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Commercial Bank Portfolio Behavior and Endogenous Uncertainty

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ABSTRACT

This paper demonstrates how Bayesian information may be analyzed as a variable input in determining an optimal bank portfolio and investigates the impact of information in a way that is statistically satisfactory. A portfolio model is developed, and the impact of information is analyzed. Information is treated as an economic input that is used up to the point where its *predicted* marginal benefit is exactly equal to its marginal cost, and, from there, the optimal demand for information is derived. A comparative-static analysis demonstrates that the reaction of optimal portfolio holdings to interest rate changes under variable uncertainty is dramatically different from portfolio behavior when uncertainty is exogenous. Finally, the elasticity of reserves with respect to scale is examined under the assumption of variable uncertainty.

IN STANDARD COMMERCIAL BANK portfolio models, the optimal levels of asset holdings are determined for a given level of uncertainty of reserve deficiencies. The intent of this paper is to demonstrate how Bayesian information may be analyzed as a variable input in determining an optimal bank portfolio and to investigate the impact of information upon bank portfolio behavior.

Aigner and Sprenkle [1], Baltensperger [3], and Baltensperger and Milde [5] were the first to develop models of the banking firm in the context of variable information, an objective this paper shares. Aigner and Sprenkle presented a very simple model of information acquisition and then examined the impact of information upon default risk and commercial bank lending behavior. The role of variable information in determining a bank's optimal precautionary reserve holdings was examined by Baltensperger [3]. While the efforts of Aigner and Sprenkle and Baltensperger were important, they failed to consider the comprehensive impact that information would have on a commercial bank's assets. Baltensperger and Milde (B and M) remedied this situation in 1976 by deriving the optimal level of reserves, loans, securities, and information simultaneously. They found that the comparative-static behavior of portfolio holdings under variable uncertainty is dramatically different from portfolio behavior when uncertainty is exogenous. However, as important as Baltensperger and Milde's results were, their usefulness is constrained by B and M's information function. In particular, B and M assumed that the mean of reserve losses never changes with the acquisition of information, only the variance of those losses. This assumption is distracting since it would seem likely that assimilated information could convince the bank that the mean of the relevant random variable has either

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increased, decreased, or remained the same. In addition, B and M had information reduce the variance of reserve losses in a fashion prescribed by an arbitrarily chosen mathematical function. This approach must be refined since the specification of how the mean and the variance of a random variable change with changes in information must be done in a fashion that is statistically satisfactory.[1]

This paper seeks to endogenize Bayesian information in a conventional bank portfolio model of the type employed by Baltensperger and Milde. The Bayesian approach to the information demand decision allows the decision maker to revise the mean and the variance of reserve losses in a statistically meaningful way. In particular, a prior probability density function is updated by the acquisition of information, and the new probability density function, the posterior, will then yield a certain mean and variance. Therefore, the Bayesian approach will allow information to convince the bank that the mean of reserve losses has increased, decreased, or remained unchanged. In addition, it becomes clear that the modification of the variance by additional information is determined by the distinction between the prior and posterior variance and not by some ad hoc mathematical function.

This paper will be divided into six sections. In Section I, I outline the portfolio model that will be used and discuss the role of information in revising the expected profit to the bank. In Section II, I derive and discuss the bank's optimal demand for financial assets. In Section III, I determine the prior expectation of the posterior expected profit so that the anticipated impact of information can be evaluated. Then I derive the optimal demand for information. The comparative-static behavior of the optimal level of information and financial holdings is examined in Section IV. In Section V, I examine the elasticity of bank reserves with respect to scale under the assumption of variable Bayesian information. The paper is summarized in the final section.

I. Expected Profit and Information

The model assumes that the bank is faced with uncertain deposit withdrawals over a set period of time but that it has no legal reserve requirements. The bank has a portfolio of an exogenously given size with which to meet these potential reserve or cash needs. The portfolio can be allocated among three financial assets that have nonrandom rates of return. If reserve needs occur during the period, assets must be sold and there are transactions costs that are proportional to the size of the sale. The bank selects its portfolio such that its expected profit will be maximized.

The following notation will be employed in determining the optimal portfolio for the bank:

Parameters

w_o = initial endowment of bank funds, the size of portfolio in dollars.

r_1, r_2, r_3 = rates of return on each asset, where $r_3 > r_2 > r_1$.

a_1, a_2, a_3 = percentage cost of converting each asset to cash, where $a_3 > a_2 > a_1$.

[1] For a more comprehensive treatment of the status of the micro bank modeling literature, see Santomero [12].

Random Variables

y = net deposit withdrawals or net reserve needs during the period.
P = profit.

Decision Variables

$x_1, x_2, x_3 (= w_o - x_1 - x_2)$ = dollar amounts held in each asset.
θ = level of information obtained.

It should be noted that the subscripts on the assets run from the most to the least liquid asset, where, for example, x_1 is the most liquid asset with the lowest rate of return and the smallest conversion cost. The prior probability density function of y is given by $f(y \mid I_\phi)$, where I_ϕ is the prior information index. If the bank decides to select its portfolio on the basis of prior information alone, then it maximizes expected profit conditional on I_ϕ:

$$\begin{aligned} E(P \mid I_\phi) = {} & r_1x_1 + r_2x_2 + r_3(w_o - x_1 - x_2) \\ & - a_1\int_0^{x_1} yf(y \mid I_\phi)\, dy - a_1x_1 \int_{x_1}^{\infty} f(y \mid I_\phi)\, dy \\ & - a_2 \int_{x_1}^{x_1+x_2} (y - x_1)f(y \mid I_\phi)\, dy - a_2x_2 \int_{x_1+x_2}^{\infty} f(y \mid I_\phi)\, dy \\ & - a_3 \int_{x_1+x_2}^{\infty} (y - x_1 - x_2)f(y \mid I_\phi)\, dy. \end{aligned} \tag{1}$$

The interpretation of equation (1) is straightforward. The first three terms on the right-hand side are the returns on the three assets available for the bank to hold. The next two terms are the expected liquidation costs of the most liquid asset x_1. The first of these two terms is a_1 times the expected net deposit withdrawals over the interval 0 to x_1, and the second term is a_1x_1 times the likelihood that net reserve needs will be greater than x_1. Similarly, the next three terms can be interpreted as liquidation costs for x_2 and x_3.

On the other hand, the bank may decide to collect additional information before determining its optimal portfolio holdings. The bank now selects a portfolio such that it maximizes the posterior expected profits given by

$$\begin{aligned} E(P \mid I_\theta) = {} & r_1x_1 + r_2x_2 + r_3(w_o - x_1 - x_2) \\ & - a_1\int_0^{x_1} yf(y \mid I_\theta)\, dy - a_1x_1 \int_{x_1}^{\infty} f(y \mid I_\theta)\, dy \\ & - a_2 \int_{x_1}^{x_1+x_2} (y - x_1)f(y \mid I_\theta)\, dy - a_2x_2 \int_{x_1+x_2}^{\infty} f(y \mid I_\theta)\, dy \\ & - a_3 \int_{x_1+x_2}^{\infty} (y - x_1 - x_2)f(y \mid I_\theta)\, dy. \end{aligned} \tag{2}$$

The interpretation of expression (2) is similar to the interpretation of (1), but now the probability density function of y, net deposit withdrawals, is given by $f(y \mid I_\theta)$, where I_θ is the posterior information index.

II. Optimal Financial Holdings

Maximizing (2) with respect to x_1 and x_2 (which implicitly determines x_3^*) yields the following optimality conditions for the bank's portfolio holdings:[2]

$$F(x_1^* \mid I_\theta) = 1 - \frac{r_2 - r_1}{a_2 - a_1} \tag{3}$$

and

$$F(x_1^* + x_2^* \mid I_\theta) = 1 - \frac{r_3 - r_2}{a_3 - a_2}. \tag{4}$$

According to (3) and (4), the optimal portfolio is obtained if x_1 and $x_1 + x_2$ are set such that their posterior cumulative distributions equal the RHS of (3) and (4), respectively.[3] The optimal conditions are simple functions of interest rates and transactions costs stated in terms of the posterior cumulatives.[4] It is interesting to note that the demand for the most liquid asset depends only on its own interest rate and transactions cost and that of the second most liquid asset. The interest rate and transactions cost for the third asset, the least liquid asset, play no role in determining demand for the first asset.[5] This "adjacency property" was first derived by Santomero [11] in the context of household demand for money.

In our commercial-banking scenario, x_1 would correspond to bank reserves, x_2 would be government securities held by the bank, and x_3 would be loans outstanding. In reality, of course, if x_1 were bank reserves, then x_1 would receive no rate

[2] The results derived in this paper are for the three-asset case, but the optimality conditions for the financial holdings could easily be generalized to the n-asset case. Second-order conditions are correct for a maximum to exist, and they imply no further restrictions on the model.

[3] Even though Baltensperger and Milde minimize a loss function to derive their optimality conditions, with just a little arithmetic, (3) can be rewritten as B and M's expression (11), and (4) can be rewritten as B and M's equation (7).

[4] The optimality conditions detailed in (3) and (4) do not preclude one asset from dominating another. For example, it is possible for $F(x_1^* + x_2^*) < F(x_1^*)$ if $\frac{r_3 - r_2}{a_3 - a_2} > \frac{r_2 - r_1}{a_2 - a_1}$, in which case x_2 would be dominated by x_1 and the inefficient or dominated asset would be removed from the bank's portfolio. For the purposes of this paper, I will assume that (3) and (4) will yield an efficient portfolio of three assets. Details of financial dominance in a portfolio-determination model can be found elsewhere.

[5] The demand for both assets depends not on levels of interest rates but on the differences. If all rates increase by a constant amount, there will be no changes in the optimal holdings of each asset. However, if all rates increase by a constant percentage, then optimal x_1 will fall and x_3 will increase, while the impact on x_2 is ambiguous. If the expected level of withdrawals increases one unit, then x_1 also increases one unit and, while x_2 is unchanged, x_3 falls by one unit. Intuitively, x_1 and x_2 increase if the standard deviation of reserve needs increases. Surprisingly, however, the elasticities of x_1 and x_2 with respect to the standard deviation are exactly one if the mean level of reserve needs is zero.

of return and the percentage conversion cost would be zero, i.e., $r_1 = a_1 = 0$. However, in deference to the possibility that one-day deposits at correspondent banks (reserves) will draw earnings and because a certain symmetry arises in the comparative statics of Section IV, the analysis in this paper will allow r_1 to be positive. The anxious reader can stylize the results by setting $r_1 = 0$ where appropriate. As a concession to reality, the parameter a_1 is set equal to zero immediately.

III. Analysis of the Demand for Information

In this section, I derive the optimal demand for information. However, for the analysis to proceed any further, I must assume that the prior probability density function of deposit withdrawals is normal with a prior mean of μ_ϕ and prior variance, σ_ϕ^2, of $1/\phi$ so that $f(y \mid I_\phi)$ is $f(y \mid \mu_\phi, \sigma_\phi^2)$. In this case, the information index is simply the mean and the variance of the probability density function since the normal distribution is completely specified by the first two moments.

Following Kihlstrom [7], the unit of information is referred to as an observation, $\tilde{z}_i$, which is normally distributed with mean μ and a variance of 1. The observation may be taken to reflect any monetary or financial variable deemed relevant to the net reserve needs of the bank.[6] Regardless of the variable chosen, the acquired information can be conveniently summarized by using the fact that the arithmetic mean,

$$\tilde{y}_\theta = \frac{1}{\theta} \sum_{i=1}^{\theta} \tilde{z}_i, \tag{5}$$

is a sufficient statistic.[7] Therefore, observing y_θ is equivalent to observing the set $\{z_1, z_2, \ldots, z_\theta\}$.

If the bank chooses to acquire θ units of information[8] and observes y_θ for the value of $\tilde{y}_\theta$, then the revised mean and variance of reserve needs y, according to Bayesian statistics, is given by (6):

$$\mu_\theta = \frac{\phi}{\theta + \phi} \mu_\phi + \frac{\theta}{\theta + \phi} y_\theta, \qquad \sigma_\theta^2 = \frac{1}{\theta + \phi}, \tag{6}$$

so that $f(y \mid I_\theta) = f(y \mid \mu_\theta, \sigma_\theta^2)$.

It must be emphasized that, when a bank makes its information-demand decision, it does not yet know which value, μ_θ, the random variable $\tilde{\mu}_\theta$ will actually realize since y_θ is unknown at this time. Hence, the bank must compute the

[6] Formally, the decision maker may choose from an infinite number of independent random variables $\{\tilde{y}_\theta\}$, $\theta \in (0, \infty)$, where $\tilde{y}_\theta$ is necessarily assumed to be correlated with net reserve needs.

[7] The realizations of the random variables $\tilde{y}_\theta$ and $\tilde{z}_i$ will be denoted as y_θ and z_i.

[8] We have referred to θ as the amount of information provided by the observation of $\tilde{y}_\theta$. A rigorous justification for this terminology is beyond the scope of this paper and is available elsewhere. (See Marschak and Miyasawa [8].) However, we can give an intuitive indication of why θ is an appropriate measure of information. First, note that, when θ and θ' are integers, if $\theta > \theta'$, then $\tilde{y}_\theta$ is more informative than $\tilde{y}_{\theta'}$ in the sense that observing $\tilde{y}_\theta$ rather than $\tilde{y}_{\theta'}$ is equivalent to observing $\theta - \theta'$ additional independent observations of $\tilde{z}_i$'s. Also, note that, as θ increases, the variance of $\tilde{y}_\theta$ falls and the variance of net reserve needs falls.

expected value of θ units of information by taking the prior expectation of the posterior expected profit. The prior expectation of the posterior profit takes account of the uncertainty of μ_θ, at the time of the information-demand decision, by weighting the bank's maximum expected profit when $\tilde{\mu}_\theta = \mu_\theta$ by the probability that $\tilde{\mu}_\theta = \mu_\theta$. The bank's maximum expected profit when $\tilde{\mu}_\theta = \mu_\theta$, $E(P(x_1^*, x_2^*) \mid I_\theta)$, can be found by substituting x_1^* for x_1 and x_2^* for x_2 in equation (2) and then using the optimality conditions for financial holdings to solve the RHS of (2). The bank's posterior expected profit when its portfolio is optimally determined is then given as

$$E(P(x_1^*, x_2^*) \mid I_\theta) = r_3 w_0 - \mu_\theta\{a_2[F(x_1^* + x_2^* \mid \mu_\theta, \sigma_\theta^2) - F(x_1^* \mid \mu_\theta, \sigma_\theta^2)] + a_3[1 - F(x_1^* + x_2^* \mid \mu_\theta, \sigma_\theta^2)]\} - \sigma_\theta^2\{a_2[f(x_1^* \mid \mu_\theta, \sigma_\theta^2) - f(x_1^* + x_2^* \mid \mu_\theta, \sigma_\theta^2)] + a_3[f(x_1^* + x_2^* \mid \mu_\theta, \sigma_\theta^2)]\}. \quad (7)$$

Equation (7) can then be evaluated to determine the prior expectation of the posterior profit, $\int_{-\infty}^{\infty} E(P(x_1^*, x_2^*) \mid I_\theta) f(\mu_\theta \mid I_\phi)\, d\mu_\theta$. Since r_i, a_i, σ_θ^2, w_0, $f(\cdot \mid \mu_\theta, \sigma_\theta^2)$, and $F(\cdot \mid \mu_\theta, \sigma_\theta^2)$ are independent of μ_θ, and since the prior expectation of μ_θ conditional on I_ϕ is μ_ϕ, we have

$$\int_{-\infty}^{\infty} E(P(x_1^*, x_2^*) \mid I_\theta) f(\mu_\theta \mid I_\phi)\, d\mu_\theta = r_3 w_0 - \mu_\phi\{a_2[F(x_1^* + x_2^* \mid \mu_\theta, \sigma_\theta^2) - F(x_1^* \mid \mu_\theta, \sigma_\theta^2)] + a_3[1 - F(x_1^* + x_2^* \mid \mu_\theta, \sigma_\theta^2)]\} - \sigma_\theta^2\{a_2[f(x_1^* \mid \mu_\theta, \sigma_\theta^2) - f(x_1^* + x_2^* \mid \mu_\theta, \sigma_\theta^2)] + a_3[f(x_1^* + x_2^* \mid \mu_\theta, \sigma_\theta^2)]\}. \quad (8)$$

The result, that the prior expectation of the posterior profit is the posterior profit after μ_ϕ is substituted for μ_θ, is important.[9] It follows from the bank's linear profit function and from assumptions about the distribution of the prior probability density function and the sample information.[10] Expression (8) yields the posterior expected profits when the stochastic aspect of μ_θ is recognized, yet it is simple enough to allow us to derive the optimal demand for information and to provide the comparative-static analysis of Section IV. This result makes an ambitious task, understanding the role of Bayesian information in a bank portfolio model, mathematically manageable.

It is convenient to rewrite (8) in terms of standardized units. Therefore, let

$$u_1 = \frac{x_1^* - \mu_\theta}{\sigma_\theta}, \quad u_2 = \frac{x_2^*}{\sigma_\theta}, \quad u_3 = \frac{x_3^*}{\sigma_\theta}, \quad \text{and} \quad u_4 = \frac{w_0}{\sigma_\theta},$$

and substitute $f(\cdot)/\sigma_\theta$ for $f(\cdot \mid \mu_\theta, \sigma_\theta^2)$ in expression (8).[11] In addition, since information becomes available only at a cost, the expected posterior profits of the portfolio must be adjusted. For simplicity, we assume that the adjustment to

[9] The parameters r_i, a_i, σ_θ^2, and w_0 are obviously independent of μ_θ. In addition, since $F(\cdot \mid \mu_\theta, \sigma_\theta^2)$ depends on the RHS of expressions (3) and (4) but not on μ_θ, it follows that $f(\cdot \mid \mu_\theta, \sigma_\theta^2)$ is independent of μ_θ.

[10] For other cases see Raiffa and Schlaifer ([10], pp. 188–207).

[11] It should be clear to the reader that dropping μ_θ and σ_θ^2 from $f(\cdot \mid \mu_\theta, \sigma_\theta^2)$ is our way of writing the probability density function of the standardized normal variate, $f(\cdot)$.

(8) is a fixed cost, q, per observation. With this in mind, equation (8) can now be written as

$$\int_{-\infty}^{\infty} E(P(x_1^*, x_2^*) \mid I_\theta) f(\mu_\theta \mid I_\phi)\, d\mu_\theta - q\theta$$
$$= r_3\sigma_\theta u_4 - \mu_\phi\{a_2[F(u_1 + u_2) - F(u_1)] + a_3[1 - F(u_1 + u_2)]\}$$
$$- \sigma_\theta\{a_2[f(u_1) - f(u_1 + u_2)] + a_3[f(u_1 + u_2)]\} - q\theta. \quad (9)$$

Expression (9), the prior expectation of the posterior profit adjusted for the cost of information, can now be maximized with respect to θ to determine the optimal demand for information in this portfolio determination model:

$$\theta^* = \left[\frac{a_2[f(u_1) - f(u_1 + u_2)] + a_3[f(u_1 + u_2)] - r_3u_4}{2q}\right]^{2/3} - \phi. \quad (10)$$

The larger $f(u_1)$ and $f(u_1 + u_2)$ in (10), the greater the demand for information. This result is particularly intuitive since large values of $f(u_1)$ and $f(u_1 + u_2)$ correspond to relatively small values of u_1 and u_2, that is, small values of x_1^* and x_2^*. Hence, there is a substitution effect between optimal financial holdings and θ^*. The bank could face the risk of reserve deficiencies by holding large amounts of the relatively liquid assets x_1^* and x_2^* and a small amount of θ^*, or the bank could hold small amounts of x_1^* and x_2^* but purchase a large amount of information. Conversion costs also have a positive impact on the demand for information.[12] Equation (10) highlights the fact that, although information is used to revise prior notions in a statistically meaningful way, the *amount* of information the bank will acquire is an economic decision. In fact, information as an economic input will be acquired up to the point where its anticipated marginal benefit is exactly equal to its marginal cost, and that level of θ is given by (10).

IV. Comparative Statics

The intent of this section is to examine the comparative-static behavior of bank portfolio holdings when variable information is gathered in a Bayesian fashion. Standardized versions of (3) and (4), along with expression (10), provided a system of optimality conditions that was differentiated with respect to various parameters to develop the comparative statics of θ^*, x_1^*, and x_2^*. In order to put the analysis *in this section* on the same basis as B and M's work, the posterior mean was set equal to zero so that u_i is necessarily positive.[13]

[12] An interior solution for the quantity of information demanded is assumed to exist in (10). However, a corner solution could be obtained, for example, when a sufficient amount of prior information exists to make any further expenditure on θ unprofitable or when r_3u_4 is a relatively large number.

[13] Setting the posterior mean equal to zero is, in effect, a stationary condition for the bank model. A nonzero posterior mean of net deposit withdrawals would indicate a known drift of reserve needs, which could be handled by an extension of transactions demand theory to a three-asset portfolio model.

Examining the comparative-static behavior of θ^* provides some interesting observations:

$$\frac{d\theta^*}{d\phi} < 0, \tag{11a}$$

$$\frac{d\theta^*}{dq} < 0, \tag{11b}$$

and

$$\frac{d\theta^*}{dr_i} = \frac{-(1/3)u_i}{q(\theta^* + \phi)^{1/2}} < 0 \quad \text{for } i = 1, 2, 3. \tag{11c}$$

The differentiation of θ^* with respect to prior information, ϕ, indicates that the smaller the variance of the prior distribution of net deposit withdrawals, the less additional information demanded; (11b) suggests that the higher the cost of acquiring information, the less demanded. The result, $d\theta^*/dr_i < 0$, indicates that, as the opportunity cost of information goes up, less information is demanded. This result is more surprising but less intuitive than it at first seems. For example, if r_3 were to increase, the reader might think that, with the concomitant increase in x_3, θ would increase to reduce the risk of a reserve deficiency as the bank assumes a relatively illiquid portfolio. However, (11c) indicates that, regardless of the assets involved, whenever the opportunity cost of buying information increases, θ^* decreases. In contrast, B and M, using their own information function, found that the demand for information fell when the return on securities, r_2, increased, while information demanded increased when r_3, the return on loans, increased.

Computation of the reaction coefficients of u_1 and u_2 to changes in r_i and use of the chain rule yields the following comparative-static results for x_1^* and x_2^*:

$$\frac{dx_1^*}{dr_1} = \frac{dx_1^*}{du_1}\frac{du_1}{dr_1} = (\theta^* + \phi)^{-1/2}\left[\frac{1}{(a_2)f(u_1)} - \frac{(1/3)u_1^2}{\gamma}\right] \gtreqless 0, \tag{12a}$$

$$\frac{dx_2^*}{dr_1} = \frac{dx_2^*}{du_2}\frac{du_2}{dr_1} = (\theta^* + \phi)^{-1/2}\left[\frac{-1}{(a_2)f(u_1)} - \frac{(1/3)u_1u_2}{\gamma}\right] < 0, \tag{12b}$$

$$\frac{dx_1^*}{dr_2} = \frac{dx_1^*}{du_1}\frac{du_1}{dr_2} = (\theta^* + \phi)^{-1/2}\left[\frac{-1}{(a_2)f(u_1)} - \frac{(1/3)u_1u_2}{\gamma}\right] < 0, \tag{12c}$$

$$\frac{dx_2^*}{dr_2} = \frac{dx_2^*}{du_2}\frac{du_2}{dr_2}$$

$$= (\theta^* + \phi)^{-1/2}\left[\frac{1}{(a_3 - a_2)f(u_1 + u_2)} + \frac{1}{(a_2)f(u_1)} - \frac{(1/3)u_2^2}{\gamma}\right] \lesseqgtr 0, \tag{12d}$$

$$\frac{dx_1^*}{dr_3} = \frac{dx_1^*}{du_1}\frac{du_1}{dr_3} = (\theta^* + \phi)^{-1/2}\left[\frac{-(1/3)u_1(u_4 - u_1 - u_2)}{\gamma}\right] < 0, \tag{12e}$$

and

$$\frac{dx_2^*}{dr_3} = \frac{dx_2^*}{du_2}\frac{du_2}{dr_3}$$

$$= (\theta^* + \phi)^{-1/2}\left[\frac{-1}{(a_3 - a_2)f(u_1 + u_2)} - \frac{(1/3)u_2(u_4 - u_1 - u_2)}{\gamma}\right] < 0, \tag{12f}$$

where $\gamma = a_2(f(u_1) - f(u_1 + u_2)) + a_3(f(u_1 + u_2)) - r_3u_4 > 0$.

Under variable information, B and M found that the interaction of bank holdings and interest rates consisted simply of a price effect and an information effect. While this paper uses a Bayesian information function with the potential of yielding vastly more complicated comparative-static behavior, the interaction of bank holdings and information is again a simple linear combination of a price effect and an information effect. The impact of the information effect is unambiguous in all six expressions recorded above. Information offsets the price effect in expressions (12a) and (12d) and augments the cross-price effect in (12b), (12c), (12e), and (12f).

Expression (12a) records the surprising result that an increase in r_1 does not necessarily mean an increase in x_1. Intuitively, the increase in r_1 increases x_1 directly by $(\theta^* + \phi)^{-1/2}\left(\frac{1}{(a_2)f(u_1)}\right)$, but this price or interest rate effect is partially, if not fully, offset by an information effect. In particular, the increase in r_1 reduces θ^*, which reduces x_1^* by $(\theta^* + \phi)^{-1/2}\frac{(-1/3)u_1^2}{\gamma}$. In (12b), the information effect reinforces the standard cross-price or cross-interest rate effect! As anticipated, an increase in r_1 reduces x_2^* directly, by $\frac{-(\theta^* + \phi)^{-1/2}}{(a_2)f(u_1)}$, since the opportunity cost of holding x_2^* instead of x_1^* has increased. In addition, the increase in r_1 reduces θ^* and consequently reduces x_2^* by $\frac{-(1/3)(\theta^* + \phi)^{-1/2}u_1u_2}{\gamma}$.

In (12c), the cross-interest rate and the information effect work in the same direction. An increase in r_2 reduces x_1^* directly and decreases θ^*, which decreases x_1^*. In expression (12d), the standard result that an increase in r_2 increases x_2^* is not obtained under endogenous information. An increase in r_2 increases u_2 and consequently increases x_2^* by $(\theta^* + \phi)^{-1/2}\left[\frac{1}{(a_3 - a_2)f(u_1 + u_2)} + \frac{1}{(a_2)f(u_1)}\right]$. But the increase in r_2 also reduces θ^* and x_2^*, to leave the net impact of an increase in r_2 upon x_2^* ambiguous. Baltensperger and Milde found that, under variable uncertainty, an increase in the return to securities rendered dx_1^*/dr_2 ambiguous and dx_2^*/dr_2 positive. The difference between their results and (12c) and (12d) is due solely to the impact of information. B and M had a positive information effect in dx_1^*/dr_2 and dx_2^*/dr_2, while the Bayesian analysis in this paper found that the reduction in information decreased bank reserves and government securities held by commercial banks.

Under exogenous uncertainty, changes in r_3 would have no impact on x_1^*.

Financial holdings of x_1 are determined by the optimality condition specified in (3), $F(x_1^* \mid I_\theta) = 1 - \frac{r_2 - r_1}{a_2 - a_1}$, which is independent of r_3. However, with variable information, the increase in r_3 reduces θ^* and, consequently, reduces x_1^* as recorded in (12e). In (12f), the cross-interest rate effect is reinforced by the information effect. A rise in r_3 directly reduces x_2^*, and the increase in r_3 decreases θ^* and reduces x_2^* indirectly. B and M also found negative information effects in dx_1^*/dr_3 and dx_2^*/dr_3, and, consequently, their results agree with (12e) and (12f). An increase in the return on loans decreases both reserves and government securities held by banks.[14]

The use of Bayesian information in analyzing bank portfolio behavior under variable uncertainty is an important improvement over B and M's approach to the problem. However, the results here generally confirm B and M's effort; portfolio adjustments to interest rate changes under variable uncertainty are complicated by an information effect. This effect either augments or moderates the traditional price effect and leads to portfolio adjustments that are not necessarily as simple or intuitive as classical analysis would suggest. In particular, (12a) and (12d) call into question the long-standing notion that a bank will necessarily increase its holdings of a given asset if the return on that asset increases. It is clear from expressions (12b), (12c), (12e), and (12f) that comparative-static analysis under constant uncertainty seriously underestimates the magnitude of commercial bank portfolio adjustments to cross-interest rate effects.

V. Stochastic Scale Economies

Banks are financial intermediaries that consolidate risks by having as assets the debt of a large number of independent borrowers and by having as liabilities the deposits of a large number of independent depositors. Stochastic scale economies emerge in this situation if the source of a change in the total volume of the bank's transactions (scale) is an increase in the number of transactions. When economies of scale exist, the increase in reserves associated with the increase in scale is less than proportionate, and banks can economize on reserves. From a stochastic point of view, larger banks are better off than smaller banks, and there could be some tendency in the banking industry toward concentration. However, econ-

[14] From (12), it is evident that the response of reserves and securities to an increase in the general level of interest rates ($dr_1 = dr_2 = dr_3$) is negative:

$$\frac{dx_1^*}{dr_1} + \frac{dx_1^*}{dr_2} + \frac{dx_1^*}{dr_3} = (\theta^* + \phi)^{-1/2}(-1/3)\gamma^{-1}[u_1^2 + u_1u_2 + u_1(u_4 - u_1 - u_2)] < 0$$

and

$$\frac{dx_2^*}{dr_1} + \frac{dx_2^*}{dr_2} + \frac{dx_2^*}{dr_3} = (\theta^* + \phi)^{-1/2}(-1/3)\gamma^{-1}[u_1u_2 + u_2^2 + u_2(u_4 - u_1 - u_2)] < 0.$$

In the two expressions above, the price and cross-price effects predictably, given the discussion in footnote 4, cancel one another out. The reduction in x_1^* and x_2^*, with an increase in the general level of interest rates, is entirely attributable to the collective impact of information. The increase in the r_i reduces θ^*, and, consequently, x_1^* and x_2^* fall.

omies of scale do not exist if changes in transaction volume are due to a change in the size of individual transactions. In this case, an increase in scale occasions a proportionate increase in reserves.[15]

Given the increasing number of small bank failures and bank consolidations in the last five years, it is interesting to investigate variable Bayesian information as an additional source of, or as a source of, economies of scale for reserve balances. Given our inflationary economy, the most likely source of a change in total transaction volume for a commercial bank would be an increase in individual transaction size. Consequently, the analysis here presumes that increases in scale, λ, are due to increases in transaction size.[16] In this case, the optimal level of information would be given by

$$\theta^* = \left[\frac{a_2\lambda[f(u_1) - f(u_1 + u_2)] + a_3\lambda[f(u_1 + u_2)] - r_3\lambda u_4}{2q}\right]^{2/3} - \phi, \tag{13}$$

where

$$u_1 = \frac{(x_1^*/\lambda) - \mu_\theta}{\sigma_\theta}, \qquad u_2 = \frac{(x_2^*/\lambda)}{\sigma_\theta}, \qquad \text{and} \qquad u_4 = \frac{(w_0)}{\sigma_\theta}.$$

Solving for reserves, with $\mu_\theta = 0$, and then taking the log will yield

$$\log x_1^* = \log u_1 + \log \lambda - \tfrac{1}{2}\log(\theta^* + \phi).$$

Therefore, the elasticity of reserves with respect to scale is given by

$$\frac{d \log x_1^*}{d \log \lambda} = 1 - \frac{1}{2}\frac{d \log(\theta^* + \phi)}{d \log \lambda}. \tag{14}$$

From (14), it is obvious that, if information were not variable, economies of scale would not exist for commercial bank reserve balances.[17] However, using the optimality conditions provided in (13), we have

$$\frac{d \log(\theta^* + \phi)}{d \log \lambda} = \frac{2}{3}.$$

Substituting this result into (14) yields an elasticity of reserves with respect to scale of ⅔.

This is a remarkable result that suggests that significant economies of scale exist for a commercial bank's reserve balances, and it may have implications for the degree of concentration in the banking industry. Scale economies in reserves emerge because there are two ways to adjust to an increase in the scale of the

[15] See Patinkin [9] and Schlesinger [13] for a more detailed discussion of the impact of alternative assumptions about scale upon reserve holdings.

[16] If the analysis assumed that increases in the bank's transaction volume were due to an increase in the number of transactions, although the exact quantitative results would change, the qualitative conclusions would not vary.

[17] An elasticity of reserves with respect to scale of one, under exogenous information, is a consequence of our assumption that scale depends on the size of individual transactions. If scale were interpreted to be the number of transactions, then the elasticity of reserves, under constant uncertainty, would be ½, Edgeworth's famous square root rule.

reserve needs of a bank. Increasing inventories of reserve assets is one way of reducing the increased risk of reserve deficiencies associated with the change in scale. Obtaining more information is another way of achieving the same goal. From (13), it is clear that an increase in scale increases the marginal benefit of information, which induces a substitution of information for reserves. Consequently, an increase in λ increases x_1^*, but the elasticity of x_1^* is less than one due to the increased reliance on information.

VI. Summary

This paper has demonstrated how Bayesian information may be analyzed as a variable input in determining an optimal bank portfolio and has investigated the impact of information in a way that is statistically satisfactory. In Section I, the portfolio model was developed, and the impact of information was analyzed. The optimal level of bank financial holdings was derived in Section II. In Section III, information was treated as an economic input that was used up to the point where its *predicted* marginal benefit was exactly equal to its marginal cost, and, from there, the optimal demand for information was derived. In Section IV, the comparative-static analysis demonstrated that the reaction of optimal portfolio holdings to interest rate changes under variable uncertainty is dramatically different from portfolio behavior when uncertainty is exogenous. Finally, in Section V, the elasticity of reserves with respect to scale was examined under the assumption of variable uncertainty.

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