THE JOURNAL OF FINANCE • VOL. XLII, NO. 4 • SEPTEMBER 1987

Maturity Intermediation and Intertemporal Lending Policies of Financial Intermediaries

GEORGE EMIR MORGAN and STEPHEN D. SMITH*

ABSTRACT

This paper considers the maturity intermediation and intertemporal lending decisions of risk-averse financial intermediaries. In particular, the maturity mismatch problem and the fixed-versus-variable-rate lending decision are modeled when the major source of risk involves uncertain future interest rates. The results imply that the strategy of matching the maturity of assets and liabilities is not generally optimal or even minimum risk. This is due primarily to the "built-in" hedge that the intermediary has as a result of rolling over short-term loans while continuing to finance long-term loans. Intertemporal dependencies between loan demand and costs (or both) also have an effect on the optimal degree of maturity mismatching and provide one rationale for making loans at rates below current marginal cost.

IN RECENT YEARS, THE problem of managing interest rate risk has taken on new importance for both financial managers of financial institutions and academics. A tremendous increase in interest rate volatility has forced managers to look more closely at both the problem of maturity mismatch (or "Gap" management) and optimal lending arrangements. Early work on this issue includes Kaufman's [10] identification of the mismatch "problem" and its implications for loan pricing. Recent work, however, has primarily focused attention on rate uncertainty and the structure of financial contracts offered by intermediaries. (See Campbell [3], Santomero [15], Deshmukh, Greenbaum, and Kanatas [4], and Ricart i Costa and Greenbaum [14].) These papers have, for the most part, focused on comparative-static results concerning the effects of increased risk on the supply of fixed-rate versus variable-rate lending.

In essence, the aforementioned papers separate the investment and financing decisions and provide a detailed analysis of the former given the latter (i.e., the funding environment is exogenous). The purpose of this paper is (a) to provide a detailed investigation of the optimal funding decision (particularly the maturity intermediation decision) and (b) to analyze the variables that cause a link between current and future decisions in a multiperiod environment.

One purpose for focusing on the financing problem is, as noted above, the greater attention that has been given to the lending decision in previous academic research. This has in turn led to confusion and a number of misconceptions concerning what the optimal funding strategy is when there are multiple sources of uncertainty. We clarify this issue by providing conditions under which a risk-

* Department of Finance, Virginia Polytechnic Institute and State University, and College of Management, Georgia Institute of Technology, respectively. Professor Smith wishes to thank the College of Business Administration, University of Texas at Austin, for research support.

averse intermediary will find it optimal, for example, to fund some of its long-term assets with short-term liabilities.

Moreover, we also show that matching the maturity of assets and liabilities is generally not the optimal, or even the minimum-risk, solution to the financing problem. In a situation where an unknown quantity of loans will be made in the future at a (currently) unknown spread and where fixed-rate, long-term loans can be financed at an unknown cost, maturity matching on long-term assets will create more risk than a better diversified funding strategy as long as future loan revenues and deposit rates are not independent. The important contributions to modeling of the intermediary's problem contained in this paper are (a) the incorporation of risk aversion and (b) the recognition that short-term loans will be made in the future (at an unknown spread) along with any necessary new financing of long-term loans.

The lending problem is not, however, ignored. We provide conditions that justify separation between the two decisions. Interestingly, we also show why, when separation fails, intermediaries may make loans at current rates that are below current marginal cost. This phenomenon, resulting from what has been called, somewhat euphemistically, the "customer relationship,"[1] is shown to be potentially optimal if (a) loan demand is not independently distributed through time and/or (b) the cost function displays intertemporal cost complementarities. (See, e.g., Baumol [2].) Furthermore, such effects also influence the size, but not the sign, of the optimal degree of maturity mismatching. These interactive influences are one positive aspect of jointly modeling the decisions.

The paper is organized as follows. In Section I, the general investment-financing model is set up in a three-date framework and the solution technique is outlined. Section II contains a discussion and analysis of the hedging, or maturity mismatch, problem. Section III provides an analysis of the lending decision. Some remarks and a summary conclude the text. The Appendix contains some of the mathematical details.

I. The Model

The model we use is two period (three date) in nature and can be thought of as an extension of the intertemporal model in Niehans and Hewson [12].[2] We assume that the intermediary can borrow short-term funds (e.g., in the negotiable CD market) in the current and future periods (where the latter cost is random now) and/or write forward contracts now (at a currently known rate) for delivery of short-term funds at the start of the second period. Since it is well known that the ability to undertake short-term transactions and forward commitments is equivalent to making long-term contracts, the forward market represents the two-period deposit market. In particular, the two-period deposit rate can be

[1] See Hodgeman [7] and Wood [18] for a detailed examination of this concept and Wood and Wood [19] for a summary.

[2] Niehans and Hewson consider a mean-variance example for intermediaries operating in perfectly competitive asset and liability markets. Furthermore, they ignore real resource costs. We relax, at different points in the analysis, all three of their simplifying assumptions.

defined as a weighted average of the current short-term rate and the forward rate. We provide a solution for this "no-arbitrage" condition in the next section.

On the asset side of the balance sheet, the intermediary makes current one- and two-period loans at known per-period rates. That is, we ignore the question of credit risk. Given that the markets for loans are neither perfectly competitive nor perfectly segmented, we would expect the current short-term lending rate to be a function of the quantity of both current short-term and long-term loans. The same logic applies to the long-term lending market. Moreover, the intermediary will also make short-term (one-period) loans next period. However, both the quantity and price of future short-term lending contracts are uncertain in the current period. The distribution of future short-term loan demand may be dependent on the current level of short-term and long-term lending, as well as future events. For the sake of generality, we allow the dependency to be of any sort.[3] By assumption, the intermediary also incurs real resource costs in the booking and servicing of loans in both the current and future periods. (See Baltensperger [1] and Santomero [16].) Thus, we take the dollar value of assets (or loans) as our output measure. (See, e.g., Sealey and Lindley [17].) In its most general form, the real resource cost function of a multiperiod firm will not be separable in its outputs. That is, the current period cost function may exhibit cost complementarities between long- and short-term loans (due, for example, to the presence of fixed costs that are jointly shared by the outputs, e.g., data-processing hardware).[4] Moreover, there is no *a priori* reason to exclude the possibility that costs in the *future* period will be a function of current long-term loans (which will still be on the books and need to be serviced), future short-term loans, and *current short-term* loans as well.

Using the balance sheet constraints to eliminate the quantity of short-term borrowings, we have the following profit functions in periods 1 and 2.

$$P_1 = (r_1 - i_1)L_1 + (r_2 - i_1)L_2 - C_1, \tag{1a}$$

$$P_2 = (r_2 - I)L_2 + (R - I)l + f(I - i_f) - C_2. \tag{1b}$$

The random variables in the problem are the future short-term loan and deposit rates (R and I, respectively) and the volume of lending in the future period, l. Therefore, C_2, which is a function of l, is also random. To anticipate the results somewhat, it can be noted that, if C_2 were known and asset markets were perfectly elastic, the uncertainty of profits in period 2 would come from (a) the level of deposit interest costs and (b) the spread earned on new loans made in period 2. If those two components of risk work in opposite directions, the intermediary has a built-in hedge on its balance sheet. Therefore, the intermediary may "over hedge" (increase its risk) by fully covering long-term loans with forward contracts.

We take the bank's objective function to be the maximization of the expected utility of profits over the two periods, or

$$\max_{L_1, L_2, l, f} \mathrm{E}[U(P_1, P_2)], \tag{2}$$

[3] We do, however, assume that the slope of R with respect to l is known today. (See the Appendix.)

[4] See Murray and White [11] and Gilligan, Smirlock, and Marshall [6] for some estimates of these interactive cost parameters for credit unions and banks, respectively.

where

$$U_i' = \frac{\partial U(P_1, P_2)}{\partial P_i} > 0, \quad \text{for} \quad i = 1, 2$$

and

$$U_i'' = \frac{\partial^2 U(P_1, P_2)}{\partial P_i^2} \leq 0, \quad \text{for} \quad i = 1, 2.$$

The problem stated in (2) can be solved using dynamic programming. We first solve for the optimal value of l, l^*, given arbitrary values of L_1, L_2, and f. We then substitute this solution into the objective function and solve for the optimal values L_1^*, L_2^*, and f^*.

The decision in the second period, when interest rates are revealed, will be made according to the first-order condition for l, which is given by

$$MR_l - MC_l - I = 0, \tag{3}$$

where $MR_l = R + \frac{\partial R}{\partial l} l$ and $MC_l = \frac{\partial C_2}{\partial l}$. The initial first-order conditions for L_1, L_2, and f are

$$MR_{L_1}^1 - MC_{L_1}^1 - i_1 = -\theta\left[\mathrm{E}(MP_{L_1}) + \frac{\mathrm{cov}\{U_2', MP_{L_1}\}}{\mathrm{E}(U_2')}\right], \tag{4a}$$

$$MR_{L_2}^1 - MC_{L_2}^1 - i_1 = -\theta\left[\mathrm{E}(MP_{L_2}) + \frac{\mathrm{cov}\{U_2', MP_{L_2}\}}{\mathrm{E}(U_2')}\right], \tag{4b}$$

and

$$i_f = \mathrm{E}(I) + \frac{\mathrm{cov}\{U_2', I\}}{\mathrm{E}(U_2')}, \tag{4c}$$

where[5]

$$\theta = \frac{\mathrm{E}[U_2']}{\mathrm{E}[U_1']}.$$

MP_{L_j} is marginal (random) profits in period 2, and $MR_{L_j}^1$ $j = 1, 2$ should be interpreted as the *net* effect of expanding one-($j = 1$) or two-($j = 2$) period loans in the current period. That is, $MR_{L_1}^1$ reflects, for example, not only the additional revenue generated from making one-period loans, but also the possible reduction in two-period loan demand associated with making more current short-term credit available.

The left-hand side of (4) can be viewed as the current known revenues and costs (over both periods) associated with a given change in short-term lending, long-term lending, and hedging (or financing) decisions, respectively. The right-

[5] The term θ implicitly represents the time discount factor for certain profits in the next period, $0 < \theta \leq 1$.

hand side represents the expected marginal revenues and costs associated with a change in a current decision variable plus a marginal risk premium. The first-order condition (3) simply represents the standard equalization of marginal revenue and costs, once uncertainty has been resolved.

In Section III, we return to an investigation of equations (4a) and (4b) (the lending decision); however, we first turn to an investigation of the maturity mismatch problem, about which equations (3) and (4c) provide a great deal of information. This follows since (4c) is related to the slope of the objective function with respect to forward financing and (3) contains the uncertainty associated with uncovered future short-term lending.

II. The Maturity Mismatch Decision

In this section, we focus attention on the optimal financing or "hedging" decision faced by the intermediary. The quantity of dollars for which the maturity of assets deviates from that of liabilities is often called the maturity "Gap" by intermediary managers. The issue of the appropriate size and direction of the Gap is similar to the hedging problem in forward markets. The formal focus is therefore on equation (4c), which is crucial for characterizing the optimal forward commitment. It is well known that the demand for forward contracts implicit in (4c) has two elements. The first is the speculative element, represented by $\mathrm{E}[I] - i_f$. When this term is negative, the intermediary has, regardless of its aversion to risk, *ceteris paribus*, an incentive to fund more of its long-term assets with short-term liabilities since the expected cost of short-term funding is lower. The converse holds when the forward rate is less than the expected cost of short-term deposits. The degree of mismatch will, however, depend on the degree of risk aversion. The speculative factor is often ignored by researchers when it is assumed that the forward rate is unbiased (i.e., the expected marginal cost of hedging is zero). This unbiasedness assumption can be partially justified when the point of interest is the "pure hedging" element of forward demand (represented by the covariance term in (4c)). That is, when forward rates are unbiased, the optimal maturity mismatch decision chosen by the intermediary will be the minimum-risk one. Because this aspect of the decision seems to dominate discussions in both academic and practitioner circles, our formal propositions are restricted to this case (i.e., $i_f = \mathrm{E}[I]$); however, we do provide some remarks on the speculative effect, i.e., cases where the expected cost/return to hedging is nonzero.

We now turn to the conditions under which the minimum-risk position will be a fully matched or mismatched one. That is, we wish to find the value of f that causes the covariance in (4c) to be zero and establish whether or not this value is greater than, equal to, or less than L_2, the optimal amount of long-term loans. Define $G \equiv f - L_2$ as the "Gap," where $f - L_2 > 0$ denotes a positive Gap, etc. Notice that profit in the second period can be rewritten as

$$P_2 = (I - i_f)G + (R - I)l + L_2(r_2 - i_f) - C_2.$$

Note that, when $G = 0$, P_2 is *not* riskless. Rewriting (1b) in the above form makes clear the fact that the intermediary has a risky profit stream because of (a) the level of deposit rates when there is a nonzero Gap and (b) uncertain future profits from short-term lending, which can be written as a function of the uncertain quantity of future lending from equation (3). At the optimum, the profit from future short-term lending (whatever its value) is positively related to profits. Therefore, if the short-term deposit rate is statistically associated with the quantity of future short-term loans, Gap management can be used to offset some of the risk associated with future lending. That is, while $G = 0$ (a zero Gap) will eliminate all interest rate risk associated with current long-term lending, it will not generally result in the overall minimum-risk position because the diversification effect outlined above is ignored. In order to formally prove this conjecture, we first need to establish the following lemma.

LEMMA: *At* $f = L_2$, $\text{cov}(U_2', I)$ *is positive (zero, negative), if future deposit rates and future loan demand are negatively (not, positively) associated, for all utility functions exhibiting risk aversion provided the slope of loan demand is known.*

Proof: See the Appendix.

Given this Lemma, we are now prepared to state and prove our major result concerning the maturity mismatch decision.

PROPOSITION 1: *When the deposit rate term structure is unbiased, the optimal (minimum-risk) Gap is negative (zero, positive) if future deposit rates and future loan demand are positively (not, negatively) associated for all utility functions exhibiting risk aversion.*

Proof: Assuming that forward contracts are normal goods, the result follows directly from the Lemma. At an unbiased forward rate structure, when deposit rates are negatively associated with loan demand, the covariance in (4c)—the slope of the objective function—is positive at $f - L_2 = 0$ (from the Lemma), and, therefore, the optimal decision must be to increase the Gap; i.e., the Gap must be positive. A similar proof is employed for the other conditions. Q.E.D.

The intuition of the maturity mismatch proposition is straightforward. Low future loan demand (and therefore a low volume of loans supplied) is associated with lower profits from future short-term lending. Risk-averse intermediaries wish to hedge against such low-profit states of the world. When deposit rates and future short-term loan demand are positively associated, risk-reducing diversification can be achieved by leaving some of the long-term assets "exposed".[6] In this case, their future cost of funding long-term loans will be lower (and profits higher) in states where uncovered short-term lending has a low profit *vis à vis* a fully matched position. Conversely, when deposit rates and loan demand are negatively associated, the cost of financing two-period lending can be reduced in states where there is a low profit from short-term lending by "locking in" a large proportion of deposits. This strategy can be contrasted with a fully matched position, which causes deposit costs on long-term (fixed-rate) loans to be invariant to the random level of profits from future short-term lending. In essence,

[6] This diversification effect between future spot lending and two-period loans (or forward commitments) is lost in a risk-neutral framework. See, for example, Ricart i Costa and Greenbaum [14].

the fully matched position concentrates the portfolio in one security, a policy that will be suboptimal under risk aversion as long as there is a statistical association between the level of future deposit costs and the profit earned on future lending. It follows that the minimum-risk position will be a fully matched one if and only if profits from future short-term lending are not related to future deposit rates. Thus, the implicit assumption behind the notion that a fully matched position is the minimum-risk one is that deposit interest rates and short-term loan demand are not associated over the business cycle.

An interesting special case involves competitive loan markets and time-separable cost functions. In this case, the Gap decision will be based simply on the association of the short-term spread, $R - I$, and the level of deposit rates, I. The analogy to portfolio theory is perhaps clearest here. If the two are securities (the spread and level, respectively) that are negatively associated, a positive Gap will be the minimum-risk solution. This result follows from the fact that high-rate environments are associated with low-profit states from future short-term lending and borrowing. The minimum-risk solution therefore involves "locking in" deposits in excess of current long-term lending in order to offset these low-spread and high-rate states. Analogous reasoning applies to the situation where the spread and level of rates are positively associated. (This case is equivalent to the minimum-risk solution in portfolio theory, which would involve shorting ($G < 0$) one of the securities when their returns are positively associated.) Only in the situation where the spread is unassociated with or "insulated" from the level of deposit rates will a zero-gap position be the minimum-risk one.[7]

Before concluding this section, we provide some brief comments on the effect of the expected cost of hedging, economies of scope, and customer relationship on the optimal funding decision. These issues are summarized by the following remarks.

1. A deposit forward rate that is an upwardly biased estimate of the expected future spot rate (i.e., the expected cost of hedging is positive) promotes, *ceteris paribus*, a negative Gap and vice versa for a forward rate that is a downwardly biased estimate of the expected spot rate.

2. When the forward rate is greater than the expected future spot rate, a fully hedged position can be optimal only if loan demand and deposit rates are negatively associated. If the forward rate is less than the expected future spot rate, a fully hedged position can be optimal only if deposit rates are positively associated with loan demand.

3. When deposit rates are not associated with loan demand, a fully hedged position can never be optimal if the forward rate is a biased estimator of the expected spot rate (i.e., if the expected marginal cost of hedging is nonzero).

Remarks 1 to 3 provide some results concerning the speculative element of the demand for fixed-rate borrowing. Remarks 1 and 2 suggest that, if the expected

[7] In an alternative framework, l could be viewed as another current control variable (e.g., current commitment contracts for future lending at a floating rate). In this case, it can be shown that the minimum-risk forward position is strictly less than $l + L_2$, total second-period lending, if asset and liability rates are positively associated. It is minimum risk to leave some commitments exposed because costs will be low (profits on committed lending will be high) when revenues on future short-term lending are low, resulting in relatively higher returns in low-loan-rate environments.

cost of funding short term is lower, then the optimal Gap will be more negative than the minimum-risk solution, while the opposite will be true if short-term funds are expected to be more costly. Remark 3 simply says that, if the minimum-risk solution is a zero Gap, then it will never be optimal unless there is a zero speculative element.

4. Intertemporal economies of scope in the production of services or intertemporal "customer relationships" in loan demand decrease the incentive to deviate from a zero Gap but do not affect its sign.

If current lending decisions decrease the marginal cost or increase the marginal return of future lending, then the slope of the objective at a zero Gap is diminished. (See the proof of the Lemma.) These factors decrease the absolute value of the sensitivity of profits to the level of interest rates, i.e., the extent to which profits are increasing in l. The dependence of costs and/or demand through time has partially offset the need for Gap management.

III. Lending Decisions

In this section, we briefly investigate the question of optimal lending. We first consider the problem when intertemporal separation is possible and then turn to the more general case. There are a number of alternative concepts of separation. We employ one from the theory of the firm literature. (See, for example, Holthausen [8] and Feder, Just, and Schmitz [5].) By separation we mean that the intermediary's current short- and long-term lending decisions can be made independently of the intermediary's subjective probability distribution of future interest rates. That is, their production decisions can be made based solely on current, known spot and forward rates and a known production technology. One immediate possibility involves the case where current and future loan rates are not a function of the quantity of loans. We call these "competitive market conditions" and utilize them in the following proposition:

PROPOSITION 2: *Separation occurs under competitive market conditions and separable cost functions if the marginal cost of two-period loans is stationary through time and/or if θ is a constant.*

Proof: See the Appendix.

As shown in the Appendix, under these conditions, loans must satisfy the equation:

$$r_2(1+\theta) - i_1 - \theta i_f = MC^1_{L_2} + \theta MC^2_{L_2}, = (1+\theta)(r_2 - i_2),$$

where the last equality follows by noting that $i_1 + \theta i_f = (1+\theta)i_2$, where i_2 is the two-period deposit rate.[8] Therefore, the optimal value of L_2 must satisfy

$$S_2 = MC^*_{L_2}, \tag{5}$$

[8] This equality can be shown from the first-order conditions when the model includes two-period deposits as well as one-period deposits and a forward contract. Notice that the equality implies that the two-period deposit rate is a weighted average (with weights $1/(1+\theta)$ and $\theta/(1+\theta)$) of the one-period rate and the forward rate.

where $S_2 = r_2 - i_2$ and $MC^*_{L_2} = (MC^1_{L_2} + \theta MC^2_{L_2})/(1 + \theta)$. Now, $MC^1_{L_2} = MC^2_{L_2}$ if marginal cost is stationary. Therefore, we have $S_2 = MC^1_{L_2}$ even if θ is not a constant.[9] The solutions for short- and long-term lending in this case are simply applications of equating marginal revenue and costs, where the marginal revenues represent spreads between the asset and liability term structures. Hence, the optimal policy is to arbitrage the segmentation between deposit and lending term structures in a manner consistent with Pringle's [13] viewpoint. The important point to be emphasized here, however, is that *both current lending decisions are made independently of the future distribution of interest rates and/or loan demand.* That is, the financing mix is immaterial to the lending decision.

Finally, we wish to consider how the random demand and cost factors influence lending decisions when separation does not occur. Our benchmark is the separable solution developed above. There are two effects to be considered. The first is the *expected* marginal profit effect of changing L_1 and/or L_2. The second is the marginal risk influence of these current decisions on second-period profits. We have the following *ceteris paribus* remarks, which apply to either L_1 or L_2. We first note that, by "direct" marginal costs and revenues, we mean those associated directly and contemporaneously with L_1 or L_2.

5. A risk-neutral intermediary with a separable cost function will expand loans beyond the equality of direct marginal revenue and costs if current and future lending are complements and vice versa if current and future lending are viewed as substitutes.

6. Under "competitive market conditions", risk-neutral intermediaries whose cost functions display intertemporal economies of scope (i.e., cost complementarities over time) will expand loans beyond the equality of direct marginal revenues and costs and vice versa if the cost function displays intertemporal diseconomies of scope.

7. The absolute deviation from the equality of direct marginal revenues and costs discussed in remarks 5 and 6 will be smaller for a risk-averse intermediary *vis à vis* a risk-neutral intermediary. In fact, the sign of the deviation for a risk-averse intermediary may be opposite to that of a risk-neutral firm.

Remark 7 points out that risk aversion may offset any customer relationship. A sufficiently large increase in risk associated with making more loans will result in fewer loans being made today despite the increase in expected profits that could be obtained.[10] This effect will be more noticeable when the level and spread between rates are *negatively* associated. In this case, the customer-relationship effect will mean making more loans next period, the profitability of which is positively associated with that on existing loans. This contrasts with Kane and Malkiel's [9] risk-aversion rationale for making more loans today in the face of a negative association between the level of interest rates and the spread. This difference is due, in part, to our consideration of long-term loans in the portfolio.

[9] Notice that θ will be a constant under various utility formulations. The risk-neutral case is obvious. However, this result will also hold if $U(P_1, P_2) = U(P_1 + \theta' P_2)$, $0 < \theta' \leq 1$. In this case, $\theta = \theta'$ for all choices of L_1, L_2, and f.

[10] Wood and Wood [19] have observed a decreasing empirical importance of the customer relationship in recent years. The model presented here suggests an alternative to the high interest rate variability explanation; the negative association between the level and spread of interest rates may have become stronger in recent years.

IV. Summary and Conclusions

In this paper, we develop a simple multiperiod model of the optimal maturity intermediation and lending decisions for financial intermediaries facing uncertain future interest rates and loan demand. We utilize the model to provide a solution for the optimal maturity intermediation (or "Gap") decision. We first consider the minimum-risk solution (that is, when the expected cost of hedging is zero) and show that, if high interest rate environments are associated with high short-term loan demand, a negative Gap (i.e., long-term assets greater than long-term liabilities) will be optimal since the intermediary wishes to hedge against low-profit states of the world. A positive Gap will be optimal if short-term loan demand is high when deposit rates are low. The traditional view that a zero-Gap position is risk minimizing holds only if loan demand and deposit rates are not associated over the business cycle—an unlikely situation from our viewpoint. These results can be contrasted with other models where risk aversion is assumed away and thus there is no incentive to diversify. We also discuss how the expected cost of hedging alters the optimal maturity mismatch decision and how intertemporally dependent loan demand and the intertemporal cost complementarities influence the size of the optimal Gap.

The lending problem is analyzed under two alternative scenarios. First, we consider environments under which the firm's optimal lending policies can be made separable from the optimal funding, or maturity intermediation, problem. That is, we provide conditions where the *actual decision concerning how loans are financed is immaterial for determining the value of long- and short-term loans.* Under separation, the appropriate "hurdle rate" for long-term lending decisions is the long-term deposit rate, regardless of what funding decisions are actually chosen. Second, we analyze the effect of intertemporally dependent demand and costs on current lending decisions when separation is not possible. We argue that positively related demand or negatively related costs cause risk-neutral intermediaries to expand current loans beyond the traditional equality of marginal revenue and costs, as earlier models would suggest. Risk aversion promotes a smaller deviation from marginal revenue/cost equality and, in some cases, may actually reverse the results since risk is also increasing as loans are expanded.

There are also a number of interesting empirical implications of the model. In addition to the obvious hypothesis that institutions where interest rates and loan demand are more positively associated will carry more negative Gaps, our results suggest that, to a first-order approximation, Gap positions and the degree of economies of scope should be negatively associated. While current empirical studies in the scope literature (i.e., Gilligan, Smirlock, and Marshall [6] and Murray and White [11]) do not investigate the relationship between size and the degree of scope economies, we conjecture that they should be positively associated. This implies that asset and Gap sizes should be negatively associated. Furthermore, our results suggest that banks that have higher degrees of "repeat business" (a proxy for the degree of customer relationship) should have more negative Gaps. Finally, due to intertemporal substitution effects, we would argue that, on a percentage basis, institutions with a higher level of long-term loans should have larger absolute Gaps, other things the same.

We have focused solely on interest rate risk and have said nothing concerning the important role of credit risk. Our feeling is that credit risk will have an asymmetric influence on the maturity intermediation decisions of intermediaries. For institutions where high interest rates correspond to lower profits, credit risk will tend to reduce its demand for long-term borrowing. Moreover, for firms where low-rate environments result in lower profits, credit risk may also cause the demand for long-term borrowing to be higher. However, a complete multi-period analysis of investment and financing decisions with credit and interest rate risk awaits future research efforts.

Appendix

Proof of Lemma: Note that, from (4c),

$$\mathrm{cov}\{U_2', I\} = \mathrm{E}_l[\mathrm{cov}(U_2', I) \mid l = L] + \mathrm{cov}_l\{\mathrm{E}[U_2' \mid l = L],\quad \mathrm{E}[I \mid l = L]\}. \tag{A1}$$

Because the first-order condition for l gives the deterministic relationship between R, I, and l, any pair of the three random variables can be employed to summarize all the uncertainty in (4c). Substituting for $R - I$ (using equation (3)) into (1b), at $f = L_2^*$ we have

$$P_2^* = -\frac{\partial R}{\partial l} l^2 + \frac{\partial C_2}{\partial l} l - C_2 + (r_2 - i_f)L_2^*. \tag{A2}$$

With additive uncertainty in R, P_2^* is random only because of the presence of l, and, therefore, the first term on the right-hand side of (A1) is zero. Assuming decreasing marginal revenue and increasing marginal cost, P_2^* is increasing in l. Given risk aversion ($U_2'' < 0$), the second term in (A1) is negative if and only if I and l are positively associated in the sense that $d\mathrm{E}[I \mid l = L]/dL > 0$. The opposite is also true. Q.E.D.

Proof of Proposition 2: The proof requires that the optimal values of L_1 and L_2 be independent of the distributions of the random variables in period 2. Under the conditions of the proposition, the right-hand side of (4a) is zero. Furthermore, $MR_{L_1}^1 = r_1$. Therefore,

$$r_1 - i_1 = S_1 = MC_{L_1}^1 \tag{A3}$$

is the optimality condition for short-term lending. Moreover, the right-hand side of (4b) is given by

$$-\theta\left(r_2 - \mathrm{E}[I] - MC_{L_2}^2 - \frac{\mathrm{cov}(U_2', I)}{\mathrm{E}[U_2']}\right).$$

Multiplying (4c) by θ and subtracting it from (4b) yields (after noting that $MR_{L_2}^1 = r_2$)

$$r_2(1 + \theta) - i_1 - \theta i_f = MC_{L_2}^1 + \theta MC_{L_2}^2, \tag{A4}$$

which is independent of θ (and therefore of the covariance of U_2' and I) if $MC_{L_2}^1 = MC_{L_2}^2$. If the latter does not hold, θ must be a constant because it would otherwise generally depend on the distribution of the random variables in period 2 and therefore on the optimal value of f.

REFERENCES

1. E. Baltensperger. "Alternative Approaches to the Theory of the Banking Firm." *Journal of Monetary Economics* 6 (January 1980), 1–37.
2. W. Baumol. "On the Proper Tests for Natural Monopoly in a Multiproduct Industry." *American Economic Review* 67 (December 1977), 809–22.
3. T. Campbell. "A Model of the Market for Lines of Credit." *Journal of Finance* 33 (March 1978), 231–44.
4. S. Deshmukh, S. Greenbaum, and G. Kanatas. "Bank Forward Lending in Alternative Funding Environments." *Journal of Finance* 37 (September 1982), 925–40.
5. G. Feder, J. Just, and S. Schmitz. "Futures Markets and the Theory of the Firm under Price Uncertainty." *Quarterly Journal of Economics* 94 (March 1980), 317–28.
6. T. Gilligan, M. Smirlock, and W. Marshall. "Scale and Scope Economies in the Multi-Product Banking Firm." *Journal of Monetary Economics* 13 (May 1984), 393–405.
7. D. Hodgeman. "The Deposit Relationship and Commercial Bank Behavior." *Review of Economics and Statistics* 43 (August 1961), 257–68.
8. D. M. Holthausen. "Hedging and the Competitive Firm under Price Uncertainty." *American Economic Review* 69 (December 1979), 989–95.
9. E. Kane and B. Malkiel. "Bank Portfolio Allocation, Deposit Variability, and the Availability Doctrine." *Quarterly Journal of Economics* 79 (February 1965), 113–34.
10. G. Kaufman. "The Thrift Institution Problem Reconsidered." *Journal of Bank Research* 3 (Spring 1972), 26–32.
11. J. Murray and R. White. "Economies of Scale and Scope in Multi-Product Financial Institutions: A Study of British Columbia Credit Unions." *Journal of Finance* 38 (June 1983), 887–902.
12. J. Niehans and J. Hewson. "The Eurodollar Market and Monetary Theory." *Journal of Money Credit and Banking* 8 (February 1976), 1–27.
13. J. J. Pringle. "Imperfect Markets Model of Commercial Bank Financial Management." *Journal of Financial and Quantitative Analysis* 9 (January 1974), 69–87.
14. J. Ricart i Costa and S. Greenbaum. "Bank Forward Lending: A Note." *Journal of Finance* 38 (September 1983), 1315–23.
15. A. Santomero. "Fixed Versus Variable Rate Loans." *Journal of Finance* 38 (December 1983), 1363–80.
16. ———. "Modeling the Banking Firm: A Survey." *Journal of Money Credit and Banking* 16 (November 1984, Part 2), 576–602.
17. C. Sealey and J. Lindley. "Inputs, Outputs, and the Theory of Production and Cost at Depository Institutions." *Journal of Finance* 32 (September 1977), 1251–66.
18. J. Wood. *Commercial Bank Loan and Investment Behavior.* London: Wiley, 1975.
19. ——— and N. Wood. *Financial Markets.* San Diego: Harcourt, Brace, Jovanovich, 1985.