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Positively Weighted Frontier Portfolios: A Note

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IN A RECENT ARTICLE, Green [2] uses duality theory to find conditions for portfolios on the minimum-variance frontier to contain positive positions in all assets. One of the main results is Theorem 2, which gives an elegant condition for no such all-positive frontier portfolio to exist.

While Theorem 1, the other main result in the paper, is proved in a straightforward manner by appeal to duality theory, the proof of Theorem 2 is relatively complicated and involves a series of technical lemmas in the Appendix. Such complexity may slow the dissemination of the result and impede further research. This note shows that, in addition to being a significant advance over previously available results, Green's Theorem 2, like Theorem 1, has the virtue of being a simple consequence of the duality theory of linear inequalities. Using Green's notation, we may state and prove the theorem as follows.

THEOREM: *There is no solution μ to*

$$\gamma_1(\mu) V^{-1}E + \gamma_2(\mu) V^{-1}q \gg 0 \tag{1}$$

if and only if there is a solution z to

$$z'[E, q] = 0, \quad Vz \geq 0, \quad Vz \neq 0. \tag{2}$$

Condition 1 says that the frontier portfolio with mean return μ has all-positive weights. Condition (2) says that z is a nontrivial hedge portfolio with zero mean return and non-negative correlation with all assets.

Proof: There is a solution z to (2) if and only if there is a solution x to

$$x' V^{-1}[E, q] \geq 0, \quad x \geq 0, \quad x \neq 0 \tag{3}$$

(simply set $x = Vz$). This is equivalent to the nonexistence of a solution y to the dual problem (cf. Gale [1], Theorem 2.9):

$$V^{-1}[E, q]y \gg 0, \quad \text{i.e.,} \quad y_1 V^{-1}E + y_2 V^{-1}q \gg 0. \tag{4}$$

If there is no solution y to (4), then clearly there is no solution μ to (1). Conversely, suppose there is a solution y to (4). Then $y_1A + y_2C = q' V^{-1}[E, q]y > 0$, so, after a positive rescaling of y, we may assume that $y_1A + y_2C = 1$. Set $\mu = y_1B + y_2A$. Then $\gamma_1(\mu) = (C\mu - A)/D = [C(y_1B + y_2A) - A(y_1A + y_2C)]/D = y_1$. A similar computation shows that $\gamma_2(\mu) = y_2$, so that μ solves (1). Q.E.D.

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