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Source: *The Journal of Finance*, Vol. 42, No. 2 (Jun., 1987), pp. 221-231
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2328250
Accessed: 26/11/2009 08:08

THE JOURNAL OF FINANCE • VOL. XLII, NO. 2 • JUNE 1987

Nonsynchronous Data and the Covariance-Factor Structure of Returns

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ABSTRACT

Evidence is presented that indicates that the standard estimator of the covariance matrix of daily returns provides a distorted view of the true covariance-factor structure. An alternative estimator, based on a model of the price-adjustment delay process, reveals roughly twice as much covariation in individual security returns. The number of factors identified also appears to increase when this estimator is employed. Since the linear space spanned by the estimated factor-loading vectors is quite sensitive to the estimator used, it is important that the consistent estimator be considered in the usual two-stage empirical investigations of the APT.

IN RECENT YEARS, THE estimator of the covariance matrix of returns has come to play an important role in financial empirical work. This is largely due to two developments. One is the increasing use of multivariate methods in tests of asset-pricing relations. Empirical applications include Gibbons [9], Stambaugh [21], Jobson and Korkie [11], and Shanken [20].[1] Another development is the growing interest in multifactor models and the related use of factor-analytic statistical methods. Much of this work has been conceptually linked to the arbitrage pricing theory of Ross [18]. Empirical investigations of this sort include Roll and Ross [17], Reinganum [15], Chen [4], Brown and Weinstein [2], and Dhrymes, Friend, and Gultekin [6]. Each of these factor-analytic studies employs daily security return data in estimating the covariance matrix of returns.[2]

The difficulties encountered in using daily data to estimate "beta" are now well known and extensively documented in the empirical literature. As Scholes and Williams [19] (henceforth SW) note, since many securities trade infrequently, accurate calculation of returns over any fixed interval is difficult.[3] The measurement problem is particularly severe when the interval is as short as a day. Nonsynchronous trading of securities thus leads to an errors-in-variables problem; in particular, the usual regression estimator of beta is biased and inconsistent. SW develop an alternative procedure for estimating beta that takes into account the problem of thin trading.

* Assistant Professor of Finance at the University of Rochester. This paper was presented at the Conference on Empirical Work on Arbitrage Pricing at U.S.C. and the Workshop on Arbitrage Pricing at the 1986 NBER Summer Institute. I am grateful to the participants and to G. Connor, G. Rubio, B. Schwert, M. Weinstein, and anonymous referees for helpful suggestions. Computational assistance was provided by A. Tajirian.

[1] These studies all employ the residual-covariance matrix of returns.

[2] The classic study of market and industry factors by King [13] uses monthly returns. Gibbons [10] also works with monthly data.

[3] Also, see Dimson [7] and Dimson and Marsh [8].

Cohen, Hawawini, Maier, Schwartz, and Whitcomb [5] (henceforth CHMSW) point to various frictions in the trading process that can lead to a distinction between "true" and observed return. Price-adjustment delays may result from (a) the actions of specialists seeking to satisfy stabilization obligations or redress inventory imbalances and (b) periodic trading by individuals due to information, decision, and transaction costs. CHMSW generalize the SW estimator to accommodate price-adjustment delays greater than a single day.

While the focus of the SW and CHMSW papers is on estimation of beta, there is some analysis of the intermediate steps of estimating covariance and variance. SW indicate that, for single securities, measured variances overstate but closely approximate true variances, while measured covariances understate in absolute magnitude true covariances. The covariance bias is greatest when one security trades very frequently while the other trades very infrequently.

The existing factor-analytic studies that use daily data all ignore the nonsynchronous return problem. In effect, they implicitly assume that the problem is not serious enough with randomly selected securities to warrant special treatment. The evidence presented below suggests, to the contrary, that the problem is substantial and should be considered in future applications that rely on an estimate of the covariance matrix of daily returns.

I. The CHMSW Model

Our empirical analysis is carried out within the analytical framework of the CHMSW paper. These authors assume that, due to price-adjustment delays, the observed return on security j over period t ($r^0_{j,t}$) can be modeled as a weighted sum of contemporaneous and lagged true returns:

$$r^0_{j,t} = \sum_{n=0}^{N} \gamma_{j,t-n,n} r_{j,t-n}, \tag{1}$$

where the gammas are random variables independent of the true returns and independent across securities. In this expression, $\gamma_{j,t-n,n}$ indicates the portion of the true return $r_{j,t-n}$ generated in period $t-n$ that is reflected n periods later in the observed return for period t. With no price-adjustment delays, $\gamma_{j,t-n,n}$ would equal zero for all positive values of n and one for $n = 0$.

Given some additional statistical assumptions, CHMSW demonstrate that

$$\text{cov}(r^0_{j,t}, r^0_{k,t}) + \sum_{n=1}^{N} \text{cov}(r^0_{j,t}, r^0_{k,t-n}) + \sum_{n=1}^{N} \text{cov}(r^0_{j,t-n}, r^0_{k,t}) = \text{cov}(r_{j,t}, r_{k,t}) \tag{2}$$

for any pair of securities j and k, $j \neq k$. It is clear from equation (2) that both contemporaneous and lagged cross-covariances of observed returns must be considered in estimating the true covariance parameter. Assuming that the true returns are independent and identically distributed over time, a consistent estimator of the true covariance may be obtained by replacing the covariances on the left side of (2) by their sample counterparts.[4] We shall refer to this statistic as the "adjusted estimator."

[4] We also assume stationarity of the gamma process and independence between gammas with second subscripts that differ by some minimum number of lags. This means that sufficiently distant true returns are assumed to work their way into observed returns independently.

II. Empirical Evidence—the Covariance Matrix

To facilitate comparison with other recent studies in the literature, our analysis is carried out over the period from July 1962 through December 1972, as in Roll and Ross [17]. Ten groups of thirty securities each are formed alphabetically. Securities with missing daily data are eliminated. If anything, this selection criterion probably biases the sample toward more actively traded firms and, therefore, against finding a significant nonsynchronous return effect. A brief exploratory analysis indicates that three lags should capture most, though not necessarily all, of the effect. Thus, all adjusted estimates below are based on $N = 3$ in (1) and (2).

Table I contains the average covariance estimates, both adjusted and unadjusted, for each of the ten subgroups. Since there are thirty securities in each group, the averages are taken over 435 ($30 \times 29/2$) distinct covariance elements. All estimates have been multiplied by 1000. The numbers are rather striking, whether we look at the individual group results or at the overall averages in the bottom row. The average magnitude of the adjusted estimate, .101, is nearly twice that of the unadjusted estimate, .052. Furthermore, over ninety-seven percent of all *individual* covariances estimates increase when the consistent estimator is used. The evidence suggests that price-adjustment delays induce substantial cross-serial correlation between individual observed security returns. While we have conducted no formal tests thus far, the consistency across the ten groups examined is compelling.

More formal evidence of the existence of cross-serial correlation is presented in Table II. For each pair of distinct securities j and k in a given subgroup, the sample covariance with k lagged one trading day is computed. Under the null hypothesis of no serial dependence in observed returns, an asymptotic standard error of this estimate and a corresponding "t-statistic" are easily obtained. (See the Appendix.) Average estimates and t-statistics for each subgroup are given in Table II, along with the proportion of statistics that are significantly positive at various levels. More than half are significant at the .05 level and more than two thirds at the .10 level, using one-sided tests.[5] The overall average estimate is .017. Thus, the average contribution of the first lag to the adjusted contemporaneous-covariance estimator is .034. This represents roughly one third (.034/.101) of the total adjusted covariation.

A quick test of the joint hypothesis that all cross-serial covariances are zero may be based on the average t-statistic over all ten subgroups. While the asymptotic variance of this cross-sectional average is fairly complicated, it cannot in any event exceed one. This conservative upper bound would be attained only if all the statistics were perfectly positively correlated. Even with this crude bound, the average, 1.86, is significantly positive at the .04 level. The true p-value is probably much lower, however. For example, assuming independence across the ten subgroups and perfect within-group correlation, the relevant

[5] Less than one percent of all t-statistics are significantly negative at these levels. The average estimates for two and three lags are .003 and .004, respectively. The corresponding average t-statistics are .33 and .47.

Table I

Average Covariance Estimates for Ten Subgroups Formed Alphabetically[a]

Group	Average Covariance Estimate[b] Unadjusted	Adjusted	Percent Increased
1	.052	.102	97
2	.057	.106	97
3	.059	.111	97
4	.054	.101	98
5	.038	.079	97
6	.054	.099	94
7	.056	.106	98
8	.049	.092	99
9	.058	.117	98
10	.047	.095	99
Averages	.052	.101	97

[a] Each group contains thirty securities from the CRSP daily return file with complete daily data from July 3, 1962, to December 31, 1972. "Unadjusted" indicates use of the usual covariance estimator, while "adjusted" indicates use of the consistent estimator based on (2). "Increased" refers to the number of covariance elements for which the adjusted estimate exceeds the unadjusted estimate.

[b] All estimates have been multiplied by 1000.

Table II

Average Cross-Serial Covariances with One Lag for Ten Subgroups Formed Alphabetically[a]

	Average		Percent Significant at Level:		
Group	Estimate[b]	t-Statistic	.10	.05	.01
1	.019	2.07	75	63	42
2	.019	1.84	66	54	36
3	.019	1.84	68	56	35
4	.016	1.82	68	58	34
5	.014	1.76	63	53	32
6	.017	1.63	60	49	28
7	.019	1.99	71	62	40
8	.015	1.83	66	55	35
9	.020	1.84	67	55	35
10	.017	1.99	71	59	40
Averages	.017	1.86	67	56	36

[a] Each group contains thirty securities from the CRSP daily return file with complete data from July 3, 1962, to December 31, 1972.

[b] Estimates are multiplied by 1000.

statistic $1.86\sqrt{10} = 5.88$ is significant at virtually any level. These observations reinforce the impressions obtained earlier from the descriptive evidence in Table I.

III. Empirical Evidence—the Factor Structure

In the next stage of our analysis, maximum-likelihood factor analysis is performed on each group of securities. Following Roll and Ross [17], the number of factors in each group is assumed to be five. The factor analysis is carried out using both adjusted and unadjusted correlation matrices.[6] Note that the CHMSW model does not provide a consistent estimator of the variance. (Equation (2) requires $j \neq k$.) Recall, however, SW's observation that nonsynchronicity of returns has a negligible effect on estimation of this parameter, at least when $N = 1$. Given this fact and lacking a superior alternative procedure, the usual variance estimator has been employed in all work described below.[7]

One way of assessing whether the adjusted covariance estimator has a significant impact on estimation of the factor structure is to look at the communalities. A communality is similar to an "R-squared" in regression analysis and indicates the proportion of security return variance that may be attributed to the factors. Average communalities for each subgroup, both adjusted and unadjusted, are contained in Table III. Also presented are the average squared loadings on each of the five factors. Since our estimation procedure ensures that the factors are orthogonal with unit variance, a security's communality is just the sum of its squared factor loadings.[8]

As with the covariances, the average communality about doubles from .16 to .33. Again, the results are consistent across the ten subgroups. In fact, all 300 individual-security communalities increase when the adjusted covariance estimator is used. It is interesting to consider the manner in which the communality increase is distributed across the five factors. We find that the average contribution of each factor roughly doubles. In particular, the contribution of the first factor rises from .10 to .18.

Recall that the statistical factor-model assumption entails the following decompositon of the (true) covariance matrix of returns:

$$V = BB' + D, \tag{3}$$

where B is a (30×5) matrix of factor loadings and D is a (30×30) diagonal matrix of residual variances. Note that the off-diagonal elements of V are

[6] Securities were eliminated from the analysis and replaced by new securities when a Heywood case arose in either the adjusted or unadjusted factor analysis. This is a situation in which some security's estimated residual variance is negative. See Johnson and Wichern [12], pp. 450, 452.

[7] Note that inconsistency of the variance estimator only affects the diagonal residual-covariance matrix of the factor model. The estimators of the factor loadings are still consistent, provided that the misspecification is not so severe as to make one or more of the diagonal elements nonpositive. In particular, if the variance is overstated, as in the SW model, this problem does not arise.

[8] Recall that the correlation matrix is factor-analyzed here. Identical communalities would be obtained using the covariance matrix although the interpretation of the squared loadings would have to be modified in this case. See Johnson and Wichern [12], pp. 450–451.

Table III

Average Communality and Squared Loading Estimates for Ten Subgroups Formed Alphabetically[a]

		Average Squared Factor Loadings				
Group	Average Communality	1	2	3	4	5
	Unadjusted					
1	.17	.11	.03	.01	.01	.01
2	.17	.12	.01	.01	.01	.02
3	.16	.10	.03	.02	.01	.01
4	.17	.07	.07	.01	.02	.01
5	.15	.10	.01	.01	.01	.01
6	.17	.09	.05	.01	.01	.01
7	.17	.10	.02	.02	.02	.01
8	.16	.11	.03	.01	.01	.01
9	.16	.12	.01	.01	.01	.01
10	.15	.10	.02	.01	.02	.01
Averages	.16	.10	.03	.01	.01	.01
	Adjusted					
1	.34	.21	.06	.02	.01	.03
2	.34	.16	.07	.03	.05	.03
3	.31	.13	.12	.02	.02	.02
4	.35	.14	.15	.02	.02	.02
5	.34	.15	.05	.07	.02	.04
6	.31	.22	.02	.03	.02	.02
7	.33	.25	.02	.02	.02	.02
8	.33	.18	.04	.06	.02	.02
9	.32	.21	.03	.02	.02	.03
10	.33	.19	.07	.02	.02	.02
Averages	.33	.18	.06	.03	.02	.02

[a] A communality indicates the proportion of a security's total variance that is "explained" by the factors. Since the correlation matrix is analyzed and the factor-covariance matrix is the identity matrix, the communality equals the sum of the squared loadings. Each group contains thirty securities from the CRSP daily return file with complete daily data from July 3, 1962, to December 31, 1972.

completely determined by B. We assume a similar decomposition for the covariance matrix of observed returns:

$$V_0 = B_0 B_0' + D_0. \tag{4}$$

The empirical findings above are consistent with a uniform attenuation of all loadings by about thirty percent, with the total variances remaining unchanged; i.e., $B_0 = .7B$, so that

$$V_0 \approx .5BB' + D_0.$$

Although the covariances and communalities are reduced, the number of factors is still five and the loading patterns are preserved.

To pursue this idea further, correlations between the thirty adjusted and unadjusted loading estimates are computed for each factor and each subgroup.

Apart from estimation error, the correlations should equal one if the loadings are uniformly attenuated. The actual results are presented in Table IV. Loadings with respect to the first factor exhibit the greatest "stability." Even in this case, though, the average correlation is only .66, indicating that less than half of the cross-sectional variation in the adjusted estimates is "explained" by their (linear) relation to the unadjusted estimates.[9] Correlations for the first factor range from .38 to .84 across the ten groups. Loadings with respect to factors 2 to 5 exhibit much less stability. In each case, there is at least one subgroup for which the correlation is actually negative. Thus, upon closer examination, the uniform-attenuation hypothesis does not appear consistent with the data. This is as expected if there is substantial heterogeneity in trading frequency (more generally, the price-adjustment delay process) for the firms in our sample.

In many asset-pricing empirical investigations, the main question of interest is whether the vector, E, of expected returns lies in the subspace spanned by the columns of B and a unit vector, i.e., whether there exists a vector λ such that

$$E = [1_N : B]\lambda.$$

Since studies such as those by Roll and Ross [17] and Dhrymes, Friend, and Gultekin [6] employ estimates of B_0, it is important to know whether the subspaces spanned by $[1_N : B]$ and $[1_N : B_0]$ are the same. While uniform attenuation is a sufficient condition for equality of these spaces, it is clearly not necessary.

To investigate this issue, for each factor and each group of thirty securities, we run a cross-sectional regression of that factor's adjusted loading vector on the five unadjusted loading vectors and a constant. If the span of the adjusted loadings equals the span of the unadjusted loadings, then we should obtain a perfect fit in each case. The actual results are presented in Table V. The average R-squared is highest for the first factor, equalling .78. The other averages are .56, .34, .35, and .43 for factors 2 to 5, respectively. Individual results are often much lower; more than a quarter of the R-squareds are less than .3, for example. Thus, the two column spaces of factor loadings look very different.

We have, of course, ignored error in estimating the loadings in this descriptive analysis. Errors of sufficient magnitude could, in principle, account for the relatively low correlations in Tables IV and V. Still, having more rigorously documented the existence of cross-serial correlation earlier, it seems unlikely that estimation error would provide a complete explanation for these results.

Previous studies, cited earlier, point to the large samples of daily data employed in the factor analysis of returns and implicitly assume that estimation error in the loadings is negligible. If this is indeed the case, then it is essential that the statistical procedure adopted provide consistent estimates, i.e., estimates that converge to the true loadings. This minimal requirement is satisfied by the adjusted estimator but is not met by the usual methods. Given the limited state of our knowledge at the present time, however, researchers may want to use both techniques and compare the results obtained.

In Table VI, we explore one final issue—the number of factors in the factor

[9] Rank correlations, not reported here, are generally lower than the reported Spearman coefficients.

Table IV

Cross-Sectional Correlations between the Thirty Adjusted and Unadjusted Loading Estimates for Each Factor and Each Subgroup[a]

	Factor-Loading Correlations				
Group	1	2	3	4	5
1	.75	.81	.63	.16	−.28
2	.38	.03	−.10	.24	.15
3	.80	.56	.67	.14	−.28
4	.51	.77	−.07	.35	.32
5	.50	.87	.36	.30	.57
6	.72	.13	.39	.31	−.30
7	.65	.60	−.14	−.41	.12
8	.71	.23	.59	.30	−.25
9	.69	−.66	.72	.07	.53
10	.85	.08	.30	−.08	.49
Averages	.66	.34	.34	.14	.11

[a] Groups are formed alphabetically, each containing thirty securities from the CRSP daily return file with complete daily data from July 3, 1962, to December 31, 1972.

Table V

R-Squareds from Cross-Sectional Regressions of a Given Factor's Adjusted Loadings on the Five Sets of Unadjusted Loadings and a Constant[a]

	Factor-Dependent Variable				
Group	1	2	3	4	5
1	.82	.73	.53	.40	.26
2	.81	.12	.44	.16	.45
3	.85	.38	.27	.40	.13
4	.70	.74	.69	.46	.62
5	.82	.57	.29	.10	.23
6	.65	.77	.21	.35	.71
7	.69	.52	.13	.24	.20
8	.84	.69	.43	.56	.77
9	.81	.66	.32	.47	.31
10	.80	.45	.12	.33	.52
Averages	.78	.56	.34	.35	.43

[a] Regressions are run for each group-factor combination. Groups are formed alphabetically, each containing thirty securities from the CRSP daily return file with complete daily data from July 3, 1962, to December, 31, 1972.

model. Chi-square tests of the null hypothesis that five factors are adequate are reported, both adjusted and unadjusted. The unadjusted results, based on the usual covariance estimator, generally support the null hypothesis. When the adjusted estimator is used, however, the test statistics rise dramatically, indicating that the five-factor model should be rejected. It would appear that more

Table VI

Chi-Square Tests of the Null Hypothesis That There Are Exactly Five Factors in the Factor Model[a]

	Unadjusted		Adjusted	
Group	Statistic	p-Value	Statistic	p-Value
1	290	.57	2430	.00
2	323	.13	2771	.00
3	330	.08	2350	.00
4	318	.17	2242	.00
5	284	.67	2185	.00
6	330	.08	2318	.00
7	316	.19	2208	.00
8	337	.05	2277	.00
9	328	.09	2324	.00
10	320	.15	2106	.00

[a] All chi-square statistics have 295 degrees of freedom. Tests are performed on ten subgroups formed alphabetically. Each group contains thirty securities from the CRSP daily return file with complete data from July 3, 1962, to December 31, 1972. "Unadjusted" indicates use of the usual covariance estimator, while "adjusted" indicates use of the consistent estimator based on (2). Minus two times the logarithm of the likelihood ratio is asymptotically distributed as chi-square with $[(N - K)^2 - N - K]/2$ degrees of freedom, where N is the number of securities and K is the number of factors.

factors are needed to explain the additional covariation captured by the adjusted estimator.

The p-values for the adjusted "chi-square statistics" must be viewed cautiously, as the use of the adjusted estimator in a statistical procedure based on the usual covariance estimator is questionable. Insofar as we have documented substantial cross-serial correlation in daily returns, though, the assumptions underlying the usual (unadjusted) chi-square test are seriously violated as well. Thus, some challenging problems of statistical inference will have to be solved before reliable inferences about the number of factors can be derived using daily data.

IV. Conclusions

The inadequacy of ordinary regression techniques for estimating "beta" with daily data has been extensively documented, and the use of alternative estimators is now fairly standard. Here we have provided evidence of a similar problem with respect to covariance estimation and the application of maximum-likelihood factor analysis to daily data. This obviously raises questions about the reliability of conclusions that have been drawn from past APT studies that use these methods. A similar concern was expressed by Roll [16] with regard to beta estimation and measurement of the "firm size effect."

A simple procedure, based on the work of Cohen, Hawawini, Maier, Schwartz, and Whitcomb [5], provides consistent estimates of the covariance-factor structure in the presence of the cross-serial correlation associated with nonsynchronous return data. Compared with standard methods, implementation of this estimation technique reveals roughly twice as much covariation in daily returns, with the explanatory power of the estimated five-factor model doubling as well. The average communality (factor-model R-squared) rises from .16 to .33 when the consistent estimator is adopted.[10] Such an increase seems reasonable in light of the observation that a single stock index typically accounts for twenty to twenty-five percent of the variation in (monthly) security returns.

More important, from the perspective of asset-pricing tests, is the fact that the linear space spanned by the estimated factor loadings is highly dependent on the estimation method employed. Thus, in principle, mean return could be (approximately) linearly related to one set of loadings yet not to the other. Whether this actually occurs in the context of any given study and associated experimental design is an empirical question. A replication of the existing literature, with consistent methods employed, would be desirable but is beyond the scope of this paper. Hopefully, the evidence presented here will convince researchers in this area of the need to re-examine their earlier findings and to adopt alternative methods in future analyses.[11]

Appendix

We shall now derive the asymptotic distribution of the sample cross-serial covariance lagged one period, under the null hypothesis of no serial dependence in observed returns. Initially, assume that expected observed returns are zero. In this case, the sample statistic is an average over T terms of the form

$$x_t \equiv r_{j,t} r_{k,t-1}.$$

The superscript "0" is omitted for convenience. We maintain the stationarity assumption of footnote 4 and further assume that all fourth-order moments are finite.

Under the null hypothesis, $\mathrm{E}(x_t) = 0$ and

$$\begin{aligned} \mathrm{E}(x_t x_{t-1}) &= \mathrm{E}(r_{j,t} r_{k,t-1} r_{j,t-1} r_{k,t-2}) \\ &= \mathrm{E}(r_{jt})\mathrm{E}(r_{k,t-1} r_{j,t-1})\mathrm{E}(r_{k,t-2}) = 0. \end{aligned}$$

Note that x_t need not actually be independent of x_{t-1}, due to the possible contemporaneous dependence of $r_{k,t-1}$ and $r_{j,t-1}$. Nonetheless, x_t and x_{t-1} are uncorrelated under the null. For $i > 1$, we do have independence of x_t and x_{t-i} since there are no overlapping terms. This is a case of "m dependence" with $m = 1$. Application of a suitable central-limit theorem (see Billingsley [1], p. 316) implies that the usual "t-statistic" computed from the x_t converges in distribution to a standard normal variate, as $T \rightarrow \infty$.

[10] This observation may have implications for the use of factor methods in event studies. See Brown and Weinstein [3].

[11] The use of monthly return data is another viable alternative although efficient use of the rich stores of daily data available is preferable. In recent work, Lehmann and Modest [14] find that their empirical conclusions are essentially the same whether daily or monthly data are used.

We have assumed that expected returns are zero. If not, replace r_{jt} by $r_{j,t} - \mathrm{E}(r_j)$ and $r_{k,t}$ by $r_{k,t} - \mathrm{E}(r_k)$ and apply the same sort of analysis. Finally, suppose we replace the expected returns by the sample means in computing the final "t-statistic." It is easily shown that the difference between this statistic and the one based on the true means converges in probability to zero, as $T \rightarrow \infty$. Thus, both have the same standard normal distribution in the limit. Details are available from the author.

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