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A Note on Optimal Credit and Pricing Policy under Uncertainty: A Contingent-Claims Approach

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INVESTMENT IN ACCOUNTS RECEIVABLE differs from investment in other assets since it is a joint decision on product pricing and credit policy. Any changes in product price, financing charges, or both will affect the quantity demanded and the investment in accounts receivable. In a world of certainty, a profit-maximizing firm would invest in accounts receivable until the marginal revenue equals the marginal cost of the product. This criterion maximizes the firm's value since no default risk in accounts receivable is considered.

In a realistic world with uncertainty, management of accounts receivable is more complicated because the quantity demanded is often stochastic and the payment by the customer may be uncertain due to default risk. A greater default risk, other things being equal, will imply a lower value of credit sale. Thus, a firm that considers the price elasticity of demand alone and that ignores the default risk and other factors will not be led to an optimal investment in accounts receivable. Product pricing decisions are not independent of the credit decisions. For example, increases in product price or financing charges may reduce the quantity demanded sufficiently to reduce the contractual revenue, but they may increase the current market value of the accounts receivable due to an increase in the payment probability.

Although the appropriateness of wealth maximization (as opposed to profit maximization) and the importance of the interdependency of the joint credit and pricing decisions have gradually been recognized in the literature, to our knowledge no work has succeeded in integrating the two. Under the assumption that the pricing decisions are given, several studies have concentrated on the credit-granting decisions and have thus assumed away the more important and interesting interactive effects between pricing and credit decisions.[1] A few recent studies have attempted to integrate these decisions. However, the criteria used in selecting the optimal policies are ad hoc and often not within a wealth-maximization framework.[2] Kim and Atkins [4] recognized some deficiencies in previous work and proposed a much improved model in which they attempt to

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[1] See, e.g., [7], [5], [10], [8], [6], and [9]. For a review and critique of the credit-granting literature after 1970, see [4] and, for that before 1970, see [2].

[2] One exception is the study by Kim and Atkins [4], who use net present value as the decision criterion.

integrate pricing and credit policies in a wealth-maximizing framework. However, the loss rate per period on credit sales is assumed to be exogenously given, and their model is essentially a certainty model. The interactive effects between pricing and credit policies have again been bypassed.

We provide herein a simple model which:

1. integrates the pricing and credit decisions under uncertainty within a wealth-maximizing framework;
2. extends the analysis to include uncertainties in both product demand and the customers' cash flows;
3. endogenizes and explicitly recognizes the interdependency between the pricing and credit policies; and
4. interprets the joint decisions as a contingent claim.

I. Uncertain Demand, Credit, and Pricing Policy

A. Impact of Stochastic Quantity Demanded on the Value of Accounts Receivable

Assume a firm is considering a credit sale to a customer (or a class of customers). Let P be the *contractual price* that includes the finance charge per unit of credit sale to be received at the end of the period. Let Q be an additive stochastic demand function as follows:

$$Q = Q(P) + X,$$

where X is normally distributed with mean zero and variance σ_X^2.[3] Thus, the revenue of the firm, θ, is also stochastic:

$$\theta = PQ(P) + PX,$$

with expected value $\bar{\theta} = P\bar{Q}$ and standard deviation $\sigma_\theta = P\sigma_X$. Furthermore, assume that the cash flow of the customer, Y, is normally distributed with mean $\bar{Y}$ and variance $\sigma_Y^2 > 0$. Y and θ are assumed to be jointly normally distributed with correlation coefficient $\rho_{Y\theta}$, covariance $\sigma_{Y\theta}$, and variance-covariance matrix given by

$$\Sigma = \begin{bmatrix} \sigma_Y^2 & \sigma_{Y\theta} \\ \sigma_{\theta Y} & \sigma_\theta^2 \end{bmatrix}.$$

In general, the expected value of the customer's cash flows can be assumed to be much greater than the contractual payment. But there may exist some states of nature in which the net cash flows of the customer are less than the contractual obligation. When that happens, the amount to be received by the firm is less than the contractual amount. Of course, the firm receives nothing if the customer's cash flow is negative. Thus, the payoff function of the credit sale at the end of the period can be expressed as $\max[\min(\theta, Y), 0]$. It is a contingent claim.

[3] A multiplicative form of a stochastic demand function with log-normal distribution can be assumed in the analysis at the expense of expositional complexity without changing the results. Furthermore, a negative value for Q has no economic meaning in either the deterministic or stochastic demand case; it has thus been ruled out for consideration.

and its value is a function of two underlying stochastic variables, Y and θ, as follows:

$$g(Y, \theta) = \begin{cases} \theta & Y \geq \theta > 0 \\ Y & 0 < Y < \theta \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Therefore, we can utilize the contingent-claim analysis of Brennan [3] and Stapleton and Subrahmanyam [11] to determine the value of accounts receivable:[4]

$$W(Y, \theta) = r^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(Y, \theta) f_{Y,\theta}(Y, \theta)\, dY\, d\theta, \tag{2}$$

where $W(Y, \theta)$ = value of the accounts receivable; $f_{Y,\theta}(Y, \theta)$ = joint modified normal density function of Y and θ with marginal means given by rV_Y and rV_θ, respectively, variance covariance matrix $\sum$, and correlation $\rho_{Y\theta}$; r = one plus the risk-free rate; and

$$rV_Y = \overline{Y} - \lambda \rho_{YM} \sigma_Y,$$

$$rV_\theta = \overline{\theta} - \lambda \rho_{\theta M} \sigma_\theta$$

are the respective certainty equivalents of the cash flows, Y, and the stochastic revenue, θ, in the CAPM framework. λ is the market price of risk, and ρ_{YM} and $\rho_{\theta M}$ are the respective correlations of Y and θ with the return on the market portfolio.[5]

The means of the joint density function in the valuation equation are the certainty equivalents of the stochastic variables Y and θ. The shifted means allow the contingent claim to be evaluated as a risk-neutral valuation relationship.[6]

Substituting (1) into (2), the value of accounts receivable becomes

$$W(Y, \theta) = r^{-1} \int_{-\infty}^{\infty} \left\{ \int_{0}^{\theta} Y f_{Y,\theta}(Y, \theta)\, dY + \int_{\theta}^{\infty} \theta f_{Y,\theta}(Y, \theta)\, dY \right\} d\theta. \tag{3}$$

The above expression has two parts. The first part represents the value of the accounts receivable when the customer's cash flow Y is not sufficient to pay for the contractual amount, while the second part represents the value when the

[4] Had we assumed continuous tradings for assets and log-normal distributions of asset prices, then Stulz's [12] complex option pricing model could have been employed to evaluate the contingent claim from credit sales. The qualitative results of the two approaches should be the same.

[5] In the case when the quantity demanded is deterministic, $Q = Q(P)$, the revenue R is deterministic. Using Brennan [3] directly, the value of the contingent claim (2) would reduce to

$$W(Y) = r^{-1} \int_{-\infty}^{\infty} g(Y) f(Y)\, dY, \tag{2'}$$

where $f(Y)$ is the modified normal probability density function with mean rV_Y and variance σ_Y^2. Note that had a log-normal distribution and continuous-time trading been assumed, then Black and Scholes' [1] options pricing model could have been employed to evaluate the contingent claim.

[6] This form of valuation relationship, where a claim in a non-risk-neutral world can be valued as if it were risk neutral, is called a risk-neutral valuation relationship. See Brennan [3] and Stapleton and Subrahmanyam [11].

cash flow is sufficient to pay for the contractual amount. Therefore, the sum of the two parts gives the value of accounts receivable to the firm.

To facilitate comparative statics analysis, equation (3) can be simplified by a change of variable. Define $Z = (\theta - rV_\theta)/\sigma_\theta$; then (3) becomes

$$W(Y, \theta) = r^{-1} \int_{-\infty}^{\infty} \left\{ \int_{0}^{z'} Y f_{Y,Z}(Y, Z)\, dY + \int_{z'}^{\infty} z' f_{Y,Z}(Y, Z)\, dY \right\} dZ, \quad (4)$$

where $z' = \sigma_\theta Z + \bar{\theta} - \lambda \rho_{\theta M} \sigma_\theta$ and $f_{Y,Z}(Y, Z)$ is the jointly normal density function of Y and Z.

With this transformation, the limits of integration and the integrand of the inner integrals are expressed in terms of cash flow distribution parameters of interest. To find the effect of changes in the expected contractual revenue on the value of accounts receivable, one can differentiate (4) with respect to $\bar{\theta}$ and obtain (the derivation is available from the authors):

$$\frac{\partial W(Y, \theta)}{\partial \bar{\theta}} = r^{-1} \int_{-\infty}^{\infty} \left\{ \int_{z'}^{\infty} f_{Y,Z}(Y, Z)\, dY \right\} dZ. \quad (5)$$

For a given level of contractual revenue, the inner integral tabulates the probability of payment ($Y > z'$) by the customer. The outer integration computes the expected value of payment for all levels of contractual revenue. Thus, a dollar increase in expected revenue ($\bar{\theta}$) does not increase the value of accounts receivable, $W(Y, \theta)$, by one dollar, due to the interactive effect of the credit and pricing policies.

We can further simplify the expression of the value of accounts receivable by standardizing Y. Define $U = (Y - rV_Y)/\sigma_Y$; then (5) can be simplified to

$$\frac{\partial W(Y, \theta)}{\partial \bar{\theta}} = r^{-1}\{1 - \bar{F}(z'')\} \geq 0, \quad (6)$$

where

$$\bar{F}(z'') = \int_{-\infty}^{\infty} \int_{-\infty}^{z''} f_{U,Z}(U, Z)\, dU\, dZ;$$

$$z'' = \frac{\sigma_\theta Z + r(V_\theta - V_Y)}{\sigma_Y};$$

and $f_{U,Z}(U, Z)$ is the standardized *joint* normal density function with zero means and unit variances.

$\bar{F}(z'')$ represents the expected probability of default and is a function of z'', which depends on the characteristics of the customer's cash flow and the stochastic revenue (V_Y, σ_Y, V_θ, σ_θ, $\rho_{Y\theta}$, ρ_{YM}, and $\rho_{\theta M}$). From (6), it is obvious that the value of accounts receivable is positively related to the expected contractual revenue, ceteris paribus. However, increasing a dollar of credit sale will not increase the value of accounts receivable by a dollar, not only because of the discount factor, but also because of the probability of default. Herein lies the interdependency between the pricing and credit policies. Given the cash flow characteristics of the customer and the correlation of this cash flow with the uncertain quantity demanded, changes in pricing policy affect not only the expected contractual revenue $\bar{\theta}$ (and thus z''), but also the probability of default, $\bar{F}(z'')$, and thus the value of accounts receivable. Since this default probability depends on both the characteristics of the customer's cash flow and the uncertain demand for the particular product, it is not difficult to see why suppliers in different industries offer different credit and pricing policies.[7]

In addition to the impact on the value of accounts receivable expressed in equation (6), changes in pricing policy may also affect the variability (σ_θ) of the contractual revenue. The impact of a change in the variability on the value of accounts receivable can be determined by taking the partial derivative of $W(Y, \theta)$ with respect to σ_θ:

$$\frac{\partial W(Y, \theta)}{\partial \sigma_\theta} = r^{-1}\{E_{u,z}(z'') - \lambda\rho_{\theta M}[1 - \bar{F}(z'')]\}, \tag{7}$$

where

$$E_{u,z}(z'') = \int_{-\infty}^{\infty} \int_{z''}^{\infty} Zf_{U,Z}(U, Z)\, dU\, dZ$$

is the partial mean of z with truncation point z''.

The total impact of a change in pricing policy on the value of accounts receivable is, therefore,

$$\frac{\partial W(Y, \theta)}{\partial P} = \frac{\partial W(Y, \theta)}{\partial \bar{\theta}} \cdot \frac{\partial \bar{\theta}}{\partial P} + \frac{\partial W(Y, \theta)}{\partial \sigma_\theta} \cdot \frac{\partial \sigma_\theta}{\partial P}. \tag{8}$$

Substituting (6) and (7) into (8) and simplifying, we obtain

$$\frac{\partial W(Y, \theta)}{\partial P} = r^{-1}[1 - \bar{F}(z'')] \cdot \left[P\frac{\partial \bar{Q}}{\partial P} + \bar{Q}\right] + r^{-1}\sigma_X\{E_{u,z}(z'') - \lambda\rho_{\theta M}[1 - \bar{F}(z'')]\}. \tag{9}$$

The first term above represents the impact of pricing policy due to changes in the expected value of the credit revenue. The second term reflects the impact of pricing policy on the value due to the ensuing changes in the variability of the credit revenue. Taken together, the impact of pricing policy on the value of accounts receivable is dependent on the cash flow characteristics of the customers, the price responsiveness of the demand, the variability of the demand, and the correlation of the revenue with the economy. (This correlation can differ from industry to industry.) Ignoring the uncertainties in both the cash flow (Y) and the quantity demanded (Q) would have distorted the optimal pricing policy for a wealth-maximizing firm. As far as we know, these interactive effects were not explicitly recognized in previous literature on pricing and credit policy.

[7] In addition to the factors determining z'', the differences in pricing and credit policies among industries may also be due to the correlation of the customer's cash flow with the economy, which will be discussed briefly below. Furthermore, we recognize that to compare practices among different industries, we also must examine the differences in credit sales provisions or indenture, a topic which will carry us too far afield.

B. The Optimal Product Pricing Policy with Credit Sale

To derive an optimal product pricing policy when the demand is uncertain, we assume that the functional form of the non-negative production cost, $C(Q)$, is known, although the realized production cost is uncertain because Q is random. Furthermore, in conjunction with the payment of a credit sale customer, the actual cash payoff to the supplier (or wage earners) of this product is assumed to depend on the random cash flows (Y) of the customer. Of course, this assumption can be easily relaxed for a big firm without any cash constraint. The payoff function to the supplier can be represented by

$$h(Y, Q) = \begin{cases} C(Q) & Y \geq C(Q) \\ Y & 0 < Y < C(Q) \\ 0 & Y \leq 0. \end{cases} \tag{10}$$

Similar to the valuation of the contingent claim on the revenue side, we can derive the value of the production cost, $H(Y, Q)$, and show that

$$\frac{\partial H(Y, Q)}{\partial \bar{Q}} = r^{-1} \frac{\partial C(Q)}{\partial \bar{Q}} [1 - \bar{F}(w)], \tag{11}$$

where

$$w = \frac{C(Q) - rV_Y}{\sigma_Y}$$

is the standardized value of the production cost. Note that $\bar{F}(w)$ represents the default probability of the firm (due to the credit sale customer) not being able to pay the production cost due to the supplier. Had we assumed that the supplier would be paid regardless of the payment of the credit sale customer, then this default probability would have been zero, and the right-hand side of (11) would have been just the expected marginal cost instead of the certainty-equivalent cost. However, (11) is more general, for it includes the classical marginal expected cost criterion as a special case.

For a value-maximizing firm, the optimality condition for pricing requires equating the marginal value of accounts receivable to the marginal value of the cost of production, i.e., $\partial W(Y, \theta)/\partial P = \partial H(Y, Q)/\partial P$. Simplifying, we obtain:

$$P^* = \left\{[1 - \bar{F}(z'')]\left[1 + \frac{1}{\bar{e}}\right]\right\}^{-1} \cdot \left\{\frac{\partial C(Q)}{\partial \bar{Q}} [1 - \bar{F}(w)] - \sigma_X E_{u,z}(z'') \frac{\partial P}{\partial \bar{Q}}\right\} + \left[1 + \frac{1}{\bar{e}}\right]^{-1} \lambda \rho_{\theta M} \sigma_X \frac{\partial P}{\partial \bar{Q}}, \tag{12}$$

where $\bar{e} = (P/\bar{Q})(\partial\bar{Q}/\partial P)$ is the expected price elasticity of demand. The optimal pricing decision is a function of the expected default probabilities, $\bar{F}(z'')$ and $\bar{F}(w)$, the expected price elasticity of demand, $\bar{e}$, the expected marginal production cost, $\partial C(Q)/\partial\bar{Q}$, and the correlation of the random demand with the return on the market portfolio. In addition, the optimal pricing decision must also include the impact of the variability of the uncertain quantity demanded (and thus the contractual revenue θ) and the correlation between the cash flow of the customer and the contractual revenue. The higher the correlation between the customer's cash flow and the contractual revenue ($\rho_{Y\theta}$), the higher the value of accounts receivable.[8] Furthermore, it can be readily seen from the definition of z'' and $\bar{F}(z'')$ that the higher the uncertainty of demand (σ_X or σ_θ), the higher the expected probability of default, ceteris paribus. This effect exists even when the cash flow of the customer is assumed to be independent of the uncertain quantity demanded, and thus it must affect the optimal pricing decision. This uncertain demand effect on pricing differs from the interactive effect discussed earlier. The latter deals with the interaction between the pricing decision and the demand uncertainty. The former deals with interaction between the uncertain quantity demanded and the customer's cash flow. These two effects taken together jointly affect, and interact with, the pricing decision. As far as we know, these interactions have not been recognized in existing literature in this area.

II. Summary and Conclusion

Existing literature on the management of accounts receivable and the optimal credit and pricing policy has recognized for some time that the product-pricing and the credit-policy decisions are interdependent and should be determined jointly. As pointed out above, however, attempts to integrate these two areas have at best been ad hoc. Either the pricing decision is assumed to be given and effort is concentrated on the credit-granting decision, or the criteria used for the decisions are at odds with those generally accepted in modern finance theory.

Applying a discrete-time contingent-claim analysis to the credit and pricing decisions under uncertainty, we have not only integrated the two decisions successfully, but have also analyzed their interactions within the wealth-maximization framework consistent with the modern theory of corporate finance. By recognizing the inherent uncertainties in both customer cash flows and quantity demanded, we have not only endogenized the default probabilities in the model explicitly, but have also captured the interactive effects between the two decisions. The analysis of the model has provided intuitive interpretations.

[8] Had we assumed continuous trading and log-normal distributions of asset prices and applied Stulz's [12] complex option pricing model in our credit sales analysis, then the partial derivative of the price of the contingent claim with respect to the correlation between the two risky assets would have been positive (see [12], p. 171).

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