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Costless Signalling in Financial Markets

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ABSTRACT

A costless, fully revealing signalling equilibrium is derived from two easily understandable conditions. The outsider-rationality condition states that the outsiders relate the price that they offer to pay for a security inversely to the supply of this security, which they interpret as a quality signal. The no-arbitrage condition requires that the marginal exchange rate for two securities be the same in both primary and secondary markets. These conditions restrict the firm's financing policy and have strong implications for the valuation of securities and of the total firm. A costless signalling equilibrium is obtained.

THERE IS LITTLE REASON to believe that information is distributed evenly across economic agents. Managers of a firm receive information on the firm's profitability earlier than outside investors. This creates a danger of the latter being exploited by the insiders through changes in the firm's financing policy or through trade of securities in the securities market. Whenever the firm's securities are mispriced, the insiders can sell overpriced and buy underpriced securities in the secondary securities market. Similarly, a change in the firm's financing policy means a substitution of potentially overpriced against potentially underpriced securities, i.e., a trade in the primary securities market.

The information asymmetry is removed if the outsiders observe insiders' decisions from which the insiders' information can be inferred. Two concepts of signalling inside information have been suggested, the first being the costly-signalling equilibrium as discussed by Spence [12], Leland and Pyle [7], Miller and Rock [8], Riley [10], Ross [11], and Talmor [13], the second being the costless-signalling equilibrium as proposed by Bhattacharya [2] and Heinkel [6] and further developed by Brennan and Kraus [3]. Signalling is costly if the production of the signal consumes resources or if the signal is associated with a loss in welfare generated by deviations from allocation and/or distribution of claims in perfect markets. Both effects are required to be absent in costless-signalling models. Costless-signalling equilibria produce first-best solutions, while costly-signalling equilibria produce second-best solutions. This paper deals with costless-signalling equilibria.

The basic idea of signalling in financial markets is that the insider managers of a firm signal their inside information by their choice of the firm's investment and financing policy. A fully revealing signalling equilibrium obtains if (a) demand equals supply for every security as in the Walrasian equilibrium, (b) the

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insiders prefer to signal their information honestly, and (c) the outsiders can infer the insiders' values of all securities unambiguously from observing their decisions. Then insiders cannot exploit the outsiders by choice of the firms' policies or by trade in the secondary market.

Bhattacharya, Heinkel, and Brennan and Kraus derive necessary properties of a costless-signalling equilibrium in two steps. (a) The insider manager of a firm acts in the interest of the old securityholders. Therefore, he or she minimizes the "true" wealth of the new outsider investors to whom he or she sells securities to raise money. (b) Necessary conditions on the outsiders' behavior required by a fully revealing signalling equilibrium are derived. The intuition behind the second step is not clear in their analysis. Therefore, the purpose of this paper is to derive a costless-signalling equilibrium in financial markets from easily understandable conditions.

The first condition is the outsider-rationality condition. It states that the price that the outsiders offer to pay for a security is negatively related to the supply of this security. The supply is perceived by the outsiders as a quality signal. A larger supply is interpreted as a signal of lower quality so that the outsiders lower their offer price. The outsider-rationality condition has nothing to do with attitudes toward risk, although risk aversion may produce a similar result. The second condition is a no-arbitrage condition. It states that the marginal exchange rates for two securities must be the same in both the primary and secondary markets.

The outsider-rationality and the no-arbitrage conditions will be used to analyze "basic financing policies." These are defined as follows. Suppose that no information asymmetry exists, but the firm's financing policy depends on its "true" total value. Then a marginal change in this value induces a marginal change in its financing policy. If the firm issues only two types of securities, then a change in its financing policy means that the outsider investors receive more securities of type 1 and less securities of type 2. The associated marginal exchange rate between these securities depends, in general, on their "true" values and the marginal changes in these values. A basic financing policy is defined such that the marginal exchange rate between both securities equals the ratio of their "true" values. Hence, it is independent of marginal changes in their "true" values. The marginal exchange rate, therefore, is the same in both primary and secondary markets. The no-arbitrage condition requires that this be true even if information asymmetries exist.

The paper shows that a costless-signalling equilibrium in financial markets can be derived for basic financing policies from the outsider-rationality and the no-arbitrage conditions. If more than two types of securities are required for signalling, their valuation functions must exhibit *constant* negative elasticity with respect to the signal. This leads to a simple rule for determining the optimal financing policy. Pure equity financing is optimal only for a firm with the lowest possible "true" total value. Finally, the paper shows that, in general, a costless-signalling equilibrium does not exist for nonbasic financing policies. The exception to this statement is provided by a situation in which there exists an equivalent basic financing policy for every nonbasic financing policy.

The paper is organized as follows. First the economic setting and then the outsider-rationality and the no-arbitrage conditions will be presented. In Section

IV, the costless-signalling equilibrium will be derived. Section V discusses the relation between total firm value and the optimal financing policy. In Section VI, it will be shown that a signalling equilibrium cannot be obtained for nonbasic financing policies unless there exists an equivalent basic financing policy for every nonbasic financing policy.

I. The Economic Setting

Assume that a firm needs currently (at date 0) the amount of money I for some purpose. The old securityholders (i.e., old investors) do not supply the money; therefore, new investors provide the money in exchange for risky securities issued by the firm.[1] For simplicity of exposition, all investors are assumed to be risk neutral. In addition, the risk-free interest rate is assumed to be zero.[2] Hence, the market value of every security equals its expected next-date market value including dividends, etc.[3] The securities market is assumed to be perfect and competitive subject to the following qualifications: (a) an information asymmetry may exist; (b) the old investors do not supply the needed money; (c) insider trading in the secondary market is restricted so that it has no appreciable impact on securities' prices; and (d) trading in the secondary market is anonymous; i.e., it is unknown whether an order is given by an insider or an outsider.

Suppose the firm's investment policy is given. The probability distribution of its date-1 market value depends on a vector of parameters, $P \equiv (p_1, \cdots, p_j, \cdots, p_J)$, that is unknown to the old and new investors. They are outsiders. Those vectors that are regarded by the outside investors as possibly "true" are the elements of the set π. These investors derive the probability distribution of the firm's market value from the parameter vector $\bar{P}$. $\bar{P}$ could be a convex combination of all $P \in \pi$, for instance. In any case, $\bar{P} \in \pi$. π is assumed to be compact. The firm's managers have inside information on the parameter vector; their vector is denoted by P^*, which is also an element of π. If a manager is also an investor, his or her insider trading has no appreciable impact on pricing in the competitive secondary market. Define

v_0, v_1 = the firm's total market value at date 0 and date 1, respectively;
$f(v_1; P)$ = the probability density function of v_1, derived from the parameter vector P;
$F(v_1; P)$ = the cumulative probability density function.

"$*$" always indicates the managers' expectations. For notational simplicity, the managers' probability beliefs and value estimates will be called "true" beliefs and "true" values, although this does not mean that the managers have perfect information on P. It only means that they have superior information. The

[1] The economic setting resembles that of Myers and Majluf [9]. They analyze investment decisions under asymmetric information.

[2] These assumptions can be removed easily in a Pareto-efficient market. See Brennan and Kraus [3, p. 43].

[3] This might be questionable if adverse selection exists. However, the economic setting and the conditions of this paper rule out adverse selection, as will be shown.

probability density functions are assumed to be differentiable and integrable with respect to v_1 and twice differentiable with respect to every parameter p_j.

With limited liability, the outsiders' estimate of the firm's market value is $v_0 = v_0(\bar{P}) = \int_0^{+\infty} v_1 f(v_1; \bar{P})\, dv_1$; the insider managers' estimate v_0^* is derived from $f(v_1; P^*)$.

As the outsiders do not know the "true" parameters, they try to infer them from the firm's financing policy. Since the amount of money I needed by the firm is given, the outsiders can infer the J "true" parameters only if the managers have at least J degrees of freedom (after correction for any restrictions to be observed) in choosing the financing policy.[4] This policy is modeled here in a simple manner. There exist $(J + 1)$ different types of securities indexed $i = 1, 2, \cdots, J + 1$. v_{0i} and v_{1i} denote the date-0 and the random date-1 market value of all securities of type i. The distribution of the total market value v_1 across security types is assumed to be independent of the financing policy. In other words, the sharing rule that determines the functions $v_{1i} = v_{1i}(v_1)$, subject to $\sum_i v_{1i} = v_1$, is assumed to be independent of the financing policy. As a consequence, the insider managers' estimates v_{0i}^* are independent of the financing policy. With limited liability and possibly positive payoffs, $v_{0i} > 0$ for every security type i.

The managers have to choose the financing policy; i.e., they choose for each type of security i the fraction α_i that the new investors receive for paying the amount I. As a consequence, the old investors receive the fraction $(1 - \alpha_i)$.

The firm's managers are assumed to maximize the "true" wealth of the old investors, which is the same as minimizing the "true" wealth of the new investors. Let $\alpha \equiv (\alpha_1, \cdots, \alpha_{J+1})$ denote the firm's financing policy. Then the managers face the following decision problem:

$$\min_\alpha \sum_i \alpha_i v_{0i}^*$$

subject to

$$\sum_i \alpha_i v_{0i}[\bar{P}(\alpha)] = I. \tag{1}$$

The constraint states that, in a competitive capital market, the market value of the securities that the new investors get must equal the money they pay. A change of the financing policy means that the new investors receive a smaller fraction of some payoffs (securities) in return for a larger fraction of other payoffs (securities). Such a change is interpreted as a signal by the outsiders. Therefore, their parameter vector $\bar{P}$ depends on α so that v_{0i} does, while the "true" value v_{0i}^* does not.

II. The Outsider-Rationality Condition

Whether a signalling equilibrium exists or not depends on the behavior of the new investors. First, suppose that the new investors do *not* consider the financing

[4] The outsiders are interested in the "true" values of the securities that they buy. Hence, if two different parameter vectors imply the same "true" values of all securities, then the outsiders are not interested in distinguishing between both vectors. Therefore, a fully revealing signalling equilibrium has been defined to reveal the "true" values of all securities.

policy as a signal. Then v_{0i} ($i = 1, \cdots, J + 1$) is independent of the financing policy. The optimal solution to the managers' minimization problem would be to sell as many overpriced securities to the new investors as possible.[5] This would imply adverse selection and force a loss on the new investors. The new investors would anticipate this and refuse to buy any securities. The market would break down (Akerlof [1]).

Therefore, suppose that the new investors regard the financing policy α as a quality signal such that the market value of type-i securities depends on α; $v_{0i} = v_{0i}(\bar{P}(\alpha))$, $\forall\ i$. More precisely, the price that the new investors are ready to pay for such a security depends on the financing policy α. This offer price equals the market price if the managers choose the financing policy α.

The function $v_{0i} = v_{0i}(\bar{P}(\alpha))$ might be complicated if α is a vector with many components. In order to retain simplicity, assume that v_{0i} depends on α_i but not on α_h ($h \neq i$). Then define $\beta_i \equiv \alpha_i v_{0i}[\bar{P}(\alpha_i)]/I$. If there exists a one-to-one correspondence between α_i and β_i, v_{0i} can be viewed equivalently as a function of β_i, $v_{0i} = v_{0i}(\beta_i)$, $\forall\ i$.

β_i is the fraction of the amount of money I to be raised through selling securities of type i. Therefore, β_i can be interpreted as the firm's relative monetary supply of type-i securities. The constraint $\sum_i \alpha_i v_{0i} = I$ translates to $\sum_i \beta_i = 1$. The firm's financing policy can be defined by α as well as by $\beta \equiv (\beta_1, \cdots, \beta_{J+1})$.

$v_{0i} = v_{0i}(\beta_i)$ means that the new investors regard the relative monetary supply β_i as a signal of the quality of security type i. Therefore, the probability density $f(v_{1i}; \bar{P})$ depends on β_i only; $f(v_{1i}; \bar{P}) = \hat{f}(v_{1i}; \beta_i)$.

Now suppose that v_{0i} strictly increases in β_i. This means that the new investors offer a higher price for a security of type i if the managers raise the monetary supply of that security. Hence, increasing the supply of overpriced securities would enlarge overpricing and thus reinforce adverse selection. Therefore, v_{0i} must be negatively related to β_i if adverse selection is to be avoided.[6] This is the essence of the outsider-rationality condition. In order to state this condition precisely, define $\beta_i \in [\underline{\beta}, \bar{\beta}]$ $\forall\ i$, with $\underline{\beta}$ and $\bar{\beta}$ being given finite numbers. In addition, the upper and lower limits of v_{0i} have to be chosen such that every possible "true" value of v_{0i} is encompassed. More precisely, the upper limit $\bar{v}_{0i}$ is defined by

$$\bar{v}_{0i} \geq \sup_{P \in \pi} v_{0i}(P);$$

the lower limit $\underline{v}_{0i}$ is defined by

$$\underline{v}_{0i} \leq \inf_{P \in \pi} v_{0i}(P).$$

Outsider-rationality condition (*ORC*): The outsiders regard an increase in the relative monetary supply of type-i securities, β_i, as a signal of lower quality of these securities and therefore reduce their price offer v_{0i}. For $\beta_i \to \underline{\beta}$, $v_{0i}(\beta_i) \to \bar{v}_{0i}$, and for $\beta_i \to \bar{\beta}$, $v_{0i}(\beta_i) \to \underline{v}_{0i}$.

[5] The managers might face a risk of being accused of fraud if they sell overpriced securities. Overpricing, however, is difficult to prove since valuation depends heavily on subjective expectations.

[6] Similarly, a specialist relates the bid-ask spread to the size of the transaction in order to protect himself or herself against losses by insider trading. See Copeland and Galai [4] and Glosten and Milgrom [5].

The outsider-rationality condition provides the outsiders with some protection against adverse selection. However, it is not sufficient for achieving a fully revealing signalling equilibrium, as will be shown now. Such an equilibrium requires that the optimal financing policy α^+ depend on the parameter vector P^*, $\alpha^+ = \alpha^+(P^*)$. Then, in equilibrium, $\sum_i \alpha_i^+(P^*)v_{0i}(P^*) = I$ must be true. Differentiate this condition with respect to P^* and obtain

$$\sum_i \left[\frac{d\alpha_i^+}{dP^*} v_{0i}(P^*) + \alpha_i^+(P^*) \frac{dv_{0i}}{dP^*}\right] = 0.$$

If $d\alpha_i^+/dP^* = 0$ for every i except h and k, then the marginal exchange rate for securities h and k equals

$$-\frac{d\alpha_h^+}{d\alpha_k^+} = -\frac{d\alpha_h^+/dP^*}{d\alpha_k^+/dP^*} = \frac{v_{0k}(P^*)}{v_{0h}(P^*)} \times \frac{1 + \dfrac{\alpha_k^+(P^*)}{v_{0k}(P^*)} \dfrac{dv_{0k}/dP^*}{d\alpha_k^+/dP^*}}{1 + \dfrac{\alpha_h^+(P^*)}{v_{0h}(P^*)} \dfrac{dv_{0h}/dP^*}{d\alpha_h^+/dP^*}}. \tag{2}$$

Consider the second fraction on the right-hand side of equation (2). This fraction will be called the "marginal parameter factor." Its size depends on dP^* and on the financing policy, defined by $\alpha_i^+(P^*)$ and $v_{0i}(P^*)$, subject to $\sum_i \alpha_i^+(P^*)v_{0i}(P^*) = I$ and $\sum_i v_{0i}(P^*) = v_0(P^*)$. A "*basic financing policy*" is defined as a policy such that $\sum_i \alpha_i^+(P^*)v_{0i}(P^*) = I$ and $\sum_i v_{0i}(P^*) = v_0(P^*)$ $\forall$ $P^* \in \pi$, and the marginal parameter factor equals 1 $\forall$ $P^* \in \pi$ and $\forall$ (h, k). The securities, being issued under a "basic financing policy," are called "basic securities."

Hence, by definition, the marginal exchange rate of any pair (h, k) of basic securities, $-d\alpha_h^+/d\alpha_k^+$, equals $v_{0k}(P^*)/v_{0h}(P^*)$. The following analysis, except for Section VI, is restricted to basic financing policies.

Now we return to the question of whether the outsider-rationality condition assures a fully revealing signalling equilibrium under asymmetric information. Consider the managers' minimization problem (1). If λ denotes the Lagrange multiplier, then necessary conditions for a minimum are

$$v_{0i}^* = \lambda\left[v_{0i}(\beta_i^+) + \alpha_i^+ \frac{dv_{0i}}{d\beta_i} \times \frac{d\beta_i}{d\alpha_i}\right], \quad \forall\ i. \tag{3}$$

These conditions can be simplified by using the elasticity $\epsilon_i(\beta_i)$ of the market value v_{0i} with respect to β_i; i.e., $\epsilon_i(\beta_i) \equiv (dv_{0i}/d\beta_i)(\beta_i/v_{0i})$. Differentiate β_i with respect to α_i and obtain

$$\frac{d\beta_i}{d\alpha_i} = \frac{v_{0i}(\beta_i)}{I} + \frac{\alpha_i}{I} \times \frac{dv_{0i}}{d\beta_i} \times \frac{d\beta_i}{d\alpha_i} = \frac{v_{0i}(\beta_i)}{I}[1 - \epsilon_i(\beta_i)]^{-1}. \tag{4}$$

Insert this into equation (3) and obtain, after a few manipulations,

$$v_{0i}^* = \lambda v_{0i}(\beta_i^+)[1 - \epsilon_i(\beta_i^+)]^{-1}; \quad \forall\ i. \tag{5}$$

Divide the type-k condition by the type-h condition and obtain

$$\frac{v_{0k}^*}{v_{0h}^*} = \frac{v_{0k}(\beta_k^+)}{v_{0h}(\beta_h^+)} \times \frac{1 - \epsilon_h(\beta_h^+)}{1 - \epsilon_k(\beta_k^+)}; \quad \forall\ (h, k). \tag{6}$$

Hence, the optimal financing policy implies that the managers' marginal exchange rate v_{0k}^*/v_{0h}^* equals that of the outsiders. The latter is the term on the right-hand side of equation (6); it equals the value relative to $v_{0k}(\beta_k^+)/v_{0h}(\beta_h^+)$ times a fraction that is called the "marginal signalling factor." It is the pendant to the marginal parameter factor in equation (2). For basic securities, the latter factor equals 1. Hence, β^+ can signal P^* correctly only if the marginal signalling factor equals 1, too. In other words, if the "true" marginal exchange rate $-(d\alpha_h^+/dP^*)/(d\alpha_k^+/dP^*)$, defined in equation (2), depends on P^* but not on dP^*, then β^+ can signal P^* correctly only if the outsiders' marginal exchange rate depends on β^+ but not on $d\beta^+$. The latter is not implied by the outsider-rationality condition. Therefore, this condition is not sufficient for a fully revealing signalling equilibrium. The additional condition that the marginal signalling factor be 1 for basic securities could be introduced simply as a requirement of informationally consistent signalling. However, perhaps this is difficult to understand. Therefore, the condition will be expressed as a no-arbitrage condition.

III. The No-Arbitrage Condition

So far, only the primary market has been considered, i.e., the choice of the firm's financing policy. The firm's securities are also traded in a secondary market. This market is anonymous, which means that an investor does not know whether the other investors trading in the secondary market are insiders or outsiders. Anonymity exists only if insider trading is not legally prohibited and if insiders are not obliged to announce trades immediately.

As the secondary market is anonymous, trades in this market do not signal inside information. Hence, given the financing policy of the firm, the marginal exchange rate between type-k and type-h securities equals $v_{0k}(\beta_k^+)/v_{0h}(\beta_h^+)$ in the secondary market. It deviates from the marginal exchange rate in the primary market if the marginal signalling factor does not equal 1. The existence of two different marginal exchange rates creates profitable arbitrage opportunities.

Suppose, for instance, that the marginal exchange rate is $v_{0k}^*/v_{0h}^* = 1$ in the primary market and $v_{0k}/v_{0h} = 0.8$ in the secondary market. The difference between these rates is caused by the marginal signalling factor being different from 1. Moreover, assume that a manager owns the fraction γ of all securities issued previously by the firm. Then he or she can change the financing policy by raising an additional dollar through selling additional type-k securities to the new investors and raising one dollar less by selling less type-h securities. This means that the old investors sell type-k securities against type-h securities in the primary market to the new investors. At the same time, the manager buys type-k securities for 0.8 γ dollars and sells type-h securities for γ dollars in the secondary market. Hence, the manager's stake in the firm remains unchanged, but he or she earns an arbitrage profit of 0.2 γ dollars.

Even if the managers did not own any securities but acted in the interest of the old investors, they could create an arbitrage profit for the old investors by changing the firm's financing policy and informing them secretly about their appropriate reaction in the secondary market.

Generally speaking, the existence of two different marginal exchange rates creates for the new investors the danger of arbitrage losses. Hence, the new investors can buy securities safely only if both marginal exchange rates are the same. This motivates the no-arbitrage condition.

No-arbitrage condition (NAC): The marginal exchange rates for any pair of securities are required to be the same in the primary and secondary markets.

Before turning to the implications of the ORC and the NAC, the assumptions of this paper will be briefly compared with those underlying the costly-signalling equilibrium as stated in Riley [10, p. 334f]. In both papers, insiders choose the signal to maximize their utility; the outsiders acting as price takers make their price offer a monotonically increasing or decreasing function of the signal (Riley's assumption (4); our ORC).

The main difference between both papers is revealed by Riley's assumption (5) versus the NAC. Riley's assumption (5) starts from the premise that a change in the signal changes the insider's utility even if outsiders do not react. Therefore, the signal is costly. Assumption (5) states that the cost of signalling a certain quality is lower, the higher the actual quality. This makes dishonest signalling unattractive. In our paper, a change in the signal alone does not affect the insider's utility as the signal is costless. The NAC, however, constrains the valuation of basic securities by outsiders such that the managers can do no better than signal honestly. This will be shown in the following section.

IV. Derivation of the Costless-Signalling Equilibrium

The NAC constrains the marginal signalling factor to be 1 for every pair of basic securities. This has quite strong implications, as revealed by Proposition 1.

PROPOSITION 1: *Suppose that the outsider-rationality condition (ORC) holds. Then the no-arbitrage condition (NAC) implies for the basic securities* $\alpha_i > 0$ *and* $\beta_i \in (0, 1)\ \forall\ i$, *a one-to-one correspondence between* α_i *and* β_i, *and*

$$\epsilon_1(\beta_1) = \epsilon_2(1 - \beta_1) < 0\ \forall\ \beta_1 \in (0, 1) \tag{7}$$

if only two types of securities exist;

$\epsilon_i(\beta_i) = c < 0$ *for every type of security if more than two types of securities exist.*

Proof: The NAC requires that the marginal signalling factor be 1 for every pair of basic securities (h, k). Hence, $\epsilon_h(\beta_h) = \epsilon_k(\beta_k)$ is required for every pair (h, k), or, equivalently,

$$\frac{dv_{0h}}{d\beta_h}\frac{\beta_h}{v_{0h}(\beta_h)} = \frac{dv_{0k}}{d\beta_k}\frac{\beta_k}{v_{0k}(\beta_k)}\ \forall\ (h, k).$$

As $dv_{0i}/d\beta_i < 0$ by the ORC and $v_{0i} > 0\ \forall\ i$, all β's must be of the same sign. $\sum_i \beta_i = 1$, then, implies that $\beta_i \in (0, 1)$, $\forall\ i$. As $\beta_i = \alpha_i v_{0i}(\beta_i)/I$, $\alpha_i > 0\ \forall\ i$ follows.

$\beta_i \in (0, 1)$ and $dv_{0i}/d\beta_i < 0$ imply that $\epsilon_i(\beta_i) < 0$, $\forall\ i$. Hence, by equation (4), $d\beta_i/d\alpha_i > 0$ so that a one-to-one correspondence between α_i and β_i exists.

If only two types of securities exist, then $\beta_1 = 1 - \beta_2$. Hence, the marginal signalling factor equals 1 only if

$$\epsilon_1(\beta_1) = \epsilon_2(1 - \beta_1)\ \forall\ \beta_1 \in (0, 1).$$

If more than two types of securities exist, then $\epsilon_h(\beta_h) = \epsilon_k(\beta_k)$ can be true for every feasible financing policy and every pair (h, k) only if $\epsilon_i(\beta_i)$ equals some constant c, $\forall\ i$. Q.E.D.

Proposition 1 states, among other things, that the new investors necessarily receive a positive fraction of every type of basic security. Otherwise, the marginal signalling factors could not be equal to 1 everywhere.
In addition, the valuation functions $v_{0i}(\beta_i)$ must have constant negative elasticity of magnitude c if more than two security types exist. This and $v_{0i}(\beta_i) \rightarrow \underline{v}_{0i}$ for $\beta_i \rightarrow 1$ imply that

$$v_{0i}(\beta_i) = \beta_i^c \underline{v}_{0i}; \quad c < 0; \quad \forall\ i. \tag{8}$$

From $\sum_i \beta_i = 1$, then, it follows for the basic securities,

$$\textstyle\sum_i [v_{0i}(\beta_i)/\underline{v}_{0i}]^{1/c} = 1, \quad \forall\ \beta. \tag{9}$$

As $v_{0i}(\beta_i)$ represents an abbreviation of $v_{0i}[P(\beta)]$, Condition (9) can be true only if

$$\textstyle\sum_i [v_{0i}(P)/\underline{v}_{0i}]^{1/c} = 1; \quad \forall\ P \in \pi. \tag{10}$$

Proposition 2 derives the optimal financing policy and shows that the ORC and NAC are sufficient to generate a fully revealing, costless-signalling equilibrium.

Proposition 2: *Suppose the ORC and the NAC hold.*

a. If there exist more than two types of basic securities, then the manager's optimal financing policy is given by

$$\beta_i^+ = [v_{0i}^*/\underline{v}_{0i}]^{1/c}, \ \forall\ i. \tag{11}$$

If there exist only two types, then the optimal financing policy is determined by equation (12):

$$v_{01}(\beta_1^+) = \frac{v_{01}^*}{v_{02}^*}\, v_{02}(1 - \beta_1^+). \tag{12}$$

b. A fully revealing, costless-signalling equilibrium obtains.

Proof:

a. Suppose $J > 1$. In order to derive the optimal financing policy, consider the managers' minimization problem;

$$\min_\alpha \textstyle\sum_i \alpha_i v_{0i}^*, \quad \text{subject to} \quad \sum_i \beta_i = 1 \quad \text{and} \quad \beta_i \geq 0\ \forall\ i.$$

From equation (8) and the definition of β_i follows

$$\alpha_i = \beta_i I/v_{0i}(\beta_i) = I\beta_i^{1-c}/\underline{v}_{0i}. \tag{13}$$

Insert this in the objective function, ignore I, and obtain

$$\min_\beta \textstyle\sum_i (v_{0i}^*/\underline{v}_{0i})\beta_i^{1-c}, \quad \text{subject to} \quad \sum_i \beta_i = 1 \quad \text{and} \quad \beta_i \geq 0\ \forall\ i.$$

As $(v_{0i}^*/\underline{v}_{0i}) > 0$ and $c < 0$, this is a strictly convex minimization problem with the unique solution

$$\beta_i^+ = [v_{0i}^*/\underline{v}_{0i}]^{1/c}/ \textstyle\sum_k [v_{0k}^*/\underline{v}_{0k}]^{1/c}, \ \forall\ i. \tag{14}$$

Equation (10) implies for $P = P^*$ that the denominator on the right-hand side of equation (14) equals 1. Hence, equation (11) follows.

The outsiders observe $\alpha_i^+ \ \forall\ i$. By equation (13), they infer β_i^+ and then v_{0i}^* from β_i^+ by equation (11), $\forall\ i$. Therefore, the equilibrium is fully revealing.

b. Suppose $J = 1$. Then $\epsilon_1(\beta_1) = \epsilon_2(1 - \beta_1)$ (Proposition 1) so that, by equation (6), $v_{01}^*/v_{02}^* = v_{01}(\beta_1^+)/v_{02}(1 - \beta_1^+)$. As v_{01}/v_{02} is strictly decreasing in β_1, this solution to the managers' problem is a unique minimum.

$d\alpha_1/d\beta_1 > 0,\ \forall\ \beta_1 \in (0, 1)$; therefore, knowledge of α_1^+ enables the outsiders to infer β_1^+ unambiguously and then v_{01}^* and v_{02}^* from the functions $v_{01}(\beta_1)$ and $v_{02}(\beta_2)$. Q.E.D.

Proposition 2 is the central message of the paper. First, it tells the managers how to derive the optimal basic financing policy. Second, it proves that the ORC and the NAC are sufficient to generate a fully revealing, costless-signalling equilibrium. These conditions are easily understandable. In contrast to Riley's costly-signalling equilibrium, no problem of unraveling exists. In addition, the number of unknown parameters is not restricted to one.

A related characterization of a costless, fully revealing signalling equilibrium can be easily derived from the preceding analysis. Suppose two firms of types 1 and 2 with different parameter vectors P_1^* and P_2^* need the same amount of money I. Each firm minimizes the "true" value of the new investors' securities. The minimal value equals the amount I in a fully revealing equilibrium. Hence, the "true" value of these securities must be smaller for firm 1 than for firm 2 at β^1, the optimal financing policy of firm 1. Similarly, the "true" value of these securities must be smaller for firm 2 than for firm 1 at β^2, the optimal financing policy of firm 2. This is illustrated in figure 1, assuming that $\beta^1 < \beta^2$.

This reproduces an important result of Bhattacharya and Brennan and Kraus. In a fully revealing costless-signalling equilibrium, the securities, sold by a firm to new stockholders, are priced on the correct supposition that they have the lowest possible "true" value. In other words, when the outsiders are offered new securities by a firm, they check for every type of firm what the "true" value of these securities would be if they were issued by that type of firm. Then they buy the securities on the supposition that they are issued by that type of firm such that they have the lowest possible "true" value. This supposition is correct in equilibrium. Therefore, it is impossible for the managers to gain anything by cheating, i.e., by signalling a parameter vector other than the "true" one.

V. Total Firm Value and Financing Policy

Proposition 2 has derived the firm's optimal basic financing policy. From equation (11), it follows that β_i^+ is the lowest for the type of security with the highest ratio v_{0i}^*/v_{0i}, and vice versa. This is not surprising, given the ORC. More interesting is the relation between the firm's total value and the optimal financing policy as revealed by Proposition 3.

PROPOSITION 3: *Assume that the ORC and the NAC hold. Then the managers of a firm with the lowest possible "true" total value sell the same fraction α^+ of all securities to the new investors.*

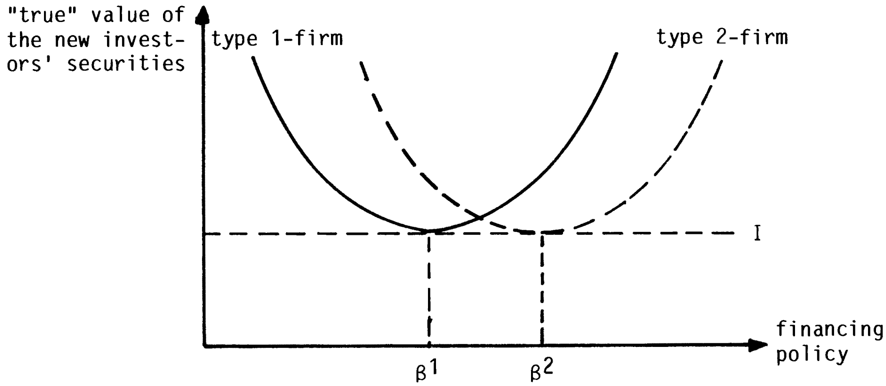

Figure 1. The "true" values of the new investors' securities are minimal at the optimal financing policies.

Proof: The "true" total value of a firm is $v_0(\beta^+) = \sum_i v_{0i}(\beta_i^+)$ in a costless-signalling equilibrium. It has to be shown that this value is minimal if $\alpha_1^+ = \cdots = \alpha_{J+1}^+$. Consider the problem $\min_\beta \sum_i v_{0i}(\beta_i)$, subject to $\sum_i \beta_i = 1$, with $v_{0i}(\beta_i)$ having the properties required by Proposition 1.

Necessary conditions for a minimum are

$$\frac{dv_{0i}}{d\beta_i} = \lambda, \quad \forall\ i.$$

Hence,

$$dv_{0i}/d\beta_i = dv_{01}/d\beta_1 \ \forall\ i, \quad \text{or}$$

$$\frac{\epsilon_i(\beta_i^+)}{\alpha_i^+} = \frac{\epsilon_1(\beta_1^+)}{\alpha_1^+}, \quad \forall\ i.$$

By Proposition 1, $\epsilon_i(\beta_i^+) = \epsilon_1(\beta_1^+)$, so that $\alpha_i^+ = \alpha_1^+ > 0$, $\forall\ i$.

If $J > 1$, then $\sum_i v_{0i}(\beta_i) = \sum_i \beta_i^{\epsilon} \underline{v}_{0i}$ is strictly convex in β ($\beta_i > 0$). Hence, the extremum is a unique minimum. If $J = 1$, then, from $\epsilon_1(\beta_1) = \epsilon_2(1 - \beta_1)\ \forall\ \beta_1 \in (0, 1)$ and $v_0(\beta_1) = v_{01}(\beta_1) + v_{02}(1 - \beta_1)$, it follows,

$$\frac{dv_0}{d\beta_1} = \frac{dv_{01}}{d\beta_1}\left[1 - \frac{\alpha_1}{\alpha_2}\right] \ \forall\ \beta_1 \in (0, 1).$$

As $dv_{01}/d\beta_1 < 0$, $dv_0/d\beta_1 <(>)\ 0$ for $\alpha_1 <(>)\ \alpha_2$. Hence, $\alpha_1 = \alpha_2$ represents a unique minimum. Q.E.D.

Proposition 3 shows that a firm with the lowest possible "true" total value sells the same fraction of all securities to the new investors. As a consequence, the new investors hold the same claims as if the firm had issued only equity capital. This result has an intuitive explanation. If a firm issues only equity capital, the managers have no degree of freedom in selecting the financing policy. Therefore, they cannot signal anything. The outsiders then suppose that there is nothing to signal; i.e., the managers have no incentive to signal a total firm value that exceeds the lowest possible value. Therefore, the outsiders are most pessimistic.

If more than two security types exist, then a similar reasoning applies to subsets of securities. If $\alpha_h^+ = \alpha_k^+$, then the firm could merge these securities and issue only the merged security instead of issuing type-h and type-k securities. The outsiders would interpret this as a partial lack of signalling and develop correspondingly pessimistic beliefs.

Conversely, firms with very high total values deviate strongly from the equal-fraction policy $\alpha_i = \bar{\alpha}\ \forall\ i$. For those firms, one of the β_i's approaches 1; approximately, these firms raise all the required money by selling just one type of basic security to the new investors.

The preceding propositions start from the assumption that the ORC and the NAC hold. This raises the question of how restrictive this assumption is. Inspection of the proof of Proposition 3 shows that the function relating the "true" total value of the firm to β^+ attains a unique minimum at $\alpha_i^+ = \alpha_1^+\ \forall\ i$; aside from that, no minimum, interior maximum, or saddle point exists. Hence, the same must be true of the function $v_0(P)$ if β^+ is required to signal P^* correctly. This proves the corollary.

Corollary: *The ORC and NAC require that the function $v_0(P)$ have no interior maximum and no saddle point; it can have only one minimum.*

The corollary reveals the restrictive content of the ORC and the NAC. A costless, fully revealing signalling equilibrium does not exist if the unknown parameters generate an interior maximum, a saddle point, or more than one minimum of the "true" total firm value. As a consequence, the ORC and the NAC restrict the nature of the unknown parameters.

VI. Analysis of Nonbasic Financing Policies

The preceding discussion has been restricted to basic financing policies. It will be shown now that, in general, nonbasic financing policies rule out a fully revealing, costless-signalling equilibrium. Before this will be proved, an intuitive explanation will be given. Suppose that the outsiders infer the "true" values of the securities from the functions $v_{0i}(\beta_i^+)$, $\forall\ i$. Then the marginal exchange rate in the primary market equals $v_{0k}(\beta_k^+)/v_{0h}(\beta_h^+)$, multiplied by the marginal signalling factor, which deviates from 1 for nonbasic securities. The marginal exchange rate in the secondary market equals $v_{0k}(\beta_k^+)/v_{0h}(\beta_h^+)$. Hence, there exist profitable arbitrage opportunities for the insider managers that rule out an equilibrium.

Proposition 4: *Suppose that the outsiders regard $v_{0i}(\beta_i^+)$ as the "true" value of the type-i securities, with β_i^+ being the relative monetary supply of type-i securities chosen by the managers, $i = 1, \cdots, J+1$. Then a fully revealing, costless-signalling equilibrium does not exist for nonbasic financing policies unless there exists an equivalent basic financing policy for every nonbasic financing policy.*

Proof: Necessary conditions for the solution of the managers' optimization problem are given by equation (5). If the outsiders correctly regard $v_{0i}(\beta_i^+)$ as the "true" value of the type-i securities, then $v_{0i}^* = v_{0i}(\beta_i^+)$, $\forall\ i$.

Hence, equation (5) yields

$$1 = \lambda[1 - \epsilon_i(\beta_i^+)]^{-1}, \ \forall\ i \quad \text{and} \quad \forall\ \beta^+, \text{ or}$$

$$1 = \frac{1 - \epsilon_h(\beta_h^+)}{1 - \epsilon_k(\beta_k^+)} \qquad \forall\ (h, k) \quad \text{and} \quad \forall\ \beta^+.$$

This is exactly the condition that the marginal signalling factors be equal to 1. Hence, basic financing policies are implied in a fully revealing equilibrium.

There exists, however, a case in which the outsiders, being unable to infer the "true" values of the nonbasic securities directly from the firm's nonbasic financing policy, can first translate this policy into an equivalent basic financing policy and then infer the "true" values.

This case obtains if

i. each nonbasic security j ($j = 1, \cdots, J + 1$) equals a portfolio of basic securities, $v_{0j} = \sum_{i=1}^{J+1} q_{ij} v_{0i}\ \forall\ j = 1, \cdots, J + 1$, with the weights q_{ij} being independent of the firm's financing policy, and
ii. the matrix of the weights q_{ij} is nonsingular.

Then, the nonbasic financing policy α_j ($j = 1, \cdots, J + 1$) translates into the unique basic financing policy $\alpha_i = \sum_{j=1}^{J+1} q_{ij} \alpha_j\ \forall\ i = 1, \cdots, J + 1$. Applying matrix algebra, it can be shown that the managers' minimization problem can be stated equally in terms of the nonbasic and in terms of the basic financing policies. Therefore, both policies are equivalent. A one-to-one correspondence between both policies exists that enables the outsiders to infer the "true" values of the nonbasic securities. Q.E.D.

VII. Conclusion

Bhattacharya, Heinkel, and Brennan and Kraus derived conditions for a costless-signalling equilibrium that are not easy to understand. This paper shows that a costless-signalling equilibrium in financial markets can be derived from an outsider-rationality condition and a no-arbitrage condition. The outsider-rationality condition is an important device against adverse selection; the no-arbitrage condition requires that marginal exchange rates be the same in the primary and secondary markets. Both conditions appear to be easily understandable. These conditions are satisfied, however, only by basic financing policies.

The implications for basic financing policies are quite strong. If more than one parameter is unknown, then the current market value of every basic security exhibits constant negative elasticity with respect to the signal. Moreover, the "true" total value of the firm is a function of the parameter vector with no interior maximum, no saddle point, and exactly one minimum. All these conditions restrict the nature of the unknown parameters and that of a sharing rule.

The analysis of this paper has been based on the assumption that the sharing rule is independent of the firm's financing policy. This assumption is not restrictive, however. If the sharing rule depends on the financing policy such that the outsiders know this dependence precisely, then the outsiders can evaluate the effects of this dependence precisely. Hence, the dependence of the sharing rule on the financing policy does not enable the managers to "cheat."

The outsider-rationality condition being used is most simple since it relates the market value of a security to only one variable of the firm's financing policy. This may be unrealistic. Therefore, empirical tests of this condition would be helpful. So far, research on costless-signalling equilibria is still in its infancy.

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