Lived Individual in a Continuous Time Model," *Journal of Financial Economics*, 2 (June), 187–203.

Rovira, Carlos E., 1983, *Tax Effects on the Term Structure of Interest Rates*, Unpublished Ph.D. dissertation, Cornell University.

United States Treasury Department, *Treasury Bulletin*, U.S. Government Printing Office, Washington 25, D.C., various issues.

Zaritsky, Howard M., 1983, *Estate and Gift Tax Guide*, Englewood Cliffs, N.J.: Prentice-Hall, Inc.

DISCUSSION

ROBERT C. WITT:[1] David Mayers and Cliff Smith (MS) have developed a very interesting, theoretical and empirical analysis of the demand for flower bonds and the incentives for holding them.[2] The term "flower bond" apparently developed because these bonds blossom or mature after death.

As MS suggest, when market interest rates exceed the coupon rate, a flower bond is equivalent to a straight bond plus a term life insurance policy. Accordingly, they argue that the market value of a flower bond should be equal to the sum of the values for a straight government bond with the same coupon and maturity plus a net single premium for the imbedded term life insurance. They suggest that the amount of life insurance imbedded in the jth flower bond is the difference between its par and market values ($P_j - F_j$). In life insurance theory, this amount would be referred to as the net amount at risk for the jth bond (NAR_j). The total amount at risk (TAR_j) would be defined as the difference between the par value of the flower bond and the value of a matching straight bond ($P_j - B_j$). By not distinguishing between these two concepts, the authors inadvertently mis-specified the equilibrium pricing relationship in Equation (3) of their paper. Fortunately, this mis-specification did not affect their statistical results, but it would raise questions about the rationality of individuals who purchase and hold flower bonds.

Due to space constraints, I will only summarize a few suggestions that I made to MS for improving their very interesting and stimulating paper. First, they ignored the effect of different marginal estate taxes among individuals with different levels of taxable wealth by assuming that estate taxes were a fixed fraction of wealth for all individuals. This is not a reasonable assumption because it ignores the *effect* of progressive estate taxes in both their theoretical and empirical analysis. Accordingly, they need to redefine their hypothesis to recognize the *effect* of estate taxes on different levels of taxable wealth. Second, they need to redefine their equilibrium pricing relationship between the net single premium for the life insurance embedded in flower bonds and the equivalent

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[2] Since flower bonds are no longer being issued by the United States Treasury and outstanding issues are slowly expiring, in jest one might say that this is a morbid topic dealing with a dying issue. Joking aside, the analysis by MS provides some significant insights about the incentives to hold flower bonds and potentially about the rationality of individuals in the market for flower bonds. All references in this comment are to the paper by Mayers and Smith entitled "Death and Taxes: The Market for Flower Bonds," which was presented in December 1986 at the meeting of the American Finance Association in New Orleans. Some constructive discussions about this paper with Harry Panjer, Seha Tinic, Donghoon Kim, and David Valaer are gratefully acknowledged.

amount of coverage in the life insurance market. Their equilibrium relationship in Equation (3) appears to be mis-specified because their insurance premium on the left hand side of the equation is based on the total amount at risk (TAR_j) or the total discount; whereas their premium for the life insurance embedded in the flower bond on the right hand side of the equation is based on the net amount at risk (NAR_j) or the net discount. They also have mis-specified the net single premium for insurance in Equation (1). Third, some empirical illustrations of the cost of life insurance embedded in flower bonds would help to clarify and enhance the relevance of their analysis. Such illustrations will be included here in order to show that the equilibrium price for life insurance was not specified properly in their paper.

The Effect of Estate Taxes

In devloping their analysis, MS ignored the effect of different marginal estate taxes among individuals with different levels of taxable wealth by assuming that estate taxes were a fixed fraction of wealth for all individuals in the economy. This is not a reasonable assumption because the partial derivative of the expected net payoff (as defined in the MS paper) with respect to the marginal tax rate is negative. This negative relationship implies that the expected net payoff would decrease as an individual's marginal estate tax rate increased. Therefore, in testing for clientele effects, the marginal estate-tax effect should be considered.

Currently, marginal estate tax rates can vary from 18 percent of the taxable estate below $10,000 to 50 percent of the estate in excess of $2,500,000. Because individuals can have dramatically different marginal estate-tax rates, the expected net payoff would vary inversely with the marginal rate. Accordingly, the authors need to revise their assertion about clientele effects in order to recognize the estate-tax effect.

The hypothesis MS developed indicated that individuals with the highest death probability would hold the deepest discount flower bonds while individuals with lower death probabilities would hold bonds with lower discounts. In order to recognize estate-tax effects, the hypothesis should be revised to state that "the individuals with the highest death probabilities and lowest marginal estate tax rates would be expected to hold the deepest discount flower bonds, while individuals with lower death probabilities and higher marginal estate taxes would be expected to hold bonds with lower discounts." If the authors recognize differences in marginal estate taxes among individuals, they will need to modify their assertions about clientele effects accordingly. Their analysis ignores the marginal estate-tax effect even though they recognize "average" estate taxes.

Redefinition of the Pricing Equilibrium

The equilibrium pricing relationship that MS specified in Equation (3) does not appear to be correct because their instantaneous insurance premium on the left hand side of the equation has to be based on the total amount at risk in a private insurance market for the jth bond (the difference between the par value of the flower bond and the value of a straight government bond with the same coupon and maturity, i.e., $TAR_j = P_j - B_j$); whereas their instantaneous premium

for the embedded life insurance in the flower bond on the right hand side of the equation is based upon the net amount at risk or the discount for the jth bondholder (the difference between the par value of the flower bond and its market value, i.e., $NAR_j = P_j - F_j$). This equilibrium cannot be attained, or exist, and therefore it needs to be redefined.

It should be emphasized that even though this equilibrium pricing relationship is not defined properly, this mis-specification does not affect the statistical results of MS because the deepest discount bonds are properly reflected by the net amount at risk rather than the total amount at risk. That is, individuals with the highest mortality rates and lowest marginal estate-tax rates would want to buy the flower bonds with the deepest discount or net amount at risk because bonds with the highest total amount at risk do not always have the highest net amount at risk, as is shown in Table 1 later.

For measurement purposes, the discount or net amount at risk is the proper value to utilize in their statistical tests. For pricing the embedded life insurance in the flower bonds, however, the net amount at risk is not the proper measure to utilize. The price of life insurance must be based on the total amount at risk rather than the net amount at risk. As shown by my illustrations in Table 1, the price of life insurance on an after estate-tax basis would exceed the amount of life insurance purchased if the net amount at risk were utilized for pricing purposes rather than the total amount at risk. This would imply that investors who purchased flower bonds are irrational with respect to the premiums they are willing to pay for the embedded life insurance. In contrast, if the total amount at risk is utilized for pricing the life insurance embedded in flower bonds, the results in Table 1 show that individuals pay a net single premium (NSP_j) which is less than the amount of insurance purchased on an after estate-tax basis.[3] Assuming bond markets are reasonably efficient and investors are rational, my illustrative calculations indicate that the total amount at risk is the proper measure to use for pricing the embedded life insurance. If the equilibrium pricing relationship were specified properly, the model would suggest that individuals who buy flower bonds are rational because the net single premium (NSP_j) they would pay would be less than the amount of life insurance (TAR_j) they purchased in all cases, which is not true if the NAR_j is utilized.

In fairness, it should be emphasized that the purpose of the MS study was not to analyze the pricing implications of their equilibrium model. However, if they had made this assessment, it would have become apparent that they would need to redefine their equilibrium pricing relationship, as shown below. The equilibrium pricing relationship MS specified in Equation (3) which was based on the

[3] It should be noted that MS have not specified the net single premium for insurance correctly in Equation (1). They used the conditional instantaneous death probability of $\pi_i(t)$ in calculating their instantaneous term insurance premium, $I_j(t)$, used in Equation (1) rather than the unconditional probability density for the time of death, $({}_tP_i)(\pi_i(t))$, where ${}_tP_i$ is the probability that a person age i survives for t units of time. The net single premium should have been specified as:

$$NSP_j = F_j - B_j = \int_0^T e^{-Rt}({}_tP_i)I_j(t)\,dt.$$

where $\ln {}_tP_i = \int_0^t \pi_i(s)\,ds$.

NAR_j after estate taxes is reproduced below (where all symbols have been defined above or by MS).

$$\begin{aligned} I_j(t) &= \pi_i(t)(P_j - F_j)(1 - \tau) \\ &= \pi_i(t)(NAR_j)(1 - \tau) \end{aligned} \tag{3}$$

The correct pricing relationship based on the total amount of risk after estate taxes should have been specified as follows.

$$\begin{aligned} I_j(t) &= \pi_i(t)[(P_j - B_j) - \tau(P_j - F_j)] \\ &= \pi_i(t)[TAR_j - \tau(NAR_j)] \\ &= \pi_i(t)[NAR_j(1 - \tau) + NSP_j] \\ &= \pi_i(t)[(P_j - F_j)(1 - \tau) + (F_j - B_j)] \end{aligned} \tag{3a}$$

where $TAR_j = NAR_j + NSP_j$.

Some Corrections and Potential Extensions

Some empirical illustrations of the cost of life insurance embedded in flower bonds would have helped to enhance the insights and relevance of the MS analysis. Such illustrations would also demonstrate that the equilibrium price for the embedded life insurance was not properly specified by MS. If the correct equilibrium pricing relationship were specified based on the total amount at risk after estate taxes, the theoretical results would be much more meaningful. In order to develop the illustrations shown in the following table, some data on flower bonds from *The Wall Street Journal* on December 19, 1986, were utilized.[4]

The empirical results presented in Table 1 for three flower bonds without call provisions clearly show that the implicit equilibrium price for the embedded life insurance in flower bonds was not properly specified by MS. If the correct measurement based on the total amount of risk after estate taxes had been utilized, the results would be much more meaningful. As the illustrations show, the price of life insurance on an after estate-tax basis would exceed the amount of life insurance if the net amount at risk, rather than the total amount at risk, after estate taxes were utilized for pricing purposes. This would imply that investors in flower bonds are irrational. When the total amount at risk is utilized for pricing the embedded life insurance in flower bonds, the illustrations show

[4] In deriving the values for comparable straight bonds, it was necessary to make some assumptions. First, it was necessary to ignore the coupon effect on fluctuations in yields. That is, in ascertaining a comparable yield for a matching straight bond it was necessary to use bonds with higher coupon rates than available on the comparable flower bond. Since there is less fluctuation in yields on bonds with higher coupon rates than in bonds with lower coupon rates, I had to abstract from this problem to develop the pricing illustrations. Second, it was necessary for me to ignore the income tax affect on bonds with larger coupons which would tend to raise their yield. In other words, bonds with larger coupon rates would tend to have more of their yield taxed at ordinary rates rather than at capital gain rates which would mean that they would have to provide a higher before tax yield in order to generate an equivalent after tax yield. I abstracted from this problem also. These two effects may offset each other as one reduces the yield and the other raises the yield, so it cannot be determined what effect that larger coupon might have on the final yield.

that individual investors pay a net single premium which is less than the amount of insurance purchased on after-estate tax basis. Thus, these results demonstrate that the total amount at risk is the proper measure to use for pricing life insurance embedded in flower bonds, assuming that bond markets are efficient and investors are rational.

In general, this is a very interesting and stimulating paper which reflects a great amount of thought and effort by the authors. They should be commended for the very interesting analysis they have developed on the incentives for holding flower bonds. My suggested changes and extensions would merely make a good paper better.

Table 1

Flower Bonds, Matching Bonds, and the Cost of Life Insurance: Some Empirical Illustrations[a]

Flower Bond Maturity	Flower Bond Coupon Rate	Yields on Flower and Matching Bonds		Prices of Flower and Matching Bonds		Net Single Premium (NSP_j) $F_j - B_j$	Net Amount At Risk (NAR_j) \$1000 $- F_j$	Total Amount At Risk (TAR_j) 1000 $- B_j$
		YF_j	YB_j	F_j	B_j			
FEB. 1990	3.5%	4.60%	6.50%	\$968.10	\$936.80	\$ 31.30	\$31.90	\$ 63.20
FEB. 1995	3.0	3.43	7.14	969.40	765.90	203.50	30.60	234.10
NOV. 1998	3.5	3.81	7.32	970.60	723.40	247.20	29.40	276.60

Price or Cost of Life Ins. Per Dollar in Flower Bonds					
		After Estate Taxes			
Before Estate Taxes		For NAR_j		For TAR_j[b]	
NSP_j/NAR_j	NSP_j/TAP_j	$\tau = 18\%$	$\tau = 50\%$	$\tau = 18\%$	$\tau = 50\%$
\$0.98	\$0.50	\$ 1.20	\$ 1.96	\$0.55	\$0.66
6.65	0.87	8.02	13.30	0.89	0.93
8.41	0.89	10.25	16.82	0.91	0.94

[a] The basic data used in these calculations were derived from "Treasury Bonds, Notes, and Bills," in the *Wall Street Journal* (December 19, 1986), p. 33.

[b] Where the after estate tax cost of life insurance based on the TAR_j is defined by:

$$(NSP_j)/[NAR_j(1 - \tau) + NSP_j].$$