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Author(s): Jonathan M. Karpoff

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A Theory of Trading Volume

JONATHAN M. KARPOFF*

ABSTRACT

A theory of trading volume is developed based on assumptions that market agents frequently revise their demand prices and randomly encounter potential trading partners. The model describes two distinct ways informational events affect trading volume. One is consistent with conjectures made by empirical researchers that investor disagreement leads to increased trading. But the observation of abnormal trading volume does not necessarily imply disagreement, and volume can increase even if investors interpret the information identically, if they also have had divergent prior expectations. Simulation tests support the model and are used to contrast the random-pairing environment with costless market clearing. Volume is lower in the costly market, and volume increases caused by an informational event persist after the event period. This is consistent with existing empirical evidence and suggests that markets do not immediately clear all orders or that investors have demands to recontract.

MOST FINANCE STUDENTS LEARN that exchange occurs when market agents assign different values to an asset. But things quickly get complicated when one looks at the *number* of assets exchanged in a market with diverse traders. Many models of financial markets implicitly assume away trading volume by assuming away heterogeneity. Yet there is active trading in most financial markets, indicating that agents are heterogeneous and operate in a changing environment.

This paper develops a theory of trading volume based on heterogeneous investors who periodically and idiosyncratically revise their demand prices. A better understanding of trading volume is important for at least three reasons. First is the inconsistency between the widespread use of the homogeneous investor assumption and observations of positive trading volume. This model presumes that investors have a demand to trade even in the absence of new information because of unique liquidity or speculative desires. Second, volume data are regularly reported in the financial media along with price data. Yet it is not clear what, if any, information is reflected by volume data. Empiricial researchers attempt to draw inferences from volume data, but others (e.g., Verrecchia [35]) argue that the link between information and volume is ambiguous.[1] This paper establishes two distinct ways information affects trading

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[1] A number of event studies test for a connection between trading volume and information flows, many of which use trading volume to determine whether an event has "information content"; see papers by Beaver [6], Kiger [24], Foster [17], Morse [26, 27, 28], Ro [32], Pincus [30], and Bamber [3, 4]. Trading volume has also been applied to an increasing number of theoretical and empirical

volume, and thus it provides a rationale for deriving the inference that an event contains information from empirical volume data. However, the ambiguity is not eliminated, as a volume increase can indicate that investors interpret the information differently or that they interpret the information identically but begin with diverse prior expectations. And third, the effects of the institutional design of the market on trading volume are not well understood.[2] These effects are explored through simulations of a continuous market with significant frictions (information and transaction costs) and a costless Walrasian call market. Trading volume is lower in the imperfect market, and information has a persistence effect on volume in the imperfect market. This is consistent with empirical evidence [6, 26] and suggests that markets do not immediately clear all demands motivated by the information or that investors make trading mistakes and have demands to recontract in subsequent periods.

While it is typically not the center of attention in asset-pricing models, trading volume has not been completely ignored, and the following section provides a brief review of theoretical models that consider trading volume. Section II develops a basic model of trading volume in a pure exchange market. The comparative statics of the model are examined in Section III. In Section IV the model is extended to multiple periods, and a comparison of market structures is made through simulations. Section V concludes the paper.

I. Previous Work on Trading Volume

Theoretical treatment of trading volume arises in the literature in at least three settings: its relation to the bid-ask spread, its relation to price changes, and its relation to information.[3] Empirical research indicates that volume is negatively related to the bid-ask spread, a finding consistent with several theoretical models. For an early survey, see Cohen, Maier, Schwartz, and Whitcomb [8]. As a later example, Copeland and Galai [10] characterize a dealer's position as a written straddle of put and call options available to informed investors. The dealer's choice of exercise prices, i.e., the bid and ask prices, weighs the cost of supplying quotes to informed traders against the revenues expected from liquidity-motivated traders. In another model, Epps [15] demonstrates that the expected number of transactions and expected volume from an exogenous shock are descreasing functions of transaction costs (including the bid-ask spread), which are exogenously determined. Epps classifies investors into two groups but requires homogeneity within each group. This is somewhat similar to assumption (A5) in this paper, which classifies market participants into two groups ("potential

questions (see Asquith and Krasker [2], Richardson, Sefcik, and Thompson [31], Grundy [18], Lakonishok and Vermaelen [25], and Winsen [36]). With the increased availability of low-cost volume data through data-retrieval services, an increase in the use of volume data in empirical studies is likely.

[2] See Ho, Schwartz, and Whitcomb [21] for an analysis of the effects of market imperfections on trading volume.

[3] These distinctions are somewhat artificial, and there is overlap between these areas. For example, the models in Tauchen and Pitts [33] and Pfleiderer [29] each have implications for both the price-volume relation and the effect of information on volume.

buyers" and "potential sellers"). However, homogeneity within groups is not required, as this model depends on the idiosyncrasies of individual investors.

Several theoretical models consider the relation of trading volume to price changes. Epps [14] derives a model in which volume on transactions in which the price ticks up is greater than volume on downticks. Copeland [9] derives a model in which a common bit of information arrives sequentially to investors. Using simulations, he shows that volume after all investors receive the information is positively related to the magnitude of the price change. This model is extended by Jennings, Starks, and Fellingham [22], who add real-world margin constraints and the possibility of short sales and come up with the additional prediction (similar to Epps') that volume is relatively heavy on transaction upticks. However, a significant feature of each of these models is a dependence on behavioral distinctions between groups of market participants, e.g., "bulls" vs. "bears" or "optimists" vs. "pessimists." The model in this paper does not rely on such behavioral distinctions.

Trading volume is also important in the "mixture of distributions" models of Clark [7], Epps and Epps [16], Tauchen and Pitts [33], and Harris [20], which provide explanations of the leptokurtosis in the empirical distributions of speculative prices. These models predict that volume is positively related to the magnitude of the corresponding price change over fixed time intervals [7, 33, 20] or on a given transaction [16]. The model by Pfleiderer [29] considers price and volume in a noisy rational expectations equilibrium. The magnitude of the price change is uncorrelated with trading by speculators with private information but is positively related to trading by liquidity-motivated investors. So the strength of the correlation between absolute price changes and volume is negatively related to the existence of private information.

Pfleiderer [29] and Varian [34] each examine the effects of private information and the market's aggregation of information on volume. Varian uses a Bayesian framework to distinguish between opinions (priors) and information (likelihoods). He argues that trading volume depends only on differences of opinion, even when investors receive different information, because the market price adjusts to reveal all information in the economy and thus negates the value of unique information to any single investor.[4] In contrast, aggregate information in the model by Pfleiderer is not fully revealed by the market price. Each investor receives information about the value of a risky asset that includes both a common and a unique component. When there is no common information error (i.e., random deviations in the information from the true price are unique to each investor), the model yields the surprising result that expected volume is a decreasing function of the variance of the idiosyncratic error. This implies that volume is a decreasing function of disagreement between investors, a result inconsistent with the conclusions of most empirical researchers on the subject.[5]

[4] One drawback of this model is that it implies that prices decrease while trading volume increases (p. 10), a prediction inconsistent with most empirical evidence. (See Karpoff [23] for a survey.)

[5] This occurs because, as private information becomes more certain, investors take larger speculative positions based on their private information. (When the common information error is positive, expected volume is at first an increasing, then a decreasing function of the precision of private information.)

This paper also examines the relation of volume to information but takes a different tack. Volume is treated simply as the number of transactions between buyers and sellers who are randomly paired in the trading period. Trading opportunities arise because both potential buyers and potential sellers revise their demand prices prior to the market period according to idiosyncratic liquidity or speculative desires, which appear random to the outside observer who does not have specific data on each agent. This approach clearly has some drawbacks. For example, it does not examine the market's role in aggregating private information and is therefore less general than the approach of Pfleiderer. The random-pairing assumption is simply untrue, as evidenced by the existence of markets that economize on the search and transaction costs of buyers and sellers who purposefully seek each other out. Another drawback of the random-pairing assumption is that the model does not yield a unique market price or a one-period equilibrium. However, the random-pairing assumption has several compensating advantages. First, it yields a simple closed-form solution for expected volume. Second, the predictions about trading volume are not driven by the random-pairing assumption, and the model's predictions about trading volume extend to multiple periods and costless (Walrasian) market clearing. (See Section V.) Third, the model avoids the more restrictive assumptions of constant absolute risk aversion or behavioral distinctions between groups of investors that are commonly employed in models of trading volume [9, 14, 20, 22, 29]. Fourth, the model is simple and yields straightforward predictions about positive trading volume in nonevent periods and the bid-ask spread, as well as the effects of information events. And fifth, the model's predictions are consistent with empirical evidence and provide a theoretical framework for evaluating volume data; information increases trading volume if it causes investors to revise their demand prices heterogeneously or if the information is partially but not homogeneously anticipated.[6]

II. The Basic Model

A. Assumptions

(A1) There exists a two-period exchange market for some asset with fixed supply I. Volume at t_1 is examined, given an initial stock equilibrium at the close of the previous trading period t_0.

(A2) Costs of transacting in this market are zero, subject to assumptions (A5) and (A6). However, transaction costs in other markets may not be zero, as implied by assumption (A3).

(A3) Market participants are heterogeneous in their personal valuation of the asset. Demand-price differentials indicate different expectations or different life-cycle-generated liquidity desires across investors.

[6] Hakansson, Kunkel, and Ohlson [19] discuss conditions for information to have positive social value in pure exchange markets. Two of their sufficient conditions (heterogeneous likelihood matrices and homogeneous prior beliefs) correspond to parts of this model (Section III, D and E), which imposes a specific trading structure to extend their insights.

(A4) Individual demand prices are revised between market periods, the revision following what appears to the outside observer to be a stochastic process with mean μ and variance σ^2.

Individual behavior is not assumed to be random; agents' willingness to hold positions in the asset is a function of their expectations or liquidity desires. However, each individual's revision appears to the outside modeler to contain a random element (the variance of which is measured by σ^2) simply because the outsider lacks specific data on each market agent. On the other hand, the modeler can be aware of market-wide influences or public information; the expected revision μ contains what is known about the individual agent.

(A5) Market participants can hold, at most, one unit of the asset and cannot take short positions.

Since the topic of this paper is exchange *volume*, agents on either end of an exchange must be isolated. Some device is necessary to distinguish the buyers from the sellers. This model uses assumption (A5) since it implies that only current asset owners can be sellers and that they can only be sellers. Similarly, current nonowners are the only potential buyers, and they can enter an exchange only as buyers. While not descriptively accurate of financial markets, this assumption avoids the problem of grouping investors along asserted behavioral characteristics.

(A6) During the market period t_1 each current asset owner randomly encounters a single, unique nonowner; an exchange occurs if the nonowner's revised demand price exceeds that of the current owner.

Assumption (A6) denies the existence of a Walrasian auctioneer. Each asset owner is paired blindly with a prospective buyer.[7] This assumption is relaxed in Section V, in which a costless market-clearing mechanism is introduced.

B. Binomial Market Process

The market consists of I potential sellers (one for each unit of the asset) and J potential buyers. The (finite) set of market participants is $S = \{I, J\}$, and assumption (A6) implies $I \leq J$. Assumption (A3) implies that, in general, $p_k \neq p_h$ for any $k \neq h$ pair such that $k \in S$ and $h \in S$. Stock equilibrium at time t_0 is characterized by $p_i \geq p_j$ for all $i \in I$ and $j \in J$. Assumptions (A1) and (A5) imply $I \cap J = \{\ \}$.

Assumption (A4) indicates that an individual's demand price p_{k0} will change from t_0 to t_1 by some amount δ_{k1}. For expositional ease, p_{k0} will refer to an agent's demand price after trading in period t_0; δ_{k1} is the change between periods; p_{k1} is the corresponding demand price before trade in period 1. Hence, p_{i1} can be less than p_{j1}, and exchanges in period 1 are possible.

[7] This trading process is similar to that of Diamond [13] and Akerlof [1]. It implies that each pairing of a prospective buyer and seller creates a temporary bilateral monopoly situation. A single market price is not determined, but rather, a set of pairwise transaction prices. Assumption (A6) implies that the number of potential buyers is greater than the number of potential sellers. The demand price of many nonowners can be zero, so this is not a restrictive assumption.

Let T_{ij} be a binary variable that indicates an exchange between an i, j pair:

$$T_{ij} = \begin{cases} 1 & \text{iff } i \in I \quad \text{and} \quad j \in J \quad \text{and} \quad p_{i1} \leq p_{j1} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

By assumption (A6), each market participant can conduct, at most, one trade. The total volume of trades is $T = \sum_i T_{ij} = \sum_j T_{ij}$.

Each i, j pairing represents an independent opportunity for an exchange. For a given i, j pair, the probability of an exchange, $\pi_{ij} = \Pr\{T_{ij} = 1\}$, is the probability that the demand-price revisions of individuals i and j are sufficient to overcome the original demand-price differential $p_{i0} - p_{j0}$. In general, π_{ij} will differ for different i, j pairs. But prior to the pairings of each asset holder with a potential buyer at the beginning of the market period, it is a random variable with mean $\pi = E(\pi_{ij})$, the expected "average" probability prior to the start of the market period.

Therefore, T is a binomially distributed random variable with parameters I, the number of independent "trials" in the market exchange process, and π, the average ex ante probability of an exchange prior to the i, j pairings. The binomial density function is

$$b(T) = T!/((I - T)!I!)\pi^T(1 - \pi)^{(1-T)} \tag{2}$$

and

$$\mu_T = E(T) = \pi I. \tag{3}$$

μ_T is a strictly increasing function of π, so, for given I, the size of π determines the level of trading volume.

C. The Probability of an Exchange and Expected Trading Volume

In this section, expressions for π and μ_T are derived. As indicated above, the demand-price revisions δ_{i1} and δ_{j1} are defined by $p_{i1} = p_{i0} + \delta_{i1}$ and $p_{j1} = p_{j0} + \delta_{j1}$. From (1), $T_{ij} = 1$ if and only if $\delta_{j1} - \delta_{i1} \geq \mathrm{p}_{i0} - \mathrm{p}_{j0}$. From assumption (A4), δ_{k1} ($k \in S$) can be written as

$$\delta_{k1} = \mu_k + \sigma\varepsilon_k, \tag{4}$$

where ε_k is a mean-zero random variable.[8] To derive closed-form solutions, ε_k is assumed to be unit normal with independence across investors:

$$E(\varepsilon_k\varepsilon_h) = 0 \quad \text{for all} \quad k \neq h.$$

In equation (4), μ_k is the expected demand-price revision from t_0 to t_1, while σ is the standard deviation of the revision process. The $\sigma\varepsilon_k$ term can be thought of as resulting from the modeler's ignorance about the changing constraints facing each individual k; σ is a measure of the degree to which individuals in this market idiosyncratically revise their demand prices in response to changed expectations or liquidity desires. By construction, these factors are unknown for any individual

[8] Equation (4) can be derived as a discrete-form diffusion process with $\Delta t = t_1 - t_0 = 1$. In a model of "representative" agents with appropriate definitions of μ_k and σ, this would imply a lognormal returns distribution.

k. On the other hand, the μ_k term represents expectations, given whatever is known about k. In the absence of known influences (e.g., new public information), μ_k is the long-term expected return from the asset.

Apply (4) to a pair of investors $i \in I$ (owner) and $j \in J$ (nonowner), and define θ as

$$\theta = \delta_{j1} - \delta_{i1} = (\mu_j - \mu_i) + \sigma(\varepsilon_j - \varepsilon_i) \tag{5}$$

$$\mu_\theta = E(\theta) = \mu_j - \mu_i \tag{6}$$

$$\sigma_\theta^2 = E(\theta - \mu_\theta)^2 = 2\sigma^2. \tag{7}$$

The probability of an exchange between any given i, j pair is

$$\pi_{ij} = \int_{p_{i0}-p_{j0}}^{\infty} f_\theta(x)\, dx = 1 - F_\theta(p_{i0} - p_{j0}), \tag{8}$$

where $f_\theta(x)$ is the normal density function with mean μ_θ and variance σ_θ^2 and $F_\theta(p_{i0} - p_{j0})$ is the associated distribution function evaluated at $p_{i0} - p_{j0}$.

As noted above, π_{ij} is specific to the difference $p_{i0} - p_{j0}$, and so it will generally vary across i, j pairs once the pairings are made. Assumption (A6) indicates that the probabilities that a given element of the set J will be paired with any single i are equal, and vice versa. So the average ex ante probability π is

$$\pi = (1/IJ) \textstyle\sum_i \sum_j \pi_{ij}. \tag{9}$$

Substituting (8) and (9) into (3),

$$\mu_T = (1/J) \textstyle\sum_i \sum_j [1 - F_\theta(p_{i0} - p_{j0})].$$

Since the ε_k are assumed normal, this is

$$\mu_T = (1/J) \textstyle\sum_{i=1}^{I} \sum_{j=1}^{J} \displaystyle\int_{p_{0i}-p_{0j}}^{\infty} [1/((2\pi)^{1/2}\sigma_\theta)] \exp-\{½[(x - \mu_\theta)/\sigma_\theta]^2\}\, dx. \tag{10}$$

III. Implications for Exchange Volume

Equation (2) describes the market process as a binomial experiment that generates a level of exchange volume, given a number of "trials" in the experiment, I, and the ex ante probability of "success" in each trial, π. Equation (10), in turn, determines the expected number of "successes" (trades). The following implications can be drawn.

A. "Normal" Trading Volume

This model provides an explanation of positive exchange volume in a pure exchange market even in the absence of exogenous shocks. Define "normal" trading volume as the state in which no unanticipated information enters the market. μ_k is the asset's expected return for all k. An outside observer, while recognizing that many demand prices will change, has no prior beliefs regarding the direction of change for any single one. Yet, even when $\mu_j = \mu_i$, a sufficient condition for $\pi_{ij} > 0$ is $\sigma > 0$. In turn, $\pi_{ij} > 0$ for *any* i, j pair insures $\pi > 0$. As long as at least one individual has idiosyncratic demand price adjustments, the expected number of exchanges is positive.

B. Market Depth

Trivially, expected exchange volume increases proportionally with the number of outstanding units of the asset (and therefore, by assumption (A5), with the number of asset holders), i.e., $\partial\mu_T/\partial I = \pi > 0$. While the proportionality is an artifact of this model, a positive relation between volume and the size of the market is supported in empirical tests (e.g., [11, 33]).

C. The Bid-Ask Spread

Momentarily relax assumption (A2) to introduce a positive transaction cost c. Equation (8) becomes

$$\pi_{ij}^{c} = \int_{p_{i0}-p_{j0}+c}^{\infty} f_\theta(x)\ dx = 1 - F_\theta(p_{i0} - p_{j0} + c),$$

where π_{ij}^{c} is the probability of an exchange, given the transaction cost. Differentiating,

$$\partial\mu_T/\partial c = -(1/J)\ \textstyle\sum_i \sum_j f_\theta(p_{i0} - p_{j0} + c) < 0. \tag{11}$$

Expected volume is a decreasing function of the transaction cost c since $\delta_{j1} - \delta_{i1}$ must now overcome the original difference $p_{i0} - p_{j0}$ *plus* the transaction cost. This is also consistent with a substantial amount of empirical evidence (e.g., [8, 11, 15]).

D. Heterogeneous Reactions to Public Information

Suppose that an information arrival has a mean effect μ on all agents' demand prices but that each investor receives slightly different information or interprets identical information differently. Then expected volume increases.

PROPOSITION 1: *If demand prices are revised by market agents in unpredictable ways that do not correlate with the subsets I and J or with investors' idiosyncratic demand-price revisions, expected trading volume increases.*

Proposition 1 is proved in the Appendix. The intuition behind it is that the information, interpreted differently by market agents, adds to the normal "jumbling up" of demand prices that comes from investors' liquidity and speculative trading. This increases the variance of the demand-price revision process, σ^2, and therefore increases σ_θ^2. The resulting reallocation of assets to higher valued owners increases the expected volume of trade.[9]

[9] A number of researchers have examined trading volume around an event to determine whether the event has "information content." The presumption, usually stated informally, is that an informational event causes more trades as investors disagree about the meaning of the information and revise their portfolios accordingly (see footnote 1). This intuition is formalized in Proposition 1.

E. Asymmetric Reactions by Buyers and Sellers

However, this is not the only mechanism by which volume increases. This model has used the artifice of assumptions (A5) and (A6) to conceptually distinguish buyers from sellers. Such a distinction can sometimes also be descriptively accurate. Events that have systematically different effects on buyers' and sellers' demand prices have volume implications since $\mu_i \neq \mu_j$ implies $\mu_\theta \neq 0$.

PROPOSITION 2: *If demand prices are revised in systematically different ways by potential buyers and sellers, expected trading volume increases if $\mu_j > \mu_i$ and decreases if $\mu_i > \mu_j$.*

The proof of Proposition 2 is in the Appendix. Cases in which $\mu_\theta \neq 0$ are very likely. Consider a partially anticipated event about which investors have different prior expectations. After t_0 but before trading at t_1, information is publicly revealed about the asset's value ζ, where ζ is a random variable with mean μ_ζ. All investors obtain the information and interpret it identically. Prior to t_1, investors' demand prices at t_0 reflect private information at t_0, which is described by $z_k = \mu_\zeta + \gamma_k$, where γ_k is a zero-mean random variable unique to investor k and z_k describes investor k's anticipation of the "true" value ζ. So $p_{k0} = g(E(\zeta \mid z_k))$, where $g' > 0$, and investors with relatively high z_k become owners of the asset at t_0. When the information is fully revealed at t_1, each investor's demand price adjusts to reflect the information: $\delta_{ik} = \mu_k + \sigma\varepsilon_k = g(\zeta) - g(E(\zeta \mid z_k)) + \sigma\varepsilon_k$. But the effect of the public release is to pull all investors' demand prices toward $g(\zeta)$, which implies $\mu_\theta > 0$. To illustrate, suppose the news is "good," e.g., a tender offer is made that was anticipated with probability less than one. Current owners are characterized as investors with relatively high z_k at t_0. They will revise their demand prices by a small amount compared with current nonowners, who were relatively "pessimistic," that is, who had demand prices at t_0 characterized by low z_k. As a result, $\mu_i < \mu_j$ and $\mu_\theta > 0$. Likewise, if the news is "bad," all expected demand-price revisions are negative. But those of current owners are, as a group, greater in absolute value, and the reversion of investors' demand prices toward the mean implies $\mu_i < \mu_j$.[10]

Note that $\mu_i > \mu_j$ implies a decrease in expected volume. This can happen if the informational event causes further divergence in the demand prices of owners and nonowners. This is a characteristic of models based on behavioral differences between buyers and sellers (e.g., Epps [14]). A more plausible story is that the information tends to confirm prior beliefs. It is not clear whether this is a common occurrence, but it is sufficient to note that information can decrease volume.

F. Simultaneous Changes in μ_θ and σ_θ

The preceding sections indicate two distinct ways trading volume can increase with an informational event—but both situations can occur simultaneously.

[10] There are other reasons why μ_θ may be different from zero. For example, information asymmetries can arise, say, if current asset owners have access to different information from potential buyers. However, this is likely only in closely held corporations where current owners also tend to be corporate insiders. In real asset markets, a difference between buyers and sellers' mean revisions can arise through government intervention. For example, federal and state housing programs often provide subsidies to home buyers but not (directly) to home sellers, and $\mu_i < \mu_j$.

Information-revealing events that induce asymmetries between group means affect μ_θ, and heterogeneity among members of the same group—either buyers or sellers—affects σ_θ. Propositions 1 and 2 indicate that simultaneous positive changes in both μ_θ and σ_θ complement each other; the effect is to increase expected volume. However, an event that simultaneously decreases μ_θ and increases σ_θ has an ambiguous effect on volume. The total differential of π_{ij} is $d\pi_{ij} = (\partial\pi_{ij}/\partial\mu_\theta)\, d\mu_\theta + (\partial\pi_{ij}/\partial\sigma_\theta)\, d\sigma_\theta$. Setting this equal to zero and substituting from equations (A1) and (A2) in the Appendix yields

$$d\mu_\theta/d\sigma_\theta\,|_{d\pi_{ij}=0} = (-1/\sigma_\theta)(p_{i0} - p_{j0} - \mu_\theta)[f_\theta(p_{i0} - p_{j0} - \mu_\theta)/f_\theta(p_{i0} - p_{j0})]. \quad (12)$$

For $p_{i0} - p_{j0} > \mu_\theta$, positive changes in μ_θ and σ_θ are substitute methods to increase π_{ij}.[11] Event studies that record positive volume reactions to new information cannot distinguish between the causal effects $\mu_\theta > 0$ and $\sigma_\theta > 0$. In contrast, observations of decreases in volume would not only imply that μ_i is greater than μ_j, but also that the difference outweighs any effect of increases in σ_θ.

IV. Extensions of the Basic Model through Simulations

Two restrictive aspects of this model are the random-pairing assumption (A6) and the two-period framework. With random pairing, some mutually preferred trades are not made because the appropriate trading partners do not "find" each other. The market generally does not clear at t_1, and the analysis would change if extended to the next period since assumption (A1) would be violated.[12] These assumptions are useful in deriving a tractable solution that yields predictions about trading volume. This section examines whether the assumptions are also the source of these predictions, by examining whether the model yields different conclusions when multiple periods and costless market clearing are introduced. Analytical solutions were not obtained, so the extensions are made through simulations.[13]

The random-pairing and costless market-clearing models represent two extremes in the trading environment. Most real-world markets fall somewhere in between. The random-pairing model approximates real-world continuous markets except that these markets provide information on current quotes and transaction prices, which makes the pairing nonrandom. On the other hand, the Walrasian costless-clearing model approximates real-world call markets except that these

[11] For $\mu_\theta = 0$, equation (12) indicates that an isoquant mapping of (μ_θ, σ_θ) combinations that yield given π_{ij} is characterized by nonparallel straight lines, each with a slope equal to the negative of the "z-score" (standardized) value of $p_{i0} - p_{j0}$ and converging to the point (μ_θ, 0) on the μ_θ axis (at which $\pi_{ij} = .5$).

[12] Akerlof [1] defends the random-pairing assumption as a reasonable approximation, given the prevalence of noncontractual relations between buyers and sellers, which indicates the absence of a Walrasian auctioneer market. However, his argument is better suited to labor and real asset markets than to financial markets.

[13] The complication in considering many periods is that the expected number of trades in period t_n depends not only on the price revisions and pairings at t_n but also on the price revisions and pairings for all previous periods. Consider trading at t_2. Since some mutually preferred trades do not occur at t_1, some initial price differences $p_{i1} - p_{j1}$ are negative, a condition previously assumed away.

markets do not operate perfectly.[14] Throughout, the predictions of the two-period, random-pairing model are compared with the simulation results under the assumption of multiple periods, costless market clearing, or both.

In each of the simulation tests reported here, the number of asset owners was set at 400 and the number of nonowners at 600. An initial (time t_0) distribution of demand prices was created by sampling from a normal distribution with initial mean price of 100 and variance of 100. The traders with the 400 highest initial demand prices were assigned ownership of the asset at t_0.[15] In subsequent trading periods, each agent's demand price was perturbed, owners were randomly paired with nonowners, and revised demand prices were compared to determine whether a trade occurred. Demand-price revisions for each trader were preserved for all subsequent trading periods.

A. Trading Volume over Many Periods, Random-Pairing Assumption

In the absence of observable outside "shocks," positive trading volume is expected as long as individuals continue to revise their demand prices for idiosyncratic liquidity or speculative reasons. This is reflected in the model as a constant value of σ_θ over time. Figure 1 displays the mean trading volume for each of fifty periods from thirty model simulations in which σ_θ^2 is set equal to

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The conditional density of the number of trades at t_2, T_2, is

$$b(T_2 \mid \Psi_1, \Theta_1) = T_2!/((I - T_2)!I!)(\pi_2 \mid \Psi_1, \Theta_1)^{T_2}(1 - (\pi_2 \mid \Psi_1, \Theta_1))^{(1-T_2)},$$

where Ψ_1 is a $1 \times I$ vector that represents the set of pairings at t_1 and Θ_1 is a $1 \times I$ vector representing the value of θ_{ij} for each i, j pair at t_1. All elements of Ψ_1 and Θ_1 are random, and Θ_1 is itself conditional on Ψ_1. There are $J!/(J - I)!$ possible sets of pairings in period 1. If each is assumed to be equally likely, the probability of a given set of pairings is $(J - I)!J!$ and the unconditional density of the number of trades at t_2 is the average of the conditional densities,

$$f(T_2) = (J - I)!/J! \sum_{n=1}^{J!/(J-I)!} \mathrm{b}(\mathrm{T}_2 \mid \Psi_{1n}, \Theta_{1n}).$$

Even in this extension to three periods (two trading periods), closed-form solutions are not forthcoming. The problem increases with each additional period considered.

[14] I thank a referee for these insights.

[15] In no cases were the results affected when any of these initial parameters were changed. Even though an algorithm that approximates the normal distribution was used to generate the initial demand prices and all demand-price revisions, no negative demand prices were generated. To preserve the variance of the cross-sectional distribution of demand prices over time, adjustments were made in the calculations of the revised demand prices. Define μ_0 and σ_0 as the mean and variance of the initial cross-sectional distribution of demand prices p_{k0} at t_0. Then

$$p_{kt} = (1/c)(p_{k,t-1} + \delta_{kt}),$$

with

$$\delta_{kt} = \mu_{kt} + \sigma_t \varepsilon_{kt},$$

$$\mu_{kt} = \mu_0(c - 1),$$

$$c = (1 + \sigma_t^2/\sigma_0^2)^{1/2}.$$

The reader can verify that the cross-sectional distribution of demand prices maintains the initial mean and variance. Deviations in the cross-sectional mean were permitted when changes in μ_θ were simulated.

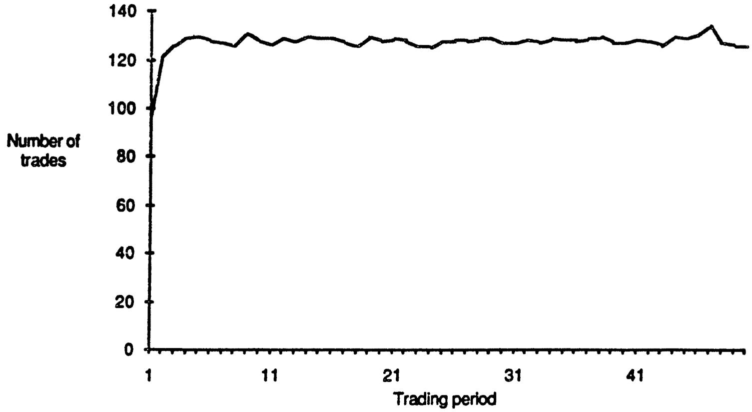

Figure 1. Mean Trading Volume for Each of 50 Trading Periods from 30 Model Simulations, Random-Pairing Model

200 for each period. The mean number of trades is lower in the first two periods than in subsequent periods. T_1 (the mean number of trades in period 1) equals 95.43, and T_2 equals 120.73; the mean of T_3 through T_{50} equals 127.91. The standard deviation of the series measured from T_3 through T_{50} is 1.61, indicating that the number of trades is significantly lower in the first two periods than in subsequent periods. (Measured from T_2, the mean and standard deviation are 127.76 and 1.64; over all periods they are 127.18 and 4.95.) This demonstrates one artifice of the random-pairing assumption when combined with the assumption that the market begins in equilibrium at t_0. When the market does not, as in all subsequent periods, the expected number of trades is higher for two reasons. First, some trades are made that could have been made at t_1 had the pairings been "better," i.e., closer to market clearing. Second, some trades made at each period t_n are undone at t_{n+1}, as some trades occur under random pairing that would not occur under a market-clearing assumption. For example, a prospective buyer with a demand price of \$95 at t_1 will buy if paired with a prospective seller who has a demand price of \$94. But at t_2, this agent, now a prospective seller, is likely to encounter a higher demand price from the prospective buyer with whom he is paired since the mean demand price of all agents is \$100.[16]

B. *Trading Volume over Many Periods, Market-Clearing Assumption*

To examine the model's reliance on the random-pairing assumption, the model was simulated after replacing assumption (A6) with a market-clearing assumption:

(A6′) The market is costlessly cleared via a Walrasian auctioneer process. In each period, the I units of the asset are allocated to the I agents with the highest demand prices.

[16] Such trading mistakes result from the random-pairing assumption, which implies that information and transaction costs are high enough to isolate each pair of potential traders from the others until the next period. Investors' demand prices should be thought of as being net of expectations of future trading partner's demand prices, so it is rational for the seller in period 1 to sell at any price above \$94.

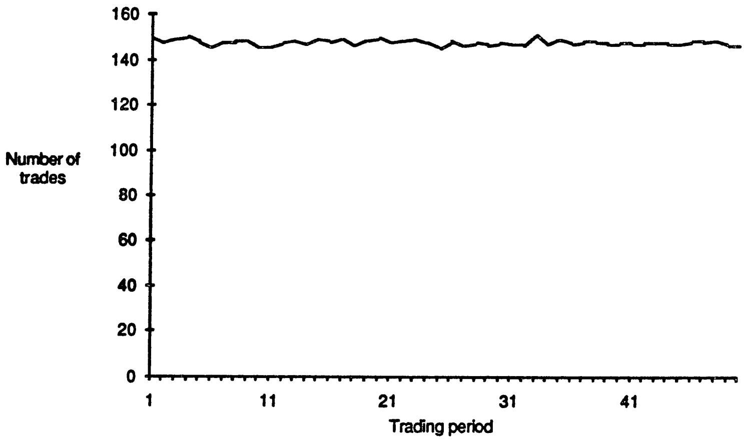

Figure 2. Mean Trading Volume for Each of 50 Trading Periods from 30 Model Simulations, Market-Clearing Model

The number of trades is still counted as the number of units of the asset that change ownership, but now all mutually preferred exchanges are made. This is similar to assuming that all search and information costs that might prevent exchange partners from finding each other are zero. Figure 2 illustrates the mean number of trades per period from thirty model simulations. It differs from Figure 1 in three ways. First, the mean number of trades is much higher: 147.40 over all fifty periods. This represents a substantial increase in allocative efficiency compared with the random-pairing case, in which roughly twenty fewer trades occur per period because some trading partners are unable to "find" each other, even though mutually preferred trades exist.[17] Second, mean volume in the first and second periods is 149.37 and 147.27, respectively, roughly the same as in subsequent periods. This occurs because the model clears each period while the conditional expectation of volume is the same for all periods. And third, the standard deviation of the number of trades is 1.27, slightly lower than with random pairing. With random pairing, the number of trades depends not only on investors' demand-price revisions, but also on the specific set of pairings. This additional source of uncertainty increases the variance of the number of trades.

C. Comparisons of Comparative Statics under Random Pairing and Market Clearing

The multiperiod model was tested under each assumption to examine its compatibility with the predictions in Section III. *Without exception and without regard to the use of the random-pairing or market-clearing assumptions, the simulations support the model's predictions*: volume increased with the number of traders or assets and decreased with the size of the bid-ask spread. Volume increased with both σ_θ and μ_θ, which represent, respectively, heterogeneous reactions to information and systematically different reactions between buyers

[17] The random-pairings case corresponds to Ho, Schwartz, and Whitcomb's [21] case in which investors face transaction-price uncertainty and submit buy and sell orders that, on expectation, will not transact. So trading volume is lower than in the absence of transaction-price uncertainty.

and sellers. These results were unaffected by the trading period examined or the assumption employed (either (A6) or (A6′)). For example, while the mean number of trades under the random-pairing assumption is lower in the first period than in subsequent periods, the predicted effects on trading volume were observed regardless of whether a change (e.g., in the bid-ask spread) was introduced in period 1 or any subsequent period. These results indicate that the restrictive two-period, random-pairing model of Sections III and IV yields predictions about trading volume that generalize when the two-period assumption, the random-pairing assumption, or both assumptions are relaxed.

Because the simulations yielded conclusions consistent with the model's predictions, the results are not belabored here. However, the random-pairing and market-clearing assumptions did generate slightly different results in one instance, which is discussed below.

D. The Persistence of Abnormal Trading Volume after an Informational Event

Propositions 1 and 2 predict that volume will increase when an event causes an increase in σ_θ or μ_θ, respectively. As a two-period model, however, the model cannot predict the volume effects in *subsequent* periods. Nevertheless, some empirical researchers have documented a persistence in abnormal trading volume after an informational event and after the price has adjusted (see Beaver [6] and Morse [26]). There are at least three ways to interpret these findings. One is that some investors are late in the informational queue. These investors adjust their holdings, ignorant of the fact that their information is "old." The second is that the information creates the desire to trade among some investors whose demands are not immediately cleared, perhaps because they face costs of coming to the market or frictions (e.g., order handling costs) once they reach the market. The third is that traders desire to recontract after the initial trading round. For example, Ho, Schwartz, and Whitcomb [21] demonstrate that trading volume generally deviates from its Pareto-optimal level when traders face transaction price uncertainty. This is because traders' buy and sell orders are affected by the price uncertainty, and it implies that some resulting trades are "mistakes" that can be corrected only in later trading.

Persistence in abnormal volume was examined by introducing a temporary, one-period increase in σ_θ^2 from 200 to 400 in trading period 11. σ_θ^2 was then set equal to 200 once again for all subsequent periods. This characterizes the effect of a public, informational event in trading period 11 to which investors react heterogeneously.

Figure 3 illustrates the volume effect under the random-pairing assumption. Volume for periods 6 through 20 is plotted. The dotted line represents "normal" trading volume, as taken from Figure 1, when σ_θ is not perturbed. Volume under the assumption of a temporary increase in σ_θ is represented by the solid line. As predicted, mean volume increases with the informational event—from 126.07 without the perturbation to 135.83 with it.[18] However, volume in the following period is also (slightly) higher than the benchmark volume. The mean abnormal

[18] The "abnormal trading volume" of 9.76 is nearly six standard deviations from the mean volume for period 11. In the simulations, the "seeds" for the random number generator used in the simulations were kept the same for the "benchmark" and "test" simulations. This insures that the increase in volume is solely attributable to the increase in σ_θ and not to randomness.

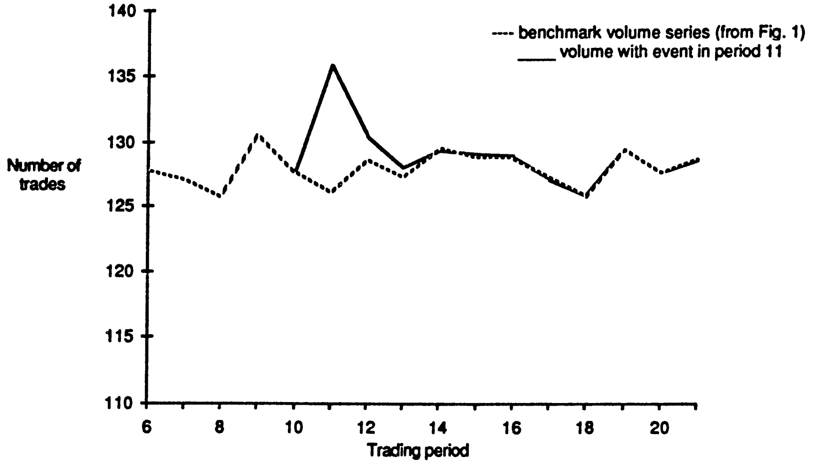

Figure 3. The Effect of an Informational Event on Trading Volume, Random-Pairing Model

volume is 1.80 trades, slightly more than one standard deviation above the benchmark. This finding is weakly consistent with the empirical findings of persistence in abnormal but dampened volume reactions after the event date.[19]

Figure 4 illustrates the effect of the same temporary increase in variance in period 11 under the assumption of costless market clearing. Once again, the dotted line indicates trading volume in the absence of the perturbation, and the solid line represents volume with the increase in σ_θ. As predicted, the abnormal trading volume is significantly greater than zero in period 11 (an increase from 144.2 to 165.2, a difference of 16.5 standard deviations). But this is the only period in which there is a volume effect. There are several, but very small, deviations from the benchmark level of volume in subsequent periods (0.1 in period 12 and −0.13 in period 13).

The only difference between the data in Figures 3 and 4 is the market-clearing mechanism. The simulation results under the random-pairing model are consistent with empirical findings of persistence in abnormal volume after informational events. Two aspects of the random-pairing model can account for this. First, the random pairing prevents all Pareto-efficient trades from being immediately cleared. This simulates frictions in real-world markets, which can prevent immediate clearing of all demands. Second, corrections of the trading mistakes that result from the random pairings mimic recontracting demands in real-world markets, as, for example, when traders face transaction-price uncertainty.[20]

E. Summary of the Simulation Results

Simulation of the model provides an opportunity to examine two extreme assumptions about the market design on trading volume. The random-pairing

[19] The size of the abnormal reaction and the persistence effect are both positively correlated with the increase in σ_θ at period 11, and the persistence effect becomes significant when σ_θ is increased enough. However, the increase in σ_θ^2 to 400 was chosen before these results were known.

[20] However, this does not constitute a test, and one cannot rule out other explanations of volume persistence. For example, information may be transmitted sequentially over several trading periods to investors who do not infer the information from trading prices.

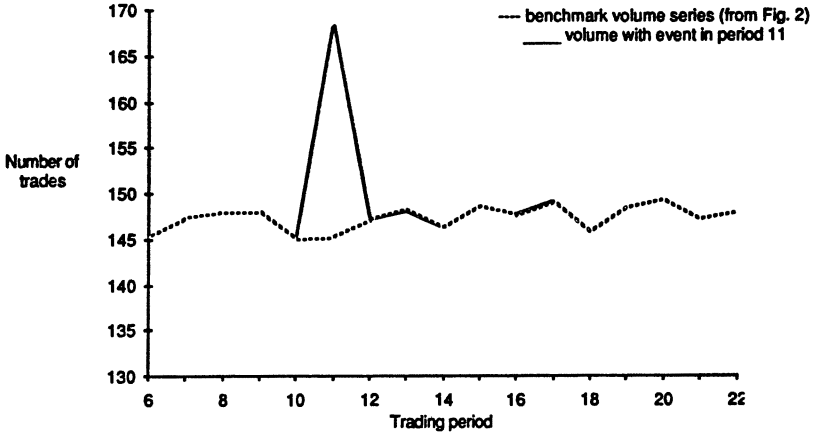

Figure 4. The Effect of an Informational Event on Trading Volume, Market-Clearing Model

assumption, while unrealistic, approximates a market in which investors face information and transaction costs and have a demand to recontract because of trading mistakes. On the other hand, the market-clearing assumption requires a costless Walrasian auction market in which all agents are able to trade at a single market-clearing price that is costlessly determined. Simulations indicate that in all cases (save one) the predictions yielded by the random-pairing assumption are qualitatively identical to the market-clearing assumption. Furthermore, all results are consistent with the model's predictions in Section IV. The two assumptions did yield different simulation results in one instance; with heterogeneous reactions to an informational event, the random-pairing model finds that trading volume is slightly higher than normal for the periods immediately after the event period. This is consistent with empirical evidence of persistence in volume reactions to informational events. The market-clearing assumption does not yield this persistence effect.

V. Conclusion

The model results and simulations are consistent with some established empirical findings: trading volume increases with the number of shares, decreases with the bid-ask spread, and is positive in nonevent periods because investors idiosyncratically revise their demand prices. The model also isolates two distinct links of volume to information and provides a rationale for the use of volume in event studies that attempt to identify whether an event has informational content. But ambiguity remains over whether a volume reaction indicates that investors *disagree* about the information. Unusually high volume can result from heterogeneous reactions to the information (σ_θ^2 increases), but it does not necessarily reflect disagreement among traders; it can also reflect consensus among traders with diverse priors (in which case $\mu_\theta > 0$). This implies that empirical researchers who attempt to infer the degree of investor consensus by examining trading volume data must first impose further restrictions. For example, consider a news release that increases trading. If only the content of the release is news and its timing has been anticipated, this is more likely to reflect agreement among traders who have taken prior speculative positions (i.e., $\mu_\theta > 0$) than if the timing of the news release is itself information.

Disentangling the two effects on volume can provide further insight. As one example, $\mu_\theta < 0$ implies a *decrease* in expected trading volume. This can be tested by identifying cases in which, *a priori*, prospective buyers and sellers can be distinguished and it is expected that $\mu_\theta < 0$. As an example, shareholders of closely held firms will be less likely to sell shares when the insiders have positive information the market does not have.

As another example, consider heterogeneous reactions to information such that σ_θ^2 increases. It is reasonable to believe that the increase is positively correlated with the "surprise content" of the information, so the increase in trading volume is positively correlated with the information "surprise." This is consistent with the empirical findings of Bamber [3, 4]. Furthermore, if the "surprise" correlates with the absolute value of the mean price revision μ_k, the correlation between volume and the absolute value of the price change is positive. This is consistent with a large amount of empirical evidence on the relation of price changes to trading volume. (See Karpoff [23].)

Finally, the model simulations indicate that the relation between information and volume is affected by the institutional design of the market. The random-pairing assumption enables the model to mimic real-world markets that do not operate perfectly and in which investors have demands to recontract. This suggests that market design would also affect the relation between price changes and volume around informational events. Markets with significant frictions (e.g., order backlogs) require some time before all trades are cleared. Over this time, volume is high and returns are serially correlated. In frictionless markets, trading is affected only in the event period, and trading and price changes in subsequent rounds are unaffected.[21]

Appendix

Proof of Proposition 1: Agent k's ($k \in S$) demand-price revision due to the information is $\mu_k = \mu + \phi_k$, where ϕ_k is the information effect idiosyncratic to agent k (and is over and above the idiosyncratic demand-price revision due to liquidity and speculative desires). To the outside observer, ϕ_k is a random variable with zero mean and variance σ_ϕ^2. Independence from the idiosyncratic liquidity or speculative demand-price revisions implies $E(\varepsilon_k \phi_h) = 0$ for all k and h.

Each agent k, whether a buyer or seller, has an expected demand-price revision $E(\delta_{k1}) = \mu$. But the variance of this process is now $E(\delta_{k1} - E(\delta_{k1}))^2 = \sigma_\phi^2 + \sigma^2$. For the parameter $\theta = \delta_{j1} - \delta_{i1}$,

$$E(\theta) = 0$$

$$E(\theta - E(\theta))^2 = 2(\sigma_\phi^2 + \sigma^2) > 2\sigma^2.$$

[21] Morse [26] finds that volume is high over the same periods that the absolute values of returns are serially correlated, but he attributes his findings to sequential information arrival rather than market structure.

So the process that describes the probability of an exchange (equation (8)) is characterized by an increased variance. Differentiating (10) with respect to σ_θ,

$$\partial\mu_T/\partial\sigma_\theta = 1/J \sum_i \sum_j (1/\sigma_\theta)(p_{i0} - p_{j0} - \mu_\theta) f_\theta(p_{i0} - p_{j0} - \mu_\theta). \tag{A1}$$

Since $p_{i0} - p_{j0} > 0$ and $\mu_\theta - \mu_i = 0$, $\partial\mu_T/\partial\sigma_\theta$ is positive, and the effect of heterogeneous reactions to new information is to increase expected volume. Q.E.D.

Proof of Proposition 2: Differentiating (10) with respect to μ_θ,

$$\partial\mu_T/\partial\mu_\theta = 1/J \sum_i \sum_j f_\theta(p_{i0} - p_{j0}) > 0. \tag{A2}$$

Q.E.D.

REFERENCES

1. George A. Akerlof. "Discriminatory, Status-Based Wages among Tradition-Oriented, Stochastically Trading Coconut Producers." *Journal of Political Economy* 93 (April 1985), 265–76.
2. Paul Asquith and William S. Krasker. "Changes in Dividend Policy, Trading Volume, and Investor Clienteles." Harvard Business School Working Paper, 1984.
3. Linda Smith Bamber. "The Information Content of Annual Earnings Releases: A Trading Volume Approach." *Journal of Accounting Research* 24 (Spring 1986), 40–56.
4. Linda Smith Bamber. "On the Relationship between Quarterly Earnings 'Surprises' and Trading Volume." University of Florida Working Paper, 1985.
5. Doyle Banks. "Information Uncertainty and Trading Volume." *Financial Review* 20 (February 1985), 83–94.
6. William H. Beaver. "The Information Content of Annual Earnings Announcements." *Empirical Research in Accounting: Selected Studies* (a supplement to the *Journal of Accounting Research*), 1968, 67–92.
7. Peter King Clark. "A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices." *Econometrica* (January 1973), 135–55.
8. Kalman Cohen, Steven F. Maier, Robert A. Schwartz, and David K. Whitcomb. "Market Makers and the Market Spread: A Review of Recent Literature." *Journal of Financial and Quantitative Analysis* 14 (November 1979), 813–35.
9. Thomas E. Copeland. "A Model of Asset Trading under the Assumption of Sequential Information Arrival." *Journal of Finance* 31 (September 1976), 1149–68.
10. Thomas E. Copeland and Dan Galai. "Information Effects on the Bid-Ask Spread." *Journal of Finance* 38 (December 1983), 1457–69.
11. Harold Demsetz. "The Cost of Transacting." *Quarterly Journal of Economics* 82 (February 1968), 33–53.
12. Douglas W. Diamond and Robert E. Verrecchia. "Information Aggregation in a Noisy Rational Expectations Economy." *Journal of Financial Economics* 9 (September 1981), 221–35.
13. Peter A. Diamond. "Aggregate Demand Management in Search Equilibrium." *Journal of Political Economy* 90 (October 1982), 881–94.
14. Thomas W. Epps. "Security Price Changes and Transaction Volumes: Theory and Evidence." *American Economic Review* 65 (September 1975), 586–97.
15. Thomas W. Epps. "The Demand for Brokers' Services: The Relation between Security Trading Volume and Transaction Cost." *Bell Journal of Economics* 7 (Spring 1976), 163–94.
16. Thomas W. Epps and Mary Lee Epps. "The Stochastic Dependence of Security Price Changes and Transaction Volumes: Implications for the Mixture-of-Distributions Hypothesis." *Econometrica* 44 (March 1976), 305–21.
17. George Foster. "Stock Market Reaction to Estimates of Earnings per Share by Company Officials." *Journal of Accounting Research* 11 (Spring 1973), 25–37.
18. Bruce Grundy. "Trading Volume and Stock Returns around Ex-Dividend Dates." Unpublished manuscript, University of Chicago, 1985.

19. Nils H. Hakansson, J. Gregory Kunkel, and James A. Ohlson. "Sufficient and Necessary Conditions for Information to Have Social Value in Pure Exchange." *Journal of Finance* 37 (December 1982), 1169–81.
20. Lawrence Harris. "The Joint Distribution of Speculative Prices and of Daily Trading Volume." Unpublished manuscript, University of Southern California, 1983.
21. Thomas S. Y. Ho, Robert A. Schwartz, and David K. Whitcomb. "The Trading Decision and Market Clearing under Transaction Price Uncertainty." *Journal of Finance* 40 (March 1985), 21–42.
22. Robert H. Jennings, Laura T. Starks, and John C. Fellingham. "An Equilibrium Model of Asset Trading with Sequential Information Arrival." *Journal of Finance* 36 (March 1981), 143–61.
23. Jonathan M. Karpoff. "The Relation between Price Changes and Trading Volume: A Survey." Forthcoming, *Journal of Financial and Quantitative Analysis* (1987).
24. Jack E. Kiger. "An Empirical Investigation of NYSE Volume and Price Reactions to the Announcement of Quarterly Earnings." *Journal of Accounting Research* 10 (Spring 1972), 113–28.
25. Joseph Lakonishok and Theo Vermaelen. "Tax-Induced Trading around Ex-Dividend Days." Center for Research in Security Prices, University of Chicago Working Paper, 1984.
26. Dale Morse. "Asymmetrical Information in Securities Markets and Trading Volume." *Journal of Financial and Quantitative Analysis* 15 (December 1980), 1129–48.
27. Dale Morse. "Price and Trading Volume Reaction Surrounding Earnings Announcements: A Closer Examination." *Journal of Accounting Research* 19 (Autumn 1981), 374–83.
28. Dale Morse. "*Wall Street Journal* Announcements and the Securities Markets." *Financial Analysts Journal* 38 (March/April 1982), 69–76.
29. Paul Calvin Pfleiderer. "The Volume of Trade and the Variability of Prices: A Framework for Analysis in Noisy Rational Expectations Equilibria." Stanford University Working Paper, 1984.
30. Morton Pincus. "Information Characteristics of Earnings Announcements and Stock Market Behavior." *Journal of Accounting Research* 21 (Spring 1983), 155–83.
31. Gordon Richardson, Stephan E. Sefcik, and Rex Thompson. "A Test of Dividend Irrelevance Using Volume Reactions to a Change in Dividend Policy." University of British Columbia Working Paper, 1984.
32. Byung T. Ro. "The Disclosure of Replacement Cost Accounting Data and Its Effect on Transaction Volumes." *Accounting Review* (January 1981), 70–84.
33. George Tauchen and Mark Pitts. "The Price Variability-Volume Relationship on Speculative Markets." *Econometrica* 51 (March 1983), 485–505.
34. Hal R. Varian. "Differences on Opinion and the Volume of Trade." University of Michigan Working Paper, 1985.
35. Robert E. Verrecchia. "On the Relationship between Volume Reaction and Consensus of Investors: Implications for Interpreting Tests of Information Content." *Journal of Accounting Research* 19 (Spring 1981), 271–83.
36. Joseph K. Winsen. "Investor Behavior and Information." *Journal of Financial and Quantitative Analysis* 11 (March 1976), 13–37.