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Author(s): Warren Bailey
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An Empirical Investigation of the Market for Comex Gold Futures Options

WARREN BAILEY*

ABSTRACT

Option-pricing models that assume a constant interest rate may misprice futures options if the interest rate fluctuates significantly or if the price of the underlying asset is correlated with the interest rate. The futures option-pricing model of Ramaswamy and Sundaresan allows for a stochastic interest rate and correlation of the underlying asset's price with the interest rate. Using a data set of daily closing prices for Comex gold futures options, this paper tests the Ramaswamy and Sundaresan model against a constant interest rate model. Results indicate that the stochastic interest rate model is a superior predictor of market prices.

THE PAST SEVERAL YEARS have witnessed many innovations in contingent-claims markets. The introduction of gold futures options by The Commodity Exchange of New York (Comex) in October 1982 has proven particularly successful. Volume and open interest have been large and growing. The combination of futures and futures options trading makes Comex the principal market for gold in North America.

The existence of gold futures options represents a dual challenge to conventional option-pricing models. First, the variability of interest rates affects futures prices and, thus, the value of futures options. Second, if the price of gold is correlated with inflation or the real interest rate, it will also be correlated with the nominal interest rate. Option-pricing models that assume that the nominal rate is constant will misprice gold futures options if these factors are significant. This paper tests whether market prices for gold futures options are best described by an alternative model that allows the price of gold to covary with the rate of interest. Section I describes Comex gold contracts and notes differences between futures option contracts and stock option contracts, which have been studied more extensively. Section II describes the data set of market prices that has been collected and tests for violations of boundaries by the prices. Section III describes and tests the competing option-valuation models. Section IV is a summary and conclusion.

I. Options on Gold Futures Contracts

Comex gold futures options are written on Comex gold futures contracts of one hundred ounces each. Options are written on contracts that mature in February, April, June, August, October, and December. The options themselves expire on

* Academic Faculty of Finance, The Ohio State University. I thank Jim Bodurtha, Bob Geske, David Hirshleifer, Francis Longstaff, Tony Sanders, Walter Torous, and the referee, Bob Whaley, for helpful discussions and comments on earlier drafts.

the second Friday of the month prior to the maturity of the underlying contract. The minimum price fluctuation for both options and futures is ten dollars a contract, that is, ten cents per ounce. When a put or call is exercised, settlement is in a futures contract priced at the option striking price. For example, an investor exercising a call receives a long position in the underlying futures contract. At the end of the day, the contract is "marked to the market," and the investor receives cash equal to the difference between the settlement price for the futures contract and the striking price of the option that was exercised. The call writer receives a corresponding short position in the futures contract and pays the difference between the settlement and exercise prices.

Among the differences between conventional stock options and futures options are two factors that are important to the pricing of gold futures options. First, futures contracts do not pay dividends to their owners. Thus, there is no need to explicitly account for dividends in deriving models or in testing boundaries and models.[1] Second, it may be rational for an American put on a futures contract to be exercised early, as is also the case for puts on stocks. However, it may also be rational to exercise an American call on a futures contract early, in contrast with the standard no-early-exercise result for American options on stocks that pay no dividend. Whaley [27] offers a proof.

II. Data and Tests of Boundary Conditions

Comex does not maintain a record of transactions prices for research purposes, so analysis must rely on closing prices. Daily prices for Comex gold futures options, matching gold futures prices, and spot gold were obtained from Comex statistical yearbooks. Matching riskless interest rates implied from Treasury-bill discounts were collected from *The Wall Street Journal.*[2] Data were collected for the period from July 1, 1983, to April 30, 1984. A total of 4915 call prices and 4441 put prices was obtained. Although options are written on contracts representing one hundred ounces of gold, prices for options and futures contracts are quoted on a per-ounce basis. Additionally, a daily sample of spot gold prices and Treasury-bill yields was collected for February 28, 1983, to June 30, 1983, to estimate parameters required by the pricing models.

Before testing the competing option-pricing models, the sample of gold futures option prices is tested against rational option boundaries. Previous authors report mixed results from such tests. Studies of Amsterdam gold options by Beckers [3] and Ball, Torous, and Tschoegl [2] show frequent violations, while Ball and Torous [1] find few violations among Comex gold options. Frequent violations of the boundaries suggest that the market for the options is inefficient or so thin that nonsynchronous prices for options and the underlying asset lead to frequent boundary violations in closing prices. Let F, C, P, and X denote the values, in dollars per ounce, of the futures price, matching American call and put options,

[1] The absence of any dividend or significant storage cost and convenience yield for gold simplifies the valuation problem even further. Fama and French [12] report that interest rates explain almost all of the basis variation of gold futures prices. Brennan [7] reports that the cost of storing an ounce of gold is about six cents a month and estimates a convenience yield of no more than plus or minus twenty cents a month per ounce.

[2] T-bills mature on Thursdays, while the options mature on Fridays, so the match is not exact.

Table I

Summary Statistics on Violations of Option Price Boundaries by Comex Gold Futures Options[a]

Boundary	Options in Sample	Bound Violated	Ex Post Violations Mean	SD	Min	Max	Ex Ante Violations Mean	SD	Min	Max
$C > F - X$	4915	85	1.25	4.57	0.0	36.3	−0.26	1.46	−11.6	1.4
$P > X - F$	4441	602	1.51	4.47	0.0	86.4	0.15	3.65	−29.1	20.9

[a] SD = standard deviation; F = futures price, \$ per ounce; C = call price, \$ per ounce; P = put price, \$ per ounce; X = striking price, \$ per ounce.

and the striking price of those options, respectively. Early exercise bounds on the option prices are

$$C > F - X, \tag{1a}$$

$$P > X - F. \tag{1b}$$

Table I presents results of the boundary-violation tests for the data set of Comex gold futures options. The table shows boundary-violation frequencies of less than two percent for early exercise of American calls but almost fourteen percent for American puts. The downward trend in gold prices during the period may explain the latter result if closing put prices occurred prior to closing futures prices.

Explicit consideration of the size of these apparent mispricings may reduce their significance. Phillips and Smith [19] and Bhattacharya [4] note that boundary violations do not necessarily represent opportunities for profitable arbitrage when closing prices are used in the tests. Furthermore, factoring in the transactions costs associated with an arbitrage transaction may wipe out any profits.[3] Returning to Table I, the summary statistics for ex post violations describe the profits a trader would earn if he or she could initiate trades at the prices that violate the boundary. The ex post cash flows are, on average, so small and highly variable that they do not represent a profitable arbitrage strategy. The ex ante cash flows summarized in the table describe the profits from a strategy with which a trader observes a boundary violation but must wait until the next trading day to initiate transactions to attempt to exploit the violation. The profits from this strategy are, on average, negative or insufficient and highly variable. The ex ante strategy does not represent an opportunity for profitable riskless arbitrage. Although simple trading strategies using closing prices are not completely realistic or representative of all possible arbitrage strategies, the results suggest that no economicallly significant simple arbitrage opportunities exist in this market. The sample of closing market prices is sufficiently accurate to be used in testing the two option-pricing models.

III. Tests of the Option-Pricing Models

Whaley [28] adapts the European futures-option model of Black [5] to the pricing of American futures options using the extrapolation technique of Geske and

[3] Bodurtha and Courtadon [6] find that apparent boundary violations obvserved in currency options prices vanish when transactions prices are used and transactions costs are accounted for.

Johnson [14]. The model assumes that the nominal interest rate is constant and that the spot price, S, of the underlying asset is lognormally distributed:

$$dS = \alpha S dt + \sigma_1 S dz_1. \tag{2}$$

Parameters α and σ_1 are the drift and volatility of the stochastic process, while dz_1 is a standard Wiener process. The two-point version of the extrapolation will serve as the constant interest rate model in the tests.[4]

Ramaswamy and Sundaresan [20] adopt a distributional assumption similar to (2) for the spot price of the asset and add an assumption about the dynamics of the rate of interest, r:

$$dr = \kappa(\mu - r)dt + \sigma_2 \sqrt{r} dz_2. \tag{3}$$

Parameters κ, μ, and σ_2 are the speed of adjustment, long-run mean, and volatility of the stochastic process, respectively, while dz_2 is a standard Wiener process. The covariance between dz_1 and dz_2 is ρdt. Assuming that the local expectations hypothesis of Cox, Ingersoll, and Ross [9] holds, the authors derive a partial-differential equation for the value, C, of a futures call option:

$$\tfrac{1}{2}\sigma_2^2 r C_{rr} + \tfrac{1}{2}\sigma_1^2 S^2 C_{SS} + \rho\sigma_1\sigma_2 S\sqrt{r} C_{rS} + \kappa(\mu - r)C_r + rSC_S - rC = C_\tau. \tag{4}$$

The equation can be solved numerically given appropriate boundary conditions. Since the potential for early exercise must be factored into the value of the call, futures prices must also be numerically computed.[5] The line hopscotch method, a finite-difference technique suggested in Siedman [22], Gordon [15], and Gourlay and McKee [16], is used to solve the partial-differential equation. The time step size is set at $1/52$. The gold price and interest rate step sizes are set to yield one hundred steps in the price direction and twenty-five steps in the interest rate direction. This relatively crude grid sacrifices little accuracy and saves considerable computational expense.[6]

Implementation of the pricing models requires estimates of the parameters κ, μ, σ_1, σ_2, and ρ. Following Dietrich-Campbell and Schwartz [10], the transformations $S' = \ln\{S\}$ and $r' = \sqrt{r}$ are used to make the system (2) and (3) homoscedastic:

$$dS' = (\alpha - \sigma_1^2/2)dt + \sigma_1 dz_1, \tag{2$'$}$$

$$dr' = [(\kappa\mu - \sigma_2^2/4)/2r') - (\kappa r'/2)]dt + (\sigma_2/2)dz_2. \tag{3$'$}$$

Because S' and r' are not continuously observable and equation (3$'$) is nonlinear, discrete first differences, $r'(t + h) - r'(t)$ and $S'(t + h) - S'(t)$, are employed in the estimation. Assuming that the drift of process (3$'$) is constant between observations, the next step is to jointly estimate the parameters by maximizing

[4] Whaley [28] finds that the two-point extrapolation is a reasonably accurate model of American futures-options prices. Whaley [27] presents an alternative approach to American futures-option valuation based on the quadratic American put approximation of MacMillan [17].

[5] The partial-differential equation for the futures price is identical to (4) except for the $-rC$ term, which is omitted.

[6] The numerical scheme reproduced the simulations published in Ramaswamy and Sundaresan [20] with errors of only a few cents. Doubling the number of price, interest rate, and time steps does not increase precision significantly and raises the cost, in CPU seconds, about 350 percent. Simulation results are available from the author.

the bivariate conditionally normal likelihood function. The locally constant drift assumption is reasonable because daily observations of the price of gold and the interest rate are used for estimation. Beginning-of-period drift (r' equals $r'(t)$) and the substitution $\beta = \kappa\mu - \sigma_2^2/4$ are used to estimate (3$'$). The resulting parameter estimates are $\hat{\sigma}_1 = 0.2435$, $\hat{\kappa} = 12.39$, $\hat{\beta} = 1.10018$, $\hat{\sigma}_2 = 0.04417$, and $\hat{\rho} = -0.1673$. The estimates imply an estimated long-run mean of $\hat{\mu} = 0.0888$.

Constant interest rate and Ramaswamy and Sundaresan model values are computed for each option in the sample. Comparisons of the theoretical values with the market values are reported in Table II for call options and Table III for put options. The tests yield an average overpricing of $2.49 per ounce for calls and $2.99 for puts by the constant interest rate model and an average underpricing of $0.19 per ounce for calls and overpricing of $0.03 for puts by the Ramaswamy and Sundaresan model. The results appear poor compared with those published in many other studies because the tests do not use implied parameters that compensate for model errors. However, the superior forecasting ability of the stochastic interest rate model is apparent.

The tables provide detailed summary statistics on price deviations by grouping the options according to time to expiration or whether an option is in or out of the money. The constant interest rate model overprices in-the-money calls by $3.59 on average, out-of-the-money calls by $1.97, in-the-money puts by $3.22, and out-of-the-money puts by $2.69. It appears to generate larger errors for longer maturity options and is less accurate for at-the-money options than for other options. The Ramaswamy and Sundaresan model overprices the in-the-money calls by $0.92 on average, underprices out-of-the-money calls by $0.70, underprices in-the-money puts by $0.66, and overprices out-of-the-money puts by $0.56. It does substantially better with the at-the-money puts and shorter maturity puts and calls. The average absolute value of the errors is $2.73 for calls

Table II

Summary Statistics on the Deviations, D, and Absolute Values of Deviations, $|D|$, of Market Prices from Model-Estimated Prices for Comex Gold Futures Call Options[a]

		Constant Interest Rate Model				Ramaswamy and Sundaresan Model			
		D		$\|D\|$		D		$\|D\|$	
	N	Mean	SD	Mean	SD	Mean	SD	Mean	SD
All	4915	−2.49	2.97	2.73	2.75	0.19	3.64	2.30	2.83
$F > X$	1591	−3.59	3.43	3.89	3.09	−0.92	4.32	3.04	3.19
$X > F$	3324	−1.97	2.56	2.17	2.39	0.70	3.17	1.97	2.58
$X \approx F$	634	−4.56	2.97	4.77	2.62	−0.78	4.23	3.18	2.92
$T < 1/8$	1239	−0.53	1.78	0.91	1.62	0.03	2.43	1.17	2.12
$1/8 < T < 1/4$	1222	−1.62	2.21	1.99	1.88	0.37	2.30	1.59	1.71
$1/4 < T < 3/8$	1027	−2.74	2.47	2.89	2.28	−0.40	3.12	2.24	2.21
$3/8 < T < 1/2$	540	−4.21	3.24	4.20	3.23	−0.17	4.58	3.24	3.25
$1/2 < T$	887	−5.10	3.03	5.17	2.90	1.42	6.52	5.47	3.82

[a] SD = standard deviation; N = number of options in the category; F = futures price, $ per ounce; X = striking price, $ per ounce; T = time, in years, until option expires.

Table III

Summary Statistics on the Deviations, D, and Absolute Value of Deviations, $|D|$, of Market Prices from Model-Estimated Prices for Comex Gold Futures Put Options[a]

		Constant Interest Rate Model				Ramaswamy and Sundaresan Model			
		D		$\|D\|$		D		$\|D\|$	
	N	Mean	SD	Mean	SD	Mean	SD	Mean	SD
All	4441	−2.99	5.24	3.44	4.95	−0.03	3.79	2.48	2.87
$F > X$	1883	−2.69	2.81	2.93	2.55	0.66	3.56	2.39	2.71
$X > F$	2558	−3.22	6.46	3.81	6.12	−0.56	3.88	2.55	2.97
$X \approx F$	634	−4.45	2.81	4.66	2.45	−0.28	4.45	3.21	3.09
$T < 1/8$	1053	−0.75	2.43	1.28	2.19	0.17	2.76	1.57	2.27
$1/8 < T < 1/4$	1137	−2.23	3.65	2.50	3.47	−0.04	2.43	1.76	1.68
$1/4 < T < 3/8$	991	−3.06	3.30	3.54	2.78	−0.84	3.70	2.77	2.60
$3/8 < T < 1/2$	519	−5.38	11.1	6.07	10.7	−0.24	5.39	3.94	3.68
$1/2 < T$	741	−5.61	3.98	5.98	3.40	1.84	6.51	5.15	4.39

[a] SD = standard deviation; N = number of options in the category; F = futures price, \$ per ounce; X = striking price, \$ per ounce; T = time, in years, until option expires.

and \$3.44 for puts using the constant interest rate model and \$2.30 for calls and \$2.48 for puts using the Ramaswamy and Sundaresan model. In comparing the two models, the absolute-value statistics reduce the relative superiority of the Ramaswamy and Sundaresan model. Nonetheless, the stochastic interest rate model produces errors with absolute values substantially less than for the competing model. The increased accuracy averages \$43 per contract for calls and \$96 per contract for puts. For at-the-money options, the improvement averages \$1.59 per ounce for calls and \$1.45 per ounce for puts, or about \$150 per contract.

Following Whaley [26], Table IV reports regressions that relate the pricing errors to parameters of the option-pricing models. The dependent variable is the difference between market and model prices, while explanatory variables are the degree to which the option is in or out of the money and the time to expiration of the option. Results indicate that the constant interest rate model's errors are strongly correlated with the time to expiration of calls and puts and the degree to which calls are in or out of the money. The Ramaswamy and Sundaresan model's errors are less strongly related to these variables, except for the association between put errors and the degree to which the put is in or out of the money. The residual mispricings must be due to model misspecification, errors in parameter estimation or numerical computations, nonsynchronous prices, or market inefficiency.

IV. Summary and Conclusion

Two competing option-pricing models were tested on daily prices for Comex gold futures options. Measured in absolute-value terms, the errors from the Ramaswamy and Sundaresan model are smaller than errors from a constant interest rate model by an average of \$43 per contract for calls and \$96 per contract for

Table IV

Regressions Relating Deviations, D, between Market and Model Prices to Model Parameters[a]

	Calls			Puts		
	α	β	RSQ	α	β	RSQ
Constant Interest Rate Model:						
$D = \alpha + \beta\,(F - X)$	−3.208	−0.022	0.168	−2.696	0.002	0.000
	(−69.7)	(−30.7)		(−35.3)	(1.03)	
$D = \alpha + \beta\,T$	0.040	−8.981	0.309	−0.222	−9.869	0.113
	(0.6)	(−45.8)		(−1.57)	(−23.2)	
Ramaswamy and Sundaresan Model:						
$D = \alpha + \beta\,(F - X)$	−0.142	−0.010	0.022	0.137	0.012	0.024
	(−2.21)	(−9.82)		(2.12)	(9.46)	
$D = \alpha + \beta\,T$	−0.218	1.563	0.005	−0.302	1.117	0.002
	(−2.16)	(4.83)		(−2.60)	(2.81)	

[a] F = futures price, \$ per ounce; X = striking price, \$ per ounce; T = time, in years, until option expires; α, β = regression coefficients. A t-test of whether the estimate is significantly different from zero is reported in parentheses beneath each estimate.

puts. Allowing for a stochastic interest rate yields a more accurate model for valuing gold futures options.

Based on these tests, several observations can be offered. First, model-free boundary tests indicate that the market for Comex gold futures options does not offer any obvious economically significant arbitrage opportunities. Second, tests of the competing pricing models suggest that the stochastic interest rate specification predicts market prices with greater accuracy. Therefore, interest rate volatility is a significant factor in the market for gold and its derivative assets. Finally, Ramaswamy and Sundaresan note that their model is very sensitive to the speed-of-adjustment parameter and the difference between the current interest rate and its long-run mean. More precise parameter estimates, a more exact numerical procedure, and the study of a variety of types of futures contracts may lead to a stronger demonstration of its superiority.

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