Tax Clienteless and Asset Pricing
Author(s): Philip H. Dybvig and Stephen A. Ross
Source: *The Journal of Finance*, Vol. 41, No. 3, Papers and Proceedings of the Forty-Fourth Annual Meeting of the America Finance Association, New York, New York, December 28-30, 1985 (Jul., 1986), pp. 751-762
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2328508
Accessed: 26/11/2009 08:30

THE JOURNAL OF FINANCE • VOL. XLI, NO. 3 • JULY 1986

Tax Clienteles and Asset Pricing

PHILIP H. DYBVIG and STEPHEN A. ROSS*

ABSTRACT

Taxation of asset returns can create various clientele effects. If every agent is marginal on all assets, no clientele effects arise. If some (but not every) agent is marginal on all assets, there arises a clientele effect in quantities but none in prices. If no agent is marginal on all assets, there arise clientele effects in both quantities and prices. In the first two cases, standard asset pricing and martingale results extend to analogous after-tax results. In the third case, linear asset pricing works only on subsets of assets, and the standard martingale results become after-tax supermartingale results.

IN THE STUDY OF investments, taxes are largely a source of embarrassment to financial economists. We know that taxes are significant, but we do not know the equilibrium effect of taxes on asset pricing and the consequent effect on portfolio choice. Even partial equilibrium models such as that of Constantinides and Ingersoll [2] lead very quickly to numerical problems which make it hard to develop general intuition and effectively preclude testability on large data sets. The purpose of this paper is to develop some general concepts, intuitions, and useful simplifications concerning the effects of taxation. It is hoped that this will lead to a better understanding of related empirical phenomena, such as the January effect and the small firm effect, and to theoretical questions such as whether leverage pays.[1]

An important concept often associated with taxation is that of a *clientele effect*, namely the notion that the behavior of different groups or *clienteles* of agents are qualitatively different. In this paper, we distinguish three different situations. The first situation, one of *no clientele effects*, is a situation in which all agents have an interior differentiable "marginalist" solution to their choice problems. This situation is the traditional situation of demand theory in the absence of taxes. In the second situation, one of a *clientele effect in quantities but not in prices*, at least some agents are on corners but all assets can be priced linearly. Consequently, there is at least a hypothetical agent on the margin. For example, there may be one marginal agent who has an interior solution, but there are other agents who are at a corner with only capital gains to the exclusion of income and vice versa. In the third or general situation, one of *clientele effects in both quantities and prices*, there are agents who face differing shadow prices of tax and income attributes because pricing is not linear. This general situation is

* Yale University School of Management. We are grateful to Jon Ingersoll and Joe Williams for useful suggestions.

[1] Our analysis is consistent with (and draws heavily upon) past work by others on the effects of taxation on asset pricing. See Brennan and Schwartz [1]. Constantinides and Ingersoll [2], Miller [6], Modigliani and Miller [7, 8], and Schaefer [11].

characterized by no agents wanting to hold all assets held by others, even at the margin, because of the differing tax implications. In this case, there can be no marginal agent. For example, if short sales are precluded, there may not be any hypothetical agent who would hold common stocks, corporate bonds, growth stocks, and municipal bonds at the same time. Wealthier investors hold municipal bonds (not taxed) and growth stocks (high capital gains and low income, taxed lightly). Simultaneously, poorer investors hold common stocks and corporate bonds (both high income and taxed heavily). In this case, as income rises, perhaps the investor switches first from common stock to growth stocks, and then from corporate to municipal bonds. No investor (real or hypothetical) would want to hold all four assets. This is a clientele effect in both pricing and quantities.

Section I develops the analysis and displays examples of these different clientele effects. In the marginalist and marginal investor cases, there exist *risk-neutral probabilities* and implicit tax rates under which all assets have the same after-tax expected return. This extends the usual no-arbitrage martingale result to a world with taxes. In the general case, no single set of risk-neutral probabilities and associated implicit tax rates can price all assets. Nonetheless, such pricing rules can price subsets, and the additional assets appear inferior. In technical terms, this gives us a *super-martingale* result that under each of these pricing rules all assets have expected return less than or equal to the shared constant. Section II extends the results to more general tax codes and discusses what happens when there are realized gains or losses from the past and each agent may have a different basis. Section III closes the paper.

I. Taxation and Asset Pricing

In our model, agents are precluded from short selling assets. This type of analysis makes it simple to study assets, like municipal bonds, which lose their tax advantage when shorted. This assumption is less restrictive than it might seem, because short sales of assets (with the same or different tax treatment) can be emulated by the introduction of new "artificial" assets, each consisting of a short position in a shorted asset plus a long position in another asset. The substantial restriction of the short sale restriction is in its interaction with the tax code. In this section, we assume that taxes are a function of an aggregate called *taxable income* and that each asset has income or losses that contribute to taxable income at a constant rate (that can vary across both states and assets).[2] Section II analyzes the more general case with multiple sources of taxed income.

Here is the basic structure of the model. While we are analyzing a one period model, we can think of the one period as being embedded in a longer intertemporal model.[3] To distinguish the price of the asset at the beginning and end of the period, we will call the beginning t and the end $t + 1$, and we define p_i^t to be the

[2] An alternative approach is to assume that principal is taxable as well as gains. That approach is just as tractable (and perhaps slightly simpler), but we feel that it is more reasonable to have the agents pay taxes only on gains and losses.

[3] In general, with many periods the value function will depend on the end-of-period holding and the basis of assets held as well as the end-of-period wealth. For now, we are ignoring bases and are assuming we will sell all assets at the end of one period.

price of asset i at the beginning and $p_{i\theta}^{t+1}$ (positive) to be the price of asset i at the end in state θ. The price at $t + 1$ is assumed to include any payout (e.g., dividend, coupon, or split) that is given by the agent, as well as capital appreciation. In state θ, a fraction $\rho_{i\theta}$ (between 0 and 1) of the gain (or loss) to holding asset i counts as ordinary taxable income (or offset). The dependence of this fraction on the state is for generality, and, for example, allows the mix between coupon income and capital gains (which are taxed at different rates) to be different in different states.

Each agent has a von Neumann-Morgenstern utility function, $u(\cdot)$ (assumed to be increasing, concave, and differentiable), and initial wealth, w_t. For economy of notation, we will not index explicitly by investor. The probability that state θ will occur is $\pi_\theta > 0$, which can be interpreted as subjective or objective. For exposition, we will assume that all agents have the same probabilities, but it will be clear from the proofs that all that is really necessary is that agents agree which states have positive probability. Let α_i be the number of shares of asset i in the portfolio. The function $t(\cdot)$ (convex and differentiable, with $t(0) = 0$ and $0 < t' < 1$) gives the amount of taxes paid as a function of taxable income. Each agent solves a problem of the following form.

PROBLEM 1

Choose α_i's (*and implicitly* c_θ's *and* y_θ's) *to maximize* $\sum_\theta \pi_\theta u[c_\theta - t(y_\theta)]$ *subject to*:

$$c_\theta = \textstyle\sum_i \alpha_i p_{i\theta}^{t+1}$$

$$y_\theta = \textstyle\sum_i \alpha_i (p_{i\theta}^{t+1} - p_i^t)\rho_{i\theta}$$

$$w_t = \textstyle\sum_i p_i^t \alpha_i$$

$$\alpha_i \geq 0.$$

The objective function is the expected utility of after-tax wealth. The first constraint defines pre-tax consumption, c_θ, as the sum of the end of period values of the assets times the number of shares. The second constraint defines taxable income as the sum across assets of the net proceeds times the fraction of the gain that is counted as ordinary income. The third constraint is the budget constraint, and the last constraint is the no-short-sales condition. To give the model some content, we will also make the following minimal rationality assumption for a single agent, which is even weaker than assuming a competitive or rational expectations equilibrium.

ASSUMPTION 1: Each asset is held by some agent.

The first order conditions for Problem 1 are given by the constraints plus the following conditions.

$$p_i^t \geq \textstyle\sum_\theta p_{i\theta}^{t+1}\lambda_\theta - \sum_\theta (p_{i\theta}^{t+1} - p_i^t)\lambda_\theta \tau_\theta \rho_{i\theta}, \quad \textit{for all } i, \tag{1}$$

with equality whenever $\alpha_i > 0$, where

$$\tau_\theta = t'(y_\theta), \tag{2}$$

$$\lambda_\theta = \frac{\pi_\theta u'(c_\theta - t(y_\theta))}{(1+r)\sum_\theta \pi_\theta u'(c_\theta - t(y_\theta))}, \tag{3}$$

and r is the agent's shadow riskless rate for a tax-exempt asset. If such an asset is actually marketed as asset 0 and is held by this agent, then $1 + r = p_{0\theta}^{t+1}/p_0^t$ for all θ, since $\rho_{0\theta} \equiv 0$. Since the marginal utilities, probabilities, and marginal tax rates are positive, so are τ_θ and λ_θ. Solving for p_i^t, (1) can be written alternatively as

$$p_i^t \geq \frac{\sum_\theta p_{i\theta}^{t+1}\lambda_\theta(1 - \tau_\theta \rho_{i\theta})}{1 - \sum_\theta \rho_{i\theta}\tau_\theta\lambda_\theta}. \tag{4}$$

The right hand side of (1) is the marginal value of holding another share of asset i, given the pre-tax cash flow and taxable income tomorrow in each state. For assets that are held this must be the price; for other assets the price is too high. The right hand side of (4) is the marginal value of holding another share of asset i, given the pre-tax cash flow tomorrow in each state and given the tax treatment; the interpretation is the same.

Equation (1) (or equivalently (4)) is the fundamental pricing equation for agents facing Problem 1. To move from Equation (1) to asset pricing, it is useful to distinguish three different cases. The first case, characterized by no clientele effects, has Equation (1) holding with equality for all agents. In this case, all agents have the same marginal tax rate in equilibrium, or, in other words, agents perform tax arbitrage until they achieve the same marginal tax rate. This is what would happen if we removed the short sales restriction. In fact, if this case arises with the short sales restriction, removing the restriction does not change anything.[4] The second case, characterized by a clientele effect in quantities but not prices, has Equation (1) holding with equality at least for some hypothetical agent. Nonetheless many (or all) agents in the economy can be at corners. The third case, characterized by a clientele effect in both quantities and prices, has Equation (1) holding with equality for no (even hypothetical) agent. In this case, there is no marginal agent who is indifferent about small changes in position in all assets, and all agents are at some corners (and do not hold all assets).

Here is a stylized example with three types of riskless assets.

EXAMPLE 1

Assume that there is only one state and that there are three riskless assets, a "coupon bond," a "discount bond," and a "municipal bond," each having different tax treatments. The coupon bond is taxed at the ordinary income rate. The discount bond is taxed at a capital gains rate less than 1 (remember, this is a *stylized* example and we are ignoring real accounting conventions). The municipal bond is not taxed. Here is a description of each of the three cases for this model.

[4] This case points up the fallacy of assuming that agents have linear schedules (with linear marginal tax rates) in a model allowing short sales. Agents having different tax rates can make arbitrage profits, obtaining arbitrarily large subsidies from the government (and therefore precluding equilibrium). Of course, governments will not really provide arbitrarily large subsidies. Realistic tax laws are non-linear. The non-linearity of the tax law in our analysis shows up in the tax schedule, $t(\cdot)$, and in the short-sales restriction. We introduce a richer source of non-linearity in Section II.

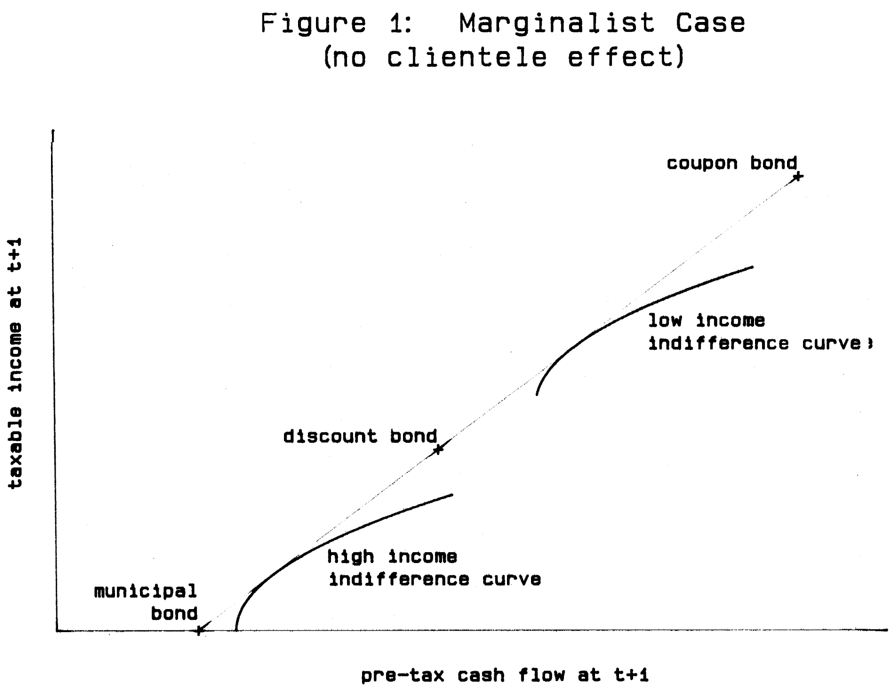

Figure 1. In the general case, no agent can be marginal on all assets, and there are clientele effects in both quantities and prices.

1. No clientele effects (marginalist case—Figure 1): In this case, all agents are characterized by equality in (1), as if they were allowed to short each asset. Because of this, (1) implies that all assets lie on a straight line in a plot of taxable income against pre-tax cash flow, both stated per dollar invested. This straight line involves an implicit tax rate at which each asset has the same return. Furthermore, given the three asset returns, we can invert (1) for each agent to conclude that all agents have the same marginal tax rate. In other words, in this marginalist equilibrium, tax arbitrage has driven all agents to the point of equal tax rates.[5]
2. Clientele effect in quantities but not in prices (Figure 2): In this case, not all agents face equality in (1), but some hypothetical representative agent does. From (1) for that agent, we again can conclude that all assets lie on a straight line in a plot of income vs. taxable income, per dollar invested, again giving an implicit marginal tax rate at which all assets have the same after-tax return. However, we cannot conclude that all agents have the same marginal tax rate, because possible inequalities in the first order conditions preclude the inversion performed in the first case. In general, then, we can see plunging into the coupon bond or the municipal bond, as well as interior solutions, as in Figure 2.

[5] This is not quite general to cases with many states, unless the states are spanned in both income and taxable income independently. Otherwise, different agents could see different implicit marginal tax rates in the same equilibrium.

Figure 2: Marginal Investor Case
(clientele effect in quantities but not in prices)

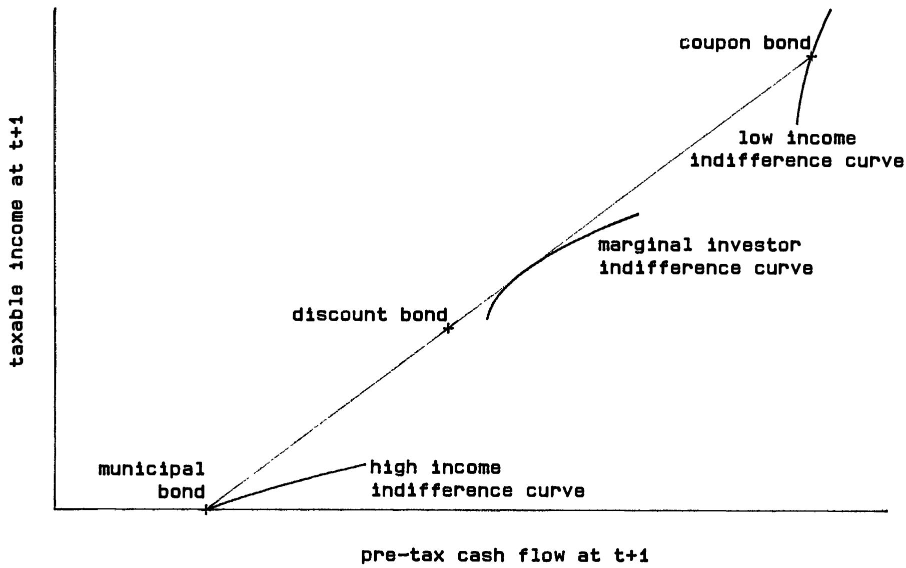

Figure 2. In the marginal investor case, some agent (real or hypothetical) is marginal on all assets. However, agents typically are not and there is a clientele effect in quantities but not in prices.

3. Clientele effect in quantities and in prices (Figure 3): In this case, no agent (real or hypothetical) can have equality in (1), and the assets cannot lie on a straight line in a plot of income versus taxable income, per dollar invested. (Otherwise, we could construct such a marginal agent.) In this case, no agent is marginal on *all* assets. Possible solutions include plunging into any of the three assets or holding the discount bond with one of the other two assets. The main restriction is that the frontier must be convex, that is, that the discount bond must not be dominated by a portfolio of the coupon bond and the municipal bond, since otherwise no agent would hold the discount bond (violating Assumption 1).

An important point in this example is that while (1) is stated in terms of price levels, it has implications for returns. This should not be surprising, in light of the fact that much of financial pricing theory is founded on inverting first order conditions to get conditions on returns. The statement about making the after-tax returns equal sounds suspiciously like the statement of the "martingale" or "risk-neutral pricing" property. This property, first described by Cox and Ross [3, 4] and later extended by Harrison and Kreps [5] and many others, is that there exists a probability reassignment that gives all assets the same expected return. In fact, here is a result that integrates the martingale or risk-neutral property with the tax model.

THEOREM 1. *Suppose that there are no clientele effects in prices, i.e., that we are either in a marginalist world or in a world with a representative individual. Then*

Figure 3: General Case
(clientele effects in quantities and in prices)

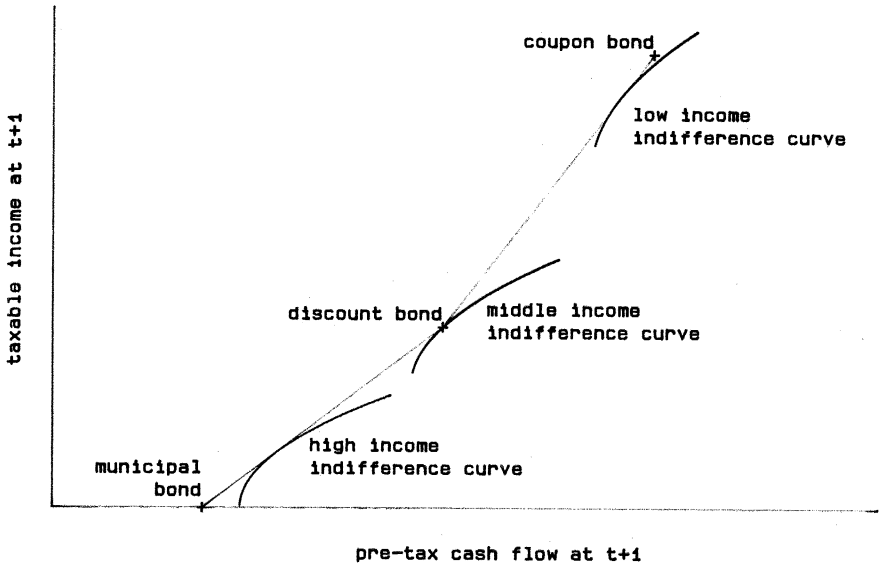

Figure 3. In the marginalist case, all agents see the same shadow prices and marginal tax rates in all states, and there are no clientele effects.

there exist state-prices λ_θ *and implicit state-tax-rates* τ_θ *such that*

$$p_i^t = \frac{\sum_\theta p_{i\theta}^{t+1}\lambda_\theta(1 - \tau_\theta\rho_{i\theta})}{1 - \sum_\theta \rho_{i\theta}\tau_\theta\lambda_\theta} \tag{5}$$

for all i. Furthermore, there exist positive risk-neutral probabilities π_θ^*, *summing to* 1, *such that all assets have the same expected after-tax return,* r^*, *under these probabilities and the tax rates,* τ_θ, *i.e.,*

$$\textstyle\sum_\theta r_{i\theta}^{t+1}\pi_\theta^*(1 - \tau_\theta\rho_{i\theta}) = r^*, \tag{6}$$

the same for all i, where

$$r_{i\theta}^{t+1} \equiv (p_{i\theta}^{t+1}/p_i^t) - 1. \tag{7}$$

Proof. Equation (5) is exactly (4) for any marginal agent. (There must be one: the second case is characterized by the existence of a marginal agent and in the first case *all* agents are marginal.) Having (6) constant across assets is derived from (1) by setting

$$\pi_\theta^* = \lambda_\theta/\textstyle\sum_\theta \lambda_\theta, \tag{8}$$

which yields

$$\textstyle\sum_\theta r_{i\theta}^{t+1}\pi_\theta^*(1 - \tau_\theta\rho_{i\theta}) = (\sum_\theta \lambda_\theta)^{-1} - 1 \equiv r^* \tag{9}$$

($\equiv r$ from (3)), which is independent of i. Q.E.D.

Theorem 1 integrates martingales and taxes for both cases in which clientele effects do not show up in prices. For the third case, in which clientele effects are present in both prices and quantities, this theorem will not go through as it stands. Instead, we have the weaker result that for each investor, Theorem 1 holds only for those assets held by the investor, while other assets are worth less. Each asset is priced exactly by some investor, by Assumption 1. In the martingale terminology, we have that after-tax returns satisfy a *super-martingale* property, since under the probability reassignment assets held have the same expected return and other assets go down, relatively speaking. (The term martingale is motivated by terminology from stochastic processes. A martingale is a process, the expected changes of which are always zero. A super-martingale is a process, the expected changes of which are always less than or equal to zero.) Here is the general result. (It is stated without reference to the three cases, since by Theorem 1 it is satisfied in a degenerate way in the first two cases.)

THEOREM 2. *Let* **I** *be the set of assets held by some agent. Then there exist state-prices λ_θ and implicit state-tax-rates τ_θ such that*

$$p_i^t \geq \frac{\sum_\theta p_{i\theta}^{t+1}\lambda_\theta(1-\tau_\theta\rho_{i\theta})}{1-\sum_\theta \rho_{i\theta}\tau_\theta\lambda_\theta}, \tag{10}$$

with equality for all i in **I**. *Furthermore, there exist positive risk-neutral probabilities, π_θ^*, such that all assets in* **I** *have the same expected after-tax expected return under these probabilities and the tax rates τ_θ, and all other assets have no larger expected return under these probabilities. I.e., there is some r^* such that after-tax returns satisfy the following martingale property.*

$$\sum_\theta r_{i\theta}^{t+1}\pi_\theta^*(1-\tau_\theta\rho_{i\theta}) \leq r^*, \tag{11}$$

with equality for all i in **I**, *where $r_{i\theta}^{t+1}$ was defined in* (7).

Proof. To have (10) hold for all i and to hold with equality for i in **I** is the first order condition (4) for the agent. The proof of the existence of r^* follows exactly the algebra of the related part of Theorem 1, the only difference being an inequality instead of an equality. The final equation (9) from the proof of Theorem 1 becomes

$$\sum_\theta r_{i\theta}^{t+1}\pi_\theta^*(1-\tau_\theta\rho_{i\theta}) \leq \left(\sum_\theta \lambda_\theta\right)^{-1} - 1, \tag{12}$$

with equality for all i in **I**. Defining r^* as in Theorem 1, the proof is complete.

Q.E.D.

A potential weakness of Theorem 2 is that, unlike the standard martingale or risk-neutral results, it is stated in terms of individual agents. There is also a related result that makes no direct reference to individual preferences.

THEOREM 3. *There exists a collection,* Π, *of risk-neutral pricing rules, each with a probability assignment, π^*, an after-tax riskless rate, r^*, and tax rates, τ_θ, such that each asset has a price at t equal to the largest discounted expected value of time $t+1$ cash flows across pricing rules*:

$$p_i^t = \max\left\{\frac{\sum_\theta \pi_\theta^*(p_{i\theta}^{t+1}-(p_{i\theta}^{t+1}-p_i^t)\tau_\theta^*\rho_{i\theta})}{1+r^*}\;\middle|\;(\pi^*, r^*, \tau_\theta^*)\in\Pi\right\}. \tag{13}$$

Furthermore, each asset has an expected return under each pricing rule that is less than or equal to r^*, with equality for some pricing rule; i.e., for each i,

$$\sum_\theta r_{i\theta}^{t+1}\pi_\theta^*(1 - \tau_\theta^*\rho_{i\theta}) \le r^* \tag{14}$$

for all pricing rules $(\pi^*, r^*, \tau_\theta^*)$ in Π, with equality for some rule in Π.

Proof. Let the set of pricing rules correspond to the set of agents. Let the risk-neutral probabilities and the tax rates corresponding to an agent be derived from the agent's first order conditions as in the proof of Theorem 1. Let the associated interest rate be

$$r^* = (\textstyle\sum_\theta \lambda_\theta)^{-1} - 1 \tag{15}$$

Then, by (1), the definition of π_θ, and the choice of r^*, (13) holds with equality for any rule corresponding to an agent holding the asset and the right hand side is smaller for any other asset. This proves the first part. The second part is just a restatement of (12) used in the proof of Theorem 2. Q.E.D.

The proof of this result uses a set of pricing rules as large as the set of agents, but of course it is most useful if there is a set which uses only a few rules. If we are in a case without clientele effects in prices, we know that we need only use a single agent.

COROLLARY 1. If there are no price clientele effects (i.e., either all agents are on the margin for all assets or some hypothetical agent is), then the collection of pricing rules in Theorem 3 can be chosen to contain a single rule. In other words, there exists probabilities π^* and a shadow after-tax riskless rate r^* such that

$$p_i^t = \frac{\sum_\theta \pi_\theta^*(p_{i\theta}^{t+1} - (p_{i\theta}^{t+1} - p_i^t)\tau_\theta\rho_{i\theta})}{1 + r^*} \tag{16}$$

and

$$\sum_\theta r_{i\theta}^{t+1}\pi_\theta^*(1 - \tau_\theta\rho_{i\theta}) = r^* \tag{17}$$

for all i.

Proof. Just follow through the proof of Theorem 3, using only a representative agent (any agent in the marginalist case). Q.E.D.

II. Extensions

The class of tax rules used in Section I is relatively simple, and may rule out many features of actual tax codes. For example, the class does not include rules which depend on whether net capital gains are positive or negative, or rules which allow offsetting gains normally taxed at one rate with losses normally taxed at another rate in some circumstances. Here is a generalization of the original model, with many types of taxable income, that can capture these other features without substantially changing the analysis. In the second case, this is essentially the model analyzed by Ross [9, 10]. The new problem is as follows, where $t(\cdot)$ is a convex (not necessarily differentiable) function of all its arguments. In the interest of brevity, we are handling this case very informally.[6]

PROBLEM 2

Choose α_i's (*and implicitly* c_θ's *and* y_θ's) *to maximize* $\sum_\theta \pi_\theta u[c_\theta - t(y_{\theta 1}, y_{\theta 2}, \cdots y_{\theta N})]$ *subject to*

$$c_\theta = \sum_i \alpha_i p_{i\theta}^{t+1}$$

$$y_{\theta n} = \sum_i \alpha_i (p_{i\theta}^{t+1} - p_i^t)\rho_{i\theta n}$$

$$w_t = \sum_i p_i^t \alpha_i$$

$$\alpha_i \geq 0.$$

This problem has almost exactly the same properties as Problem 1. In terms of the results in this paper, the main difference is that there is an implicit tax rate for each type of income (instead of just one implicit tax rate). For example, the first order conditions in (1) and (4) become the following.

$$p_i^t \geq \sum_\theta p_{i\theta}^{t+1}\lambda_\theta - \sum_\theta \sum_n (p_{i\theta}^{t+1} - p_i^t)\lambda_\theta \tau_{\theta n}\rho_{i\theta n}, \quad \textit{for all } i, \tag{18}$$

with equality whenever $\alpha_i > 0$.

$$p_i^t \geq \frac{\sum_\theta p_{i\theta}^{t+1}\lambda_\theta(1 - \sum_n \tau_{\theta n}\rho_{i\theta n})}{1 - \sum_\theta \sum_n \rho_{i\theta n}\tau_{\theta n}\lambda_\theta}. \tag{19}$$

As an example, here is how Theorem 3 would look under Problem 2.

THEOREM 4 (Theorem 3 adapted to Problem 2). *There exists a collection,* Π, *of risk-neutral pricing rules, each with a probability assignment,* π^*, *an after-tax riskless rate,* r^*, *and tax rates,* $\tau_{\theta n}$, *such that each asset has a price at t equal to the largest discounted expected value of time t + 1 cash flows across pricing rules. In other words,*

$$p_i^t = \max\left\{\frac{\sum_\theta \pi_\theta^*(p_{i\theta}^{t+1} - (p_{i\theta}^{t+1} - p_i^t)\sum_n \tau_{\theta n}^*\rho_{i\theta n})}{1 + r^*} \;\middle|\; (\pi^*, r^*, \tau_{\theta n}^*) \in \Pi\right\}. \tag{20}$$

Furthermore, each asset has an expected return under each pricing rule that is less than or equal to r^*, *with equality for some pricing rule; i.e., for each i,*

$$\sum_\theta r_{i\theta}^{t+1}\pi_\theta^*(1 - \sum_n \tau_{\theta n}^*\rho_{i\theta n}) \leq r^* \tag{21}$$

for all pricing rules $(\pi^*, r^*, \tau_{\theta n}^*)$ *in* Π, *with equality for some rule in* Π.

Proof. Similar to that of Theorem 3, only using (18) and (19) in place of (1) and (4). Q.E.D.

Most of our previous results and intuitions are robust to this case. Interestingly enough, if we allow short sales in this case (or have a marginal agent), it still is the case that we cannot have a clientele effect in prices. Intuitively, this is because

[6] For example, we are not providing sufficient conditions for the marginal after-tax value of pretax income to be positive (although it is easy to do so). For the first order conditions, differentiability of $t(\cdot)$ is not needed, since in general we can use an element of the derivative correspondence.

tax arbitrage will eliminate the discrepancy in marginal tax rates. Note that we have not yet made enough assumptions to ensure that an equilibrium exists, and that there has to be some more structure put on the tax code to preclude arbitrarily large subsidies. Nonetheless, the tax code in Problem 2 gives the opportunity to prevent such subsidies in any of a number of ways.

As a final note in this section, the first order condition in (1) can easily be adjusted to reflect any gains or losses an agent may have as of t. Such gains or losses are important since they affect the cost to the agent of holding another unit of the asset. In equation (1), having gains or losses effectively raises or lowers the p_i^t on the left hand side of the equation. Similarly, if the basis going into $t + 1$ is different from p_i^t, this has the same effect as changing the p_i^t on the right hand side of (1). Of course, if we do not know what gains, losses, or basis various agents might have, all of our equilibrium results (such as Theorem 3) become vacuous. In particular, the difference in basis may drive a wedge between the reservation prices of those holding the asset and those not holding the asset, and the current market price may lie anywhere between the reservation prices of those owning the asset and those not owning the asset. This is one perspective on why equilibrium intertemporal asset pricing in the presence of taxes is so difficult.

III. Conclusion

We have derived results that integrate the theory of arbitrage and risk-neutral pricing with the theory of taxation. If we have a marginalist solution or at least a hypothetical marginal individual, we obtain a linear tax pricing rule as in Ross [10]. More generally, there are many pricing rules and the asset price is the largest price across potential pricing rules. Furthermore, we have shown that the model can easily handle multiple types of taxable income.

Much work remains. In particular, little of practical use is known about the general case (discussed at the end of Section II) in which each agent may have a different basis for assets already owned. This case has some interesting features and may very well be useful for explaining tax-related empirical anomalies, provided we can develop a tractable and reasonable model.

REFERENCES

1. Michael J. Brennan and Eduardo S. Schwartz. "Corporate Income Taxes, Valuation, and the Problem of Optimal Capital Structure." *Journal of Business* 51 (January 1978), 103–114.
2. George M. Constantinides and Jonathan E. Ingersoll, Jr. "Optimal Bond Trading with Personal Taxes." *Journal of Financial Economics* 13 (September 1984), 299–335.
3. John C. Cox and Stephen A. Ross. "A Survey of Some New Results in Financial Option Pricing Theory." *Journal of Finance* 21 (May 1976), 383–402.
4. ———. "The Valuation of Options for Alternative Stochastic Processes." *Journal of Financial Economics* 3 (June 1976), 145–166.
5. J. Michael Harrison and David Kreps. "Martingales and Arbitrage in Multiperiod Securities Markets." *Journal of Economic Theory* 20 (June 1979), 381–108.
6. Merton H. Miller. "Debt and Taxes." *Journal of Finance* 33 (May 1977), 261–275.
7. Franco Modigliani and Merton H. Miller. "The Cost of Capital, Corporation Finance and the Theory of Investment." *American Economic Review* 48 (June 1958), 261–297.

8. Franco Modigliani and Merton H. Miller. "Corporate Income Taxes and The Cost of Capital: A Correction." *American Economic Review* 53 (June 1963), 433–443.
9. Stephen A. Ross. "Debt and Taxes and Uncertainty." *Journal of Finance* 40 (July 1985), 637–657.
10. ———. "Arbitrage and Martingales with Taxation." Working paper, Yale School of Management (1985).
11. Stephen A. Schaefer. "Taxes and Security Market Equilibrium." In William F. Sharpe and Cathryn M. Cootner, eds., *Financial Economics: Essays in Honor of Paul Cootner.* Englewood Cliffs, N.J.: Prentice-Hall (1982), 159–178.

DISCUSSION

JOSEPH WILLIAMS*: Dybvig and Ross reexamine the relationship between taxes, tax clienteles, and asset prices. Their analysis is crisp, and their results are provocative. Assuming progressive personal taxes and fixed supplies of securities, the authors identify three possible equilibria characterized by either no clientele effects, clienteles in quantities but not prices, and clienteles in both quantities and prices. For the first two cases, they construct risk-neutral probabilities and implicit tax rates under which all assets have equal expected returns. As the authors indicate, "This extends the usual no-arbitrage martingale result to a world with taxes." In the third case, multiple pricing rules are possible. Nevertheless, the authors still derive "a supermartingale result that under each of these pricing rules all assets have expected returns less than or equal to the shared constant."

For these results, Dybvig and Ross initially require that each investor pays taxes only on his aggregate taxable income. Subsequently, they extend their results to more realistic tax codes with piecewise-differentiable tax functions defined over multiple categories of taxable income. This interesting extension permits distinct tax brackets with different tax rates assessed on, for example, net capital gains versus net losses.

Dybvig and Ross reach their results roughly as follows. Because all assets are supplied exogenously, prices are determined in equilibrium by the solution to the representative investor's portfolio problem. For a risk-averse investor facing progressive personal taxes, the solution to this portfolio problem is uniquely characterized by Kuhn-Tucker conditions. If these conditions hold with equality for all assets, then convenient normalizations of the Kuhn-Tucker multiplier produce both the linear pricing relationships for all assets and the equality of expected returns net of the representative investor's taxes. Following the authors' terminology, such situations exhibit at most a clientele effect in quantities. In the remaining case with clientele effects in both prices and quantities, i.e., inequalities in at least some Kuhn-Tucker conditions, the same normalizations of the multiplier produce both lower bounds on asset prices and upper bounds on expected returns after personal taxes. Moreover, similar Kuhn-Tucker conditions and resulting pricing relationships must hold for any piecewise-differential tax function defined over multiple categories of taxable income.

* New York University.