DISCUSSION

DAVID FELDMAN*: Gennotte [8], in an interesting and well presented work, addresses important issues of investment decisions in a Merton [11]/Cox Ingersoll and Ross [2] production economy. In this economy, identical individuals maximize expected lifetime utility of consumption by continuously allocating their wealth to consumption, locally risk free borrowing and lending, and several realized outputs which are noisy functions of unobservable productivity factors. The paper's stated objectives are: (1) to determine the estimation procedure, (2) to establish the separation between filtering and control, and (3) to investigate the impact of the estimation error on portfolio rules. The first two objectives were previously pursued to different ends, e.g., Feldman [6], Dothan and Feldman [4], and Detemple [3].[1] These works, using results from Liptser and Shiryayev [10], identified a Markovian system of diffusion equations (the nonlinear filter) which describes the posterior distribution of the unobservable factor, and the first two works further solved the optimal control problem in a general equilibrium setting. Gennotte uses these works' framework and results to pursue the third objective. To meet this objective would be new and interesting.

The paper's contribution to the third objective consists of two interesting observations. The first one is that an increase in the uncertainty regarding instantaneous expected returns of a certain production process will reduce its weight in the portfolio. This observation is consistent with the static analysis of Klein and Bawa [9]. Within a dynamic framework, however, the effect of increased uncertainty might be complex depending on the signs and the sizes of systematic and nonsystematic sources of covariance. That is, when the realizations of the investment opportunity set are unobservable, investors use realized returns to learn about it through two sources of covariance. The systematic source of covariance conveys the direct effect of the unobservable productivity factor on realized returns. The nonsystematic source of covariance represents the mutual dependency of the productivity factor and realized returns on the underlying uncertainty in the economy. Under complete information, equilibrium demands are functions of the variance of the investment opportunities; however, under incomplete information, when investment opportunities are unobservable, equilibrium demands are functions of the variance of the estimated investment opportunities. This latter variance is an average of the two sources of covariance, dynamically weighted by the estimation error. Thus, when a certain portfolio weight is decreasing in the variance of the estimated investment opportunities, it is not necessarily decreasing in the estimation error. Depending on the time path of the estimation error, the variance might be an increasing, decreasing, or decreasing-increasing function of the estimation error. Furthermore, the estimation error might assume (temporarily or in a steady state) a value where the two sources of covariance offset each other. In this case, realized returns become uninformative with respect to the productivity factors and consequently investors cease to update stochastically their demands even though they know that the

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[1] See Duffie and Huang [5] and Admati and Pfleiderer [1] as representatives of two other important directions of recent research in incomplete and differential information issues.

underlying investment opportunities fluctuate randomly. None of the parameter sets leading to each of these specific cases can be rejected offhand and it would be interesting to identify the types of economies that give rise to each set and its likelihood.

Gennotte's second observation regarding portfolio rules is that the decisions of investors with logarithmic preferences, given their estimates of expected returns, are independent of the estimation error; therefore, they behave as if the conditional means are the true ones. However, even logarithmic investors use the estimation error while continuously updating their portfolios. Moreover, they endogenously determine the conditional means as functions of the estimation error (in a Bayesian fashion starting at the initial condition), hence, conditioning on these endogenous estimates already captures the inference process. In fact, in this incomplete information economy the informational requirements of these investors are identical to those of "non-myopic" investors and this led Feldman [7] to propose a generalized definition of myopia.

There are ample avenues for further research of which the following are just a few. While a conditionally Gaussian information structure is sufficient for a finite Markovian filter and an induced tractable optimal control problem, it is not necessary; identifying the necessary conditions would be most illuminating. One could conjecture that a posterior distribution with a stochastic second moment will also be sufficient as long as the posterior mean is a sufficient statistic. Solving such a model would allow the investigation of more realistic situations since a deterministic estimation error is not genrally plausible. The effect of the dynamic quality of information on prices of contingent claims should be studied, and lastly, time series tests that condition the distribution of the observations on past and present realizations, as the nonlinear filter does, will generate interesting insights regarding nonstationary variables in asset pricing models.

REFERENCES

1. A. R. Admati and P. Pfleiderer. "Viable Allocations of Information in Financial Markets." Research Paper No. 847, Graduate School of Business Administration, Stanford University, November 1985.
2. J. C. Cox, J. E. Ingersoll and S. A. Ross. "An Intertemporal General Equilibrium Model of Asset Prices." *Econometrica* 53 (January 1985), 363–383.
3. J. Detemple. "Asset Pricing in a Production Economy with Incomplete Information." *Journal of Finance* 41 (June 1986), 383–392.
4. M. Dothan and D. Feldman. "Equilibrium Interest Rates and Multiperiod Bonds in a Partially Observable Economy." *Journal of Finance* 41 (June 1986), 369–382.
5. D. Duffie and C. F. Huang. "Martingales and Multiperiod Security Markets with Differential Information: Martingales and Resolution Times." Research Paper No. 812, Graduate School of Business Administration, Stanford University, March 1985.
6. D. Feldman. *A Theory of Asset Prices and the Term Structure of Interest Rates in a Partially Observable Economy.* Ph.D. Dissertation: Northwestern Unversity, August 1983.
7. ———. "Logarithmic Preferences, Myopic Decisions, and Incomplete Information." Working Paper No. 36-85/86, Graudate School of Industrial Administration, Carnegie-Mellon University, September 1985.

8. G. Gennotte. "Optimal Portfolio Choice Under Incomplete Information." *Journal of Finance* 41 (July 1986), 733–749.
9. R. W. Klein and V. S. Bawa. "The Effect of Estimation Risk on Optimal Portfolio Choice." *Journal of Financial Economics* 3 (June 1976), 215–231.
10. R. S. Liptser and A. N. Shiryayev. *Statistics of Random Processes II.* New York: Springer-Verlag, 1978.
11. R. C. Merton. "An Intertemporal Capital Asset Pricing Model." *Econometrica* 41 (September 1973), 867–887.