects the fact that they receive a constant fraction of earnings in excess of *OF*, the par value of debt outstanding after the repurchase.

We illustrate the possibilities for value-revealing equilibria under first-degree stochastic-dominance orderings by two examples. In the first example, there are only two possible firm types; in the second example, there is a continuum of types. In both examples, the firm has debt outstanding at time 0 and the amount of debt repurchased in the financing is a bullish signal of firm type. The reason for this is that debt retirements increase the firm's payouts to bondholders for low levels of earnings realizations by making the payout independent of the subsequent earnings realization; investors correctly infer from this that low levels of earnings are less probable. For example, a firm with debt that is, in fact, riskless can costlessly retire it at par. Since this would be costly for a firm with debt that is risky, the retirement is bullish information.

Example 1: The data for this example are given in Table I. At time 0, the firm has an opportunity to invest 10. The distribution of returns on the investment depends on the current state of the world; this state, denoted A or B, is private information to the firm. Under either state, the earnings of the firm at time 1 follow a symmetric two-point distribution; the distribution in state A dominates the distribution in state B. Since valuation is by expected value, the investment opportunity has a full information net present value of $150 - 120 - 10 = 20$ in state A, and of $137.5 - 120 - 10 = 7.5$ in state B. Part 3 of the table gives the full-information values of the firm's debt and equity when the investment policy and state are known but before the financing is completed.

Consider now the following (fully) revealing equilibrium. If state A prevails, the firm undertakes the following worst-case financing: it retires the debt at its

Table 1

A Revealing Equilibrium for Two Possible States

1. Distribution of Earnings at Time 1

	Probability = ½	Probability = ½	Expected Value
With No Investment	100	140	120
With Investment of 10:			
In State A	100	200	150
In State B	80	195	137.5

2. Initial Capital Structure

100 face value of debt maturing at time 1; 40 shares of equity.

3. Full-Information Security Values at Time 0

	Debt	Equity	Total
With No Investment	100	20	120
With Investment of 10:			
In State A	100	40	140
In State B	90	37.5	127.5

4. Worst-Case Financings

In State A: Retire debt at 100
Issue 110 shares at 1.00 per share
In State B: Issue 10.67 shares at 0.937 per share

full-information value of 100 and raises 110 to pay for this and the 10 of new investment by the sale of 110 new shares at the full-information value of 1.00 per share; the true, full-information, value of the original equity is then 40. If state B prevails, the firm raises the 10 of new investment by the sale of 10.67 new shares at their full-information value of 0.937 per share; the true, full-information, value of the original equity is then $(40/50.67) \times 47.5 = 37.5$. This equilibrium is possible only if there is no incentive for the firm to misrepresent the state that prevails at time 0 by adopting the financing we have proposed for the other state. If the true state is A, the firm has no incentive to adopt the simple equity financing since $(40/50.67) \times 50 = 39.47 < 40$. If the true state is B and the firm adopts the financing we have proposed for state A, the original shareholders will own 40/150 of the equity, which will be worth $(40/150) \times 137.5 = 36.67$. Since this is less than the 37.5 they obtain with the simple equity financing, there is no incentive for misrepresentation and the equilibrium is fully revealing.

This example, in which the firm repurchases debt when the better state prevails, is consistent with the finding of Masulis and Korwar that firms that use part of their equity issues to finance debt repurchases have higher abnormal returns than do firms that use the whole of the equity issue for capital expenditure.[12]

[12] −2.52 percent versus −3.75 percent for industrial firms. See Masulis and Korwar [14], Table 6. This difference is statistically significant on the usual criteria.

Example 2: For this example, we assume that the probability density of firm returns at time 1, $f(x; t)$, is given by

$$f(x;t) = \begin{cases} \pi, & x = X, \\ (1-\pi)/d, & t \le x \le t + d \le X, \\ 0 & \text{otherwise}, \end{cases} \tag{11}$$

so that t, which is private information, orders the distributions by first-order stochastic dominance. The distribution may be thought of as resulting from a situation in which, if the firm experiences no adversity, the return is X, but there is a probability $(1 - \pi)$ that adversity will be experienced, and conditional on this, the returns are distributed uniformly on the interval $[t, t + d]$. The firm is assumed to have an outstanding bond issue with face value B_0 that matures at time 1. Financings, which take place at time 0, are restricted to equity issues combined with partial debt retirements, so that each possible financing may be described by a pair $z = (\alpha, B_1)$, where α is the fraction of the equity owned by the new shareholders and B_1 is the par value of the debt outstanding after the financing is completed.

Let $V(\alpha, B_1; t)$ denote the true, full-information, value of the financing. It is shown in the Appendix that, for $t \le B_1$,

$$\begin{aligned} V(\alpha, B_1; t) = {} & \alpha\pi(X - B_1) \\ & + (1-\pi)\,[\alpha(t + d - B_1)^2 + (B_0 - B_1)\,(B_0 + B_1 - 2t)]/2d \\ & - B_0 + B_1, \end{aligned} \tag{12}$$

and, for $t \ge B_1$,

$$\begin{aligned} V(\alpha, B_1; t) = {} & \alpha\pi(X - B_1) + (1-\pi)\alpha(t + d/2 - B_1) \\ & - B_0 + B_1 + (1-\pi)(B_0 - t)^2/2d. \end{aligned} \tag{12$'$}$$

For a revealing equilibrium, a firm of type t^i must find a worst-case financing (α^i, B^i) such that

$$V(\alpha^i, B_1^i; t^i) = K \tag{13}$$

and

$$t^i = \underset{\{t \in T\}}{\operatorname{argmin}}\; V(\alpha^i, B_1^i; t). \tag{14}$$

In order to find the worst-case financing for a given firm, equations (13) and (14) are solved for (α, B_1) under an assumption about the relative values of t and B_1 that dictates the use of (12) or (12′). If the resulting value of B_1 violates the initial assumption, the alternative assumption is made; if the new value of B_1 violates this assuption, then no worst-case financing exists.[13]

[13] It is quite possible that only a subset of firms will be able to find worst-case financings; the remaining firms will be valued at one of two values of t according to whether the intrinsic values of their financings are increasing or decreasing in t. These two values of t will depend upon investors' prior distributions over possible values of t. Investigation of such semipooling equilibria is beyond the scope of this paper.

Instead of solving directly for the worst-case financing for each value of t and K, we shall consider the inferences about t that investors will draw from a given financing (α, B_1).

We know from Theorem 1 that $\hat{t}(\alpha, B_1)$ in a value-revealing equilibrium is the type that minimizes the true, full-information, value $V(\alpha; B_1; t)$. Thus, setting the derivatives of (12) and (12′) with respect to t equal to zero[14] and solving for $\hat{t}$, we have

$$\begin{aligned} \hat{t} &= (B_0 - B_1)/\alpha + B_1 - d, && \text{for } \alpha > (B_o - B_1)/d, \\ \hat{t} &= B_0 - \alpha d, && \text{for } \alpha \le (B_o - B_1)/d. \end{aligned} \tag{15}$$

Equation (15) implies

$$\partial \hat{t}/\partial \alpha < 0, \tag{16}$$

$$\frac{\partial \hat{t}}{\partial B_1} \begin{array}{ll} \le 0 & \text{for } \alpha > (B_o - B_1)/d, \\ = 0 & \text{for } \alpha \le (B_o - B_1)/d. \end{array} \tag{17}$$

Equation (16) is consistent with empirical finding by Masulis and Korwar of a negative association between announcement returns and the proportional size of common stock offerings. As we have already mentioned, condition (17) is consistent with the finding, also by Masulis and Korwar, of a significantly smaller announcement-related price drop for firms that use part of their equity issues to retire debt than for those that make a simple equity issue; moreover, the signal of firm value, $\hat{t}$, is nondecreasing in the amount of debt retired for a given initial debt level.

It might appear that our results are inconsistent with the empirical findings of Masulis [13] that, for pure exchange offers, announcement returns are negatively associated with the amount of debt retired. However, it should be clear that our theory relates only to firms that wish to make a financing to raise capital. In a revealing equilibrium, there is nothing to be gained by a pure exchange offer since the true, full-information, values of the securities issued and retired must be identical. Thus, a theory of exchange offers must rely on considerations not encompassed by our model.

Table II shows, for a specific example, the value of t signalled and the amount of capital raised by different financings. Note that the marginal signalling effect of debt repurchases becomes zero once a threshold is reached and that, despite the negative signal associated with an increase in α, the amount of capital raised is increasing in α.

III. Mean-Preserving-Spread Orderings

In this section, we consider the possibility that the information asymmetry concerns the "riskiness" of the distribution of returns. While the riskiness of firm returns is not relevant for valuation of the firm itself, given our assumption

[14] It may be verified that the second-order conditions for a minimum in (12) and (12′) are satisfied.

Table 2

Firm Type and Capital Raised under Alternative Financings in a Revealing Equilibrium[a]

B_1	$\alpha = 0.1$	0.2	0.3	0.4	0.5	0.6
975	925, 34	800, 88	758,140	734,192	725,243	717,295
950	970, 12*	900, 72	817,128	775,182	750,235	733,289
925	970, −11*	940, 52*	875,113	813,171	775,227	750,282
900	970, −33*	940, 32*	910, 96*	850,158	800,217	767,274
850	970, −78*	940, 8*	910, 61*	880,128*	850,194	800,258
800	970,−123*	940,−48*	910, 26*	880, 98*	850,169*	820,238*
750	970,−168*	940,−88*	910, −9*	880, 68*	850,144*	820,218*

[a] Each cell contains ($\hat{t}$, $V(\alpha, B_1; \hat{t})$. Initial data: $X = 2000$, $\pi = 0.5$, $d = 300$, $B_0 =$ 1000.

* The remaining debt outstanding is riskless ($\hat{t} > B_1$).

that financial assets are valued at the expected value of their payoffs, it is relevant for separate valuation of the constituent securities of the firm's capital structure. Therefore, if a firm has pre-existing debt in its capital structure, resolution of the information asymmetry about the riskiness of its returns is necessary for the proper pricing of new security issues.

THEOREM 4: *If the firm's type, t, orders the family of possible probability density functions of the firm's total returns $\{f(\cdot, t)\}$ by mean-preserving spread, then a value-revealing equilibrium requires that the net payoff function chosen by firm t in the interior of $\{T\}$ be neither convex nor concave.*

Proof: If $y(x; z)$ were convex or concave, $V(z, t)$ would be monotone in t and no minimum would exist in (1) for interior values of t.[15] Q.E.D.

The intuition underlying this theorem is analogous to that of Theorem 3. If the payoff function were, say, convex and the probability distributions differed only in, say, the variance, the financing would have least value if made by the lowest variance firm. In a revealing equilibrium, investors will always expect this to be the firm that made the financing; therefore, it cannot be made by any other firm if the equilibrium is indeed to be revealing.

Examples of financings that may lead to a revealing equilibrium for a mean-preserving spread ordering include issues of convertible bonds, junior bonds, or packages of bonds and warrants. We shall illustrate the possibility of achieving a revealing equilibrium when firm returns are ordered by mean-preserving spread by means of an example in which the firm issues a subordinated convertible bond. Such securities are commonly issued, yet there is no widely accepted explanation for their use.[16]

Example 3: For this example, we assume that the probability density of firm returns at time 1 is uniform on the interval $(m - t, m + t)$. At time 0, when a

[15] Note that, if the private information related to a second-degree stochastic-dominance ordering were only upward-sloping concave or downward-sloping convex, payoff functions could be excluded from a revealing equilibrium a priori.

[16] Green [9] argues that convertibles may reduce agency costs.

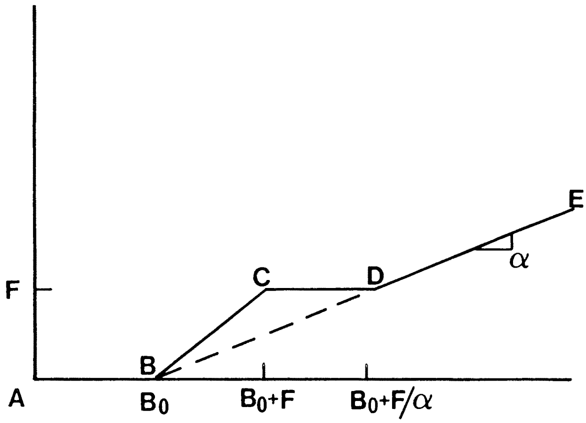

Figure 3. Payoff function for a subordinated convertible bond. B_0, face value senior debt; F, face value of convertible bond; α, conversion ratio.

financing must be made, m is common knowledge, but t, which orders the distributions by mean-preserving spread, is private information to the firm. The firm is assumed to have an outstanding bond issue with face value B_0 that matures at time 1. Financings are restricted to subordinated convertible bonds so that each possible financing may be described by a pair $z \equiv (\alpha, F)$, where F is the face value of the covertible and α is the fraction of the firm's equity into which it is convertible. The payoff function for the convertible is marked *ABCDE* in Figure 3.

Inspection of Figure 3 reveals that a necessary condition for the true, full-information, value of the convertible to achieve a minimum for an interior value of t is that there be a positive probability that the subordinated convertible bond will be converted at time 1:

$$t \geq B_0 - m + F/\alpha. \tag{18}$$

We assume that it is common knowledge that the outstanding bond is not riskless: $t \geq m - B_0$. Then the full-information value of the convertible, $V(\alpha, F; t)$ is given by

$$V(\alpha, F; t) = \{F^2(1-\alpha)/\alpha + \alpha(m + t - B_0)^2\}/4t. \tag{19}$$

A revealing equilibrium requires that

$$\min_{\{t \in T\}} V(\alpha, F; t) = K, \tag{20}$$

where K is the amount of capital to be raised. Using (19), the first-order condition from (20)[17] implies that, under the worst-case financing for a firm of type t, the bond is convertible into a fraction $\alpha(t)$ of the equity where

$$\alpha(t) = 2K/(m + t - B_0). \tag{21}$$

Thus, a revealing equilibrium is possible only if the set, T, of possible firm types is such that $t > 2K + m - B_0$ for $t \in T$.

[17] It may be verified that the second-order conditions for a minimum are satisfied.

Solving (19) and (20) for $F(t)$, the face value of the bond under the worst-case financing is given by

$$F(t) = \sqrt{\frac{\alpha}{1-\alpha}\,[4Kt - \alpha(m + t - B_0)^2]}, \tag{22}$$

it being understood that the terms of the financing satisfy condition (18).

To find $\hat{t}(\alpha, F)$, the firm type that investors infer from the financing (α, F), we eliminate K between (21) and (22) to obtain

$$\hat{t}(\alpha, T) = \sqrt{(m - B_0)^2 + F^2(1-\alpha)/\alpha^2}. \tag{23}$$

This implies for $\alpha < 1$ that

$$\partial\hat{t}/\partial\alpha < 0, \quad \partial\hat{t}/\partial F > 0. \tag{24}$$

Since the payoff on equity is a convex function of the firm return, this model predicts that the announcement-period stock returns will be negatively associated with the conversion ratio, α, and positively associated with the convertible-debt par value, F. While there is as yet no formal test of this prediction, it is interesting to note that Dann and Mikkelson [6] report a positive association between the increase in the book debt ratio and the stock announcement-period returns for convertible debt issues.[18]

IV. Multiparameter Private Information and the Prospects for Financial Efficiency

We have seen that, when the information asymmetry can be described in terms of a single parameter, it may nevertheless be possible for each firm type to make a worst-case financing so that new financings are properly priced and Pareto efficiency is achieved. When the information asymmetry concerns more than one parameter of the probability distribution of future earnings, the principles remain the same; full Pareto efficiency and correct pricing will follow if a worst-case financing can be constructed for each firm type.

However, when private information is characterized by several parameters, the aggregate net payoff function of a worst-case financing satisfying (9) becomes considerably more complex. For example, if the earnings of the firm follow a uniform distribution, both parameters of which are private information, then it can be shown that the aggregate net payoff function of the worst-case financing must have a section with a shape similar to that of a capital W. Such a payoff function could result, for example, from a financing in which one convertible bond was replaced by another and simultaneously one warrant issue was replaced by another.

Although such financings are possible, they do not seem to correspond with what we observe in practice. There are several alternative conclusions that could

[18] The twenty issues with the largest increase in the debt ratio were associated with announcement-period stock returns of −1.82 percent. The corresponding figure for the twenty issues with the lowest increase was −2.85 percent. See Dann and Mikkelson [6], Table 10.

be drawn from this. First, it is possible that multiparameter private information of the type we describe is rare. Second, it may be that we must broaden our definition of financings if the full range of signalling possibilities is to be understood. For example, firms enter into long-term contracts with customers and suppliers, and these contracts often contain complex provisions for setting prices and quantities that, taken in conjunction with the contracts agreed to with capital suppliers, may serve to reveal the private information.[19] Such long-term supply contracts often play an integral role in the financing of natural-resource projects.

A third possibility is that our assumed objective function of maximizing the true, full-information, value of the outstanding shares is not the appropriate one. An obvious alternative assumption that is commonly employed is that the managers of the firm act to maximize their own pecuniary and nonpecuniary rewards. Vermaelen [20] uses this assumption as the basis for a signalling model of the stock-repurchase decision. The analytical advantage of this assumption is that contracts for management remuneration are often highly complex and therefore possess the richness necessary to reveal multiparameter private information. However, in view of the disparate interests of different members of management and the board of directors, it is by no means clear that corporate financings can be better understood in terms of some composite management utility function than in terms of the more traditional objective of share-price maximization, and, for this reason, we have chosen to explore the implications of the classical assumption of value maximization.

V. Conclusion

In this paper, we have characterized the conditions under which the adverse-selection problem that may prevent a firm from issuing securities to finance an otherwise profitable investment may be costlessly overcome by an appropriate choice of financing strategy. The conditions require a certain compatibility between the nature of the information asymmetry and the set of financing strategies available to the firm, which may depend upon its pre-existing capital structure. What is required is that the economic value of the net claim issued by the firm take on its lowest possible value when issued by that firm; the firm has to be the worst possible issuer of that claim from the viewpoint of the investor. This requirement imposes conditions on the function relating the payoff on the net claim to the future earnings of the firm; for example, if the information asymmetry is about the mean of the earnings distribution, the payoff function must be V-shaped.

The conditions on the net payoff function required for a revealing equilibrium are not, in general, sufficient to yield a unique financing strategy; on the other hand, depending on the nature of the information asymmetry and the firm's prior capital structure, there may exist no strategy that satisfies the conditions. As examples of financings that resolve particular types of information asymmetry,

[19] However, unless the provisions of contacts with customers and suppliers are public information, investors will not be able to infer the firm's private information.

we considered equity issues combined with debt retirements, as well as junior convertible bond issues. For these financings, our theory provides a consistent rationale for the stock price reactions that have been found to be associated with their announcement.

Appendix

We assume that $t \leq B_0 \leq t + d$. The full-information discount from face value of the original debt is $(1 - \pi)(B_0 - t)^2/2d$. After the debt repurchase, the full-information value of the remaining debt is $B_1 - (1 - \pi)(B_1 - t)^2/2d$ if $t < B_1$ and B_1 otherwise. If $t < B_1$, the full-information value of the equity issue is $\alpha\pi(X - B_1) + (1 - \pi)\alpha[t + d - B_1]^2/2d$. The value of the debt repurchase is the difference between the discounted value of the original debt and the full-information value of the debt remaining after the repurchase. Combining these expressions for the equity issue and the debt repurchase yields (12). If $t \geq B_1$, the full-information value of the equity issue is $\alpha\pi(X - B_1) + (1 - \pi)\alpha[t + d/2 - B_1]$. Combining this with the full-information value of the debt repurchase yields (12′).

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