



American Finance Association

Collateral and Competitive Equilibria with Moral Hazard and Private Information

Author(s): Yuk-Shee Chan and Anjan V. Thakor

Source: *The Journal of Finance*, Vol. 42, No. 2 (Jun., 1987), pp. 345-363

Published by: Blackwell Publishing for the American Finance Association

Stable URL: <http://www.jstor.org/stable/2328256>

Accessed: 26/11/2009 08:08

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/action/showPublisher?publisherCode=black>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.



Blackwell Publishing and American Finance Association are collaborating with JSTOR to digitize, preserve and extend access to *The Journal of Finance*.

<http://www.jstor.org>

Collateral and Competitive Equilibria with Moral Hazard and Private Information

YUK-SHEE CHAN and ANJAN V. THAKOR*

ABSTRACT

The authors examine equilibrium credit contracts and allocations under different competitiveness specifications and explain the economic roles of collateral under these specifications. Both moral hazard and adverse selection are considered. The principal message is that how a competitive equilibrium is conceptualized significantly affects the characterization of equilibrium credit contracts. Specifically, some well-known results in the rationing literature are shown to rest delicately on the adopted equilibrium concept. Two somewhat surprising results emerge. First, high-quality borrowers with unlimited collateral may be priced out of the market despite the bank having idle deposits. Second, high-quality borrowers may put up more collateral.

THIS PAPER EXPLORES THE allocational consequences of alternative credit market competitiveness specifications and explains the different economic roles of collateral under these specifications. We model banks that lend to borrowers financing projects with payoffs depending on *a priori* unknown borrower attributes and *ex post* unobservable borrower actions. Borrowers are assumed to have unconstrained access to collateral.

Our analysis is based on two concepts of competitive equilibrium. Under both, banks compete in the sense that they earn zero expected profits. However, under the first concept of equilibrium (T1), as in Jaffee and Russell [11], all rents accrue to borrowers, whereas, under the second concept (T2), all rents accrue to depositors, as in Stiglitz and Weiss [17]. Our research makes two main points. First, we demonstrate that the manner in which a competitive equilibrium is conceptualized importantly influences the characterization of equilibrium credit allocations. Specifically, one of the findings is that the rationing equilibria of Stiglitz and Weiss [17] and others should be viewed as rather delicate things in that they are very sensitive to the competitiveness specification used. For instance, under the T1 notion of a competitive equilibrium, unlimited collateral eliminates credit rationing despite moral hazard and private information, whereas, under the T2 equilibrium concept, there is rationing. Our second main point is that the widespread use of collateral can be rationalized on the grounds that it often efficiently resolves moral hazard and adverse selection problems.

* Chan is a Visiting Associate Professor of Finance and Business Economics, School of Business, University of Southern California, and an Associate Professor of Finance, J. L. Kellogg Graduate School of Management, Northwestern University. Thakor is an Associate Professor of Finance, School of Business, Indiana University. We thank an anonymous referee and an associate editor for very useful comments. We have also benefited from the comments of seminar participants at University of Southern California, Indiana University, University of Pennsylvania, and University of California, Berkeley. All remaining errors are our own.

We begin in Section I with a model description and a characterization of first-best equilibria under both competitive equilibrium specifications. Second-best T1 and T2 equilibria with moral hazard are also derived. There are few surprises here. Moral hazard alone causes no welfare loss under *either* competitiveness notion because of unlimited collateral availability. Further, in a T1 equilibrium with both moral hazard *and* private information, full collateralization helps avoid any welfare loss, including rationing.

In Section II, we study T2 equilibria with private information and moral hazard. Here we find that, even though full collateralization is feasible, the juxtaposition of private information with moral hazard alters the (first-best) equilibrium attained with moral hazard alone. Two key assumptions determine the equilibrium. The first concerns the relationship between the borrower's "quality" and the marginal gross return to the borrower's effort, where a "higher quality" borrower is one whose payoff distribution stochastically dominates that of other borrowers. The second is about the reservation utilities of different borrower types. These reservation levels are assumed to vary systematically with borrower types. We explore equilibria under different combinations of these assumptions and find that the identity of those who are rationed when there is rationing, as well as the question of who puts up more collateral and who pays a higher loan rate, depends on these two assumptions. For instance, we find two surprising results. First, it is possible for high-quality borrowers to be priced out of the market, and this rationing may occur even though the bank has idle funds. Second, high-quality borrowers may put up more collateral. This clarifies that the results obtained by Besanko and Thakor [2], Stiglitz and Weiss [17], and others regarding the effect of collateral, the identity of those who are denied access to credit, etc., are outcomes of what these papers implicitly introduce in terms of these two assumptions *and* how they conceptualize equilibrium, rather than generic credit market properties.¹ Section III concludes.

Our paper is related to the rationing literature as well as that on collateral. The latter literature is scant. Barro [1] assumes asymmetric borrower-lender collateral valuation and studies the impact of collateral on loan rates. The borrower's loan is secured by collateral with stochastic terminal value, prompting repayment when collateral value exceeds the loan repayment. However, Barro focuses only on the loan market supply side and does not develop equilibrium implications. Moreover, informational problems are ignored in Barro's analysis. Later papers have addressed credit market-related informational issues involving either moral hazard or adverse selection.² Most notably, in their rationing analysis, Stiglitz and Weiss [17] note in passing that adding collateral to loan contracts based on interest rate alone may exacerbate adverse selection problems. On the other hand, Besanko and Thakor [2] and Chan and Kanatas [5] show

¹ Our finding that rationing could occur in a T2 equilibrium even with *unconstrained* collateral may be viewed as strengthening the Stiglitz and Weiss [17, 18] justification for excluding collateral in much of their formal analysis, although our analysis does indicate that the Stiglitz and Weiss [17] argument that collateral could worsen incentive problems does not apply generally.

² See, e.g., Besanko and Thakor [2], Chan and Kanatas [5], Milde and Riley [12], Stiglitz and Weiss [17], and Wette [20].

that collateral can be used to sort when there is precontract private information about borrower quality or when there is asymmetric project valuation.

Our paper differs from these in the following ways. First, our primary focus is on *contrasting* the implications of the two different equilibrium concepts used in this literature. Second, unlike Barro [1] and Besanko and Thakor [2],³ we assume no bank-borrower disparity in collateral valuation. Thus, we show that collateral is useful *regardless* of whether valuation disparities exist. Moreover, collateral-related transaction costs are absent, in contrast to Chan and Kanatas [5]. Assuming that collateral is worth less to the bank—perhaps due to transaction costs—than it is to the customer would change our results. Even in a T1 equilibrium, some informationally induced welfare losses may be tolerated to limit the loss due to asymmetric collateral valuations. Third, unlike Barro [1], collateral value is nonstochastic. Again, making collateral value random would not affect our model. Fourth, unlike all the other papers, the borrower's collateral is unconstrained, which permits exclusive focus on informationally created distortions. Fifth, in order to focus on aspects of collateral other than its obvious ability to provide risk sharing, we assume that borrowers are risk neutral with respect to the project payoff. However, reservation utilities may vary with borrower attributes. This distinction between our work and the existing literature is significant; we show that the cross-sectional distribution of reservation utilities materially affects equilibrium. Finally, to the best of our knowledge, other than a recent paper by Stiglitz and Weiss [19], ours is the only competitive equilibrium analysis of collateral and rationing that considers both private information and moral hazard.⁴

I. The Model and Some First-Best Solutions

The credit market has many risk-neutral banks and many risk-neutral borrowers. At the beginning of the period, a borrower requires a \$1 loan, repayable at the end of the period, to finance an investment that yields a payoff of $\$R \in [0, K] \subseteq \mathcal{R}_+$ at the end of the period if \$1 is invested initially.⁵ R is a random variable with distribution $H(\cdot; a, b)$ and probability density function $h(R; a, b)$; the scalar

³ Since Besanko and Thakor [2] also study both competitive equilibria, a brief discussion of the relationship of their results to ours is appropriate. First, Besanko and Thakor consider only precontract private information and not nonpecuniary moral hazard. Second, Besanko and Thakor assume *identical* borrower reservation utilities and show that only the low-quality borrowers are rationed in a T2 equilibrium, whereas we have shown this to be just one special case of our model. With an alternative specification involving differential reservation utilities, we have shown that even high-quality borrowers can be rationed. Third, Besanko and Thakor show that the presence of an endowment constraint on collateral could trigger rationing in a T1 equilibrium. Not surprisingly, the absence of such a constraint in our model precludes rationing in a T1 equilibrium. Fourth, unlike our paper, the Besanko and Thakor paper is mainly concerned with the existence and stability of T1 equilibria—both Nash and non-Nash—when the endowment constraint is binding.

⁴ The Stiglitz and Weiss [19] paper came to our attention while revising this paper. Their paper examines credit rationing with moral hazard and adverse selection. Heterogeneity in wealth among borrowers (hence, their ability to offer collateral) is shown to be sufficient in some cases to generate credit rationing.

⁵ This means that we focus on indivisible projects with a fixed investment scale.

b is exogenously given for any borrower and parameterizes a family of density functions $h(\cdot; \cdot, \cdot)$, whereas $a \in [0, 1]$ is an endogenous action choice by the borrower. Each borrower's b is potentially different, with $b \in B$. We assume

$$H_a \leq 0 \quad \text{and} \quad H_b \leq 0 \quad \forall R \in [0, K]; \quad (1)$$

i.e., a higher effort, a higher borrower quality, or a combination of both improves the return distribution in a first-order stochastic dominance sense. Each borrower's utility is the *net* expected payoff from the project less the effort disutility.

Denote by i the interest factor (one plus the interest rate) on the loan and by C the dollar value of collateral. Borrowers have unlimited collateral available for the \$1 loan.⁶ The pair (i, C) is called a *credit contract*. The incremental payoffs to the lender and the borrower are:

	<u>Lender</u>	<u>Borrower</u>	<u>Total</u>	<u>Payoffs</u>
If $R > i - C$	i	$R - i$	R	
$R \leq i - C$	$R + C$	$-C$	R	

Note that these payoffs are incremental in the sense that they have been defined relative to zero. If the project is successful (i.e., $R > i - C$ and the lender is paid in full), it produces an incremental wealth impact of $R - i$ for the borrower. If the project is unsuccessful, the borrower suffers a negative incremental wealth impact of C by losing the collateral.⁷

The expected utility of a borrower of type b is defined as

$$U(i, C, a; b) = -CH(i - C; a, b) + \int_{i-C}^K [R - i]h(R; a, b) dR - S(a), \quad (2)$$

where $H(i - C; a, b) = \int_0^{i-C} h(R; a, b) dR$ and $S(a)$ is the borrower's effort disutility function with $S'(a) > 0$, $S''(a) > 0$. Similarly, the expected loan payoff of the bank, for a given credit contract, is

$$V(i, C, a; b) = \int_0^{i-C} [R + C]h(R; a, b) dR + i[1 - H(i - C; a, b)]. \quad (3)$$

We first consider a competitive credit market with no moral hazard or private information. Two notions of competition are introduced. *T1 competition* involves banks competing for loans, with each bank facing a perfectly elastic deposit supply schedule at some (exogenously given) market-determined bank borrowing rate. Competition for loans produces credit contracts that maximize borrower expected utilities subject to the constraint that banks break even, given the cost of deposit funding and informational constraints, if any. This notion of competition seems appropriate for describing an environment in which there is free entry and where bank liabilities and other financial instruments are close substitutes. Thus, in a *T1 competitive equilibrium*, in the absence of any infor-

⁶ We assume that collateral-eligible assets are physically tied to other uses, so that (premature) liquidation at the initial time is costly. This means that these assets cannot be optimally liquidated to finance the project and avoid the bank loan.

⁷ We assume that the collateral is symmetrically valued by the bank and the borrowers, thus abstracting from issues explored by Barro [1] and Besanko and Thakor [2].

mational constraints, a bank designs the credit contract for a type- b borrower by solving⁸

$$\begin{array}{ll} \text{maximize} & U(i, C, a; b) \\ a \in [0, 1], i > 0, C \in [0, i] \end{array} \quad (4)$$

$$\text{subject to} \quad V(i, C, a; b) = \rho, \quad (5)$$

where ρ is one plus the interest rate at which the bank funds its deposits.⁹

We define *T2 competition* as one that involves banks competing for a limited quantity of deposits. Each bank designs a set of credit contracts to offer to its borrowers subject to their reservation-utility constraint (banks are deposit rate setters) and computes the maximum interest rate it can promise depositors that is consistent with non-negative expected profits for the bank. We assume that free entry causes each (perfectly competitive) bank to earn zero expected profit in equilibrium. Also, since we want to focus on competition among banks for a *limited* quantity of deposits, we assume that deposit supply is sufficiently constrained so that it is always either equal to or exceeded by the number of prospective borrowers. For some equilibrium credit contracts, credit demand may exceed deposit supply; i.e., the bank may ration credit among borrowers applying for the same contract. Formally, in a *T2 competitive equilibrium*, with *no* informational constraints, a bank designs the credit contract for a type- b borrower by solving

$$\begin{array}{ll} \text{maximize} & V(i, C, a; b) \\ a \in [0, 1], i \geq 0, C \in [0, i] \end{array} \quad (6)$$

$$\text{subject to} \quad U(i, C, a; b) = \phi(b), \quad (7)$$

where $\phi(b)$ is the type- b borrower's reservation utility. The solution to (6)–(7), denoted by $V^o(b)$, represents the bank's equilibrium expected return from its type- b borrower loan. The equilibrium deposit rate the bank offers is a cross-sectionally weighted average of its equilibrium expected returns on all loans, and it makes the bank's expected profit zero.

We study these competitive equilibria for two reasons. First, each seems a reasonable description of the way *some* credit markets work. Second, both have

⁸ Throughout our analysis, collateral is constrained only to be non-negative and no greater than the interest factor, i . With nonstochastic collateral (unlike Barro [1]) and no divergence in bank-borrower evaluation of collateral (unlike Besanko and Thakor [2]), both constraints are necessary to ensure that only economically meaningful credit contracts are admissible. Note also that, for the maximization problem (4)–(5), the reservation expected utility of every borrower is implicitly assumed to be satisfied at the maximum; i.e., every borrower chooses to undertake the project.

⁹ Bank deposits may be fully insured, in which case ρ will equal one plus the riskless interest rate. Regardless of the insurance coverage, deposits are always priced to make the depositors' expected payoff one plus the riskless rate. Moreover, (5) says that, in the absence of informational constraints, the bank breaks even on every borrower, which is a stronger requirement than Jaffee and Russell's [11] stipulation that the bank breaks even *across* all borrowers. Although (5) is imposed presently under symmetric information, it also can be imposed when the bank does not know any borrower's b *a priori* if borrowers' types are fully revealed in equilibrium. (See footnote 13.) In the T1 equilibria we consider here, (5) is satisfied (trivially) even under asymmetric information. (See Proposition 3.) For a more substantial illustration in a differential model setting, see Milde and Riley [12] and Besanko and Thakor [3], where the bank does not know any borrower's attribute and yet offers a different contract to each borrower type, breaking even on each contract.

been deployed in the literature without any demonstration of either their possible equivalence or the inherent superiority of one as a description of competitive credit markets. For instance, the T1 notion has been used in Besanko and Thakor [2, 3], Jaffee and Russell [11], and Milde and Riley [12]. The T2 notion appears in at least three important rationing papers: the original single-period Stiglitz and Weiss [17] model, the two-period sequel [18], and Wette's [20] extension of [17]. Moreover, our analysis generates striking contrasts between the two competitive notions in terms of their predictions about the relationship between collateral, interest rates, and rationing.

The advantage of the T2 notion over the T1 notion is that the deposit interest rate is endogenously determined. Thus, studying T2 competition should illuminate competitive processes in markets where deposit availability is sufficiently constrained relative to loan demand to make competition for deposits significantly impact the deposit rate. T2 competition should result when a bank, perhaps because of its position in a segmented credit market, need not compete for loans but rather for deposits obtained from a common pool shared with other banks. An example would be a bank (or an S&L) that, under liability management, competes with banks from various geographical areas for deposits but faces less competition for loans in its own area due to branching restrictions. This resembles the traditional nature of bank competition, and viewing competition this way seems consistent with regulatory attempts to limit borrower surplus extraction, both directly (usury ceilings) and indirectly through the placement of upper bounds on how much surplus can be passed on to depositors (Regulation-Q ceilings). On the other hand, the weakness of the T2 concept is that the "captive-borrower-pool" assumption takes one away from interesting issues related to possible nonexistence of equilibria with Nash or non-Nash competition for borrowers (Besanko and Thakor [2] and Milde and Riley [12]). The advantage of the T1 concept is that it is consistent with the standard notion of a firm operating in a perfectly competitive product (loan) market and obtaining elastically supplied funds (deposits) from a capital market in which it is an atomistic price taker.

The following three propositions are immediate and intuitive results from our model and are thus presented without formal proofs.

PROPOSITION 1: *When borrower types are known a priori and their action choices are observable ex post, each borrower granted a loan in the competitive equilibrium (T1 or T2) chooses the first-best action. However, given the optimal action choice, if conditions (5) and (7) are satisfied in the T1 and T2 equilibrium, respectively, then there will be a multiplicity of (i, C) contracts for both the T1 and T2 equilibria. In a T1 equilibrium, every borrower is funded since there is a perfectly elastic supply of deposits. In a T2 equilibrium, the borrowers are ranked and granted credit according to their expected yield to the bank, $V^o(b)$. Those with lower yields are rationed with their identities chosen randomly because the deposit supply is constrained.*

Multiple equilibria here are not surprising. Given full information, risk neutrality, and symmetric collateral valuation, the tradeoff between the loan interest

rate and collateral is the same for the bank and the borrower. Consequently, there are numerous interest rates, each of which can be perturbed with an accompanying opposite movement in collateral in a manner that leaves unchanged the borrower's utility as well as the bank's. The subsequent introduction of moral hazard will eliminate multiplicity of equilibria, and the addition of private information will fundamentally alter the equilibrium.

Even in this setting, collateral may play an interesting role if institutional constraints disallow otherwise feasible loan rates. For instance, if usury laws restrict "*i*," collateral will help preserve the borrower welfare implied by Proposition 1.

We now turn to the nature of equilibrium with moral hazard.

PROPOSITION 2: *When borrower types are known a priori but their action choices are unobservable ex post, both T1 and T2 competitive equilibria involve fully collateralized loans and are unique. The banks' credit-granting decision and the action choices of borrowers who are granted credit are the same as in Proposition 1.*

The intuition is that the risk-neutral borrower is willing to fully absorb the risk associated with *R*. Thus, it negotiates a credit contract that makes the bank's payoff certain ($i = C$) and *costlessly* resolves moral hazard. (See also Harris and Raviv [9].) In the absence of collateral, then, moral hazard would reduce borrower welfare—without any improvement in the bank's welfare—in a T1 equilibrium, and it would impair the bank's welfare in a T2 equilibrium without improving the borrower's lot.

We now introduce private information with moral hazard by assuming that only the borrower knows its own *b*. Banks are aware only of the cross-sectional distribution of borrower types. We initially examine the T1 equilibrium with an easily interpretable proposition. The T2 equilibrium is examined in the next section.

PROPOSITION 3: *A T1 competitive equilibrium with private information and moral hazard is unique and identical to the T1 competitive equilibrium with only moral hazard.*

Precontract private information does not alter the T1 equilibrium because the optimal contract for *all types* with *only* moral hazard involves $i^* = C^* = \rho$ and is thus invariant to the borrower's type. Even with private information, therefore, collateral continues to serve a useful purpose. There is also no credit rationing, an outcome directly attributable to collateral. Much of the rationing literature ignores collateral. (See, for example, Jaffee and Modigliani [10], Jaffee and Russell [11], and Stiglitz and Weiss [18].) The exceptions are a section in Stiglitz and Weiss [17] and a recent note by Wette [20]. Both claim that collateral, instead of eliminating rationing, could deter borrowing by the "safer" borrowers. The Stiglitz and Weiss [17] result depends on borrower risk aversion, whereas the key assumption in Wette [20] is that collateral can vary but that the interest

rate is fixed, which rules out any positive sorting effects of collateral and thus eliminates a powerful source of collateral value. (See Besanko and Thakor [2].)¹⁰

II. T2 Equilibrium with Moral Hazard and Private Information

We now examine T2 equilibria when borrowers' actions are unobservable *ex post* and borrowers' types are private information. In a T2 equilibrium, banks compete for a limited supply of deposits. We assume, for simplicity, that deposit supply per bank is perfectly inelastic with a fixed quantity $\bar{D} \leq M$, where M is the total number of prospective borrowers for each bank.¹¹ To ease geometric presentation, we assume that there are two types of borrowers: τM are type- b_2 borrowers, and $(1 - \tau)M$ are type- b_1 borrowers. (This simplifying assumption is not critical to our results; see the discussion in Section III.) Let $b_2 > b_1$, so that type- b_2 borrowers are higher quality borrowers (cf. (1)). Note, however, that high-quality borrowers may have higher reservation utility, i.e., $\phi(b_2) \geq \phi(b_1)$. Furthermore, we also allow borrowers to differ in their propensity to work harder, depending on their marginal return to effort. This condition is discussed later.

Given a loan contract (i, C) , each borrower maximizes expected utility by choosing an optimal action $a^o(i, C, b_j)$, $j = 1, 2$. $a^o(\cdot, \cdot, \cdot)$ is assumed to satisfy the first-order condition¹² (cf. (2)):

$$-CH_a + \int_{i-C}^K [R - i]h_a dR = S'(a^o), \quad (8)$$

where $H \equiv H(i - C; a, b_j)$, for $j = 1, 2$.

The impact of precontract informational asymmetry on equilibria is now studied. Myerson [13] and others have shown that direct, truthful revelation mechanisms cannot be dominated in asymmetric-information settings.¹³ Thus,

¹⁰ The absence of rationing in a T1 equilibrium depends on the assumption that feasible collateral levels are unconstrained by borrower resources. If the full collateral solution is infeasible because of borrower endowment constraints, both moral hazard and private information will become relevant concerns.

¹¹ The perfect inelasticity assumption can be supplanted with the assumption that deposit supply is imperfectly elastic; our results will remain unchanged in spirit.

¹² Grossman and Hart [8] have shown that the first-order approach is not generally valid in the sense that the agent's optimal action choice may lie outside the set of actions that satisfy the first-order condition. Grossman and Hart show that the first-order-condition approach is valid if the CDFC (concavity of distribution function condition) holds. In our context, this means that, for a fixed $b \in B$, some $\lambda \in [0, 1]$, and $a, \hat{a}$ and $a' \in [0, 1]$ such that

$$S(a) = \lambda S(\hat{a}) + [1 - \lambda]S(a'),$$

we must have

$$H(\cdot; a, b) \leq \lambda H(\cdot; \hat{a}, b) + [1 - \lambda]H(\cdot; a', b).$$

We assume that this condition holds.

¹³ Myerson's [13] "revelation principle" asserts that direct, truthful revelation mechanisms—in which the informed knows the allocation he or she will receive for every possible report of his or her private information to the uninformed—are as good as any other mechanism. That is, without loss of generality, the bank can instruct the borrower to directly report his or her b honestly. The intuition

the bank asks each borrower to truthfully report his or her type and awards a credit contract contingent on the report. This means that an incentive-compatibility constraint (in addition to (8)) is required. In equilibrium,

$$U(i_j^o, C_j^o, a_j^o; b_j) \geq U(i_k^o, C_k^o, a_j^k; b_j), j, k = 1, 2, \quad (9)$$

where (i_j^o, C_j^o) is the credit contract designed for a type- b_j borrower, a_j^o is the action choice of a type- b_j borrower taking the contract (i_j^o, C_j^o) , and a_j^k is the action of a type- b_j borrower taking the contract designed for a type- b_k borrower. In order to offer higher deposit rates, banks attempt to extract all possible surplus from prospective borrowers with contracts satisfying the incentive-compatibility constraint (9). The effects of credit-contract parameters on the borrower's action are given below.

LEMMA 1: $\partial a^o / \partial C \geq 0$ and $\partial a^o / \partial i \leq 0$. That is, *ceteris paribus*, higher collateral induces higher effort level, but a higher interest rate induces a lower effort level.

Proof: See the Appendix.

The intuition is as follows. Given a contract (i, C) , a borrower selects the optimal level of effort such that his or her marginal expected payoff is equal to the marginal cost of effort. The bank's contract does not affect the borrower's effort-disutility function, so the impact of alternative contracts must be assessed on the basis of their effect on the borrower's marginal (net) expected payoff. When more collateral is posted, it increases the borrower's loss (reduces his or her marginal payoff) in those states in which there is project failure and collateral transfers to the bank. The borrower can cut his or her expected loss (and increase the net expected payoff) by reducing the failure probability. This can only be achieved, for a fixed b , by increasing effort. In other words, higher collateral makes failure (default) more costly for the borrower, thereby making failure avoidance more attractive. Consequently, higher effort is supplied. On the other hand, a higher interest rate reduces the borrower's net payoff in states in which

behind it is as follows. Suppose the bank chose some general credit policy other than the one prescribed by the revelation principle. For each value of $b \in B$, let $\xi(b)$ be the report of b that the borrower would submit under this policy if his or her true attribute were b . That is, $\xi(b)$ maximizes the borrower's expected utility when facing this credit policy and when the borrower's true attribute is b . Consider now the following new credit policy. Ask the borrower to report his or her attribute b . Then compute $\xi(b)$. Finally, implement the credit allocations that would have been implemented in the *original* credit policy if $\xi(b)$ had been reported there. It is clear that the borrower does not have an incentive to lie to the bank in the new policy, or else the borrower would have had some incentive to lie to himself or herself in the originally given policy. Hence, a (truthful) "direct-report" policy does at least as well as any other.

This implies that, in a competitive, asymmetrically informed credit market, each competing bank knows that it cannot outperform any other bank by adopting a strategy other than the one that evokes direct and truthful revelation by each loan applicant. Consequently, every competing bank will optimize by adopting a credit policy consistent with the revelation principle. For a more extended application of the revelation principle in a competitive credit market context, see Besanko and Thakor [2]. The revelation principle has also been applied in other areas. (See, for example, Rothschild and Stiglitz [15].)

the project *succeeds* and the loan is repaid. Increasing the likelihood of success, therefore, becomes less attractive for the borrower, and effort supply is curtailed.

Define (cf. (2))

$$W(i, C; a, b) = -CH(i - C; a, b) + \int_{i-C}^K [R - i]h(R; a, b) dR \quad (10)$$

as the expected gross return to the borrower of type b choosing action a when given a contract (i, C) . We assume diminishing marginal return to effort for all borrower types, i.e., $\partial^2 W / \partial a^2 < 0 \quad \forall (i, C), b$. A condition linking the return to effort with the borrower's type is given below.

Define increasing and diminishing marginal returns in borrower's quality respectively as

$$\frac{\partial}{\partial b} \left(\frac{\partial W}{\partial a} \right) \geq 0 \quad \forall (i, C), b \text{ (IMRQ)}, \quad (11)$$

$$\text{and } \frac{\partial}{\partial b} \left(\frac{\partial W}{\partial a} \right) \leq 0 \quad \forall (i, C), b \text{ (DMRQ)}. \quad (12)$$

Condition IMRQ says that the marginal gross return to effort is larger for higher quality types. Since $S(a)$ is the same for all borrowers, it implies that high-quality borrowers are more willing to supply effort. Condition DMRQ is simply the reverse of IMRQ. We study equilibrium under both assumptions to highlight the impact of this "cross-partial" assumption on equilibrium. We start with IMRQ.

Case A: IMRQ

LEMMA 2: *IMRQ implies that (i) $\partial a^o / \partial b \geq 0$ and (ii) $\partial (di/dC | \bar{v}) / \partial b > 0$.*

Proof: See the Appendix.

Lemma 2 indicates that, given IMRQ, the high-quality borrowers expend more effort for any (i, C) and also have a flatter indifference curve in the (i, C) space. (See Figure 1.) The intuition for result (i) is clear. With IMRQ, an increase in quality (b) implies an increase in the expected marginal return to effort. Thus, higher quality borrowers optimally choose higher effort. The intuition for result (ii) is that, since $\partial a^o / \partial b \geq 0$ and $H_b \leq 0$, the gross return distribution, $H(\cdot; a^o(b), b)$, of the high-quality borrowers dominates that of the low-quality types in the first-order (stochastic dominance) sense, which implies that the high-quality borrowers face a lower probability that failure will occur and collateral will transfer to the bank. Thus, for these borrowers, the expected marginal loss from a given increase in collateral is smaller than that of the low-quality borrowers. Therefore, they require a smaller interest rate deduction to remain equally well off, relative to the low-quality types. That is, their indifference curves are flatter in the (i, C) space.

Let ϕ_j be the reservation utility for borrower of type $j = 1, 2$. When $\phi_1 = \phi_2 = \phi$, as shown in Figure 1, the bank can offer either (i) a pair of (separating) contracts such that (i_2^o, C_2^o) (fully collateralized) is attractive to type-2 borrowers

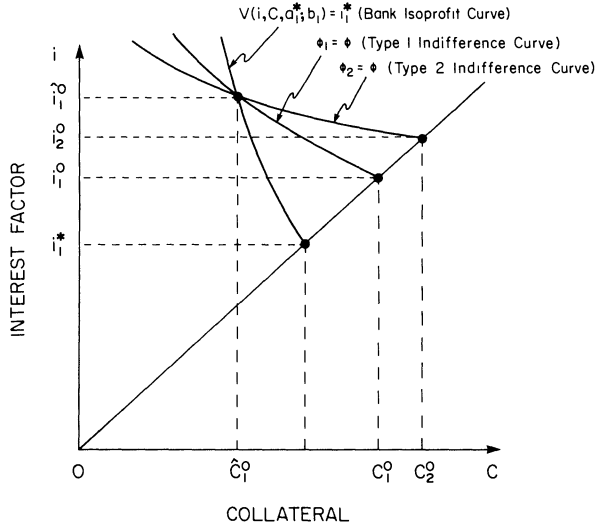


Figure 1. T2 Equilibria: IMRQ and $\phi_1 = \phi_2$

and $(\hat{i}_1^o, \hat{C}_1^o)$ is attractive to type-1 borrowers or (ii) a (pooling) contract (i_1^o, C_1^o) (fully collateralized) to both types.¹⁴ These contracts are incentive compatible and are designed to maximize the rate the bank can pay the $\bar{D}$ depositors. Consider the pair of separating contracts first. The bank will realize i_2^o per unit of loan to the type-2 borrowers. The loan to the type-1 borrowers, however, is not fully collateralized since $\hat{C}_1^o < \hat{i}_1^o$. Consequently, the bank faces moral hazard since the borrower's action is contract dependent and affects the bank's payoff [cf. (8)]. The bank's certainty-equivalent return from type-1 borrowers, i_1^* , can be imputed from its expected-utility function; i.e., i_1^* is such that $V(i_1^*, C_1^*, a_1^*; b_1) = V(\hat{i}_1^o, \hat{C}_1^o, \hat{a}_1^o; b_1)$, where $i_1^* = C_1^*$ and a_1^* is the optimal (first-best) action when there is no moral hazard. With the separating contracts, the bank can offer the depositors a weighted average of i_2^o and i_1^* , the (certainty-equivalent) returns from type-2 and type-1 borrowers, respectively. It can be shown that¹⁵ $i_1^* < i_1^o$, where i_1^o is the realized return per unit of loan when the pooling contract (i_1^o, C_1^o) is offered to both types of borrowers. With a pooling contract (i_1^o, C_1^o) , the bank can offer a deposit rate i_1^o . The deposit rate that the bank offers in equilibrium depends on the available deposit supply $\bar{D}$ as described below.

¹⁴ It is possible that the relevant pooling contract to compare with the pair of separating contracts is a point off the 45° line and on the type-1 indifference curve through (i_1^i, C_1^i) ; i.e., the pooling contract may involve less than full collateral. A sufficient condition that ensures that (i_1^i, C_1^i) is the optimal pooling contract is that the bank's indifference curve on type-2 loans is steeper than the indifference curve of type-1 borrowers. This condition can be shown through tedious algebra to be

$$\partial H[i - C; a^0(b), b]/\partial b > HaVa/\{S''(a_2^0) - \partial^2 W/\partial a^2\}$$

for all (i, C) , b . The interpretation is that a bank will demand full collateral—and thus the first-best effort level—if the marginal “cost” of inducing effort does not increase too rapidly ($S''(a^o)$ is small), the marginal gain from increased effort is high (H_a has a high absolute value), or both. Note that $H_a \leq 0$.

¹⁵ This result is implied if the bank's indifference curve on type-1 loans in the (i, C) space is steeper than that of the type-1 borrower, as shown in the Appendix.

PROPOSITION 4: (cf. Figure 1) *When IRMQ holds and all borrowers have identical reservation utility ϕ , the following observations hold in a T2 equilibrium with private information and moral hazard:*

1. (Separating Equilibrium I) *If $\bar{D} \leq \tau M$, banks offer the high interest rate and fully collateralized contract (i_2^*, C_2^*) such that $U(i_2^*, C_2^*, a_2^*; b_2) = \phi$. $\tau M - \bar{D}$ type-2 borrowers are rationed randomly. All type-1 borrowers are priced out of the market. In equilibrium, all borrowers' surplus is extracted.*
2. (Separating Equilibrium II) *If $\tau M < \bar{D} \leq \min(\hat{D}, M)$, where $\hat{D} \equiv \tau M[i_2^* - i_1^*]/[i_1^* - i_1^*]$, then a separating equilibrium exists in which banks offer contracts (i_2^*, C_2^*) and $(\hat{i}_1^*, \hat{C}_1^*)$ to type-2 and type-1 borrowers, respectively. Further, $M - \bar{D}$ type-1 borrowers are rationed randomly. In equilibrium, all borrowers' surplus is extracted.*
3. (Pooling Equilibrium) *If $\hat{D} < \bar{D} \leq M$ (i.e., $\min(\hat{D}, M) = \hat{D}$), banks offer the pooling contract (i_1^*, C_1^*) and ration $M - \bar{D}$ borrowers (type 1 or 2) randomly. In equilibrium, only the type-2 nonrationed borrowers attain higher expected utility than ϕ .*

$\hat{D}$ is the critical deposit supply level at which the average expected return per unit of deposit is the same for the separating (II) and pooling equilibria.¹⁶ Note that the (high-quality) type-2 borrowers take the fully collateralized contract $[(i_2^*, C_2^*) \text{ or } (i_1^*, C_1^*)]$ in all cases, while the type-1 borrowers take the partially collateralized contract $(\hat{i}_1^*, \hat{C}_1^*)$ in case (2). Further, in case (2), $i_2^* < \hat{i}_1^*$ and $C_2^* > \hat{C}_1^*$; that is, the type-2 borrowers' contract has higher collateral but a lower interest rate than the type-1 borrowers' contract. It is surprising that borrowers reporting themselves to be of high quality are asked for higher collateral.¹⁷ The intuition is that the IMRQ condition implies a higher propensity to expend effort on the part of the higher quality borrowers, which makes it optimal for the bank to elicit higher effort from such borrowers by demanding higher collateral (cf. Lemma 2).

Proposition 4 is based on identical reservation utilities for all borrowers. Generally, reservation utilities may vary across borrower types. In a partial-equilibrium model such as ours, it is hard to ascertain the precise functional relationship between borrower quality and reservation utility. A plausible assumption is that high-quality borrowers have higher reservation utilities than low-quality borrowers, i.e., $\phi_2 > \phi_1 = \phi$. (For a recent paper that assumes a similar link between (unobservable) borrower quality and reservation utility, see Bhattacharya and Pfleiderer [4].) The interpretation is that high-quality borrowers have better alternatives to bank credit than low-quality borrowers.¹⁸ The point

¹⁶ That is, $\hat{D}$ is such that $i_1^* \hat{D} = \tau M i_2^* + [\bar{D} - \tau M] i_1^*$.

¹⁷ Assuming asymmetric project valuation, Chan and Kanatas [5] show that underrated borrowers may offer higher collateral because there is a lower probability of loss. Our result is similar in a sense to the Ross [14] result that higher valued firms signal with more debt because they can better afford the (unlikely) bankruptcy costs.

¹⁸ This is *not* to imply that borrowers' types are observable to credit sources other than banks. We can visualize alternative credit sources as also designing revelation mechanisms under asymmetric information such that $\phi(b)$ is the expected utility of a type- b borrower self-selecting the contract designed for that borrower.

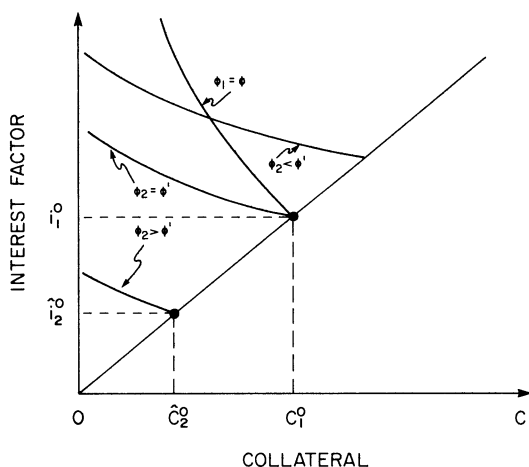


Figure 2. T2 Equilibria: IMRQ and $\phi_1 < \phi_2$

of the analysis that follows is that systematic (quality-based) cross-sectional variations in reservation utilities have an important bearing on credit market equilibrium.¹⁹

With this assumption, we see that a higher ϕ_2 shifts the indifference curve of the type-2 borrower towards the origin. (See Figure 2.) When $\phi_2 = \phi'$, the indifference curves of the two types intersect at (i_1^0, C_1^0) . As long as $\phi_2 < \phi'$, the equilibria take the same form as in Figure 1. When $\phi_2 > \phi'$, the indifference curves of the two borrower types ($\phi_1 = \phi$ and $\phi_2 > \phi'$) do not cross, and the incentive-compatibility condition cannot be satisfied with multiple loan contracts; i.e., the bank cannot offer two different contracts that just exactly satisfy borrowers' reservation utilities (fully extract their surplus) such that each contract will attract only one type of borrower. The reason is that $\phi_2 > \phi'$ lies below $\phi_1 = \phi$; thus, the type-1 borrowers always prefer the type-2 borrowers' contract. This means that the bank must offer either (i) a contract that attracts type-1 borrowers but not type-2's (that is, the type-2's are priced out of the market) or (ii) a contract such that pooling occurs. The bank will select the optimal contract to maximize the depositors' surplus. These results are summarized as follows.

PROPOSITION 5: (cf. Figure 2) *When IMRQ holds and the type-2 borrowers have higher reservation utility $\phi_2 > \phi_1 = \phi$, in a T2 equilibrium with private information and moral hazard the following observations hold:*

1. *If $\phi < \phi_2 < \phi'$, where ϕ' is the critical level of utility such that the borrowers' indifference curves cross at (i_1^0, C_1^0) , the equilibria take the same form as those in Proposition 4, with a downward shift of the type-2 borrower's indifference curve reflecting a higher level of reservation utility.*
2. *If $\phi_2 \geq \phi'$, only fully collateralized loan contracts are offered. These contracts are as follows. (See Figure 2 for a graphical description of the contracts.)*
 - (a) *(Separating Equilibrium I) If $\bar{D} \leq [1 - \tau]M$, banks offer (i_1^0, C_1^0) and ration $\bar{D} - [1 - \tau]M$ type-1 borrowers randomly. All type-2 (high-quality) borrow-*

¹⁹ See the following analysis, which leads to Propositions 5 and 6 and footnotes 22 and 24.

ers are priced out of the market. In equilibrium, all borrowers' surplus is extracted.

- (b) (*Separating Equilibrium II*) If $(1 - \tau)M < \bar{D} < \min(\hat{D}', M)$, where $\hat{D}' \equiv (1 - \tau)Mi_1^*/i_2^*$, banks offer (i_1^*, C_1^*) to all type-1 borrowers and keep idle $\bar{D} - [1 - \tau]M$ units of deposits. All type-2 borrowers are priced out of the market. In equilibrium, all borrowers' surplus is extracted.
- (c) (*Pooling Equilibrium*) If $\hat{D}' < \bar{D} \leq M$ (i.e., $\min(\hat{D}', M) = \hat{D}'$), banks offer the pooling contract $(\hat{i}_2^*, \hat{C}_2^*)$ and ration $M - \bar{D}$ borrowers (type-1 or 2) randomly. In equilibrium, only the type-1 nonrationed borrowers attain higher expected utility than ϕ .

$\hat{D}'$ is the critical deposit supply level at which the average return per unit of deposit is the same for the separating (II) and the pooling equilibria.²⁰ Two points are noteworthy when the type-2 borrowers' reservation utility is sufficiently high ($\phi_2 \geq \phi'$). First, fully collateralized contracts are optimal, providing an immediate dissolution of moral hazard. Second, type-2 borrowers are priced out of the market in both of the separating equilibria (even though only one contract (i_1^*, C_1^*) is offered), including the case in which banks have idle funds.²¹ The intuition behind the somewhat surprising result that high-quality borrowers are excluded is as follows. When these borrowers have a sufficiently high reservation utility, even the contract that fully extracts their surplus has a relatively low collateral requirement and a relatively low interest rate. While the bank would not mind offering this contract to a high-quality borrower, it does not wish to offer the same to a low-quality borrower. Because of the private information problem, however, the bank cannot distinguish between borrowers and must, therefore, offer incentive-compatible contracts. If it offers the contract that gives the high-quality borrowers *exactly* their reservation utility, the low-quality borrowers covet the contract. If it attempts to restore incentive compatibility by increasing the loan rate or collateral requirement, the reservation-utility constraint for the high-quality borrowers is violated. The bank, thus, has two options. It can offer just the contract that fully exploits the high-quality borrowers and let the low-quality borrowers retain some surplus by taking that contract. Alternatively, it can offer a contract that no high-quality borrower is willing to take but that appropriates all surplus from the low-quality borrowers. The proposition identifies the conditions under which the second alternative produces higher expected profit for the bank. Note that this is more than just a question of allocating scarce deposits to the highest return loans. The bank may even prefer to keep some funds idle and price out high-quality borrowers because any attempt to loan out those idle funds at terms acceptable to the high-quality borrowers will precipitate an incentive-compatibility problem.²² This may be viewed as another illustration of market

²⁰ That is, $\hat{D}'$ is such that $i_2^*\hat{D}' = [1 - \tau]Mi_1^*$, as stated in the proposition.

²¹ The observation that high-quality borrowers could be priced out of the market is novel. Assuming identical reservation utilities for all borrowers, Besanko and Thakor [2] show that, in a T2 competitive equilibrium, only the low-quality borrowers are excluded.

²² Note that, if we had assumed that the low-quality borrowers have higher reservation utility than the high-quality borrowers, the equilibria would be of the form described in Proposition 4 and Figure 1. (Shifting type 1's indifference curve downwards does not alter the nature of equilibria.)

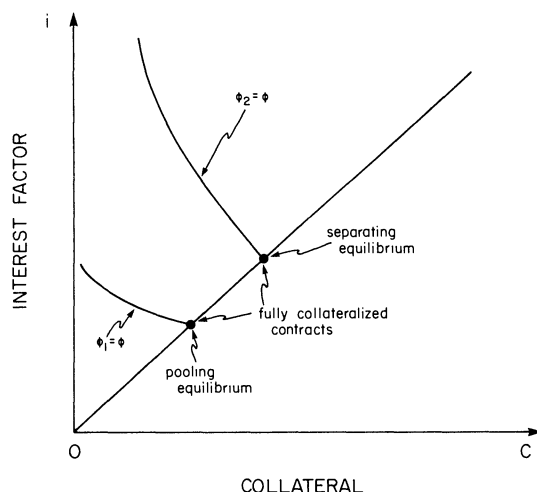


Figure 3. T2 Equilibria: DMRQ and $\phi_1 = \phi_2$

failure—credit supply beyond a point is zero despite funds availability—with imperfect information.

Case B: DMRQ

When DMRQ holds, the analysis is more complicated as the relative slopes of the indifference curves of the type-1 and type-2 borrowers cannot be determined unambiguously.²³ Our results under IMRQ will still hold if the type-2 borrowers have flatter indifference curves. On the other hand, if the type-1 borrowers have flatter indifference curves, the equilibrium results change substantially. Since the analysis is straightforward once the relative slopes are signed, we now simply contrast—without formal proof—the results under DMRQ in the present setting with the previous results.

PROPOSITION 6: *If DMRQ holds and type-1 borrowers have flatter indifference curves in the (i, C) space, then*

1. *When all borrowers have identical reservation utilities, in a T2 equilibrium only fully collateralized contracts are offered. However, contrary to Proposition 5, the type-1 borrowers are priced out of the market in the separating equilibria (even when banks have idle funds). (See Figure 3.)*
2. *When type-2 borrowers have sufficiently higher reservation utilities, separating or pooling equilibria of the forms described in Proposition 4 result. However, contrary to Proposition 4, the type-1 borrowers are offered the higher collateral-*

²³ From the proof of Lemma 1,

$$\frac{\partial}{\partial b} \left(\frac{di}{dC} \bigg|_v \right) = \frac{-1}{(1-H)^2} \left\{ H_a \frac{\partial a^o}{\partial b} + H_b \right\}.$$

DMRQ implies that $\partial a^o / \partial b \leq 0$; since $H_a, H_b \leq 0$, the sign of change in slope is indeterminant.

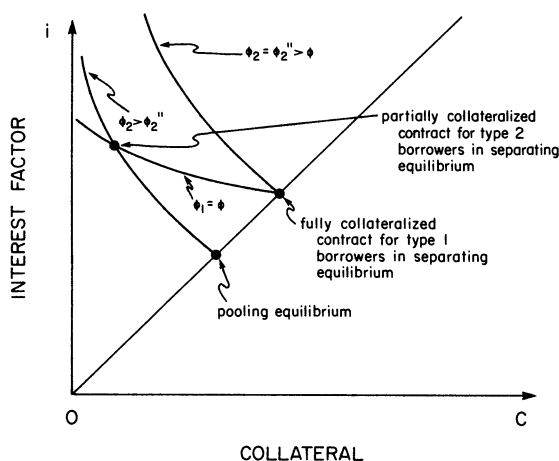


Figure 4. T2 Equilibria: DMRQ and $\phi_1 < \phi_2$

lower interest rate (fully collateralized) contract, and the type-2 borrowers are priced out of the market in the separating equilibria. (See Figure 4.)²⁴

III. Concluding Remarks

We have highlighted the sensitivity of results in the credit allocation literature to the equilibrium concept employed. The two competitive equilibrium concepts defined here have both been used in some important papers in this literature. Further, we have explained the economic role of collateral under moral hazard and private information. Both the manner in which collateral is utilized and its efficacy in eliminating credit rationing depend on *how* a competitive equilibrium is defined.

Our principal results can be reiterated as follows.

1. The rationing explanations in the existing literature do not appear robust. That is, rather than explain what may be taken as a generic property of credit markets hampered by private information, moral hazard, or both, existing explanations are shown to depend heavily on assumptions about the notion of competition, cross-sectional variation in reservation utilities, interactions (if any) between private information and moral hazard, etc.
2. In a T1 competitive equilibrium, there is neither rationing nor any other distortion even with private information and moral hazard, as long as collateral is unconstrained. This result rests on the assumption that collateral is symmetrically valued by the bank and the borrower. If, as in Barro [1], for example, the bank values collateral less highly than the borrower, then using enough collateral to completely dissolve moral hazard and private information concerns will not generally be optimal. Efficient credit contracts will trade off

²⁴ Note that, if we had assumed here that the low-quality borrowers have higher reservation utility than the high-quality borrowers, the equilibria would be of the form described in Proposition 6 (1) and Figure 3.

collateral-related costs for the bank against the distortions arising from informational problems.

3. In a T2 competitive equilibrium with private information and moral hazard, allocation distortions—including rationing—may emerge. High-quality borrowers may be excluded from the market despite their unlimited ability to post collateral and the bank's access to idle deposit funds.
4. In a T2 competitive equilibrium with private information and moral hazard, high-quality borrowers may post more collateral.

The simplifying assumption in Section II that there are only two quality types does not sacrifice much generality. The results with a continuum of types will be mostly similar to those obtained here. For example, in those cases in which high-quality borrowers are denied access to credit in the two-type model (cf. Proposition 5), there will be a subset (of nonzero measure) of highest quality types who are all denied access to credit, conditional on the reservation-utility function displaying sufficient convexity with respect to quality. The key determinant of the extent to which the continuum results will mirror the two-type results is the cross-sectional distribution of types, where a “type” is taken to be a vector consisting of all of the borrower's payoff-relevant attributes. However, a caveat to our analysis deserves mention. The static nature of our model does not allow us to capture the value of ongoing relationships in mitigating moral hazard and misrepresentation incentives. Consequently, we may have overemphasized the advantage of collateral. However, a borrower's concern with the longevity of his or her bank relationship is unlikely to completely ameliorate informational problems.

Our paper is part of an emerging literature on credit arrangements involving informationally disadvantaged lenders. Other examples are the mortgage-contracting models of Chari and Jagannathan [6] and Dunn and Spatt [7]. One interesting extension of our work would be to examine collateral in a multiperiod setting in which an applicant's past credit history affects future contracts.²⁵ Another would be to allow collateral to have a random terminal value dependent on the borrower's action. This would accord well with many borrower-lender relationships, such as automobile loans and home mortgages, where a borrower's endogenously chosen level of “care” affects collateral value.

Appendix

Proof of Lemma 1: The optimal action $a^o(\cdot, \cdot, \cdot)$ is determined through (cf. (8))

$$\partial W / \partial a = S'(a^o), \quad (\text{A1})$$

where $W(\cdot)$ is defined in (10). Partially differentiating (A1) with respect to i :

$$\frac{\partial a^o}{\partial i} = \frac{-\int_{i-C}^K h_a dR}{S''(a^o) - (\partial^2 W / \partial a^2)} = \frac{H_a}{S''(a^o) - (\partial^2 W / \partial a^2)} \leq 0. \quad (\text{A2})$$

²⁵ See Spatt [16] for a model in which a borrower's decision to default or repay signals that borrower's future creditworthiness.

Partially differentiating (A1) with respect to C :

$$\frac{\partial a^o}{\partial C} = \frac{-H_a}{S''(a^o) - (\partial^2 W / \partial a^2)} > 0. \quad \text{Q.E.D.} \quad (\text{A3})$$

Proof of Lemma 2: Partially differentiating (A1) with respect to b :

$$\frac{\partial a^o}{\partial b} = \frac{\frac{\partial}{\partial b} \left(\frac{\partial W}{\partial a} \right)}{S''(a^o) - (\partial^2 W / \partial a^2)}. \quad (\text{A4})$$

Therefore, (i) of Lemma 2 holds. (Note that DMRQ implies that $\partial a^o / \partial b \leq 0$.) To prove (ii), first note that, for an indifference curve, $U = \bar{U}$. Thus,

$$U_C dC + U_i di + U_a da = 0. \quad (\text{A5})$$

At $a^o(i, C, b)$, $U_a = 0$; therefore, at $U = \bar{U}$,

$$\left. \frac{di}{dC} \right|_{\bar{U}} = -\frac{U_C}{U_i} = -\frac{H(i - C, a^o, b)}{1 - H(i - C, a^o, b)}. \quad (\text{A6})$$

Letting $H \equiv H(i - C, a^o, b)$,

$$\begin{aligned} \frac{\partial}{\partial b} \left(\left. \frac{di}{dC} \right|_{\bar{U}} \right) &= \frac{-1}{[1 - H]^2} \left\{ [1 - H] \left[H_a \frac{\partial a^o}{\partial b} + H_b \right] + H \left[H_a \frac{\partial a^o}{\partial b} + H_b \right] \right\} \\ &= \frac{-1}{[1 - H]^2} \left\{ H_a \frac{\partial a^o}{\partial b} + H_b \right\}. \end{aligned} \quad (\text{A7})$$

Since $H_a, H_b \leq 0$ and $\partial a^o / \partial b \geq 0$ from (i), (ii) holds. Q.E.D.

Proof that the banks' indifference curve is steeper in (i, C) space than that of the borrower: Note that, for bank's indifference curve, $V = \bar{V}$. Thus,

$$V_i di + V_C dC + V_a da = 0 \quad (\text{A8})$$

and

$$\begin{aligned} V_a &= \int_0^{i-C} (R + C) h_a dR + i(-H_a) \\ &= i - \int_0^{i-C} H_a dR > 0 \end{aligned} \quad (\text{A9})$$

since $H_a \leq 0$. Therefore,

$$\left. \frac{di}{dC} \right|_{\bar{V}} = -\frac{H + V_a(\partial a^o / \partial C)}{1 - H + V_a(\partial a^o / \partial i)}. \quad (\text{A10})$$

From (A2) and (A3),

$$\left. \frac{di}{dC} \right|_{\bar{V}} < -\frac{H}{1 - H} = \left. \frac{di}{dC} \right|_{\bar{U}}. \quad (\text{A11})$$

Thus, the bank's indifference curve is steeper in the (i, C) space. Q.E.D.

REFERENCES

1. R. J. Barro. "The Loan Market, Collateral and Rates of Interest." *Journal of Money, Credit and Banking* 8 (November 1976), 439-56.
2. D. Besanko and A. V. Thakor. "Competition, Collateral and Rationing: Models of Sorting Equilibria in the Credit Market." BRC Working Paper No. 96, Northwestern University, 1985.
3. ———. "Competitive Equilibrium in the Credit Market under Asymmetric Information." Forthcoming, *Journal of Economic Theory*.
4. S. Bhattacharya and P. Pfleiderer. "Delegated Portfolio Management." *Journal of Economic Theory* 36 (June 1985), 1-25.
5. Y. Chan and G. Kanatas. "Asymmetric Valuations and the Role of Collateral in Loan Agreements." *Journal of Money, Credit and Banking* 17 (February 1985), 84-95.
6. V. V. Chari and R. Jagannathan. "Adverse Selection and a Model of Real Estate Lending." Northwestern University Working Paper, June 1985, presented at the American Finance Association Meeting, Dallas, December 1984.
7. K. Dunn and C. Spatt. "Risk Sharing and Incentives: Implications for Mortgage Contract Terms and Pricing." Paper presented at the American Finance Association Meeting, Dallas, December 1984.
8. S. Grossman and O. D. Hart. "An Analysis of the Principal-Agent Problem." *Econometrica* 51 (January 1983), 7-46.
9. M. Harris and A. Raviv. "Optimal Incentive Contracts with Imperfect Information." *Journal of Economic Theory* 20 (April 1979), 231-59.
10. D. Jaffee and F. Modigliani. "A Theory and Test of Credit Rationing." *American Economic Review* 59 (December 1969), 850-72.
11. D. Jaffee and T. Russell. "Imperfect Information, Uncertainty, and Credit Rationing." *Quarterly Journal of Economics* 90 (November 1976), 651-66.
12. H. Milde and J. Riley. "Signalling in Credit Markets." Working Paper No. 34, Department of Economics, UCLA, January 1984.
13. R. Myerson. "Incentive Compatibility and the Bargaining Problem." *Econometrica* 47 (January 1979), 61-74.
14. S. A. Ross. "The Determination of Financial Structure: The Incentive Signalling Approach." *Bell Journal of Economics* (Spring 1977), 23-40.
15. M. Rothschild and J. E. Stiglitz. "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information." *Quarterly Journal of Economics* 90 (November 1976), 629-49.
16. C. Spatt. "Credit Reputation Equilibrium and the Theory of Credit Markets." GSIA Working Paper No. 40-81-82, Carnegie-Mellon University, 1982.
17. J. E. Stiglitz and A. Weiss. "Credit Rationing in Markets with Imperfect Information." *American Economic Review* 71 (June 1981), 393-410.
18. ———. "Incentive Effects of Termination: Applications to the Credit and Labor Markets." *American Economic Review* 73 (December 1983), 912-27.
19. ———. "Credit Rationing with Collateral." Bell Communication Research, Inc., Economics Discussion Paper #12, December 1985.
20. H. Wette. "Collateral in Credit Rationing in Markets with Imperfect Information: Note." *American Economic Review* 73 (June 1983), 442-45.