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A Multiproduct Cost Study of Savings and Loans

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ABSTRACT

This paper investigates the cost structure of savings and loans. Most studies of financial institutions have failed to take into account the multiproduct nature of these institutions. Using the multiproduct approach, the existence of subadditivity, multiproduct global and product-specific economies of scale and scope, and substitutability between inputs is investigated. A translogarithmic cost function is estimated using 1982 data on California savings and loans. Restrictive functional forms also estimated are rejected. Standard errors for the statistics calculated are estimated, and various statistical tests are conducted. Previous authors have not calculated standard errors for these statistics. No evidence of subadditivity in the industry is found.

IN ORDER TO INVESTIGATE the impact of deregulation on the financial services industry, it is necessary to know the cost structure of the industry. Optimal firm size and product mix can be determined once the structure is known. The presence of scale economies would imply that smaller firms or entering firms that operate at a small scale would be at a cost disadvantage compared with larger, established firms. A finding of economies of scope (i.e., economies of joint production) would imply that firms that were specialized would be at a cost disadvantage and that regulations that restrict the outputs a firm is allowed to produce may lead to inefficient production.

Although there have been many previous studies of the cost structure of financial firms, most have suffered from at least one of the following problems:

1. utilization of a single-product framework,
2. assumption of restrictive functional forms for the production technology,
3. restriction of the operating cost of a branch to a constant percentage of the firm's total operating cost,
4. failure to compute standard errors for all the statistics that characterize the cost function,
5. incomplete or careless formulation of hypothesis tests.

This study is an attempt to solve the above problems in determining the cost structure of savings and loans. It explicitly takes into account the multiproduct nature of the firm by using the theory developed by Baumol, Panzar, and Willig.[1] Most of the previous studies have either proxied the multiple outputs by creating

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[1] The author knows of no other published multiproduct cost study of S&Ls.

a composite commodity using scalar indices, chosen one of the firm's outputs to represent all outputs of the firm, or treated the firm as a collection of separate production functions. By treating a vector of outputs as a single composite commodity, the proportions of the various outputs that make up the composite are held fixed, so that only changes in which outputs change by the same percentage can be considered. Also, relative marginal costs of any two outputs are forced to be independent of input price (i.e., the cost function is separable in output). When separate cost functions are estimated for each output, it is implicitly assumed that the marginal cost of producing one output is independent of another output. If significant cost complementarities exist between various outputs, one could erroneously conclude that scale economies exist. Since these methodologies confuse the issues of economies of scale and economies of scope, the conclusion reached by most of these studies—that significant scale economies exist in both commercial banking and the savings and loan (S&L) industry—is called into question.

In the present study a translogarithmic cost function is estimated. This functional form has substantive flexibility, and the only a priori restrictions imposed are those that make the cost function consistent with the existence of a dual production transformation. Previous S&L studies and most of the previous commercial bank studies have estimated functions such as Cobb-Douglas or constant elasticity of substitution, which are not consistent with economies and diseconomies of scale and scope or with super- and subadditivity costs over all relevant output ranges.[2] Results are thus biased from the outset when inflexible forms are estimated. According to a study by Guilkey, Lovell, and Sickles [15], the translog functional form performed as well as the extended generalized Cobb-Douglas and better than the generalized Leontief (two other flexible forms). However, in a recent paper by Barnett and Lee [1], the minflex Laurent functional form was shown to have, in general, a larger regular region than the generalized Leontief or the translog.[3] A problem with the translog, which will be dealt with in Section IV below, is that cost must equal zero when any output level is equal to zero.[4] Despite these drawbacks, the flexibility of the translog has made it a frequent choice in other multiproduct cost studies, and it is estimated here to facilitate the comparison of the results of this study with those of previous studies of financial firms.[5]

Two different models of branching are estimated, and, in both, the branching variable is allowed to interact with *all* other variables instead of being restricted

[2] These concepts are defined in Section I.

[3] The regular region is that region over which the cost function is well defined, i.e., nondecreasing and concave in input prices. Barnett and Lee [1] are concerned with the estimation of indirect utility functions, and so the regular region is defined as the region over which the indirect utility function is monotonically decreasing and quasiconvex in prices. Their results should follow through with cost functions.

[4] Note that, when the minflex Laurent cost function is specified in logarithms, it also suffers from this problem.

[5] The hybrid translog function, which is obtained by substituting $(y_i^\lambda - 1)/\lambda$ for $\ln y_i$, $\forall\ i$, in the translog function, was also estimated. With the hybrid translog, cost is not necessarily zero when any output level is zero. But this functional form is more difficult to analyze because many of its cost properties cannot be represented as tractable functions of its parameters.

to enter the cost function linearly as in other studies. Thus, the cost of a branch is not restricted to be a constant percentage of the firm's total operating cost.

An important improvement of this study over previous multiproduct studies of financial and other types of firms is that approximate standard errors are calculated for *all* the statistics given to characterize the cost function, not only for statistics that are linear functions of the estimated parameters. Since previous studies omitted standard errors and based some conclusions on numerical values alone, their results cannot be considered reliable. Although Murray and White [20] calculate standard errors for their measure of cost complementarity, their measure may be inaccurate, as discussed below. They do not give standard errors for the elasticities of substitution they report. Gilligan, Smirlock, and Marshall [14] and Gilligan and Smirlock [13] measure the degree of scope economies but do not give the standard errors of their measurements. Gilligan, Smirlock, and Marshall [14] also report the elasticities of cost with respect to the various outputs and marginal costs without their standard errors. Marginal costs also appear in Benston, Berger, Hanweck, and Humphrey [5] without standard deviations, as do the measures of cost complementarity between the output pairs.

This paper attempts to test for subadditivity of the cost structure in a more comprehensive way than has previously been done. Rather than only measuring relationships between pairwise combinations of outputs, output combinations are evaluated together, and conditions that imply subadditivity are tested at more than one point on the cost surface. Some of the tests used in previous studies to characterize the cost structure were incorrect. As discussed below, the test used in Gilligan, Smirlock, and Marshall [14] and Gilligan and Smirlock [13] and the test used in Murray and White [20] for economies of scope are improper. Also, the conditions reported as sufficient for subadditivity in Gilligan, Smirlock, and Marshall [14] are incorrect.

In this study, no evidence in support of natural monopoly is found. In Section V the results of this paper are compared with results of four previous studies of depository institutions that utilize the multiproduct approach: Gilligan, Smirlock, and Marshall [14], Gilligan and Smirlock [13], and Benston, Berger, Hanweck, and Humphrey [5] for commercial banks and Murray and White [20] for credit unions.[6]

The rest of this paper is divided into five sections. Section I discusses the multiproduct cost concepts utilized. Section II describes the models and data; Section III discusses the estimation procedure. Empirical results are given in Section IV, and Section V is the conclusion.

I. Cost Concepts for the Single and the Multiproduct Firm

S&Ls are assumed to choose output quantities and input quantities simultaneously, taking the production technology and input prices as given, to maximize profits. The choice can be viewed as a two-stage process. The firm considers any

[6] Subsequent to the completion of the study described in this paper, three additional multiproduct studies of financial firms have appeared. These are Berger, Hanweck, and Humphrey [6] and Lawrence and Shay [16] on commercial bank costs and LeCompte and Smith [17] on savings and loan costs.

level of output and chooses inputs to minimize the cost of producing that output level. It then chooses output to maximize profits. A standard result of duality theory is that the properties of the production transformation can be derived from the cost function. Thus, the minimum efficient scale of the industry can be determined by analyzing the cost function.

A cost function $C(\mathbf{y})$ is said to be strictly subadditive at output vector $\mathbf{y}$ if, for all quantities $\mathbf{y}^1, \cdots, \mathbf{y}^k, \mathbf{y}^j \neq \mathbf{y}, j = 1, \cdots, k$ such that $\sum_{j=1}^{k} \mathbf{y}^j = \mathbf{y}$, $C(\mathbf{y}) < \sum_{j=1}^{k} C(\mathbf{y}^j)$.[7] That is, it is more expensive for two or more firms to supply $\mathbf{y}$ than for one firm. If its cost function is subadditive over the relevant range of outputs, the industry is a natural monopoly; i.e., it is most efficient for one firm to supply the industry output.

When the cost structure of a *single*-product firm is analyzed, emphasis is placed on the degree of scale economies that exist. Increasing returns to scale at a point are sufficient for subadditivity at a point in the single-product case. This is significant since subadditivity is a *global* concept.[8] That is, to determine whether a cost function is subadditive at y, it is necessary to know the value of the function at *all* $y^* < y$. However, determination of the degree of scale economies at y requires knowledge of the function only in a neighborhood of y. In the multiproduct case, however, there are no necessary and sufficient conditions for subadditivity that are easier to check than the definition of subadditivity itself. To determine subadditivity at $\mathbf{y}$, information on the cost function for all $\mathbf{y}^* < \mathbf{y}$ is needed.

Multiproduct cost functions do not have a natural scalar quantity over which costs may be averaged, but the analogue to average cost is given by *ray average cost* (RAC), which gives the average cost of a proportionate increase in all outputs. The degree of multiproduct scale economies defined over the entire product set $(y_1, \cdots, y_n) = \mathbf{y}$ is given by $S(\mathbf{y}) = C(\mathbf{y})/\mathbf{y} \times \nabla C(\mathbf{y}) = C(\mathbf{y})/\sum_{i=1}^{n} y_i C_i(\mathbf{y})$, where $C_i(\mathbf{y}) = \partial C(\mathbf{y})/\partial y_i$.[9] Returns to scale are increasing, constant, or decreasing as S is greater than, equal to, or less than 1. Increasing returns to scale at $\mathbf{y}$ implies decreasing ray average costs (DRAC) at $\mathbf{y}$, analogous to the single-product case. These concepts concern proportional increases in all outputs. Changes in the quantities of a subset T of the N outputs holding other output quantities constant must also be considered. The degree of scale economies specific to product set T is defined as $s_T(\mathbf{y}) = IC_T(\mathbf{y})/\sum_{j\in T} y_j C_j(\mathbf{y})$, where $IC_T(\mathbf{y})$ is the incremental cost of producing the outputs in T, i.e., the rise in cost when these outputs are produced in positive quantities. Product-specific economies of scale are increasing, constant, or decreasing as $s_T(\mathbf{y})$ is greater than, equal to, or less than 1.

In addition to changes in the quantities of outputs produced, the firm must also select which outputs to produce. Economies of scope measure economies of joint production. The degree of economies of scope at $\mathbf{y}$ relative to product set T is given by

[7] In the following discussion, the input price vector will be suppressed and the cost function written as $C(\mathbf{y})$. All definitions are from Baumol, Panzar, and Willig [2].

[8] Baumol, Panzar, and Willig [2, p. 171].

[9] $\nabla C(\mathbf{y}) \equiv (\partial C(\mathbf{y})/\partial y_1, \cdots, \partial C(\mathbf{y})/\partial y_n)$ is the gradient of C.

$$SC_T = \frac{C(\mathbf{y}_T) + C(\mathbf{y}_{N-T}) - C(\mathbf{y})}{C(\mathbf{y})},$$

where $\mathbf{y}_T$ is the vector with a zero component in place of y_i for all $i \notin T$ and $\mathbf{y}_{N-T}$ is the vector with a zero component in place of y_i for all $i \in T$. It measures the relative increase in cost if $\mathbf{y}$ were produced in two groups T and $N - T$. There exist economies of scope if $SC_T > 0$ and diseconomies of scope if $SC_T < 0$. Economies of scope are necessary for subadditivity. However, increasing returns to scale together with economies of scope are not sufficient for subadditivity.

Conditions sufficient for subadditivity can be obtained by combining economies of scale with forms of cost complementarity stronger than economies of scope. One set of sufficient conditions for subadditivity at $\mathbf{y}$ that can be tested is the existence of (nonstrict) transray convexity along any one hyperplane through $\mathbf{y}$ and strictly DRAC up to the hyperplane.[10] A cost function is transray convex at $\mathbf{y}$ if there is at least one negatively sloping cross-section such that costs toward the interior (costs of mixtures of output) are less than or equal to costs at the extremes (costs of producing the commodities separately).[11] Subadditivity at $\mathbf{y}$ is also implied by the existence of multiproduct economies of scale together with cost convexity since DRAC is implied by economies of scale and transray convexity is implied by cost convexity.[12]

[10] The authors' claim in Gilligan, Smirlock, and Marshall [14, p. 395] that DRAC together with economies of scope is sufficient for subadditivity is incorrect. Decreasing *average incremental cost* and economies of scope imply subadditivity. DAIC, unlike DRAC, in a product line implies that that product line must be monopolized to minimize costs.

[11] These conditions can be explained intuitively. DRAC at $\mathbf{y}$ implies that it is not possible to divide the firm's output vector $\mathbf{y}^0$ proportionately among several firms without increasing total costs. But consider other partitions $\mathbf{y}^1$, $\mathbf{y}^2$ such that $\mathbf{y}^1 + \mathbf{y}^2 = \mathbf{y}^0$ with $\mathbf{y}^1$ and $\mathbf{y}^2$ on different rays. If $\mathbf{y}^1$ and $\mathbf{y}^2$ are expanded proportionately to the hyperplane, costs are lowered because of DRAC, and then, if $\mathbf{y}^0$ is obtained by taking a weighted average of these expanded vectors, costs are lowered or remain constant because of transray convexity. Thus, no partitioning would lower costs, and the cost function is subadditive at $\mathbf{y}$. This proof is in Baumol, Panzar, and Willig [2, p. 178–79].

[12] When the cost function is a function of two outputs (nonstrict) transray convexity is implied by

$$C_{ii} \geq 0, \quad C_{jj} \geq 0, \quad C_{ij} \leq 0, \qquad i = 1, \; j = 2,$$

i.e., by increasing incremental costs and weak cost complementarities. Transray convexity (nonstrict) is also implied by

$$C_{ii} \leq 0, \quad C_{jj} \leq 0, \quad C_{ij} = C_{ji} < 0, \quad C_{ij} \leq -\,(C_{ii}C_{jj})^{1/2}, \qquad i = 1, \; j = 2.$$

If the cost function is a function of more than two outputs, either set of conditions implies (nonstrict) pairwise transray convexity between outputs i and j.

Evans and Heckman [10] have suggested an alternative test for subadditivity that involves comparing the cost of single-firm production with two-firm production at output levels within an "admissible region." If two-firm production is less costly than single-firm production, there cannot be subadditivity. Evans and Heckman indicate that their criterion tests only necessary conditions for subadditivity as long as the admissible region is a proper subset of the region of possible output levels. However, the test is not sufficient even within the admissible region, because a finding that single-firm production is less costly than two-firm production does not necessarily imply that single-firm production is less costly than n-firm production, $\forall\, n > 2$.

II. Model Specification and Data

The cost function of S&Ls is assumed to be of the translog form. Data from the semiannual financial reports of 149 S&Ls operating in California in 1982 are utilized.[13] Of these 149 S&Ls, two are state-chartered mutuals, eighty-nine are state-chartered stock firms, forty-one are federally chartered mutuals, and seventeen are federally chartered stock firms. Average asset size is $683,008,775. Total costs, C, include all labor and capital expenses and interest payments net of service charges, i.e., total operating and interest costs. As S&Ls are financial intermediaries, it is assumed that economic output is best measured by the dollar value of earning assets of the firm, with inputs being labor, capital, and deposits.[14] Three outputs are considered: y_1 = mortgage loans, y_2 = other loans, and y_3 = cash, securities, and real estate investments in excess of the minimum liquidity requirements. Each y_i is measured as the average of its dollar amount at the end of 1982 and the dollar amount at the end of 1981. Four inputs are considered: labor, capital, passbook and NOW accounts (demand deposits), and accounts earning in excess of the regular rate (term deposits).[15] An S&L-specific wage w_1 is proxied by $\sum_i (b_i/\sum b_j)\omega_i$, where b_i = number of branches the S&L has in county i, $\sum b_j$ = total number of branches the S&L has in all counties, and ω_i = wage rate in county i.[16] A crude unit price of capital, w_2, is constructed as rent, depreciation, utilities, equipment, and furniture expenditure divided by average dollars of deposits during 1982.[17] Prices for the two deposit inputs are the effective rates of interest paid on each dollar of deposits. The price of passbook and NOW accounts, w_3, is total interest and commissions paid on these accounts, minus service charges paid on NOW accounts, divided by average dollars of deposits in these accounts during 1982.[18] The price of accounts earning greater than the

[13] The S&Ls included are those that operated the entire year. The seventeen S&Ls that began or ceased operating during 1982 are excluded. Also, five S&Ls are excluded because of large discrepancies between data provided by the Federal Home Loan Bank Board and data provided by the Federal Home Loan Bank of San Francisco.

[14] Sealey and Lindley [21] advocate this approach, and Gilligan and Smirlock [13] attain the best fit using this definition of bank output, although they omit deposits as inputs. Other studies (e.g., Benston, Berger, Hanweck, and Humphrey [5] and Gilligan, Smirlock, and Marshall [14]) treat deposits as outputs of the banking firm and exclude interest expenses when measuring costs.

[15] This degree of aggregation for outputs and inputs is chosen to conserve on degrees of freedom. For example, if an additional output were included, the number of unrestricted parameters in model I, described below, would increase from 37 to 46.

[16] Ideally the price of labor should be a wage rate that takes into account the different types of labor utilized at the S&L (full and part time, salaried and per hour) and includes employee benefits as well as explicit wages. S&Ls that have more than one office may operate in more than one market, making determination of a single wage rate difficult. Data were limited to labor expenses over all branches. No data on number of employees per S&L were available. The weighting scheme used tries to take into account multiple market operations and assumes that employment is more closely related to number of branches than, say, to number or size of accounts.

[17] A preferable measure of w_2 would have been average rental cost per square foot of office space over all markets in which the S&L operates. However, this was available for only seven metropolitan areas of California.

[18] Service charges were imposed by 134 of the S&Ls on NOW accounts. No service charges were imposed on other accounts. On average, for the S&Ls that collected these charges, they were less than 0.38% of the interest paid out on passbook and NOW accounts. When costs and price of passbook and NOW accounts were calculated ignoring service charges and when the cost functions were reestimated, none of the results reported below was altered.

regular rate, w_4, is total interest paid on these accounts divided by average dollars of deposits in these accounts during 1982.[19] A variable *GROWTH*, defined as ln[assets in 1982 divided by assets in 1981], is included to control for any short-run disequilibrium effects on cost behavior.[20] Table I gives the minimum, maximum, and mean values of the variables for the sample.

Two different models for branching are estimated.[21] In model I, branching is treated as a technological condition of production, i.e., a characteristic of the firm's technology. The branching variable, *BR*, is allowed to interact with all other variables in the cost function. In this model, statistics characterizing the relationship between costs, outputs, and input prices are measured holding the number of branches constant. The elasticity of cost with respect to the number of branches can be calculated. In model II, branching is viewed as a function of the levels of outputs produced. The S&L can increase output by opening more branches. The structural model consists of equation (1) in Section III and the equation $\ln BR = q_0 + \sum_{i=1}^{3} q_i \ln y_i$. Substituting this branching equation into the cost equation (1) yields a reduced-form equation that is estimated. In this model, the number of branches is not held constant when relationships between costs, outputs, and input prices are calculated.[22]

III. Estimation Procedure

Maximum-likelihood estimates are obtained by estimating the cost equation with three of the four cost-share equations using the iterative seemingly unrelated regression equations (SURE) technique. The share equation corresponding to the fourth input is omitted.[23] Restrictions implying homogeneity of degree one in input prices and symmetry are imposed.

Model I:

$$\begin{aligned}\ln C = {} & A_0 + \textstyle\sum_{i=1}^{3} A_i \ln y_i + \sum_{j=1}^{4} B_j \ln w_j + \tfrac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} S_{ij} \ln y_i \ln y_j \\ & + \tfrac{1}{2} \textstyle\sum_{j=1}^{4} \sum_{k=1}^{4} G_{jk} \ln w_j \ln w_k + \sum_{i=1}^{3} \sum_{j=1}^{4} D_{ij} \ln y_i \ln w_j + E \ln BR \\ & + \textstyle\sum_{i=1}^{3} F_i \ln y_i \ln BR + \sum_{j=1}^{4} H_j \ln BR \ln w_j + \tfrac{1}{2} K \ln BR \ln BR \qquad (1) \\ & + L_1\, GROWTH + u.\end{aligned}$$

$$S_j = B_j + \textstyle\sum_{k=1}^{4} G_{jk} \ln w_k + \sum_{i=1}^{3} D_{ij} \ln y_i + H_j \ln BR + u_j \quad j = 1, 2, 3,$$

[19] The fact that some S&Ls may operate in different markets and therefore face different explicit interest rates cannot be incorporated into the deposit rate proxies because of lack of data on the volume of deposits an S&L obtains from each market.

[20] An anonymous referee has suggested that *GROWTH* may be a proxy for regulatory interference. The inclusion of dummy variables to distinguish between different charter types doubles the number of parameters, making the estimates very imprecise. In current research, I am using broader categories for outputs and inputs in order to test whether S&Ls with different types of charters have different production technologies.

[21] See Section III for the equations estimated.

[22] An anonymous referee has suggested that, if deposits were being treated as an output, the structural model could consist of equation (1) and an equation relating deposits to the number of branches and factor prices.

[23] A share equation must be omitted; otherwise, the variable-covariance matrix is singular because the shares sum to one. Iterative SURE dominates noniterative SURE because maximum-likelihood estimates are obtained and the estimates are invariant to which share equation is omitted.

Table I

Values of Variables

Variable	Minimum Value	Mean Value	Maximum Value
C Total Cost	548972.00	59251890.00	1053003211.00
S_1 Labor Share	.04574	.145	.52
S_2 Capital Share	.01002	.065	.19
S_3 Passbook and NOW Account Deposit Share	.01505	.081	.19
S_4 In-Excess-of-Regular-Rate Account Deposit Share	.3424	.709	.90
y_1 Mortgage Loans	1533497.00	514407504.963	9369926413.00
y_2 Other loans	8649.00	15763372.503	338024580.00
y_3 Cash, Securities, Real Estate Investments	727168.60	54933794.289	1324985942.50
w_1 Price of Labor	4.77	7.511	10.69
w_2 Price of Capital	.00144	.010	.04
w_3 Price of NOW and Passbook Account Deposits	.01389	.050	.14
w_4 Price of In-Excess-of-Regular-Rate Account Deposits	.02902	.133	.26
BR Number of Offices	1.0	17.362	184.00
$GROWTH$ Disequilibrium	−.097	.439	3.32

with

$$\left.\begin{aligned} S_{ij} &= S_{ji} \quad i, j = 1, 2, 3 \\ G_{jk} &= G_{kj} \quad j, k = 1, 2, 3, 4 \end{aligned}\right\} \text{symmetry}$$

$$\left.\begin{aligned} &\textstyle\sum_{j=1}^{4} B_j = 1 \\ &\textstyle\sum_{k=1}^{4} G_{jk} = 0 \quad j = 1, 2, 3, 4 \\ &\textstyle\sum_{j=1}^{4} D_{ij} = 0 \quad i = 1, 2, 3 \\ &\textstyle\sum_{j=1}^{4} H_j = 0 \end{aligned}\right\} \text{homogeneity}$$

where

C = total cost
S_1 = labor share
S_2 = capital share
S_3 = passbook and NOW account share
S_4 = accounts earning in excess of the regular rate share
y_1 = mortgage loans
y_2 = other loans
y_3 = cash, securities, and real estate investments
w_1 = price of labor
w_2 = price of capital
w_3 = price of passbook and NOW accounts
w_4 = price of accounts earning in excess of the regular rate
BR = number of offices
$GROWTH$ = measure of disequilibrium (as defined in Section II).

Model II:

Model II is model I with the additional restrictions:

$$F_i = 0 \qquad i = 1, 2, 3$$

$$E = 0$$

$$K = 0$$

$$H_j = 0 \qquad j = 1, 2, 3, 4$$

(Greek letters comparable to the letters used to specify the parameters in model I will refer to the parameters of model II.)

By placing additional restrictions on the parameters of the translog, it can be made to correspond to a more restrictive production technology. The restrictive forms can be estimated, and, because maximum-likelihood estimates are obtained, the likelihood-ratio test can be used to test the restricted vs. unrestricted technology. Five hypotheses are tested for each model: homotheticity (marginal rates of substitution in production are independent of scale effects and depend only on relative prices), homogeneity with respect to output (a t-fold increase in all inputs leads to a t^k increase in output where k is the degree of homogeneity), unitary elasticity of substitution between inputs, generalized Cobb-Douglas (unitary elasticity of substitution together with homogeneity), and globally constant returns to scale.

IV. Empirical Results

The estimated parameters and their standard errors and t-statistics are given in Table II for model I and Table III for model II. Goodness-of-fit measurements for the cost equation and the three estimated share equations are given in Table IV for both models.[24]

The structural tests of the cost function of both models I and II indicate that each restricted functional form of the production technology can be rejected, and quite strongly. (See Table V.) Thus, use of the flexible function form to estimate the cost structure is appropriate.

Using the parameter estimates of the cost function, it is possible to investigate the production structure of the S&L industry. Since homotheticity was rejected, the relationship between costs, outputs, input prices, and branches cannot be characterized globally. The effects on costs of changes in the variables usually differ depending on the levels of the variables. The measures of these effects are evaluated at the point consisting of the sample means of the output, input price, branch, and *GROWTH* variables (the "typical" firm), the point consisting of the minimum values of output variables with mean values of input price, branch,

[24] The low R^2 on the demand-deposit share equation may indicate that passbook and NOW accounts should be treated as separate outputs; however, disaggregation leads to too large a loss of degrees of freedom. Alternatively, it may indicate that the interest rate proxy is inaccurate. However, as explained above, data limitations make this the only available proxy.

Using the Goldfeld-Quandt test and ordering the observations by the size of mortgage loans, the null hypothesis of homoscedastic disturbances cannot be rejected.

Table II
Model I Parameter Estimates

Parameter	Variable[a]	Estimate	Approximate Standard Error	T-Ratio	Approximate Prob > $\lvert T \rvert$
A_0	Constant	−2.3007	4.7042	−.49	.6257
A_1	$\ln y_1$	.76590	.66959	1.14	.2549
A_2	$\ln y_2$	.36144	.30748	1.18	.2421
A_3	$\ln y_3$	.18391	.42479	.43	.6658
B_1^*	$\ln w_1$	.79111	.10271	7.70	.0001
B_2^*	$\ln w_2$	.19146	.024936	7.68	.0001
B_3	$\ln w_3$	.095565	.065141	1.47	.1447
B_4 $(= 1 - B_1 - B_2 - B_3)$	$\ln w_4$	−.078125	.11161	−.70	.4840
S_{11}	$\ln y_1 \ln y_1$	.10508	.063805	1.65	.1021
S_{12}	$\ln y_1 \ln y_2$	−.0093747	.027636	−.34	.7325
S_{13}^*	$\ln y_1 \ln y_3$	−.10089	.038607	−2.61	.0101
S_{22}	$\ln y_2 \ln y_2$	.0019589	.019390	.10	.9186
S_{23}	$\ln y_2 \ln y_3$	−.012694	.023919	−.53	.5966
S_{33}^*	$\ln y_3 \ln y_3$	.12539	.046724	2.68	.0083
G_{11}^*	$\ln w_1 \ln w_1$	.041805	.017448	2.40	.0180
G_{12}	$\ln w_1 \ln w_2$	.0057757	.0038770	1.49	.1386
G_{13}^*	$\ln w_1 \ln w_3$	.020438	.0084852	2.41	.0173
G_{14}^* $(= -G_{11} - G_{12} - G_{13})$	$\ln w_1 \ln w_4$	−.068019	.016304	−4.17	.0000
G_{22}^*	$\ln w_2 \ln w_2$	.051117	.0021490	23.79	.0001
G_{23}^*	$\ln w_2 \ln w_3$	−.0092905	.0026046	−3.57	.0005
G_{24}^* $(= -G_{12} - G_{22} - G_{23})$	$\ln w_2 \ln w_4$	−.047602	.0040372	−11.79	.0000
G_{33}	$\ln w_3 \ln w_3$	.0085480	.0077873	1.10	.2743
G_{34}^* $(= -G_{13} - G_{23} - G_{33})$	$\ln w_3 \ln w_4$	−.019696	.0095326	−2.07	.0384
G_{44}^* $(= G_{11} + G_{22} + G_{33} + 2G_{12} + 2G_{13} + 2G_{23})$	$\ln w_4 \ln w_4$	.13532	.020332	6.66	.0001
D_{11}^*	$\ln y_1 \ln w_1$	−.051828	.0070522	−7.35	.0001
D_{12}^*	$\ln y_1 \ln w_2$	−.0042190	.0020485	−2.06	.0414
D_{13}	$\ln y_1 \ln w_3$	.0042825	.0043339	.99	.3249
D_{14}^* $(= -D_{11} - D_{12} - D_{13})$	$\ln y_1 \ln w_4$	.051765	.0061615	8.40	.0000
D_{21}	$\ln y_2 \ln w_1$	−.0015873	.0029949	−.53	.5970

D_{22}	$\ln y_2 \ln w_2$	−.0009829	.00096437	−1.02	.3100
D_{23}	$\ln y_2 \ln w_3$	.0007076	.0024640	.29	.7744
D_{24} $(= -D_{21} - D_{22} - D_{23})$	$\ln y_2 \ln w_4$	.0018625	.0023632	.79	.4296
D_{31}	$\ln y_3 \ln w_1$	.0074082	.0043919	1.69	.0940
D^*_{32}	$\ln y_3 \ln w_2$	.0045033	.0016009	2.81	.0056
D^*_{33}	$\ln y_3 \ln w_3$	−.031685	.0042630	−3.21	.0017
D_{34} $(= -D_{31} - D_{32} - D_{33})$	$\ln y_3 \ln w_4$	.0017735	.0044567	.40	.6892
E	$\ln BR$	−.19110	.76678	−.25	.8036
F_1	$\ln y_1 \ln BR$	.0033647	.058558	.06	.9543
F_2	$\ln y_2 \ln BR$	.019190	.026650	.72	.4728
F_3	$\ln y_3 \ln BR$	−.021682	.039605	−.55	.5851
H^*_1	$\ln BR \ln w_1$	.037473	.0074007	5.06	.0001
H_2	$\ln BR \ln w_2$	.0008100	.0021356	.38	.7051
H^*_3	$\ln BR \ln w_3$	.010728	.0051714	2.07	.0400
H^*_4 $(= -H_1 - H_2 - H_3)$	$\ln BR \ln w_4$	−.049011	.0087055	−5.63	.0000
K	$\ln BR \ln BR$	.022870	.070857	.32	.7474
L_1	$GROWTH$	−.0096999	.022304	−.43	.6644

[a] y_1 = mortgage loans; y_2 = other loans; y_3 = cash, securities, and real estate investments; w_1 = price of labor; w_2 = price of capital; w_3 = passbook and NOW (demand deposit) account price; w_4 = in-excess-of-regular-rate (term deposit) account price; BR = number of offices; $GROWTH = \ln \dfrac{\text{total assets in 1982}}{\text{total assets in 1981}}$.

* Significant at .05 level.

Table III
Model II Parameter Estimates

Parameter	Variable[a]	Estimate	Approximate Standard Error	T-Ratio	Aproximate Prob > $\|T\|$
α_0	Constant	1.0095	1.31536	.77	.4442
α_1^*	$\ln y_1$	.51494	.24203	2.13	.0353
α_2	$\ln y_2$	.11471	.15586	.74	.4631
α_3	$\ln y_3$	.29496	.24970	1.18	.2387
β_1^*	$\ln w_1$	.43835	.083664	5.24	.0001
β_2^*	$\ln w_2$	.16138	.018850	8.56	.0001
β_3	$\ln w_3$	.0069270	.046259	.15	.8812
β_4^* $(= 1 - \beta_1 - \beta_2 - \beta_3)$	$\ln w_4$	.39334	.081997	4.78	.0000
σ_{11}^*	$\ln y_1 \ln y_1$	.10417	.043955	2.37	.0193
σ_{12}	$\ln y_1 \ln y_2$	.0005262	.02534	.02	.9835
σ_{13}^*	$\ln y_1 \ln y_3$	−.098259	.036740	−2.67	.0084
σ_{22}	$\ln y_2 \ln y_2$	−.0073061	.020098	−.36	.7168
σ_{23}	$\ln y_2 \ln y_3$	.0000265	.023012	.00	.9991
σ_{33}^*	$\ln y_3 \ln y_3$	.10222	.047009	2.17	.0315
γ_{11}	$\ln w_1 \ln w_1$	.015846	.017573	.90	.3688
γ_{12}^*	$\ln w_1 \ln w_2$	.010015	.0037496	2.67	.0085
γ_{13}	$\ln w_1 \ln w_3$	.0083807	.0084043	1.00	.3204
γ_{14}^* $(= -\gamma_{11} - \gamma_{12} - \gamma_{13})$	$\ln w_1 \ln w_4$	−.034241	.016644	−2.06	.0394
γ_{22}^*	$\ln w_2 \ln w_2$	.053570	.0019695	27.20	.0001
γ_{23}^*	$\ln w_2 \ln w_3$	−.0091561	.0026110	−3.51	.0006
γ_{24}^* $(= -\gamma_{12} - \gamma_{22} - \gamma_{23})$	$\ln w_2 \ln w_4$	−.054429	.0040162	−13.55	.0000
γ_{33}	$\ln w_3 \ln w_3$	.0075678	.0077957	.97	.3334
γ_{34} $(= -\gamma_{13} - \gamma_{23} - \gamma_{33})$	$\ln w_3 \ln w_4$	−.0067924	.0097216	−.70	.4840
γ_{44}^* $(= \gamma_{11} + \gamma_{22} + \gamma_{33} + 2\gamma_{12} + 2\gamma_{13} + 2\gamma_{23})$	$\ln w_4 \ln w_4$	.095462	.02088	4.57	.0000
δ_{11}^*	$\ln y_1 \ln w_1$	−.015108	.0037271	−4.05	.0001
δ_{12}^*	$\ln y_1 \ln w_2$	−.0035207	.0016669	−2.11	.0365
δ_{13}^*	$\ln y_1 \ln w_3$	.0086744	.0037701	2.30	.0229
δ_{14}^* $(= -\delta_{11} - \delta_{12} - \delta_{13})$	$\ln y_1 \ln w_4$	.0099543	.0019913	5.00	.0000

δ^*_{21}	$\ln y_2 \ln w_1$	−.0068276	.0023730	−2.88	.0047
δ_{22}	$\ln y_2 \ln w_2$	−.0013466	.0010032	−1.34	.1817
δ_{23}	$\ln y_2 \ln w_3$	−.00067923	.0025807	−.26	.7928
δ^*_{24} $(= -\delta_{21} - \delta_{22} - \delta_{23})$	$\ln y_2 \ln w_4$	.0088534	.0029306	3.02	.0025
δ_{31}	$\ln y_3 \ln w_1$	.0032259	.0029832	1.08	.2814
δ^*_{32}	$\ln y_3 \ln w_2$	.0053680	.0015318	3.50	.0006
δ^*_{33}	$\ln y_3 \ln w_3$	−.0077596	.0038038	−2.04	.0433
δ_{34} $(= -\delta_{31} - \delta_{32} - \delta_{33})$	$\ln y_3 \ln w_4$	−.0008343	.0045572	−.18	.8572
λ_1	*GROWTH*	−.0056689	.022855	−.25	.8045

[a] y_1 = mortgage loans; y_2 = other loans; y_3 = cash, securities, and real estate investments; w_1 = price of labor; w_2 = price of capital; w_3 = passbook and NOW (demand deposit) account price; w_4 = in-excess-of-regular-rate (term deposit) account price; $GROWTH = \ln \frac{\text{total assets in 1982}}{\text{total assets in 1981}}$.

* Significant at .05 level.

Table IV

Models I and II Goodness-of-Fit Measurements

	Sum of Squared Errors (SSE)	Degrees of Freedom (DF)	Mean Square Error (MSE = SSE/DF)	R^2
Model I				
Cost Equation	2.85594	123	.023219	.9940
Labor Cost Share	.336665	145	.00232183	.5783
Capital Cost Share	.0187681	145	.000129435	.9010
Passbook and NOW Account Cost Share	.143822	145	.000991877	.0604
Model II				
Cost Equation	3.5041	129	.0271636	.9927
Labor Cost Share	.388518	146	.00266108	.5134
Capital Cost Share	.0202783	146	.000138892	.8930
Passbook and NOW Account Cost Share	.149226	146	.0010221	.0251

Table V

Structural Tests

	Determinant of Disturbance Covariance Matrix $\|\Omega\|$	Test Statistic $-2 \ln LR = -2 \ln\left(\frac{\|\Omega_R\|}{\|\Omega_U\|}\right)^{-T/2}$ where $T = 149$	Degrees of Freedom	Critical Value at .001 Confidence Level
Model I				
Unrestricted Cost Function	$.3943 \times 10^{-11}$			
Homotheticity	$.8737 \times 10^{-11}$	117.7	12	32.909
Homogeneity in Output	$.9760 \times 10^{-11}$	134.1	21	46.797
Unitary Elasticity of Substitution	$.2290 \times 10^{-10}$	260.4	6	22.457
Generalized Cobb-Douglas	$.6031 \times 10^{-10}$	406.41	27	55.476
Global Constant Returns to Scale	$.6899 \times 10^{-11}$	82.79	9	27.877
Model II				
Unrestricted Cost Function	$.5999 \times 10^{-11}$			
Homotheticity	$.1247 \times 10^{-10}$	108.4	12	32.909
Homogeneity in Output	$.1381 \times 10^{-10}$	123.4	18	42.312
Unitary Elasticity of Substitution	$.4050 \times 10^{-10}$	282.7	6	22.457
Generalized Cobb-Douglas	$.7895 \times 10^{-10}$	384.0	24	51.179
Global Constant Returns to Scale	$.9183 \times 10^{-11}$	63.03	8	26.125

and *GROWTH* variables (the "small" firm), and the point consisting of the maximum values of output variables with mean values of input price, branch, and *GROWTH* variables (the "large" firm).[25] Most of the statistics calculated are not linear functions of the estimated parameters, so exact standard deviations

[25] Alternatively, if the production structure were found to be homothetic within various size categories, the cost effects of changes in the variables could be measured for these endogenously determined categories. I would like to thank an anonymous referee for this suggestion.

cannot be calculated. Following Fuller [11], the approximate standard deviation of a statistic that is a nonlinear function of the estimated parameters is obtained by expanding the statistic in a Taylor series of the parameters, dropping terms of order of two or more, and solving for the standard deviation of this approximation. The approximate variance-covariance matrix of a set of statistics $x_1, \cdots, x_m$ that are functions of the estimated parameters $a_1, \cdots, a_r$ is given by DVD', where

$$D = \begin{bmatrix} \partial x_1/\partial a_1 & \cdots & \partial x_1/\partial a_r \\ \vdots & & \vdots \\ \partial x_m/\partial a_1 & \cdots & \partial x_m/\partial a_r \end{bmatrix}$$

evaluated at the point estimates of $a_1, \cdots, a_r$, and V is the variance-covariance matrix of the parameters.

A. Subadditivity

In model I, with branching held constant, the degree of global economies of scale, S, is given by $S = 1/\sum_{i=1}^{3} (A_j + \sum_{k=1}^{3} S_{ik}\ln y_k + \sum_{j=1}^{4} D_{ij}\ln w_j + F_i\ln BR)$.[26] There is evidence that S increases as firms grow larger, with "small" firms exhibiting diseconomies and "typical" and "large" firms exhibiting economies.[27] (See Table VI.) Only the "typical" firm's S is significantly greater than one at the .05 level.[28] Considering the wide range in output levels among S&Ls in the sample, any difference in scale economies for small and large S&Ls is very slight, implying no significant cost advantage due to the overall scale of operation. In model II, $S = 1/\sum_{k=1}^{3} (\alpha_i + \sum_{k=1}^{3} \sigma_{ik}\ln y_k + \sum_{j=1}^{4} \delta_{ij}\ln w_j)$, and the degree of scale economies decreases as firms grow larger, with all size firms exhibiting economies. Only the "typical" firm's S is significantly different from one at the .05 level, and, again, any difference between S's for different size firms is very slight compared with the range of output. Thus, even when expansion by branching is taken into account, there appear to be only small gains possible from increasing the scale of operation.

Cost convexity is a sufficient condition for the existence of transray convexity and can be determined by examining the matrix of second derivatives of the cost function with respect to output $[C_{ij}]$. Cost convexity exists if $[C_{ij}]$ is positive definite. Table 9 in Mester [19] gives the matrix with standard errors for models I and II, for "small," "typical," and "large" firms. Only the C_{11} and C_{33} entries in

[26] S = 1/sum of individual cost elasticities. Gilligan, Smirlock, and Marshall [14, p. 403] claim that the sum of individual cost elasticities is a deficient measure for scale economies compared with the elasticity of RAC. This is incorrect since the sum of the cost elasticities is equal to 1 + elasticity of RAC.

[27] The degree of scale economies was also computed for "small" firms operating with the minimum number of branches and "large" firms operating with the maximum number of branches. The pattern is the same as when the mean number of branches is used.

[28] Since $1/S$ is a linear function of the parameters, its standard deviation is exact, while the standard deviation of S is approximate; t-tests involving $1/S$ rather than S are used because they are more accurate. If the t-statistic is calculated using S, then S is insignificantly different from 1 at the .05 level.

Table VI

Degree of Scale Economies S

	S	Approximate Standard Deviation of S[a]	$1/S$	Standard Deviation of $1/S$	$\frac{1 - 1/S}{\text{Standard Deviation of } 1/S}$
Model I					
"Small" Firm[b]	.9597	.1852	1.0420	.2011	−.2088
"Typical" Firm[c]	1.0963	.0506	.9121	.0421	2.0866
"Large Firm"[d]	1.1453	.1666	.8731	.1270	.9993
"Small Firm" with Few Branches[e]	.9620	.08085	1.0395	.08736	−.4520
"Large Firm" with Many Branches[f]	1.1426	.09522	.8752	.07293	1.7117
Model II					
"Small" Firm[b]	1.0730	.05317	.9319	.04618	1.4741
"Typical" Firm[c]	1.0685	.02155	.9359	.01888	3.3954
"Large Firm"[d]	1.0564	.0432215	.9466	.03873	1.3791

[a] Standard deviation of $S \approx S^2 \times$ standard deviation of $1/S$.

[b] S evaluated at mininum values of outputs, mean values of input prices and *GROWTH* (and mean value of *BR* for model I).

[c] S evaluated at mean values of outputs, input prices and *GROWTH* (and *BR* for model I).

[d] S evaluated at maximum values of outputs, mean values of input prices and *GROWTH* (and mean value of *BR* for model I).

[e] S evaluated at minimum values of outputs, mean values of input prices and *GROWTH*, and minimum value of *BR*.

[f] S evaluated at maximum values of outputs, mean values of input prices and *GROWTH*, and maximum value of *BR*.

model II for the "small" firm are significantly different from zero. Thus, neither model shows any output pair to be cost complements for the "small," "typical," or "large" firm. Positive definiteness of each matrix can be checked by testing the joint hypothesis that all principal minors have positive determinants, i.e.,

$$C_{11} > 0$$

$$\text{and} \quad C_{11}C_{22} - C_{12}^2 > 0$$

$$\text{and} \quad C_{11}(C_{22}C_{33} - C_{23}^2) - C_{12}(C_{12}C_{33} - C_{13}C_{23}) + C_{13}(C_{12}C_{23} - C_{13}C_{22}) > 0.$$

Joint hypothesis testing of this sort is difficult. A test of the joint hypothesis is stronger than a separate test of each condition of the hypothesis with a decision to reject the entire hypothesis if any condition can be rejected. When each part of the hypothesis is viewed alone, the other restrictions of the hypothesis are not imposed. Thus, if the hypothesis were rejected by testing each of its parts separately, it would probably be rejected when the joint hypothesis was tested. However, if the separate test accepted the hypothesis, the joint test could still reject it. For both models, on the basis of the separate tests, the joint hypothesis can be rejected at the .05 level of significance. Therefore, the hypothesis that the matrix $[C_{ij}]$ is positive definite can be rejected. So cost convexity does not exist for any size firm in model I or II, and no evidence of transray convexity is found.[29]

To summarize, neither model I, in which branching is treated as a technological condition of production, nor model II, in which branching is a function of output level, supports the hypothesis that the S&L industry is a natural monopoly. This result implies that S&Ls would not gain a cost advantage from increased size given the set of outputs produced. Since there might still be economies associated with production of a particular output or specific output combinations, product-specific economies of scale and economies of scope are investigated.

B. Product-Specific Economies of Scale

In order to analyze product-specific scale economies, the level of fixed costs associated with production must be determined. A problem in specifying the translog as the functional form of the cost function is that cost must equal zero when any output level is equal to zero. Here, fixed costs are estimated by evaluating the cost function at $\mathbf{y}^* = (y_1^*, y_2^*, y_3^*)$, where $y_i^* = 10\%$ of y_i at the mimimum of the sample. Thus, $\mathbf{y}^* = (153349.7365, 864.900, 72716.867)$. Table VII gives the values at the sample mean and standard deviations of total cost, incremental cost of each output, average incremental cost of each output, and degree of product-specific scale economies for each output, s_i $(i = 1, 2, 3)$.

In model I, none of the s_i is significantly different from zero. Thus, from the standpoint of costs alone, the typical S&L would not benefit from changing the levels of loans it produces or investments it holds. In model II, s_1 is significantly greater than one at the .05 level, but s_2 and s_3 are not, indicating that the S&L would gain by increasing the level of mortgage loans while leaving other loans and investments at their current levels. Recall that, when s_1 is calculated in model II, the number of branches is allowed to vary since the cost function estimated is the reduced form, but, when s_1 is calculated in model I, the number of branches is held constant at its mean value. This may account for the difference in s_1 between the two models.

C. Global and Product-Specific Economies of Scope

Table VIII gives the global degree of scope economies, SC, and product-specific degrees of scope economies, SC_i $(i = 1, 2, 3)$, for each model. In both models, global scope economies are positive but insignificantly so. The same is true for the degrees of product-specific scope economies with respect to output 1 and output 3. The degree of product-specific economies with respect to output 2 is negative but insignificantly different from zero. The lack of scope economies is further evidence against subadditivity. The fact that there are no production cost advantages to producing the three goods jointly does not imply that there are no economic reasons for joint production. If one considers the consumers' demand for services, it is easy to formulate a model with transportation costs (but no production cost advantages to joint production) that has a unique equilibrium

[29] A separate test of the conditions given in footnote 12 rejects the existence of pairwise transray convexity between any output pair.

Table VII

Product-Specific Scale Economies at Sample Means

Model I

$c(\mathbf{y}, \mathbf{w}, \mathbf{t})^a =$ 67336978.29* = cost at means of each output, input price, and branch variable
(31750276.26)

	y_1 (Mortgage Loans)	y_2 (Other Loans)	y_3 (Cash, Securities, and Real Estate Investments)
Incremental Cost i^a	61488267.42*	12303152.56	−312869162.60
	(23875631.09)	(49861244.39)	(398211348.99)
Average Incremental Cost i^a	.1195*	.7805	−5.6954
	(.04641)	(3.1631)	(7.2489)
Elasticity of Cost wrt i^b	.7275*	.03031	.1543*
	(.04902)	(.02497)	(.04786)
Marginal Cost i^a	.09523*	.1295	.1892*
	(.030921)	(.1167)	(.09526)
s_i = Product-Specific Economies i^a	1.2552*	6.0287	−30.1096
	(.2603)	(24.8350)	(37.9100)
$\frac{s_i - 1}{\text{Standard Deviation of } s_i} = T$	.9804	.2025	−.8206
Approximate Prob > $\lvert T \rvert$	.3270	.8414	.4124

Model II

$c(\mathbf{y}, \mathbf{w}, \mathbf{t})^a =$ 50476244.64* = cost at means of each output and input price
(13335904.41)

	y_1 (Mortgage Loans)	y_2 (Other Loans)	y_3 (Cash, Securities, and Real Estate Investments)
Incremental Cost i^a	48146643.87*	7795582.84	−122443222.73
	(12620482.2)	(40108812.31)	(177629037.48)
Average Incremental Cost i^a	.09360*	.4945	−2.2289
	(.02453)	(2.5444)	(3.2335)
Elasticity of Cost wrt i^b	.8017*	−.01874	.1529*
	(.0491)	(−.03056)	(.04754)
Marginal Cost i^a	.07867*	−.06000	.1405*
	(.02091)	(.10073)	(.05971)
s_i = Product-Specific Economies i^a	1.1898*	−8.2417	−15.8616
	(.1006)	(46.1198)	(23.2821)
$\frac{s_i - 1}{\text{Standard Deviation of } s_i} = T$	1.8856**	.2004	−.7242
Approximate Prob > $\lvert T \rvert$	.0594	.8414	.2358

[a] Approximate standard deviation in parentheses.
[b] Exact standard deviation in parentheses.
* Significant at .05 level.
** Significant at .10 level.

involving joint production. Another reason firms might produce products jointly is that diversification across different types of assets can reduce risk.

A sufficient condition for economies of scope at output vector $\mathbf{y}^*$ is for the cost function to exhibit weak cost complementarities between each pair of outputs at all $\mathbf{y}$ up to $\mathbf{y}^*$. That is, $\partial^2 C(\hat{\mathbf{y}})/\partial \hat{y}_i \partial \hat{y}_j \leq 0$, $i \neq j$, for all $\hat{\mathbf{y}}$ such that $0 \leq \hat{\mathbf{y}} \leq \mathbf{y}^*$,

Table VIII

Global and Product-Specific Economies of Scope at Sample Means[a]

Model I

SC = degree of global economies of scope = 1.0574 (3.4121) $\dfrac{SC}{\text{Standard Deviation of } SC} = T = .3099$

Approximate Prob > $|T|$ = .7566

	Output		
	y_1 (Mortgage Loans)	y_2 (Other Loans)	y_3 (Cash, Securities, and Real Estate Investments)
SC_i = degree of economies of scope specific to output i	1.1085 (3.5050)	−.1806 (.7440)	4.6800 (5.6223)
$\dfrac{SC_i}{\text{Standard Deviation } SC_i} = T$	.3163	.2427	.8327
Approximate Prob > $\|T\|$	.7490	.8104	.4066

Model II

SC = degree of global economies of scope = 1.9432 (4.9600) $\dfrac{SC}{\text{Standard Deviation of } SC} = T = .3918$

Approximate Prob > $|T|$ = .6966

	Output		
	y_1 (Mortgage Loans)	y_2 (Other Loans)	y_3 (Cash, Securities, and Real Estate Investments)
SC_i = degree of economies of scope specific to output i	1.9478 (5.0134)	−.1536 (.8031)	2.4664 (3.4748)
$\dfrac{SC_i}{\text{Standard Deviation } SC_i} = T$	.3885	−.1913	.7098
Approximate Prob > $\|T\|$	.6966	.8494	.4778

[a] Approximate standard deviations in parentheses.

with strict inequality holding over a set of nonzero measure. Gilligan, Smirlock, and Marshall [14] and Gilligan and Smirlock [13] estimate a translog cost function with two outputs. In both papers, the authors test for economies of scope by imposing the restriction $\partial^2 C/\partial y_1 \partial y_2 = 0$ and performing a χ^2 test on the restricted vs. unrestricted regression. Although this is a proper test for jointness, it is not a proper test for economies of scope because a rejection of the restricted form might imply $\partial^2 C/\partial y_1 \partial y_2 > 0$ somewhere in the data. Consider the Cobb-Douglas cost function $C(\mathbf{w}, y_1, y_2) = b(\mathbf{w}) y_1^{a_1(\mathbf{w})} y_2^{a_2(\mathbf{w})}$. Then, $\partial^2 C/\partial y_1 \partial y_2 = a_1(\mathbf{w}) a_2(\mathbf{w}) C/y_1 y_2$, which is greater than zero if marginal costs and outputs are positive.

Murray and White [20] utilize an appropriate test of cost complementarity between output pairs derived by Denny and Fuss [8], which is independent of output and input price levels. The values they obtain are supposed to approximate cost complementarities across the entire cost manifold. The translog is a quadratic approximation around the expansion point $(y_1, \cdots, y_n, w_1, \cdots, w_m) = (1, \cdots, 1)$ to an arbitrary cost function of the form $\ln C = f(\ln y_1, \cdots, \ln y_n, \ln w_1, \cdots, \ln w_m)$.[30] For the translog, cost complementarities exist when (for $i \neq k$), $\sigma_{ik} + (\alpha_i + \sum_{j=1}^{n} \sigma_{ij} \ln y_j + \sum_{j=1}^{m} \delta_{ij} \ln w_j) \times (\alpha_k + \sum_{j=1}^{n} \sigma_{kj} \ln y_j + \sum_{j=1}^{m} \delta_{kj} \ln w_j) < 0$. Evaluated at $(1, \cdots, 1)$, this is $\sigma_{ik} + \alpha_i\alpha_k < 0$. Murray and White [20] test for cost complementarities by testing whether $\sigma_{ik} + \alpha_i\alpha_k < 0$ at the ordinary least-squares (OLS) estimates.[31] However, White [23] shows the severe limitations of using this "approximations" approach when OLS estimation is used.[32] The parameters estimated by OLS need not correspond to the quadratic expansion even at the point of expansion, nor do the slopes necessarily coincide at any point. Thus, evaluating the cost complementarities expression at the expansion point $(1, \cdots, 1)$ does not yield a good approximation to cost complementarities across the cost surface. A better test is to evaluate the expression $\partial^2 C/\partial y_i \partial y_k$ at different output levels and input prices across the surface.

D. Substitutability between Inputs and Branching Costs

To investigate the substitutability between inputs, the elasticities of demands for inputs, ε_{jh}, and Allen partial elasticities of substitution between inputs, Ω_{jh}, were calculated.[33] (See Table IX.) Both models indicate that capital and labor are strong substitutes. Strong substitutability is also indicated between labor and demand deposits. These results are consistent with those obtained by Murray and White [20]; however, they cannot check significance since they do not compute standard errors. Where Murray and White [20] find a negative elasticity between demand and term deposits, the results of this study indicate that they are substitutes. This seems reasonable since the S&L is using both types of deposits to finance its loans, cash, securities, and real estate investments.

Model I, which includes the branching variable, allows computation of elasticity of cost with respect to branching. The value at the sample mean is 0.011862, with standard deviation 0.048840 and t-statistic 0.242879. The hypothesis that the elasticity is zero cannot be rejected. Thus, there is no evidence that there is a cost advantage to having a large network of branches at typical output levels rather than a small network of branches at these *same* output levels.

V. Conclusions

This study of a multiproduct cost function for S&Ls in California found no evidence to support the existence of a natural monopoly. This is at odds with the

[30] Denny and Fuss [8, p. 404].

[31] Other cost studies that utilized this approach include Denny and Pinto [9], Spady and Friedlaender [22], and Fuss and Waverman [12].

[32] Although Byron and Bera (*Int. Econ. Rev.*, Vol. 24, No. 1) found numerical errors in White's example, this does not invalidate White's point.

[33] Elasticity of demand for input j is given by $\varepsilon_{jj} = \partial \ln x_j/\partial \ln w_j = (1/S_j)(\partial S_j/\partial \ln w_j) + S_j - 1$; the cross elasticity of demand between input j and h is given by $\varepsilon_{jh} = \partial \ln x_j/\partial \ln w_h = (1/S_j)(\partial S_j/\partial \ln w_h) + S_h$; the Allen partial elasticities of substitutions are given by $\Omega_{jj} = \varepsilon_{jj}/S_j$ and $\Omega_{jh} = \varepsilon_{jh}/S_h$, where x_j = demand for input j, w_j = unit price of input j, and S_j = cost share of the jth input. Note that $\Omega_{jh} = \Omega_{hj}$ but that $\varepsilon_{jh} \neq \varepsilon_{hj}$.

Table IX

Elasticities of Substitution (ε_{jh}) and Allen Partial Elasticities of Substitution (Ω_{jh}) at the Sample Means[a]

$j =$	Labor		Capital		Passbook and NOW Account Deposits		In-Excess-of-Regular-Rate Account Deposits	
	ε_{jh}	Ω_{jh}	ε_{jh}	Ω_{jh}	ε_{jh}	Ω_{jh}	ε_{jh}	Ω_{jh}
Model I								
$h =$								
Labor	−.4750*	−4.8529*	.1768*		.3597*		.0073	
	(.1826)	(1.8089)	(.0564)		(.1145)		(.0222)	
Capital	.1322*	1.8058*	−.2287*	−3.1234*	−.0458		.0098**	
	(.0377)	(.5314)	(.0220)	(.3224)	(.0372)		(.0055)	
Passbook and NOW	.2869*	3.6750*	−.0488	−.6254	−.8124*	−10.4084*	.0518*	
	(.0844)	(1.1372)	(.0387)	(.5148)	(.0993)	(1.5299)	(.0129)	
In Excess of Regular	.0559	.0745	.1008**	.1342**	.4985*	.6639	−.0689*	−.0918*
Rate	(1.697)	(.2262)	(.0567)	(.0751)	(.1220)	(.1619)	(.0273)	(.0364)
Model II								
$h =$								
Labor	−.7458*	−6.8138*	.2433*		.2123*		.0628*	
	(.1607)	(1.4900)	(.0532)		(.1057)		(.0228)	
Capital	.1663*	2.2227*	−.2093*	−2.7970*	−.0376		.0007	
	(.0328)	(.4491)	(.0206)	(.2901)	(.0351)		(.0055)	
Passbook and NOW	.1580*	1.9398*	−.0409	−.5018	−.8256*	−10.1340*	.0722*	
	(.0754)	(.9409)	(.0375)	(.4736)	(.0955)	(1.3908)	(.0132)	
In Excess of Regular	.4214	.5739*	.0069	.0094	.6509*	.8865*	−.1357*	−.1849*
Rate	(.1511)	(.2059)	(.0542)	(.0738)	(.1202)	(.1619)	(.0202)	(.0383)

[a]Approximate standard deviation in parentheses.
* Significant at .05 level.
** Significant at .10 level.

finding of significant scale economies by Bell and Murphy [3] and Benston [4] in the commercial banking industry and McNulty [18] and Carron [7] in the S&L industry. These studies, however, estimated a series of separate cost functions, one for each output, or chose one output to represent all other outputs of the firm, and thus ignored the possibilities of economies of scope. Four previous studies of the cost structures of depository financial institutions that used the multiproduct approach include Gilligan, Smirlock, and Marshall [14], Gilligan and Smirlock [13], and Benston, Berger, Hanweck, and Humphrey [5] for commercial banks and Murray and White [20] for credit unions.

Both Gilligan, Smirlock, and Marshall [14] and Gilligan and Smirlock [13] find slight economies of scale for small banks (with less than $25 million in deposits) and slight diseconomies of scale for large banks. Benston, Berger, Hanweck, and Humphrey [5] find constant returns to scale at all size firms when the numbers of branches are allowed to vary as in model II of this paper. When branches are held constant, they find slight scale economies for small firms and constant returns to scale for larger firms. Murray and White [20] find increasing returns to scale over their entire sample. The finding in this paper of constant returns to scale (except at the mean of the data set where there are slight economies of scale) agrees most closely with the results of Benston, Berger, Hanweck, and Humphrey [5].

It is more difficult to compare the results concerning economies of scope and cost complementarities of this paper with the results of the other papers. The two studies that report the degree of scope economies (Gilligan, Smirlock, and Marshall [14] and Gilligan and Smirlock [13]) do not report its standard error. Also, Benston, Berger, Hanweck, and Humphrey [5] report numerical values for cost complementarity between output pairs but no standard errors. Murray and White [20] approximate cost complementarity between each pair of outputs and find that mortgage loans and other loans are complements. However, it is not clear that the approximation is representative of the sample.

The findings in this study indicate that savings and loans exhibit neither global nor product-specific economies of scope and that no two outputs are cost complements. Non-(production) cost motivations such as customer convenience and diversification to reduce risk may explain why S&Ls provide these outputs jointly.

If the S&L industry exhibited tendencies toward natural monopoly, this could explain the tendency exhibited by S&Ls that are not failing toward mergers and acquisitions. Based on evidence found here, the tendency toward large, more diversified S&Ls must be based on something other than production cost considerations on the part of the firm. The results of constant returns to scale and scope in this study imply that the small, undiversified S&L is not at a cost disadvantage compared with a large, diversified S&L.

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