Orthogonal Frontiers and Alternative Mean-Variance Efficiency Tests: Discussion
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DISCUSSION

SHMUEL KANDEL*: This paper by Professor Lehmann [5] presents an interesting alternative approach to testing mean-variance efficiency of a given portfolio. Mean-variance efficiency of a given portfolio, p, implies that all zero-cost portfolios that are uncorrelated with p (zero-beta) have zero expected return. Hence, the mean-variance efficiency of p can be tested using returns on such a portfolio. At least two difficulties arise in the implementation of this straightforward approach. First, the betas are unobservable and have to be estimated, of course with errors. Second, even if the betas are known, there are many such portfolios, and the choice of any particular one may determine the power of the test.

Different tests with different solutions to the errors-in-variables problem have been suggested in the literature. The analysis in the first part of Lehmann's paper is based on the following important observation: most of the test statistics used can be interpreted as functions of the sample moments of the return on a portfolio that is approximately uncorrelated with the tested portfolio. Lehmann shows that the vector of weights of this portfolio is the solution of a constrained quadratic programming problem, "the minimum-norm problem", where the portfolio's cost, beta, and mean return are the three constrained values.

Different tests of mean-variance efficiency correspond to different sets of parameters employed in the minimum-norm problem. The differences are reflected in at least one of the following characteristics of the optimization problem: (1) the sample moments used in the calculations of the mean return and beta (same period or previous period), (2) the constrained value of beta (zero or some correction of the measurement error), (3) the constrained cost (zero or one), (4) the matrix employed in the objective function (the identity matrix, a diagonal matrix, a sample covariance matrix, etc.), and (5) the dimension of the problem (individual assets or portfolios).

Following Gibbons [1] and Stambaugh [9], most of the recent empirical studies of the CAPM have employed multivariate tests. Shanken [8] shows that all of these test statistics are transformations of each other. They are functions of the return on a particular zero-cost portfolio. This portfolio has the maximum sample Sharpe measure among all zero-cost portfolios having zero sample correlation with the tested portfolio p. Using Lehmann's framework, these tests correspond to a minimum-norm problem with the following characteristics: (1) the same period is used for both the test and the estimation of the moments (one-step test), (2) the value of the beta is constrained to be zero, (3) the cost of the portfolio is constrained to be zero, and (4) the full sample covariance matrix of the market model residuals is employed in the objective function. Of course, this last characteristic restricts the multivariate tests to a subset of the assets, where the number of assets is less than the number of the time-series observations. Potentially, this reduces the power of the resulting tests.

The main contribution of this paper is the introduction of an alternative procedure that allows the researcher not to be restricted by the number of time-series observations and to use returns on many individual assets instead of

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forming portfolios. It is shown that for *any* positive definite matrix, if the following assumptions are satisfied, then any feasible solution of the minimum norm problem is a zero-cost portfolio with unconditional beta of zero. The assumptions are: (1) assets' returns are serially independent, (2) the sample moments used in the calculation of the weights are based on an estimation period prior to the test period (two-step test), (3) the constrained cost is zero, (4) the constrained mean return is non-zero, and (5) the constrained value of the beta is an unbiased estimator of the corresponding population measurement error, estimated independently of all other estimators. If the given portfolio p is efficient, then all feasible solutions of the minimum norm problem, including the optimal one, are portfolios with unconditional zero mean return. This hypothesis can be tested for the unique optimal solution with the usual asymptotic t statistic.

A major difficulty in applying this procedure is the construction of unbiased estimates of the measurement error corrections. To the best of my knowledge, this is close to impossible. The errors-in-variables adjustments suggested in Proposition 3 are similar to those proposed by Litzenberg and Ramaswamy [6] and modified by Shanken [7]. Lehmann "hedges" against the event that the used estimates are biased, and suggests to employ the optimal portfolio, together with another standard portfolio he derives, in Shanken's [8] CSR and joint F tests. It seems to me that it is assumed implicitly that the incorporation of these two portfolios in these tests is preferred, in some sense, on applying these tests with other portfolios. More elaboration is needed on the relations between the alternative approach presented here and these tests.

The researcher has to decide which matrix to employ when applying Lehmann's procedure. The choice of a particular positive definite matrix is related to the efficiency of the estimates and the power of the tests. Assuming the market model is a one-factor model, Litzenberger and Ramaswamy [6] used a similar procedure with a diagonal matrix, the elements of which are the estimates of the variances of the market model residuals. The market model, however, is not a one factor model. Industry and size related covariations of market model residuals have been documented by King [4] and Huberman, Kandel and Karolyi [3]. The power and efficiency of the alternative procedure may be improved by accounting for these and other empirical regularities of cross sectional correlations.

Another related question is when should a researcher use the alternative procedure with individual assets instead of a multivariate test with grouping of stocks? An advantage of the multivariate tests over the alternative procedure is that exact and approximate finite sample distributions of the test statistics have been calculated. On the other hand, forming portfolios for the multivariate tests may result in a reduction in efficiency due to a loss of information.

Many of the multivariate tests, as well as the tests suggested by this paper's alternative approach, do not test the pricing model against any specific alternative hypothesis about deviations from the pricing model. Gibbons, Ross, and Shanken [2] imply that using grouping in a multivariate test may not result in a loss in power if the grouping variable is motivated by a specific alternative hypothesis. But, if a specific alternative hypothesis is used as a basis of grouping, then a test with a specific alternative hypothesis might be preferred to any test with unspecified alternative hypothesis.

Finally, I would like to note that mean-variance efficiency of one or more benchmark portfolios is implied not only by the CAPM, but also by other asset pricing models, such as the APT and the Consumption-based CAPM. On the one hand, applying this paper's approach to tests of these theories may require some modifications, such as estimating the weights of the mimicking portfolios for a large subset of the assets. On the other hand, the additional structure assumed by these models may help the researcher to determine which of the tests should be employed and which matrix should be used.

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