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On Timing and Selectivity

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ABSTRACT

The dichotomy between timing ability and the ability to select individual assets has been widely used in discussing investment performance measurement. This paper discusses the conceptual and econometric problems associated with defining and measuring timing and selectivity. In defining these notions we attempt to capture their intuitive interpretation. We offer two basic modeling approaches, which we term the *portfolio approach* and the *factor approach*. We show how the quality of timing and selectivity information can be identified statistically in a number of simple models, and discuss some of the econometric issues associated with these models. In particular, a simple quadratic regression is shown to be valid in measuring timing information.

THE STUDY OF INVESTMENT performance has long sought to draw the distinction between the ability to "time" the market and the ability to forecast the returns on individual assets.[1] This distinction has been viewed as a useful device for attribution of a manager's performance, i.e., superior performance is due to either timing or selection ability or some combination of the two. Indeed, portfolio managers often characterize themselves as market timers or stock pickers. An ability to distinguish between these two sources of superior performance may allow more accurate measures of the value of the services provided by managers. At the same time, the distinction between these two types of performance may have intrinsic interest. The distribution of prices and observed trades may be very different depending on whether private information in the economy is predominantly about market aggregates or is predominantly firm-specific.

The active nature of portfolio management is often based on the claim of superior information. If this is so, then the main problem in performance evaluation is inferring the *quality* of private information possessed by a portfolio manager. Many tests do not successfully separate measures of quality from measures of aggressiveness. By trading aggressively, a manager with little information may be able to mimic certain aspects of the performance of a manager having more precise information. Since a client can always alter the aggressiveness of a managed portfolio on his own account, aside from possible savings in transactions costs that may make mutual funds superior to individual investment, the value added by a portfolio manager is a function solely of the precision of his

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[1] A partial list of references includes Treynor and Mazuy [13], Fama [6], Jensen [8], Kon [9], Kon and Jen [10], Merton [12], Henriksson and Merton [7], and Connor and Korajczyk [3].

information. Empirical tests of timing and selectivity should be designed to recover some measure of the quality of the information a manager has, as distinct from measuring the investment strategies of the manager (such as the level of changes in the beta of his portfolio). We show that this can be done for a number of interesting specifications. Related work can be found in Admati and Ross [2].

Our focus in this paper is mainly on the conceptual and econometric problems involved in distinguishing timing and selection ability. Although the terms timing and selectivity have been used quite often, providing satisfactory definitions is not an easy task. Timing is often characterized (see, e.g., Jensen [8]) by the *response* of the informed manager to the private information he or she obtains. In the language we will use, there is a small set of portfolios, called *timing portfolios*, which essentially provide separating funds for the managed portfolio—the change in the portfolio's composition in response to timing information always involves only shifting funds among the timing portfolios. Selectivity is usually thought of as the ability of a manager to pick individual assets. Two intuitive characterizations of this notion are: (i) as distinct from timing, selectivity information is *statistically independent* of the returns on the timing portfolios,[2] and (ii) different types or coordinates of selectivity information pertain to different assets. We first observe that these two characterizations may be inconsistent with each other. Indeed, they are, strictly speaking, inconsistent with each other if asset returns are normally distributed, as we assume for most of our analysis.[3]

In the next section we discuss two approaches to defining selectivity, which are motivated by the observation above. We call the first the *portfolio approach*, and the second the *factor approach*. In the portfolio approach we use the first characterization of selectivity. Specifically, for a given set of timing portfolios, selectivity information pertains to the residuals in the regression of asset returns on the returns of the timing portfolios. For example, if timing portfolios are the riskless asset and the market portfolio of risky assets, then selectivity information pertains to the residuals from the standard market-model regression. This definition is consistent with many of the empirical studies in the literature, but it is somewhat awkward and not easily motivated. In particular, it is difficult to reconcile the information structures that arise in this formulation with the information acquisition technology that gives rise to the existence of this information.

In what we call the *factor approach*, a factor generating process is postulated for asset returns, and timing and selectivity information are interpreted in terms of their statistical relation to the factors and to the idiosyncratic terms in the generating process. This approach is consistent with the above definition of

[2] This characterization has been important in the recent controversy with regard to the use of the securities market line to measure performance (see Mayers and Rice [11] and Dybvig and Ross [4, 5]). It was argued that using the securities market line is justified if there is no timing information. Private information is assumed to be independent of the returns on the index. This is the main motivation behind the portfolio approach developed below.

[3] To see this most clearly, note that in this case, if there are L timing portfolios and N assets, and if selectivity information is independent of the returns on the timing portfolios, then it can be summarized by an $N - L$ dimensional sufficient statistic. It follows that if selectivity information satisfies (i) above, then it is impossible that there are N independent selectivity signals, one for each asset.

timing, i.e., if information pertains to a set of K factors, then there are (at least) K timing portfolios. But now the selectivity information, which pertains to the idiosyncratic terms in the factor model, is truly information about individual assets that is statistically independent (given the factors) of returns on other assets, and is, further, independent of the factor information that gives rise to the timing portfolios. What is not true anymore is that selectivity information is distinct from timing in the sense that this information is independent of the returns on the timing portfolios. Thus, this definition captures the second characterization of selectivity mentioned above, but not the first. It is, in our opinion, more natural and easier to motivate.

One result we derive, which is important to the ability to study timing and selectivity empirically is that, surprisingly, the composition of the timing portfolios obtained in the factor approach is *independent of the quality of the private information.* For example, if two managers each have timing information (information that pertains to some K factors of the distribution of returns), then no matter what the precision of the private information is, the two managers have the same timing portfolios, i.e., they shift the composition of their portfolios within the same set of separating portfolios. This result, however, depends on the manager possessing no selectivity information. Information about the idiosyncratic terms affects the composition of the timing portfolios, so that although the response to the timing information is such that L timing portfolios are used, these portfolios would be different for two managers possessing selectivity information where the quality of information is different. In particular, if the quality and nature of private information are unknown, then it is not possible to identify ex ante the timing portfolios and use them in a market-type regression. One must instead use the data to infer the identity of the timing portfolios.

The above conceptual distinctions between timing and selectivity are discussed in Section I. We then examine the possibilities for attributing the performance of a managed fund to one activity or the other when only ex post returns are observed. Contrary to Jensen [8], we can show that it is possible to disentangle timing and selectivity empirically. In Section II we discuss the statistical properties that are special to the factor approach with regard to measuring performance. In Section III we focus on the portfolio approach, where statistical methods for measuring performance may be different. The tests are essentially extensions of the approach first suggested in Treynor and Mazuy [13] and developed further in Jensen [8]. Essentially it is assumed that the manager's optimal portfolio position is linearly related to his expectations of the returns to be earned on assets and timing portfolios. This gives rise to an estimable relation between the returns earned on the managed fund and a quadratic function of observables. Although the factor approach seems to be superior to the portfolio approach on conceptual grounds, we find that it presents more difficulties in testing. Section IV provides concluding remarks.

I. The Distinction Between Timing and Selectivity

In this section we present a general definition of timing and then discuss two approaches to the definition of selectivity. The first approach, which we call the *portfolio approach*, is implicit in much of the previous literature but suffers from some conceptual problems. The second, the *factor approach*, is not subject to these problems and seems to us to be more in the spirit of the intuitive interpretation of these terms. In both approaches, the distinctions are ultimately statements about the type of information which managers receive.

For most of our analysis, we assume that information signals and asset returns are distributed multivariate normal.[4] The information signals that managers receive are, if they are informative, correlated with some or all of the returns to be realized on the individual assets. Under what conditions can a signal be called a timing signal? In the approach taken here, a signal is a timing signal if the appropriate response to *any* realization of the signal involves only the shifting of funds among timing portfolios. Timing information is therefore characterized by the limited nature of the response it produces.

To make this precise, assume that there are $N + 1$ assets in the economy and that the return on the ith asset is $\tilde{R}_i$. Assume also that a riskless asset exists, which is labeled as the 0'th asset. (All of the concepts we present here are easily extended to settings in which no riskless asset exists.) A portfolio α, is an $(N + 1)$-vector with α_i equal to the weight placed on asset i and $\sum_{i=0}^{N} \alpha_i = 1$. We assume that the portfolio manager receives a signal, $\tilde{Y}$, that is informative about the returns to be earned on some or all of the $N + 1$ assets, and that this information is used to construct his or her optimal portfolio, $\alpha^*(\tilde{Y})$. Let $\{\alpha_0^0, \alpha_1^0, \ldots, \alpha_N^0\}$ be a set of $N + 1$ linearly independent portfolios. The set $\{\alpha_0^0, \alpha_1^0, \ldots \alpha_N^0\}$ is a basis for $\mathscr{R}^{N+1}$ and any portfolio α can be constructed by combining portfolios in the basis, i.e., $\alpha = \sum_{n=0}^{N} \gamma_n \alpha_n^0$ for some γ_n, $\sum_{n=0}^{N} \gamma_n = 1$. Thus, when the portfolio is chosen in response to the information signal $\tilde{Y}$, we can write, without loss of generality, $\alpha^*(\tilde{Y}) = \sum_{n=0}^{N} \gamma_n(\tilde{Y})\alpha_n^0$. Let us consider a particular set of basis portfolios and call the first $L + 1$ portfolios timing portfolios. Then, if the signal $\tilde{Y}$ is a timing signal, the optimal portfolio $\alpha^*(\tilde{Y})$ can be constructed as a combination of the timing portfolios $\{\alpha_0^0, \alpha_1^0, \ldots, \alpha_L^0\}$.

$$\alpha^*(\tilde{Y}) = \sum_{l=0}^{L} \gamma_l(\tilde{Y})\alpha_l^0. \tag{1}$$

Upon receiving timing signals, a manager shifts his funds among the first $L + 1$ portfolios and never needs to choose portfolios that are not spanned by these $L + 1$ funds. A simple example is *market timing*, where only two portfolios are used, the riskless asset and the market portfolio of risky assets, i.e.,

$$\alpha_1^0 = \begin{pmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{pmatrix}; \ \alpha_2^0 = \begin{pmatrix} 0 \\ \alpha_1^m \\ \cdot \\ \cdot \\ \cdot \\ \alpha_N^m \end{pmatrix}, \tag{2}$$

where α_i^m is the value weight of the i'th risky asset in the portfolio of all risky assets. More generally, we might have timing defined over industry portfolios. In

[4] This makes the calculation of the optimal response to information and the econometric analysis in the next section more tractable.

this case a portfolio in $\{\alpha_1^0, \alpha_2^0, \ldots, \alpha_L^0\}$ would be the value weighted portfolio of risky assets corresponding to firms in a particular industry. There clearly are as many ways to define specific timing abilities as there are ways to identify timing portfolios. Presumably, the timing distinction is meaningful only when $L + 1$, the number of timing portfolios, is substantially less than $N + 1$, the number of assets. Note also that once the timing portfolios are specified, there are still many different signals that are timing signals, i.e., that produce a timing response with regard to these portfolios.

Now consider a manager whose information is exclusively timing information. Let $\tilde{R}$ be the vector of returns on the $N + 1$ assets. Then the return on the manager's portfolio is

$$(\alpha(\tilde{Y}))'\tilde{R} = \textstyle\sum_{l=0}^{L} \gamma_l(\tilde{Y})(\alpha_l^0)'\tilde{R} = \sum_{l=0}^{L} \gamma_l(\tilde{Y})\tilde{R}_l^0, \tag{3}$$

where $\tilde{R}_l^0$ is the return on the l'th timing portfolio. The timing information, $\tilde{Y}$, is valuable if the $\gamma_l(\tilde{Y})$ and the $\tilde{R}_l^0$ are correlated. If a time series of the $\gamma_l(\tilde{Y})$ and the $\tilde{R}_l^0$ is observable, one can in principle detect timing information by looking at the joint distribution between $\gamma_l(\tilde{Y})$ and $\tilde{R}_l^0$. Under the assumption that $\tilde{R}$, the vector of returns, is (unconditional on any private information) independently and identically distributed over time, only an informed manager could choose γ_l, $(l = 0, \ldots, L)$ to be correlated with $\tilde{R}_l^0$, $l = 0, \ldots, L$. In general we would like not only to detect timing information but also to measure its precision. To do this requires that we specify in some detail the manner in which the manager reacts to his information, i.e., the functional form of $\gamma_l(\tilde{Y})$. Examples are found in the next two sections.

In contrast to timing information, which pertains to portfolios or aggregates, selectivity information is usually considered to be information about individual assets. This implies that no restrictions can be placed on the space spanned by the optimal portfolios of a manager receiving selectivity information. Indeed, for a manager who receives selectivity information about each individual asset, the set of all possible portfolio responses, $\{\alpha^*(\tilde{Y})\}$, spans $\mathbb{R}^N$. Thus it appears that we cannot easily define selectivity in terms of the portfolio space in which the manager reacts to his information. In the end, signals about the $L + 1$ timing portfolios and about the $N + 1$ assets are melded together to produce at most an $(N + 1)$-dimensional response.

How then do we define selectivity information once we have specified what we mean by timing? We discuss here two approaches to this problem. To illustrate these two we consider a very simple example. Assume there are three assets ($N = 2$) with asset 0 being riskless. Let asset 1 and asset 2 be risky with identically and normally distributed returns (unconditional on any private information). The following three portfolios span the portfolio space.

$$\alpha_0^0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix};\ \alpha_1^0 = \begin{pmatrix} 0 \\ 1/2 \\ 1/2 \end{pmatrix};\ \alpha_2^0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \tag{4}$$

Let us define timing in terms of the first two portfolios in the basis (i.e., the riskless asset and the equally–weighted portfolio of risky assets). Thus a manager who receives only timing information has an optimal portfolio of the form

$$\alpha^*(\tilde{Y}) = \begin{pmatrix} 1 - \gamma(\tilde{Y}) \\ \tfrac{1}{2}\gamma(\tilde{Y}) \\ \tfrac{1}{2}\gamma(\tilde{Y}) \end{pmatrix}. \tag{5}$$

In the portfolio approach to distinguishing selectivity from timing we define a selectivity signal to be one that is (potentially) informative about the returns on some of the assets but uninformative about the returns on the timing portfolios. If $\tilde{Y}^S$ is selectivity information, then $\mathrm{E}(\tilde{R}_l^0 \mid \tilde{Y}^S) = \mathrm{E}(\tilde{R}_l^0)$ for $l = 0, 1, 2, \ldots, L$ or in the context of our particular example, $\mathrm{E}(\tfrac{1}{2}(\tilde{R}_1 + \tilde{R}_2) \mid \tilde{Y}^S) = \mathrm{E}(\tfrac{1}{2}(\tilde{R}_1 + \tilde{R}_2))$. To see what form such a signal might take, consider our example. We can imagine regressing the returns of the second and third assets on the returns of the second portfolio in the basis. We have

$$\tilde{R}_1 = \mathrm{E}(\tilde{R}_1) - \frac{\mathrm{Cov}(\tilde{R}_1, \tilde{R}_1^0)}{\mathrm{Var}(\tilde{R}_1^0)}\,\mathrm{E}(\tilde{R}_1^0) + \frac{\mathrm{Cov}(\tilde{R}_1, \tilde{R}_1^0)}{\mathrm{Var}(\tilde{R}_1^0)}\,\tilde{R}_1^0 + \tilde{\nu}_1$$

$$\tilde{R}_2 = \mathrm{E}(\tilde{R}_2) - \frac{\mathrm{Cov}(\tilde{R}_2, \tilde{R}_1^0)}{\mathrm{Var}(\tilde{R}_1^0)}\,\mathrm{E}(\tilde{R}_1^0) + \frac{\mathrm{Cov}(\tilde{R}_2, \tilde{R}_1^0)}{\mathrm{Var}(\tilde{R}_1^0)}\,\tilde{R}_1^0 + \tilde{\nu}_2 \tag{6}$$

where $\tilde{\nu}_1$ and $\tilde{\nu}_2$ are independent of $\tilde{R}_1^0$. Information about the realizations of $\tilde{\nu}_1$ or $\tilde{\nu}_2$ is therefore informative about the returns on the individual assets but reveals nothing about the return to be earned on the timing portfolio, $\tilde{R}_1^0$. Note, however, that since $\tfrac{1}{2}(\tilde{R}_1 + \tilde{R}_2) = \tilde{R}_1^0$, $\tfrac{1}{2}(\tilde{\nu}_1 + \tilde{\nu}_2)$ must equal zero for all realizations of $\tilde{\nu}_1$ and $\tilde{\nu}_2$. This means that $\tilde{\nu}_1$ and $\tilde{\nu}_2$ must be perfectly correlated. Clearly it is impossible to have independent information about $\tilde{R}_1$, $\tilde{R}_2$, and $\tilde{R}_1^0$. Selectivity information, when it is defined in this way, cannot be information exclusively about each of the N individual risky assets. It must also be noted that the random variables, $\tilde{\nu}_1$ and $\tilde{\nu}_2$, do not have a natural interpretation in this approach. That is, what does it mean for a manager to gather information about $\tilde{R}_1$ and $\tilde{R}_2$ which happens to be uninformative about the return on the portfolio, $\tfrac{1}{2}(\tilde{R}_1 + \tilde{R}_2)$? We do not have an appealing story about the source of this information. Thus, the approach seems artificial. Nevertheless, this approach to defining selectivity is consistent with much of the literature about timing and selectivity. For example, in many empirical tests designed to detect and distinguish timing and selectivity, portfolio returns are regressed on a constant term and terms which control for timing, i.e., which are a proxy for the interaction of $\gamma_l(\tilde{Y})$ and $\tilde{R}_l^0$. It is claimed that the constant term in these regressions is related to the selectivity performance of the portfolio manager. This will be true if we define selectivity in the manner just proposed, as will be shown in Section III. The optimal portfolio, $\alpha^*(\tilde{Y})$, of a manager having only selectivity information is independent of the returns on the timing portfolios, $\tilde{R}_l^0$. Thus in the regression described, only the constant term can be affected by the manager's actions.

The second approach to defining selectivity is, we believe, more in the spirit of the intuitive interpretation of this term and the concept of timing. We call this the factor approach. The terms macro-forecasting and micro-forecasting are often used in place of timing and selectivity. These suggest that timing involves predicting those factors that affect the aggregate, while selectivity is based on forecasting firm-specific determinants of asset returns. Assume that in our example the returns on assets 1 and 2 can be represented in terms of the following very simple factor model[5]:

$$\tilde{R}_1 = E + \tilde{\delta} + \tilde{\epsilon}_1 \tag{7}$$
$$\tilde{R}_2 = E + \tilde{\delta} + \tilde{\epsilon}_2,$$

with $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$ independently and identically distributed. In this factor model, it is natural to consider information about $\tilde{\delta}$ to be timing information and information about $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$ to be stock specific or selectivity information. Note that a manager observing only information about $\tilde{\delta}$ will indeed form portfolios spanned by α_0^0 and α_1^0 in (4). The manager will engage only in timing. In contrast to the first approach, however, a manager observing selectivity information, i.e., information about either $\tilde{\epsilon}_1$ or $\tilde{\epsilon}_2$, does learn something about the return on the timing portfolio, $\tilde{R}_1^0$. Thus, the portfolio selections of a manager receiving only selectivity information may be weakly correlated with the return on the timing portfolio, $\tilde{R}_1^0$. They are, however, independent of the realization of the factor, $\tilde{\delta}$. Empirical tests designed to detect timing ability as defined in the factor approach will therefore look for evidence of correlation between a manager's portfolio positions and the factor realizations (rather than with returns on timing portfolios).

In a model with more than one factor, it is natural to define several timing portfolios, one for each factor. A manager with only timing information will shift his funds only among these factor-timing portfolios. Even when there are several factors, it is still possible to define N distinct selectivity signals corresponding to the N asset-specific disturbances, $\tilde{\epsilon}_i$, $i = 1, 2, \ldots, N$.

The factor model resolves several conceptual problems inherent in the portfolio approach. First, selectivity information can be truly asset specific. In the factor approach, there are as many degrees of freedom to define selectivity signals as there are risky assets. This is not so in the portfolio definition. Second, and more importantly, in the factor approach the information signals potentially correspond to economically meaningful sources of uncertainty. As argued above, there is no satisfactory way to give such an interpretation to the selectivity signals in the portfolio definition.

Note that, under joint normality, with N risky assets there always exists a sufficient statistic whose dimension is at most N. A manager may observe (under the factor definition) K timing signals and N asset–specific selectivity signals but these can be collapsed into N signals. The same is true for managers receiving timing and selectivity signals under the portfolio interpretation. In other words, any jointly normal signal can be written as $\tilde{Y} = C\,\tilde{R} + \tilde{\eta}$ for some matrix, C. It thus seems that the only distinctions we can make between different types of information signals can concern either the matrix, C, or the variance-covariance matrix of the errors, $\tilde{\eta}$. Nevertheless, the two approaches discussed above are distinct in some important ways. First, the distinction on the individual level comes in when we consider statistical tests of performance, since the set of observables and the statistical hypotheses that are tested would be different in

[5] We choose here a particularly simple representation of a factor model to illustrate the approach. Obviously the definition can be applied to any factor model. This is done in the next section.

the two formulations. Second, the portfolio and factor specifications are different on the equilibrium level. If, for example, we postulate that a large number of investors collect timing and selectivity signals of some specific nature, and that the private information is diverse in the sense that individual signals are conditionally independent of each other (give some sufficient statistic), then the aggregation of these diverse pieces of information in asset equilibrium prices may give rise to observationally distinct economies.[6]

II. A factor model of timing and selectivity

In this section we develop an explicit model of timing and selectivity under the factor approach. This requires that we specify the nature of the information received by a portfolio manager. We are mainly interested in determining whether it is possible to identify empirically the quality of a particular manager's timing and selectivity information from a time series of the returns earned on a managed portfolio. We show that in principle this is always possible but in practice it may not be feasible.[7] This leads us to consider in the next section a simpler case in which the problem is to determine the quality of only the timing information. Rather than measuring the quality of the selectivity information, we simply test to see if such information is present. These more modest goals seem to be achievable with the limited data on mutual fund returns that is usually available.

Assume that the N risky asset returns are generated by a factor model. The returns earned in period t are given by

$$\tilde{R}_t = \hat{E} + B\tilde{\delta}_t + \tilde{\epsilon}_t, \tag{8}$$

where $\tilde{R}_t$ is the vector of the N risky asset returns, $\tilde{\delta}_t$ is a K-dimensional random vector, $\tilde{\epsilon}_t$ is an N-dimensional random vector, and B is an $N \times K$ dimensional matrix of the factor loadings. We can normalize the factors so that $\mathrm{Var}(\tilde{\delta}_t) = I$ where I is the K dimensional identity matrix. Let D be the variance of $\tilde{\epsilon}_t$ and assume that $\tilde{\delta}_t$ and $\tilde{\epsilon}_t$ are uncorrelated and distributed multivariate normal. Since we do not require that D be diagonal, (8) is completely general. Of course, if we want to think of the $\tilde{\delta}_t$ as being meaningful factors, then we will want to assume that D is nearly diagonal so that most sources of common variation are due to the $\tilde{\delta}_t$ terms.

There are two types of signals in our model. Each signal is assumed to be jointly normally distributed with $\tilde{\delta}_t$ and $\tilde{\epsilon}_t$. Timing signals have the form $\tilde{Y}_t^T = \tilde{\delta}_t + \tilde{\eta}_t$; correspondingly, selectivity signals are written as $\tilde{Y}_t^S = \tilde{\epsilon}_t + \tilde{\theta}_t$. It is assumed that the vectors $\tilde{\eta}_t$ and $\tilde{\theta}_t$ are uncorrelated and that each is independent of $\tilde{\delta}_t$ and $\tilde{\epsilon}_t$. The assumption that $\tilde{\eta}_t$ and $\tilde{\theta}_t$ are independent is made to simplify the analysis. Since $\tilde{\eta}_t$ corresponds to the errors made in forecasting the factors and $\tilde{\theta}_t$ is related to the errors made in forecasting the firm-specific disturbances, it seems sensible to assume that they are independent. We let $\mathrm{Var}(\tilde{\eta}_t) = \Sigma$ and

[6] To see this, one needs to examine the measurability requirement of prices. When the aggregate of private information is observationally distinct, then the equilibrium results would also be distinct. This argument is similar to that in Admati [1], section 6.

[7] See Admati and Ross [2] for related results on the identifiability of performance parameters in a similar model.

$\mathrm{Var}(\tilde{\theta}_t) = \Omega$ and assume that these are both positive definite. The quality of a manager's information is completely determined by Σ and Ω and it is these two variance matrices that we wish to identify.

It is clearly not possible to recover Σ and Ω from a time series of portfolio returns unless some restrictions are placed on the way the manager responds to his information. One of the simplest assumptions specifies a linear response to forecasts. This is consistent with the manager acting as if he were maximizing the expected utility of someone with constant absolute risk tolerance. With this assumption, the optimal portfolio of risky assets is

$$\alpha^*(\tilde{Y}_t^T, \tilde{Y}_t^S) = \rho(\mathrm{Var}(\tilde{R}_t \mid \tilde{Y}_t^T, \tilde{Y}_t^S))^{-1}(\mathrm{E}(\tilde{R}_t \mid \tilde{Y}_t^T, \tilde{Y}_t^S) - R_0\mathbf{e}), \tag{9}$$

where ρ is a measure at risk tolerance, $\mathrm{E}(\tilde{R}_t \mid \tilde{Y}_t^T, \tilde{Y}_t^S)$ is the vector of conditional expected returns given the manager's information, and $\mathbf{e}$ is the vector of ones. It is easily shown, using the theory of normal variables, that

$$\mathrm{E}(\tilde{R}_t \mid \tilde{Y}_t^T, \tilde{Y}_t^S) = \hat{E} + B(I + \Sigma)^{-1}(\tilde{\delta}_t + \tilde{\eta}_t) + (D + \Omega)^{-1}D(\tilde{\epsilon}_t + \tilde{\theta}_t), \tag{10}$$

and

$$\mathrm{Var}(\tilde{R}_t \mid \tilde{Y}_t^T, \tilde{Y}_t^S) = B(I + \Sigma^{-1})^{-1}B' + (D^{-1} + \Omega^{-1})^{-1}. \tag{11}$$

Let $E = \hat{E} - R_0\mathbf{e}$ be the expected excess return vector. Then the optimal portfolio, $\alpha^*(\tilde{Y}_t^T, \tilde{Y}_t^S)$ is

$$\rho(B(I + \Sigma^{-1})^{-1}B' + (D^{-1} + \Omega^{-1})^{-1})^{-1}(E + B(I + \Sigma)^{-1}(\tilde{\delta}_t + \tilde{\eta}_t) + (D + \Omega)^{-1}D(\tilde{\epsilon}_t + \tilde{\theta}_t)) \tag{12}$$

First note that the information signal, $\tilde{Y}_t^T$, is truly a timing signal. The portfolio position of a manager receiving only timing information is

$$\alpha^*(\tilde{Y}_t^T) = \rho(B(I + \Sigma^{-1})^{-1}B' + D)^{-1}(E + B(I + \Sigma)^{-1}(\tilde{\delta}_t + \tilde{\eta}_t)). \tag{13}$$

Thus the response to the information signal, $\tilde{\delta}_t + \tilde{\eta}_t$, is simply to shift funds among K timing portfolios in the span of

$$(B(I + \Sigma^{-1})^{-1}B' + D)^{-1}B(I + \Sigma)^{-1}. \tag{14}$$

From (14) it might appear that the set of timing portfolios depends upon Σ, the nature of the timing information received. This is not true. It is shown in the appendix that the span of $(B(I + \Sigma^{-1})^{-1}B' + D)^{-1}B(I + \Sigma)^{-1}$ is independent of Σ. Thus all managers who receive only timing information shift funds among the same set of timing portfolios.

If a fund manager receives both timing and selectivity information, it is still true that the responses to timing information are restricted to a set of K portfolios. However, unlike the situation when only timing information is received, it is not true that this set of portfolios is independent of the quality of the information received. The timing response portfolios now lie in the span of

$$(B(I + \Sigma^{-1})^{-1}B' + (D^{-1} + \Omega^{-1})^{-1})^{-1}B(I + \Sigma)^{-1}, \tag{15}$$

which does depend on the value of Ω, the quality of the selectivity information. Thus two managers who receive the same timing signals but selectivity information of differing qualities will respond to the timing signals in different fashions. This makes the disentanglement of timing and selectivity much more cumbersome than it would otherwise be.

Despite the fact that the timing portfolios depend upon the quality of the selectivity information, it is in principle possible to identify both Σ and Ω, even if the parameter ρ, which measures the aggressiveness of the manager's response to information, is unknown. Using (12) it can be shown that the realized excess return on the managed portfolio is

$$\begin{aligned}\tilde{R}_t^P &= \alpha^*(\tilde{Y}_t^T, \tilde{Y}_t^S)'(\tilde{R}_t - R_0\mathbf{e})\\ &= \rho[E'\Psi^{-1}E + E'\Psi^{-1}B(I+\Sigma)^{-1}(\tilde{\delta}_t + \tilde{\eta}_t)\\ &\quad + E'\Psi^{-1}(D+\Omega)^{-1}D(\tilde{\epsilon}_t + \tilde{\theta}_t)\\ &\quad + \tilde{\delta}_t'B'\Psi^{-1}E + \tilde{\delta}_t'B'\Psi^{-1}B(I+\Sigma)^{-1}(\tilde{\delta}_t + \tilde{\eta}_t)\\ &\quad + \tilde{\delta}_t'B'\Psi^{-1}(D+\Omega)^{-1}D(\tilde{\epsilon}_t + \tilde{\theta}_t)\\ &\quad + \tilde{\epsilon}_t'\Psi^{-1}E + \tilde{\epsilon}_t'\Psi^{-1}B(I+\Sigma)^{-1}(\tilde{\delta}_t + \tilde{\eta}_t)\\ &\quad + \tilde{\epsilon}_t'\Psi^{-1}(D+\Omega)^{-1}D(\tilde{\epsilon}_t + \tilde{\theta}_t)],\end{aligned} \tag{16}$$

where $\Psi = \mathrm{Var}(\tilde{R}_t \mid \tilde{Y}_t^T, \tilde{Y}_t^S) = B(I+\Sigma^{-1})^{-1}B' + (D^{-1}+\Omega^{-1})^{-1}$. If one assumes that E, B and D are known parameters of the generating process, then it is possible to determine (ex post) the realizations of $\tilde{\delta}_t$ and $\tilde{\epsilon}_t$. The factor realizations can be obtained by the cross sectional regression of $\tilde{R}_t - \hat{E}$ on B. If N is sufficiently large (and the elements of D are bounded), $\tilde{\delta}_t$ can be recovered with negligible error. The idiosyncratic terms are then recovered: $\tilde{\epsilon}_t = \tilde{R}_t - B\tilde{\delta}_t$.

Consider now the time series regression of $\tilde{R}_t^P$ on a constant, $\tilde{\delta}_t$, $\tilde{\epsilon}_t$, and the distinct terms in the matrices $\tilde{\delta}_t\tilde{\delta}_t'$ ($K(K+1)/2$ terms), $\tilde{\epsilon}_t\tilde{\epsilon}_t'$ ($N(N+1)/2$ terms), and $\tilde{\epsilon}_t\tilde{\delta}_t'$ (NK terms). The constant term is easily shown to be a consistent estimator of $\rho E'\Psi^{-1}E$. For the other terms we have

$$\mathrm{plim}(\text{coefficients on } \tilde{\delta}_t) = \rho E'\Psi^{-1}B(I + (I+\Sigma)^{-1}) \tag{17}$$

$$\mathrm{plim}(\text{coefficients on } \tilde{\epsilon}_t) = \rho E'\Psi^{-1}(I + (D+\Omega)^{-1}D) \tag{18}$$

$$\mathrm{plim}(\text{coefficients on } \tilde{\delta}_t\tilde{\delta}_t') = \rho(\mathrm{sym}(B'\Psi^{-1}B(I+\Sigma)^{-1})) \tag{19}$$

$$\mathrm{plim}(\text{coefficients on } \tilde{\epsilon}_t\tilde{\epsilon}_t') = \rho(\mathrm{sym}(\Psi^{-1}(D+\Omega)^{-1}D)) \tag{20}$$

$$\mathrm{plim}(\text{coefficients on } \tilde{\epsilon}_t\tilde{\delta}_t') = \rho(\Psi^{-1}B(I+\Sigma)^{-1} + \mathrm{D}(\mathrm{D}+\Omega)^{-1}\Psi^{-1}\mathrm{B}), \tag{21}$$

where sym(A) refers to the $M(M+1)/2$ distinct terms in the symmetric version[8] of the $M \times M$ matrix A.

If we include the constant term, we have a total of $(K^2+3K)/2 + (N^2+3N)/2 + NK + 1$ estimated coefficients. The number of unknown parameters in Σ, Ω, and ρ is $(K^2+K)/2 + (N^2+N)/2 + 1$. It appears that we potentially have $K + N + NK$ overidentifying restrictions. In fact there are many more than this. First there are the restrictions required for Σ and Ω to be positive definite. Added to

[8] The symmetric version of a square matrix A is $(A + A')/2$.

these are restrictions related to the information contained in the disturbance terms in the proposed regression. These terms are heteroskedastic with the variance related in a known way to ρ, Σ and Ω.[9] Given the nonlinearities involved and the distributional assumptions made, an attractive estimation procedure would be the maximum likelihood approach which explicitly accounts for these additional restrictions. Whether this procedure is used or not, it is possible to recover the appropriate measures of the quality of both the timing information and the selectivity information. Note also that to identify the information quality of a manager using the procedure just discussed, it is not necessary to assume that any particular model of asset pricing holds.

Although both Σ and Ω (as well as ρ) can in principle be recovered, in practice this may generally prove too difficult or impossible. The reason is that the number of regressors in (17)–(21) is extremely large and most likely exceeds the number of time series observations obtainable for any fund. In the next section we look at a much simpler problem based on the portfolio approach and show that recovery of at least the quality of timing information will generally be feasible.

III. A Portfolio Model of Timing and Selectivity

In the last section it was shown that when the factor approach is used to define timing and selectivity, it is possible to identify both the quality of timing information and the quality of selectivity information. This can be done even if the only observable effect of the manager's information in each period is the total return earned on the managed fund; for identification of the quality of information it is not, for example, necessary to observe portfolio positions. Unfortunately, because of the large number of interactions between information signals and asset returns, it was necessary to include an extremely large number of regressors in the regression proposed in the last section. For this reason, estimation of the quality parameters of the timing and selectivity information seems to be possible under the assumptions of the last section only when the number of time series observations is impractically large.

In this section we pursue the more modest goal of estimating the quality of only the timing information. The testing procedure proposed will only be capable of determining whether selectivity information is or is not present; the quality of the selectivity information will not be identified. By reducing the number of parameters to be estimated, we obviously simplify the estimation problem. It should be noted, however, that the complexity of the estimation procedure discussed in the last section is due to more than just the large number of parameters that must be estimated. Much of the difficulty can be attributed to the fact that under the factor definition the timing response portfolios cannot be identified without knowing the quality of the selectivity information. Although managers receiving only timing information respond to their information using

[9] The error term is $\rho(\tilde{\epsilon}_t'\Psi^{-1}B(I+\Sigma)^{-1}\tilde{\eta}_t + \tilde{\epsilon}_t'\Psi^{-1}(D+\Omega)^{-1}D\tilde{\theta}_t + \tilde{\delta}_t'B'\Psi^{-1}B(I+\Sigma)^{-1}\tilde{\eta}_t + \tilde{\delta}_t'B'\Psi^{-1}(D+\Omega)^{-1}D\tilde{\theta}_t + \tilde{\epsilon}_t'\Psi^{-1}B(I+\Sigma)^{-1}\tilde{\eta}_t + \tilde{\epsilon}_t'\Psi^{-1}(D+\Omega)^{-1}D\tilde{\theta}_t)$.

the same set of timing portfolios, this is not the case for managers who receive both timing and selectivity information. (The span of (15) is not independent of Ω.) Because the set of portfolios used to respond to factor information cannot be defined without knowledge of the quality of selectivity information, the estimation of information quality becomes much more complex than it would otherwise be.

In contrast to the factor approach, the portfolio approach to defining timing and selectivity does allow one to specify the timing response portfolios without knowing the quality of the selectivity information. This is because selectivity information under the portfolio approach is by definition completely uninformative about the returns to be earned on the timing portfolios. The estimation problem is thus simplified if selectivity information is assumed to be independent of the return earned on the timing portfolios. Despite the conceptual problems inherent in the portfolio approach, or perhaps because of them, identification of information quality is, given certain assumptions, easier under this approach than it is under the factor approach.

To show how estimation of the quality of timing information can be accomplished under the portfolio approach we consider a simple case where there is only one timing portfolio. Let there be N risky assets and let the variance of their returns be $\hat{V}$. The timing portfolio can in principle be any portfolio of the N risky assets. In almost all of the previous discussions of timing in the literature, the timing portfolio is assumed to be the market portfolio. We will not make this particular assumption here but the reader is free to think of the timing being discussed as market timing. Let the timing portfolio be α^T and let its variance be $\sigma^2 = (\alpha^T)'\hat{V}\alpha^T$. It is possible to define $N - 1$ portfolios, α_j^S, $j = 1, \ldots, N - 1$, such that

$$(\alpha_j^S)'\mathbf{e} = 1$$

$$(\alpha^T)'\hat{V}\alpha_j^S = 0 \tag{22}$$

for each j. We can define N new assets with the first being the timing portfolio and the remaining $N - 1$ being a set of portfolios which satisfy (22). In other words we can rotate assets so that the first is the timing portfolio and all the other portfolios are uncorrelated with the first. Let $\tilde{R}_t^T$ be the return on the timing portfolio and $\tilde{R}_{i,t}^S$ be the return on the portfolio with weights α_j^S. Then

$$\operatorname{Var}\begin{pmatrix} \tilde{R}_t^T \\ \tilde{R}_{1,t}^S \\ \vdots \\ \tilde{R}_{N-1,t}^S \end{pmatrix} = \begin{pmatrix} \sigma^2 & O_{N-1}' \\ O_{N-1} & V \end{pmatrix}, \tag{23}$$

where O_{N-1} is the $N - 1$ vector of zeroes and V is the variance-covariance matrix of the $N - 1$ portfolios that have returns orthogonal to the return of the timing portfolio. In the portfolio approach we define the timing signal to be $\tilde{Y}_t^T = \tilde{R}_t^T + \tilde{\eta}_t$ and the $N - 1$ possible selectivity signals to be of the form $\tilde{Y}_{i,t}^S = \tilde{R}_{i,t}^S + \tilde{\theta}_{i,t}$. We assume as before that returns and the information errors $\tilde{\eta}_t$ and $\tilde{\theta}_t$ are jointly normally distributed and that $\tilde{\eta}_t$ and $\tilde{\theta}_t$ are independent. The parameters of interest are again $\mathrm{Var}(\tilde{\eta}_t) = \sigma_\eta^2$ and $\mathrm{Var}(\tilde{\theta}_t) = \Omega$. We will be able to estimate σ_η^2 but given our more limited goals we will only test whether Ω is finite or not, i.e., whether selectivity information exists or not.

As in the previous section it is necessary to specify how the manager reacts to his information. Again we make the assumption that the response is linear and therefore consistent with maximization of utility with constant absolute risk aversion. The position taken in the N portfolios by the manager is given by

$$\rho\begin{pmatrix} \left(\dfrac{1}{\sigma^2}+\dfrac{1}{\sigma_\eta^2}\right) & O'_{N-1} \\ O_{N-1} & (V^{-1}+\Omega^{-1})^{-1} \end{pmatrix}^{-1} \begin{pmatrix} \dfrac{\sigma_\eta^2}{\sigma^2+\sigma_\eta^2}\mathrm{E}(\tilde{R}_t^T) + \dfrac{\sigma^2}{\sigma^2+\sigma_\eta^2}\tilde{Y}_t^T - R_0 \\ (I_{N-1}-(V+\Omega)^{-1}V)\mathrm{E}(\tilde{R}_t^S) \\ \quad + (V+\Omega)^{-1}V\tilde{Y}_t^S - R_0\mathbf{e} \end{pmatrix}, \tag{24}$$

where R_0 is the riskless rate.

The return realized on the managed portfolio in period t, $\tilde{R}_t^P$, is consequently equal to

$$\rho\left(\frac{\mathrm{E}(\tilde{R}_t^T) - R_0}{\sigma^2} - \frac{R_0}{\sigma_\eta^2}\right)\tilde{R}_t^T + \frac{\rho}{\sigma_\eta^2}(\tilde{R}_t^T)^2 + \frac{\rho}{\sigma_\eta^2}\tilde{\eta}_t\tilde{R}_t^T + \tilde{u}_t \tag{25}$$

where

$$\tilde{u}_t = \rho(\tilde{R}_t^S)'(V^{-1}+\Omega^{-1})((I_{N-1}-(V+\Omega)^{-1}V)\mathrm{E}(\tilde{R}_t^S) + (V+\Omega)^{-1}V\tilde{Y}_t^S - R_0) \tag{26}$$

and $(\tilde{R}_t^S)' = (\tilde{R}_{1,t}^S, \tilde{R}_{2,t}^S, \ldots, \tilde{R}_{N-1,t}^S)$.

Now consider the regression of the managed portfolio's return on a constant term, the return on the timing portfolio and the squared return on the timing portfolio:

$$\tilde{R}_t^P = \Gamma_0 + \Gamma_1\tilde{R}_t^T + \Gamma_2(\tilde{R}_t^T)^2 + \tilde{\omega}_t \tag{27}$$

Since $\tilde{u}_t$ and $\tilde{\eta}_t$ are by assumption independent of $\tilde{R}_t^T$, Γ_2 is a consistent estimator of ρ/σ_η^2. As we argued above, the parameter of interest is σ_η^2. We need therefore to get an estimate of ρ. This can be done using the residuals of the regression. The true disturbance terms are heteroskedastic and equal to $(\rho/\sigma_\eta^2)\tilde{\eta}_t\tilde{R}_t^T + \tilde{u}_t - \mathrm{E}(\tilde{u}_t)$. Consider regressing $\tilde{\omega}_t^2$ on a constant and the square of $\tilde{R}_t^T$.

$$\tilde{\omega}_t^2 = \Lambda_0 + \Lambda_1(\tilde{R}_t^T)^2 + \tilde{\xi}_t \tag{28}$$

Under the distributional assumptions we have made, $\mathrm{plim}(\Lambda_1) = \rho^2/\sigma_\eta^2$. Since we have estimates of ρ/σ_η^2 and ρ^2/σ_η^2, we can recover both σ_η^2 and ρ.

The two step procedure just discussed clearly does not produce the most efficient estimates of the information quality parameter, σ_η^2. More efficient estimates can be obtained by taking into account the heteroskedasticity of the disturbance terms. One possible approach would use the estimates of ρ and σ_η^2 obtained in the inefficient manner just described to predict the variance of the residuals. A GLS procedure could then be used to obtain better estimates of the coefficients in the two regressions. As before, a maximum likelihood approach which completely accounts for the structure of the model is probably most appropriate. It is beyond the scope of this paper to discuss in detail such refinements.

Having shown that the quality of timing information can be identified, we now turn to the detection of selectivity information. Responses to selectivity information affect only the distribution of $\tilde{u}_t$ in (25) and Γ_0 can be shown to be a consistent estimator of $\mathrm{E}(\tilde{u}_t)$. It is not possible, however, to infer anything from the value of $\mathrm{E}(\tilde{u}_t)$ unless further assumptions are made about asset pricing. Assume, for example, that the CAPM holds and $\tilde{R}_t^T$ is the return on the market portfolio. The portfolios α_i^S are constructed to have returns orthogonal to $\tilde{R}_t^T$. This means that $\mathrm{E}(\tilde{R}_{i,t}^S) = R_0$, the riskless rate, for all i. A manager who does not have any selectivity information will not be able (in expectation) to earn more than the riskless rate by shifting funds among the portfolios α_i^S. If a manager does have valuable selectivity information (finite Ω) and if he responds linearly to this information as is assumed in (26), then he will earn (in expectation) more than the riskless rate and this will be reflected in a higher value of $\mathrm{E}(\tilde{u}_t)$. It is clear that we cannot learn from $\mathrm{E}(\tilde{u}_t)$ all that we need to know to value the manager's selectivity information. The value of the selectivity information depends on the matrix, Ω, and is not summarized in the scalar, $\mathrm{E}(\tilde{u}_t)$. Nevertheless, it is still encouraging that the simple regression procedure described above can be used to identify fully the quality of timing information and to detect the existence of selectivity information.

IV. Conclusions

The important problem in performance evaluation is the measurement of the quality of the information possessed by a portfolio manager. Many tests proposed in the literature do not successfully separate the aggressiveness of a manager from the quality of the information he possesses. To measure the quality of information it is necessary to specify both the nature of the information received and the nature of the manager's reaction to it. For this the distinction between timing information and selectivity information may be useful. We have argued that the distinction between timing and selectivity is not straightforward and we have examined two approaches that can be used to make the distinction precise. Under the factor approach timing information is information about the realizations of factors which affect the returns of many assets. Selectivity information is then defined to be information about the asset-specific determinants of asset returns. Under the portfolio approach, timing information is information about the returns to be earned on a pre-specified set of timing portfolios. Selectivity information under this definition is any information that is uninformative about the timing portfolio returns but informative about some asset returns.

These two approaches to defining timing and selectivity are not equivalent. They are observationally distinct both in their implications for the joint distribution of returns on assets and managed funds and in their implications for asset pricing. We have shown that under each interpretation it is possible to identify empirically some measure of the quality of information possessed by a fund manager. This is possible even when all that is observed are the ex-post returns earned on the managed fund. It is important to note, however, that the specification of these tests depends in critical ways on the approach taken to define timing and selectivity. Although the factor approach has conceptual advantages over the portfolio approach, testing may be more feasible if the information received by managers is consistent with the portfolio approach.

It should be noted finally that our discussion has abstracted from an equilibrium model explaining asset returns. In discussing empirical tests we have made assumptions of independence (and normal distributions) that are consistent with most of the empirical literature. However, such assumptions have not been justified by a theoretical intertemporal asset pricing model which explicitly admits asymmetric information. If returns are determined in a world of homogenous beliefs, only an insignificant portion of the market can possess any superior information. The importance of measuring performance in such a world is questionable.

Appendix

In this appendix we show that in the factor model developed in section II, the factor timing portfolios used by managers receiving timing information do not depend upon the quality or nature of that information. This requires that we show that the span of the matrix

$$(B(I + \Sigma^{-1})^{-1}B' + D)^{-1}B(I + \Sigma)^{-1} \tag{29}$$

does not depend on Σ. To do this we will show that

$$\text{span}[(B(I + \Sigma^{-1})^{-1}B' + D)^{-1}B(I + \Sigma)^{-1}] = \text{span}[(BB' + D)^{-1}B] \tag{30}$$

for all Σ. Let F and G be two $N \times K$ matrices with $N > K$. Then span(F) = span(G) if and only if there exists a $K \times K$ matrix H such that $F = GH$. Since $(I + \Sigma)$ is invertible, (30) is established if it is shown that there exists H that solves

$$(B(I + \Sigma^{-1})^{-1}B' + D)^{-1}BH = (BB' + D)^{-1}B. \tag{31}$$

Let $Q = (BB' + D)^{-1}B$. Then $B = (BB' + D)Q$ or $B - BB'Q = DQ$. Substituting Q into (31) and rearranging gives us $BH - B(I + \Sigma^{-1})^{-1}B'Q = DQ$. Combining these two we obtain

$$B(I - B'Q) = B(H - (I + \Sigma^{-1})^{-1}B'Q), \tag{32}$$

which means that H must satisfy

$$I - B'Q = H - (I + \Sigma^{-1})^{-1}B'Q. \tag{33}$$

A matrix H which solves (31) therefore exists and is given by

$$\begin{aligned} H &= I - (I - (I + \Sigma^{-1})^{-1})B'Q \\ &= I - (I - (I + \Sigma^{-1})^{-1})B'(BB' + D)^{-1}B. \end{aligned} \tag{34}$$

Since all matrices $(B(I + \Sigma^{-1})^{-1}B' + D)^{-1}B$ have the same span as $(BB' + D)^{-1}B$ and the spanning relation defines equivalence classes, we have proved the result. Q.E.D.

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DISCUSSION

ROBERT E. VERRECCHIA*: Throughout the portfolio performance evaluation literature, attempts are made to distinguish between market timing ability and forecasting ability (as it relates to the returns on individual assets). But these attempts have not been altogether satisfactory, in large part because the notions of "timing" and "selectivity" that have been introduced into the literature have usually been defined in terms of their regression characteristics, as opposed to primitive economic elements (see, for example, Grinblatt-Titman [1], which also provides an excellent synthesis of the relevant literature). To the extent that the paper on timing and selectivity by Admati, Bhattacharya, Pfleiderer, and Ross (ABPR henceforth) grapples with various interpretations of these phenomena, it fills an important void in the literature. The controversy is whether one should be *en*couraged or *dis*couraged about the possibility of identifying timing information and detecting the existence of selectivity information on the basis of the evidence put forth in the ABPR analysis.

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