THE JOURNAL OF FINANCE • VOL. XLII, NO. 2 • JUNE 1987

Taxable vs. Tax-Exempt Bonds: A Note on the Effect of Uncertain Taxable Income

CHRISTOPHER D. PIROS*

THE CONVENTIONAL WISDOM WITH respect to an investor's choice between taxable and tax-exempt bonds can be summarized as follows:

> For an investor to be indifferent between a municipal security and, say, a corporate security of equal risk, the following would need to hold for par value bonds:
>
> $$R_{TE} = R(1 - T'),$$
>
> where
>
> R_{TE} = the yield on the municipal security,
> R = the yield on the corporate security, and
> T' = the marginal tax rate of the investor.
>
> If R_{TE} were greater than $R(1 - T')$, rational investors would invest in municipal securities; if it were less, they would invest in corporates.[1]

According to this proposition, a rational investor would always select one of three positions: 100% taxable bonds, 100% tax-exempt bonds, or an arbitrary combination of taxable and tax-exempt bonds. He or she would never express a preference for a specific mixture of taxable and tax-exempt bonds. This standard view rests upon the implicit assumption that the investor knows his or her marginal tax rate with certainty at the time he or she makes his or her bond portfolio decision.[2] While this may be true for some investors, particularly near the end of the tax year, many investors face an uncertain marginal tax rate due to uncertainty with respect to their taxable income. The purpose of this note is to examine the implications of allowing for this uncertainty.

It will be shown that the investor selects a unique combination of taxable and tax-exempt bonds. Furthermore, a risk-averse investor will not hold tax-exempt bonds unless they offer a yield greater than the expected after-tax return on taxable securities. An approximate expression for the required risk premium is also presented. These results are derived in the next section using a simple model of the investor's choice between taxable and tax-exempt bonds with uncertain taxable income. Some implications are outlined in Section II.

* Fuqua School of Business, Duke University. I thank Douglas Breeden, Robert Dammon, Robert Piros, and James Van Horne for helpful comments and discussions. The Business Associates Fund provided financial support.

[1] Van Horne [8, pp. 227–28]. Van Horne's notation has been replaced by the notation to be used in this note. Similar statements can be found throughout the literature, e.g., Skelton [5, pp. 343–44] and Trzcinka [7, p. 908].

[2] The conventional view also assumes that the investor's tax rate does not depend upon his or her bond portfolio. However, no new insights arise from relaxing this assumption if the marginal tax rate is known.

I. Choosing a Bond Portfolio with Uncertain Taxable Income

Consider a taxable investor who wishes to maximize his or her expected utility of after-tax income, $E\{U(X)\}$. In order to focus upon the investor's optimal mixture of taxable and tax-exempt bonds and to avoid unnecessary complexity, it will be assumed that: (a) the investor has already decided to invest $\$W$ in bonds, (b) all bonds are pure discount securities maturing at the end of the period, (c) tax-exempt and taxable bonds are identical in every respect except tax status, (d) bonds may not be sold short,[3] (e) the only source of uncertainty is the value of the investor's taxable income arising from sources other than interest on taxable bonds, e.g., stocks, employment, etc., and (f) the investor's marginal tax rate is strictly increasing in taxable income.

If the investor selects a bond portfolio with a fraction α in tax-exempt bonds and a fraction $(1 - \alpha)$ in taxable bonds, then his or her after-tax income will be

$$X = Y + (1 - \alpha)WR + \alpha WR_{TE} - T(Y + (1 - \alpha)WR), \tag{1}$$

where

Y = uncertain taxable income exclusive of interest,
R = yield on taxable bonds,
R_{TE} = yield on tax-exempt bonds,
$T(\cdot)$ = tax function such that $T' > 0$ and $T'' > 0$,

and X and W are as defined above. The first-order necessary condition for a maximum of $E\{U(X)\}$ with respect to α is

$$E\{U'[R_{TE} - R(1 - T')]\} \begin{cases} \geq 0 \text{ if } \alpha = 1 \\ = 0 \text{ for } 0 < \alpha < 1 \\ \leq 0 \text{ if } \alpha = 0. \end{cases} \tag{2}$$

It can easily be verified that the second-order sufficient condition is satisfied; i.e., expected utility is strictly concave in α. Thus, the investor expresses a preference for a unique combination of taxable and tax-exempt bonds.

Let us now consider the conditions under which the investor will hold at least some tax-exempt bonds, i.e., $0 < \alpha \leq 1$. Using a property of the covariance, the relevant cases in equation (2) can be rewritten as

$$\{R_{TE} - (1 - E\{T'\})R\} \geq \frac{\text{cov}[U', (1 - T')R]}{E\{U'\}} \geq 0, \tag{3}$$

where the second inequality reflects the fact that marginal utility is nonincreasing in taxable income while the marginal tax rate is an increasing function of taxable income. This inequality is strict for a risk-averse investor. The first inequality is an equality if the investor holds both taxable and tax-exempt bonds. The left-hand side of equation (3) is the difference between the return on tax-exempt bonds and the expected after-tax return on taxable bonds. Our main result follows immediately from this equation.

[3] Selling taxable bonds short would amount to borrowing for the purpose of holding tax-exempt bonds. This is made unattractive by the fact that interest on such a loan is not tax deductible. Selling municipal bonds short is difficult if not impossible in practice.

PROPOSITION 1: *A risk-averse investor whose taxable income is uncertain will not hold tax-exempt bonds unless they offer a sufficient premium over the expected after-tax return on taxable bonds.*

The intuition behind this result is simple. Due to the progressive tax schedule, the after-tax return on taxable bonds, $[(1 - T')R]$, is positively correlated with the marginal utility of after-tax income (consumption), while the return on tax-exempt bonds is uncorrelated with marginal utility. Thus, taxable bonds offer a hedge against fluctuations in after-tax income (consumption), which tax-exempt bonds do not provide. If taxable and tax-exempt bonds offered the same expected after-tax return, then a risk-averse investor would strictly prefer taxable bonds. The investor can be induced to hold tax-exempt bonds only if they offer a premium sufficient to compensate for the foregone hedge.

The basic logic of this result still holds if bond returns are uncertain. With stochastic returns, equation (3) becomes

$$E\{R_{TE}\} - E\{R(1 - T')\} \geq \frac{\text{cov}[U', R(1 - T')] - \text{cov}[U', R_{TE}]}{E\{U'\}}. \quad (4)$$

Thus, the validity of Proposition 1 depends upon taxable bonds providing a *better* hedge than tax-exempt bonds. The key determinant of their relative hedging capabilities is their different tax treatment since, by assumption, the bonds are otherwise identical. The progressivity of the tax code still works in favor of taxable bonds. However, if the pre-tax returns, R and R_{TE}, have a very strong negative correlation with other income, Y, then tax-exempt bonds may provide a better hedge. To see this, assume that $\text{cov}[R_{TE}, Y] = \text{cov}[R, Y] < 0$ and ignore the tax progressivity. Then

$$\text{cov}[R_{TE}, Y] < (1 - T')\text{cov}[R, Y] = \text{cov}[R(1 - T'), Y] < 0. \quad (5)$$

Although the bonds have equal covariance with Y on a pre-tax basis, the tax factor $(1 - T')$ reduces the after-tax hedging effectiveness of taxable bonds. If this effect is strong enough, it could outweigh the effect of progressivity. In that case, the investor would demand a premium to hold *taxable* bonds. On the other hand, if pre-tax bond returns are positively correlated with other income, then this effect reinforces the advantage of taxable bonds over tax-exempt bonds. Skelton [6] found that taxable and tax-exempt bonds have positive betas of the same magnitude. Since beta can be taken as a proxy for the relevant covariances above, this evidence indicates that Proposition 1 holds with stochastic bond returns.

Returning to the original model, an expression for the risk premium required on tax-exempt bonds can be derived by approximating both the numerator and denominator on the right-hand side of equation (3) by first-order Taylor series expansions around the expected value of Y. This yields

$$\frac{\text{cov}[U', (1 - T')R]}{E\{U'\}} \approx \left[\frac{-U''}{U'}\right] T''[(1 - T')R]\sigma_Y^2. \quad (6)$$

Thus, the risk premium is an increasing function of absolute risk aversion, the progressivity of the tax schedule, the after-tax return on taxable bonds, and the

variance of taxable income. Breaking the after-tax return on taxable bonds into two parts, it follows that the risk premium increases with the level of interest rates, R, and decreases with the investor's marginal tax rate evaluated at the expected value of Y.

II. Summary and Implications

It has been shown that, faced with uncertain taxable income and progressive taxation, the investor will express a preference for a unique combination of taxable and tax-exempt bonds. A risk-averse investor will not hold tax-exempt bonds unless they offer a return greater than the expected after-tax return on similar taxable securities since taxable bonds provide a hedge against fluctuations in after-tax income (consumption), which tax-exempt bonds do not offer. To a first approximation, the required risk premium depends upon risk aversion, the progressivity of the tax schedule, the after-tax return on taxable bonds, and the variance of taxable income.

The fact that risk-averse investors must be offered a premium to induce them to hold tax-exempt bonds has three immediate implications. First, holders of taxable bonds and holders of tax-exempt bonds cannot be distinguished on the basis of expected (much less *ex post* realized) marginal tax rates alone. Second, the "implicit tax rate on tax-exempt bonds," τ, defined by

$$R_{TE} = (1 - \tau)R, \tag{7}$$

is a downwardly biased measure of the marginal investor's expected marginal tax rate. Third, unless supply is perfectly elastic, the implicit tax rate on tax-exempt bonds will be lower than in the absence of uncertainty.[4] Equivalently, the spread between taxable and tax-exempt yields will be narrower. It is well known that the implicit tax rate on long-term tax-exempt bonds is consistently far below the top personal tax rate and indeed well below the corporate tax rate. This phenomenon is usually ascribed to market segmentation.[5] The major institutional holders of tax-exempt bonds, banks and property-casualty insurance companies, tend to concentrate in short maturities. In order for individual investors to absorb the supply of long-term tax-exempt bonds, the implicit tax rate must be such that even investors in the middle tax brackets find them attractive. As shown here, the implicit tax rate must be low enough to provide risk-averse investors in the middle brackets with a risk premium on tax-exempt bonds.

[4] The Miller Hypothesis [3] implies that the relative supply of taxable and tax-exempt bonds will be perfectly elastic and the implicit tax rate on tax-exempt bonds always equal to the corporate tax rate. However, Ross [4] has shown that Miller's argument breaks down under uncertainty. The empirical evidence, though mixed, also favors rejecting the hypothesis. See Ang, Peterson, and Peterson [1], Kidwell and Koch [2], Skelton [5, 6], and Trzcinka [7].

[5] Trzcinka [7] suggested an alternative explanation. He accepts the Miller Hypothesis and attributes the difference between the implicit tax rate and the corporate rate to a risk premium on tax-exempt bonds arising from differential risk not captured by standard bond ratings. Skelton [6] found no evidence of differential risk in *pre-tax* returns. While, as shown here, differential taxation gives rise to a risk premium on tax-exempt bonds, this cannot reconcile the Miller Hypothesis with observed yield spreads because the spread is completely supply-determined in the Miller world.

The risk premium required on tax-exempt bonds also lends support to the frequently debated but as yet unadopted proposal that the tax-exempt status of municipal bonds be replaced by a direct subsidy or rebate from the federal government. Under this proposal, municipal borrowers would raise funds in the taxable debt market, and the federal government would rebate all or part of the taxes levied on municipal interest payments. By issuing taxable securities, the municipalities would avoid paying the premium induced by tax rate uncertainty. In addition, they would not have to share the federal subsidy with investors. Under the current system, a (possibly large) portion of the revenue foregone by the Treasury accrues to inframarginal investors in the form of tax-exempt yields in excess of the after-tax returns they could obtain on taxable securities of similar default risk.

REFERENCES

1. James Ang, David Peterson, and Pamela Peterson. "Marginal Tax Rates: Evidence from Nontaxable Corporate Bonds: A Note." *Journal of Finance* 40 (March 1985), 327–32.
2. David S. Kidwell and Timothy W. Koch. "Market Segmentation and the Term Structure of Muncipal Yields." *Journal of Money, Credit and Banking* 15 (February 1983), 40–55.
3. Merton H. Miller. "Debt and Taxes." *Journal of Finance* 32 (May 1977), 261–75.
4. Stephen A. Ross. "Debt and Taxes and Uncertainty." *Journal of Finance* 40 (July 1985), 637–58.
5. Jeffrey L. Skelton. "Banks, Firms and the Relative Pricing of Tax-Exempt and Taxable Bonds." *Journal of Financial Economics* 12 (November 1983), 343–55.
6. ———. "Relative Risk in Municipal and Corporate Debt." *Journal of Finance* 38 (May 1983), 625–34.
7. Charles Trzcinka. "The Pricing of Tax-Exempt Bonds and the Miller Hypothesis." *Journal of Finance* 37 (September 1982), 907–23.
8. James C. Van Horne. *Financial Markets Rates and Flows.* Englewood Cliffs, NJ: Prentice-Hall, 1984.